

# Math Simulation of Slightly Compressible Fluid

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## Overview

Math simulation is a powerful tool in reservoir description. There are various math modeling methods can be applied. However, in oil and gas industry, finite difference method is most frequently used. In this project, I will apply this method to 8.6-chapter project to numerically simulate slightly compressible fluids (single phase) in A-1 reservoir.

## Reservoir Model

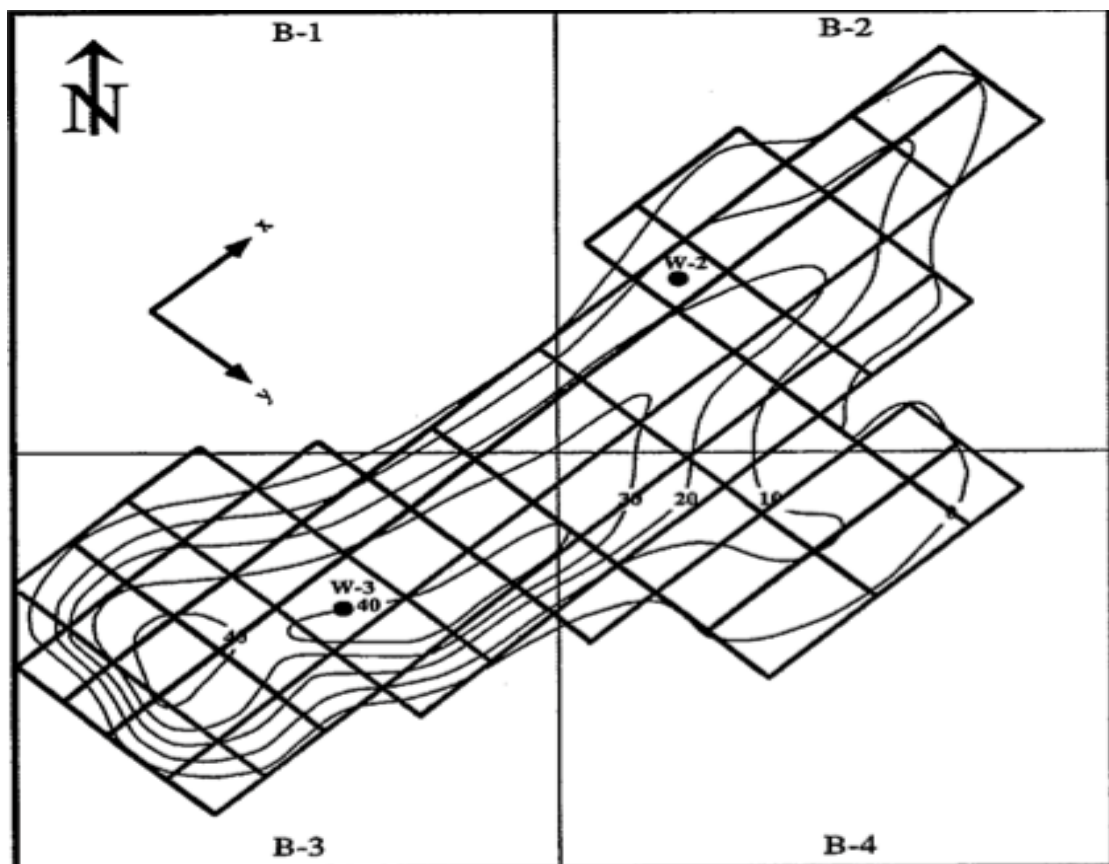


Figure1. A-1 Single-phase reservoir with only two wells

Well	Block	Type	Radius (ft)	Specification
W-2	(9,3)	Producer	0.25	$q_{sc} = -650.0$ STB/D
W-3	(4,4)	Producer	0.25	$p_{wf} = 5,600.0$ psia

Figure2. Well information for A-1

$c_o, \text{psi}^{-1}$	$9.0 \times 10^{-6}$
$\rho_o, \text{lbm/ft}^3$	52.4
$B_o, \text{RB/STB}$	$1/[1 + 9.0 \times 10^{-6}(p - 14.7)]$
Oil Viscosity Data	
Pressure (psia)	Viscosity (cp)
4,500	0.9180
5,000	0.9200
5,500	0.9243
6,000	0.9372
6,500	0.9494
7,000	0.9650
7,500	0.9812
8,000	1.0019

