Euler's Method:

$$\vec{x}_{Eu} = \vec{x}(0) + \Delta t \vec{v}(0)$$

$$\vec{v}_{Eu} = \vec{v}(0) + \Delta t \vec{a}(0, \vec{v}_{Eu}) = \vec{v}(0) + \Delta t \frac{\vec{F}(0, \vec{v}_{Eu})}{m}$$

Heun's Method: A Solution of Order 2

$$\vec{x}_{He}(\Delta t) = \vec{x}(0) + \Delta t \frac{\vec{v}(0) + \vec{v}_{Eu}}{2}$$

$$\vec{v}_{He}(\Delta t) = \vec{v}(0) + \Delta t \frac{\vec{a}(0, \vec{v}_{Eu}) + \vec{a}(\Delta t, \vec{v}_{Eu})}{2}$$

$$= \vec{v}(0) + \Delta t \frac{\vec{F}(0, \vec{v}_{Eu}) + \vec{F}(\Delta t, \vec{v}_{Eu})}{2m}$$

Local Truncation Error:

$$LTE = \left|\left|\vec{x}_{Eu} - \vec{x}_{He}\right|\right| + \Delta t \left|\left|\vec{v}_{Eu} - \vec{v}_{He}\right|\right| = C\Delta t^2$$

To keep LTE in tolerance:

$$C\Delta t'^2 \leq tolerance$$

Thus:

$$\frac{\Delta t'^2}{tolerance} \leq \frac{\Delta t^2}{LTE}$$

$$\Delta t' \leq \Delta t \sqrt{\frac{tolerance}{LTE}}$$