



E stands for earth (blue object)  
M stands for moon (pink object)  
S stands for satellite (green object)

For S, according to Newton's Second Law and Newtonian Gravity:

$$m_s \vec{a}_s = \frac{Gm_E m_s}{d_{ES}^2} \vec{e}_{SE} + \frac{Gm_M m_s}{d_{MS}^2} \vec{e}_{SM}$$

Where  $m_S$ ,  $m_E$ ,  $m_M$  are the mass of satellite, earth, moon respectively,  $G$  is the gravitational constant,  $d_{ES}$ ,  $d_{MS}$  are the distance from earth to satellite and distance from moon to satellite,  $\vec{a}_s$  is the vector of acceleration of satellite,  $\vec{e}_{SE}$ ,  $\vec{e}_{SM}$  are the unit vectors from satellite to earth and to moon.

$$\vec{e}_{SE} = \frac{-\vec{x}_S}{d_{ES}} = \frac{-\vec{x}_S}{||\vec{x}_S||}$$

$$\vec{e}_{SM} = \frac{\vec{x}_M - \vec{x}_S}{||\vec{x}_M - \vec{x}_S||}$$

Finally:

$$\frac{d^2 \vec{x}}{dt^2} = \vec{a}_s = G \left[ \frac{-m_E}{||\vec{x}_S||^3} \vec{x}_S + \frac{m_M}{||\vec{x}_M - \vec{x}_S||^3} (\vec{x}_M - \vec{x}_S) \right]$$

Obviously, this PDE is non-linear.