Figure3. Slightly compressible fluid properties

BLOCK DIMENSION IN x DIRECTION (ft)											
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0
439.0	509.0	491.0	596.0	526.0	561.0	912.0	807.0	439.0	877.0	544.0	772.0

BLOCK DIMENSION IN y DIRECTION (ft)											
474.0	474.0	474.0	474.0	474.0	474.0	474.0	474.0	474.0	474.0	474.0	474.0
404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0
386.0	386.0	386.0	386.0	386.0	386.0	386.0	386.0	386.0	386.0	386.0	386.0
491.0	491.0	491.0	491.0	491.0	491.0	491.0	491.0	491.0	491.0	491.0	491.0
404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0	404.0
316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0
316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0	316.0
421.0	421.0	421.0	421.0	421.0	421.0	421.0	421.0	421.0	421.0	421.0	421.0
526.0	526.0	526.0	526.0	526.0	526.0	526.0	526.0	526.0	526.0	526.0	526.0

Figure4. Gridblocks' dimensions along x and y directions (A-1)

# DEPTHS TO THE STRUCTURE TOP (ft)

```

***** 9342.0 9345.0 9347.0 *****
9341.0 9327.0 9330.0 9338.0 9333.0 ***** 9311.0 9310.0 *****
9336.0 9319.0 9316.0 9322.0 9325.0 9315.0 9299.0 9300.0 9299.0 9297.0 9297.0 9305.0
9340.0 9326.0 9316.0 9308.0 9310.0 9313.0 9297.0 9296.0 9295.0 9293.0 9292.0 9295.0
9342.0 9332.0 9323.0 9305.0 9298.0 9298.0 9296.0 9292.0 9291.0 9288.0 *****
***** 9315.0 9297.0 9295.0 9292.0 9289.0 9289.0 9287.0 *****
***** 9294.0 9290.0 9286.0 *****
***** 9289.0 9281.0 9282.0 *****
***** 9290.0 9280.0 9278.0 *****

```

# BLOCK THICKNESSES (ft)

```

***** 10.0 12.0 5.0 *****
8.0 35.0 30.0 15.0 6.0 ***** 4.0 5.0 *****
14.0 44.0 36.0 30.0 22.0 16.0 12.0 14.0 15.0 11.0 6.0 3.0
20.0 34.0 35.0 40.0 34.0 32.0 29.0 25.0 22.0 18.0 10.0 3.0
5.0 12.0 12.0 40.0 44.0 42.0 32.0 20.0 16.0 10.0 *****
***** 10.0 19.0 27.0 24.0 10.0 6.0 3.0 *****
***** 4.0 10.0 6.0 *****
***** 8.0 7.0 3.0 *****
***** 4.0 5.0 2.0 *****

```

Figure5. Gridblocks' depths and thickness (A-1)

# PERMEABILITY IN x DIRECTION (md)

```

***** 275.0 270.0 252.0 *****
267.0 274.0 280.0 265.0 253.0 ***** 259.0 270.0 *****
265.0 280.0 289.0 278.0 271.0 271.0 270.0 269.0 270.0 279.0 283.0 275.0
258.0 271.0 295.0 297.0 282.0 280.0 281.0 276.0 290.0 293.0 279.0 270.0
253.0 259.0 275.0 285.0 290.0 280.0 289.0 277.0 290.0 280.0 *****
***** 272.0 276.0 273.0 288.0 281.0 274.0 268.0 *****
***** 265.0 280.0 290.0 *****
***** 270.0 280.0 270.0 *****
***** 260.0 268.0 260.0 *****

```

# PERMEABILITY IN y DIRECTION (md)

```

***** 220.0 216.0 201.6 *****
213.6 219.2 224.0 212.0 202.4 ***** 207.2 216.0 *****
212.0 224.0 231.2 222.4 216.8 216.8 216.0 215.2 216.0 223.2 226.4 220.0
206.4 216.8 236.0 237.6 225.6 224.0 224.8 220.8 232.0 234.4 223.2 216.0
202.4 207.2 220.0 228.0 232.0 224.0 231.2 221.6 232.0 224.0 *****
***** 217.6 220.8 218.4 230.4 224.8 219.2 214.4 *****
***** 212.0 224.0 232.0 *****
***** 216.0 224.0 216.0 *****
***** 208.0 214.4 208.0 *****

```

# POROSITY (fraction)

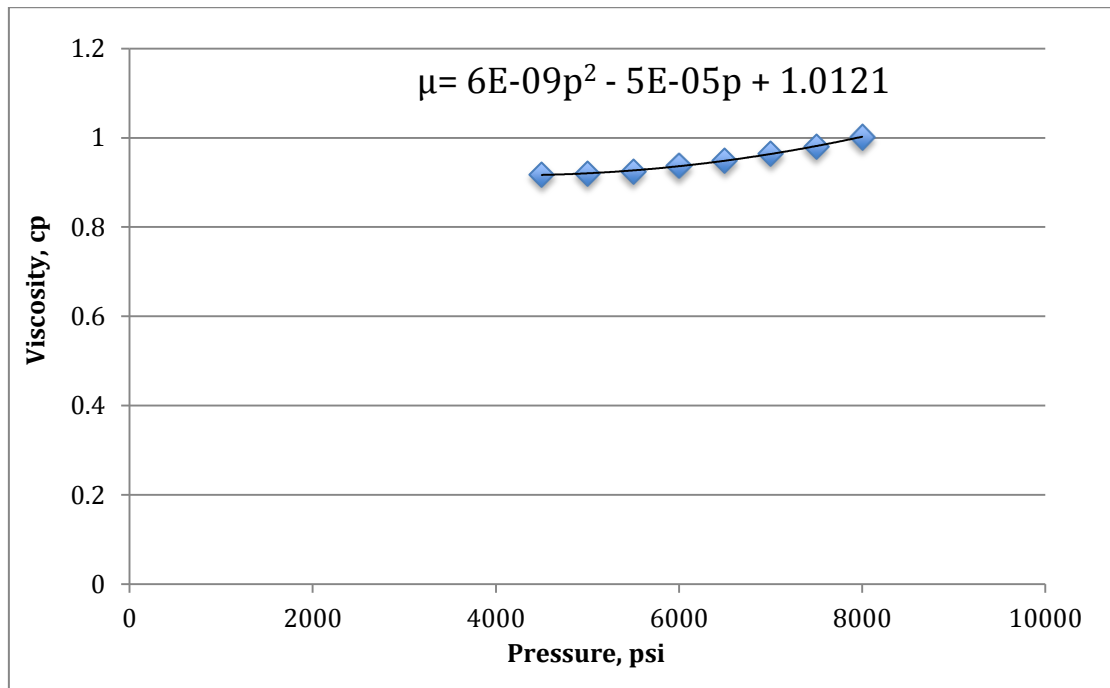
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***** 0.192 0.197 0.202 *****
0.190 0.195 0.200 0.204 0.207 ***** 0.215 0.205 *****
0.190 0.196 0.205 0.207 0.210 0.216 0.220 0.223 0.215 0.210 0.203 0.200
0.185 0.195 0.205 0.213 0.216 0.221 0.225 0.226 0.220 0.215 0.207 0.200
0.183 0.195 0.205 0.212 0.218 0.225 0.232 0.232 0.225 0.219 *****
***** 0.210 0.219 0.226 0.235 0.230 0.220 0.216 *****
***** 0.225 0.235 0.230 *****
***** 0.232 0.226 0.217 *****
***** 0.229 0.220 0.217 *****

```

Figure6. Gridblocks' permeabilities and porosities (A-1)

By figure3, viscosity versus pressure can be plotted



**Figure7. Plotting viscosity vs. pressure**

A-1 reservoir grid blocks can be simplified as following:

-	1	2	3	-	-	-	-	-	-	-	-
4	5	6	7	8	-	-	-	9	10	-	-
11	12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33	34
35	36	37	38	39	40	41	42	43	44	-	-
-	-	-	45	46	47	48	49	50	51	-	-
-	-	-	-	-	52	53	54	-	-	-	-
-	-	-	-	-	-	55	56	57	-	-	-
-	-	-	-	-	-	58	59	60	-	-	-

## Methodology

For slightly compressible flow (single phase)

$$\begin{aligned} \frac{\partial}{\partial x} \left[ \beta_c \frac{A_x k_x}{\mu_l B_l} \left( \frac{\partial p}{\partial x} - \gamma_l \frac{\partial Z}{\partial x} \right) \right] \Delta x + \frac{\partial}{\partial y} \left[ \beta_c \frac{A_y k_y}{\mu_l B_l} \left( \frac{\partial p}{\partial y} - \gamma_l \frac{\partial Z}{\partial y} \right) \right] \Delta y + q_{lsc} &= \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left( \frac{\phi}{B_l} \right) \\ \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left( \frac{\phi}{B_l} \right) &\cong \frac{V_b}{\alpha_c \Delta t} \left[ \left( \frac{\phi}{B_l} \right)^{n+1} - \left( \frac{\phi}{B_l} \right)^n \right] = \frac{V_b}{\alpha_c \Delta t} \left[ \frac{\phi^{n+1} - \phi^n}{B_l^{n+1}} + \left( \frac{1}{B_l^{n+1}} - \frac{1}{B_l^n} \right) \phi^n \right] \\ \rho_l &= \rho_l^o [1 + c_l(p - p^o)] \\ B_l &= \frac{B_l^o}{[1 + c_l(p - p^o)]} \\ \phi &= \phi^o [1 + c_\phi(p - p^o)] \end{aligned}$$

So

$$\left[ \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left( \frac{\phi}{B_l} \right) \right]_{i,j} \cong \left[ \frac{V_b}{\alpha_c \Delta t} \left( \frac{\phi^o c_\phi}{B_l^{n+1}} + \frac{\phi^n c_l}{B_l^o} \right) \right]_{i,j} (p_{i,j}^{n+1} - p_{i,j}^n)$$

If we set

$$\Gamma_{li,j}^{n+1} = \left[ \frac{V_b}{\alpha_c} \left( \frac{\phi^o c_\phi}{B_l^{n+1}} + \frac{\phi^n c_l}{B_l^o} \right) \right]_{i,j} = \left[ \frac{V_b \phi c_l}{\alpha_c B_l^o} \right]_{i,j}$$

Then

$$\left[ \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left( \frac{\phi}{B_l} \right) \right]_{i,j} \cong \frac{\Gamma_{li,j}^{n+1}}{\Delta t} (p_{i,j}^{n+1} - p_{i,j}^n)$$

Finite difference approximations for slightly compressible fluid flow equation can be derived as:

$$\begin{aligned} W_{i,j}(p_{i-1,k} - p_{i,j}) + E_{i,j}(p_{i+1,k} - p_{i,j}) + S_{i,j}(p_{i,k-1} - p_{i,j}) + N_{i,j}(p_{i,k+1} - p_{i,j}) \\ = \frac{\Gamma_{li,j}^{n+1}}{\Delta t} (p_{i,j}^{n+1} - p_{i,j}^n) - q_{lsci,j} + Q_{Gi,j} \end{aligned}$$

Transmissibility  $(T_{ix})_{i+1/2,j} = (\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x})_{i+1/2,j}$

While  $w_{i,j} = (T_{ix})_{i-1/2,j}$ ;  $E_{i,j} = (T_{ix})_{i+1/2,j}$ ;  $N_{i,j} = (T_{ix})_{i,j+1/2}$ ;  $S_{i,j} = (T_{ix})_{i,j-1/2}$

Assume the consecutive grid blocks have the same properties. For simplification, I use arithmetic averaging to calculate the average value of viscosity and FVF of fluid

$$\bar{\mu}_{l_{i+1/2,j}} = \left( \frac{\mu_{l_{i,j}} + \mu_{l_{i+1,j}}}{2} \right); \bar{B}_{l_{i+1/2,j}} = \left( \frac{B_{l_{i,j}} + B_{l_{i+1,j}}}{2} \right)$$

To calculate transmissibility, I use harmonic averaging:

$$\left( \frac{A_x k_x}{\Delta x} \right)_{i+1/2,j} = \left( \frac{2A_{xi,j}A_{xi+1,j}k_{xi,j}k_{xi+1,j}}{A_{xi,j}k_{xi,j}\Delta x_{i+1,j} + A_{xi+1,j}k_{xi+1,j}\Delta x_{i,j}} \right)$$

$$(T_{ix})_{i+1/2,j} = \beta_c \left( \frac{2}{\mu_{l_{i,j}} + \mu_{l_{i+1,j}}} \right) \left( \frac{2}{B_{l_{i,j}} + B_{l_{i+1,j}}} \right) \left( \frac{2A_{xi,j}A_{xi+1,j}k_{xi,j}k_{xi+1,j}}{A_{xi,j}k_{xi,j}\Delta x_{i+1,j} + A_{xi+1,j}k_{xi+1,j}\Delta x_{i,j}} \right)$$

In figure3,  $B_o = \frac{1}{1+9.0*10^{-6}(p-14.7)}$  is given

In figure4, by fitting the trend line, I can get

$$\mu_o = 6 * 10^{-9} * p^2 - 5 * 10^{-5} * p + 1.0121$$

In this case, explicit transmissibility and implicit approximation method can be applied, so the overall equation can be derived

$$W_{i,j}^n p_{i-1,j}^{n+1} + S_{i,j}^n p_{i,j-1}^{n+1} + E_{i,j}^n p_{i,j+1}^{n+1} + N_{i,j}^n p_{i,j+1}^{n+1} + C_{i,j}^{n+1} p_{i,j}^{n+1} = Q_{i,j}$$

1. For gridblocks without wells and for gridblocks hosting rate-specified wells

$$C_{i,j}^{n+1} = -(W_{i,j}^n + S_{i,j}^n + E_{i,j}^n + N_{i,j}^n + \frac{\Gamma_{li,j}^{n+1}}{\Delta t}); \quad Q_{i,j} = -(\frac{\Gamma_{li,j}^{n+1}}{\Delta t} p_{i,j}^n + q_{lsci,j}^{n+1} - Q_{Gi,j}^n)$$

2. For gridblocks hosting pressure-specified wells

$$C_{i,j}^{n+1} = -(W_{i,j}^n + S_{i,j}^n + E_{i,j}^n + N_{i,j}^n + \frac{\Gamma_{li,j}^{n+1}}{\Delta t} + J_{wi,j}^n); \quad Q_{i,j} = -(\frac{\Gamma_{li,j}^{n+1}}{\Delta t} p_{i,j}^n + J_{wi,j}^n p_{wfi,j} - Q_{Gi,j}^n)$$

For  $Q_{Gi,j}^n$  term

$$Q_{Gi,j} = W_{Gi,j} Z_{i-1,j} + S_{Gi,j} Z_{i,j-1} + E_{Gi,j} Z_{i+1,j} + N_{Gi,j} Z_{i,j+1} + C_{Gi,j} Z_{i,j}$$

Where

$$W_{Gi,j} = 0.00694 \rho_{li-1/2,j} W_{i,j} \quad ; \quad S_{Gi,j} = 0.00694 \rho_{li,j-1/2} S_{i,j} \quad ; \quad E_{Gi,j} = 0.00694 \rho_{li+1/2,j} E_{i,j} \quad ;$$

$$N_{Gi,j} = 0.00694 \rho_{li,j+1/2} N_{i,j}; \quad C_{Gi,j} = -(W_{Gi,j} + S_{Gi,j} + E_{Gi,j} + N_{Gi,j})$$

All the equations have been provided, and code can be developed via MATLAB to simulate this process. By doing some iterative calculation and matrix manipulation in the computer, result can be achieved.

## Result

	PI 19	PI 26	P 19	P 26	SFP 19	SFP 26	q 19	q 26	q total	q cum
t=1	4.81	13.48	7648	7025	7512	5600	650	19209	19859	19859
t=2	4.83	13.79	7574	6787	7440	5600	650	16369	17019	36878
t=3	4.84	13.87	7489	6635	7354	5600	650	14361	15011	51889
t=4	4.86	13.92	7392	6517	7258	5600	650	12771	13421	65310
t=5	4.87	13.96	7289	6421	7156	5600	650	11454	12104	77414
t=6	4.89	13.99	7184	6339	7051	5600	650	10337	10987	88401
t=7	4.90	14.01	7078	6269	6945	5600	650	9375	10025	98426
t=8	4.92	14.03	6974	6208	6842	5600	650	8535	9185	107611
t=9	4.93	14.04	6873	6155	6741	5600	650	7794	8444	116055
t=10	4.94	14.06	6776	6108	6644	5600	650	7134	7784	123839
t=11	4.96	14.07	6683	6065	6551	5600	650	6540	7190	131029
t=12	4.97	14.08	6594	6026	6463	5600	650	6003	6653	137682
t=13	4.98	14.09	6510	5991	6380	5600	650	5514	6164	143846
t=14	4.99	14.09	6431	5959	6301	5600	650	5066	5716	149562
t=15	4.99	14.10	6357	5930	6227	5600	650	4654	5304	154866
t=16	5.00	14.11	6287	5903	6157	5600	650	4274	4924	159790
t=17	5.01	14.11	6221	5878	6092	5600	650	3923	4573	164363
t=18	5.01	14.12	6160	5855	6030	5600	650	3597	4247	168610
t=19	5.02	14.12	6102	5833	5973	5600	650	3295	3945	172555
t=20	5.02	14.12	6049	5813	5919	5600	650	3014	3664	176219
t=21	5.03	14.13	5998	5795	5869	5600	650	2753	3403	179622
t=22	5.03	14.13	5951	5778	5822	5600	650	2509	3159	182781
t=23	5.04	14.13	5907	5761	5778	5600	650	2283	2933	185714
t=24	5.04	14.14	5866	5747	5737	5600	650	2072	2722	188436
t=25	5.04	14.14	5828	5733	5699	5600	650	1875	2525	190961
t=26	5.04	14.14	5792	5720	5664	5600	650	1691	2341	193302
t=27	5.05	14.14	5759	5707	5630	5600	650	1520	2170	195472
t=28	5.05	14.15	5728	5696	5599	5600	650	1360	2010	197482
t=29	5.05	14.15	5699	5686	5570	5600	650	1212	1862	199344
t=30	5.05	14.15	5672	5676	5543	5600	650	1073	1723	201067
t=31	5.05	14.15	5647	5667	5518	5600	650	943	1593	202660
t=32	5.06	14.15	5623	5658	5495	5600	650	823	1473	204133
t=33	5.06	14.15	5601	5650	5473	5600	650	710	1360	205493
t=34	5.06	14.16	5581	5643	5452	5600	650	605	1255	206748
t=35	5.06	14.16	5562	5636	5433	5600	650	507	1157	207905
t=36	5.06	14.16	5544	5629	5416	5600	650	416	1066	208971
t=37	5.06	14.16	5528	5623	5399	5600	650	331	981	209952
t=38	5.06	14.16	5512	5618	5384	5600	650	251	901	210853
t=39	5.06	14.16	5498	5613	5369	5600	650	177	827	211680
t=40	5.06	14.16	5484	5608	5356	5600	650	108	758	212438
t=41	5.06	14.16	5472	5603	5343	5600	650	44	694	213132
t=42	5.06	14.16	5460	5599	5332	5600	650	0	650	213782



t=43	5.06	14.16	5449	5595	5321	5600	650	0	650	214432
t=44	5.07	14.16	5439	5591	5311	5600	650	0	650	215082
t=45	5.07	14.16	5430	5588	5301	5600	650	0	650	215732
t=46	5.07	14.16	5421	5585	5293	5600	650	0	650	216382
t=47	5.07	14.17	5413	5582	5284	5600	650	0	650	217032
t=48	5.07	14.17	5405	5579	5277	5600	650	0	650	217682
t=49	5.07	14.17	5398	5576	5270	5600	650	0	650	218332
t=50	5.07	14.17	5391	5574	5263	5600	650	0	650	218982
t=51	5.07	14.17	5385	5572	5257	5600	650	0	650	219632
t=52	5.07	14.17	5379	5569	5251	5600	650	0	650	220282
t=53	5.07	14.17	5374	5568	5246	5600	650	0	650	220932
t=54	5.07	14.17	5369	5566	5241	5600	650	0	650	221582
t=55	5.07	14.17	5364	5564	5236	5600	650	0	650	222232
t=56	5.07	14.17	5360	5562	5232	5600	650	0	650	222882
t=57	5.07	14.17	5356	5561	5227	5600	650	0	650	223532
t=58	5.07	14.17	5352	5560	5224	5600	650	0	650	224182
t=59	5.07	14.17	5348	5558	5220	5600	650	0	650	224832
t=60	5.07	14.17	5345	5557	5217	5600	650	0	650	225482

## Pressure distribution with time

	p(t=10)	p(t=20)	p(t=30)	p(t=40)	p(t=50)	p(t=60)
block 1	6282	5893	5720	5635	5592	5572
block 2	6279	5892	5720	5635	5593	5572
block 3	6275	5893	5721	5636	5593	5573
block 4	6278	5891	5719	5635	5593	5572
block 5	6264	5880	5709	5624	5582	5562
block 6	6258	5879	5709	5625	5584	5563
block 7	6253	5882	5712	5628	5587	5566
block 8	6305	5905	5718	5625	5579	5556
block 9	6828	6085	5699	5506	5410	5363
block 10	6889	6128	5730	5531	5433	5384
block 11	6269	5886	5716	5631	5590	5569
block 12	6251	5872	5703	5619	5578	5557
block 13	6232	5865	5700	5618	5577	5557
block 14	6210	5860	5698	5617	5578	5558
block 15	6325	5911	5714	5617	5568	5544
block 16	6467	5971	5728	5608	5548	5518
block 17	6648	6043	5739	5588	5513	5476
block 18	6769	6080	5728	5552	5465	5422
block 19	6776	6049	5672	5484	5391	5345
block 20	6882	6123	5727	5529	5431	5383
block 21	6929	6153	5745	5542	5441	5391
block 22	6971	6179	5761	5552	5448	5397



block 23	6263	5884	5715	5631	5590	5569
block 24	6243	5872	5705	5623	5582	5562
block 25	6201	5853	5695	5616	5577	5558
block 26	6108	5813	5676	5608	5574	5557
block 27	6319	5905	5706	5607	5559	5534
block 28	6478	5974	5727	5604	5543	5513
block 29	6651	6043	5738	5586	5511	5473
block 30	6771	6086	5735	5561	5474	5431
block 31	6819	6096	5723	5536	5444	5398
block 32	6886	6129	5735	5538	5440	5392
block 33	6927	6152	5745	5542	5441	5392
block 34	6967	6175	5757	5549	5445	5394
block 35	6266	5888	5718	5635	5593	5573
block 36	6252	5879	5712	5629	5587	5567
block 37	6233	5871	5705	5622	5581	5561
block 38	6231	5865	5693	5607	5565	5543
block 39	6361	5920	5707	5601	5549	5523
block 40	6493	5977	5723	5597	5534	5503
block 41	6664	6049	5740	5586	5509	5472
block 42	6774	6091	5742	5569	5483	5440
block 43	6825	6107	5737	5552	5461	5415
block 44	6883	6131	5739	5544	5447	5399
block 45	6283	5893	5708	5616	5570	5548
block 46	6395	5937	5715	5605	5550	5523
block 47	6517	5988	5728	5598	5534	5502
block 48	6673	6054	5744	5589	5512	5474
block 49	6778	6096	5749	5576	5491	5448
block 50	6825	6111	5743	5560	5469	5424
block 51	6883	6133	5743	5548	5452	5404
block 52	6567	6012	5738	5601	5534	5500
block 53	6705	6071	5752	5593	5514	5475
block 54	6789	6105	5757	5584	5498	5456
block 55	6754	6094	5762	5596	5514	5473
block 56	6810	6117	5765	5590	5503	5460
block 57	6834	6130	5772	5594	5506	5462
block 58	6796	6115	5771	5600	5515	5473
block 59	6830	6128	5772	5594	5506	5463
block 60	6843	6134	5774	5595	5506	5462

## Reference

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