Simultaneous identification of a system model and disturbance using a Kalman filter

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***Abstract*— This report includes a method to simultaneous identify a system model and disturbance using a Kalman filter [1]. We pick Project idea 1.2, that is, "Extend the algorithm proposed in [1] to make it applicable to a building with multiple rooms, by dividing the building into multiple sections (depending on its geometry perhaps) and then estimate the model and the disturbance for each section." [2]. A full order RC model is used to simulate a building with multiple sections. State space model and Kalman filter is used to identify the model and the disturbance.**

***Index Terms*— Kalman Filter, RC model, system identification, disturbance, thermal model.**

1. INTRODUCTION

The goal of this project is to find a thermal dynamic model for a building with multiple rooms. The challenge comes from a few aspects;

1. A proper model needs to be found.
2. The disturbance which cannot be modeled or measured needs to be estimated so we can have a more accurate model.
3. All parameters in the model need to be estimated by the experiment data comes from sensors.
4. The model should be tested and the result needs to be analyzed.

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A.Choose a model

A proper model which is given in [3] is shown below:

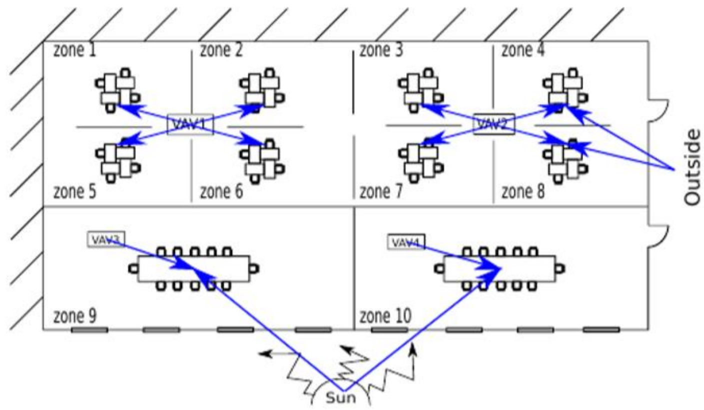

Figure 1 Building layout for the multi-zone simulation data.

We can see the model have 10 separate zones and each of them has their own temperature and will have thermal interact with certain other zones. The temperature of the environment outside this building only affect zone 4, 8 and 10 while the solar radiation only affect zone 9 and 10. Heating, ventilation, and air conditioning (HVAC) system will provide inputs to control the indoor temperature. These inputs are labeled as VAV1---VAV4 and the zones that can be influenced by them can be seen in Figure 1.

From Figure 1 we can get a full order RC model (which is also given in [3]) to represent the building:

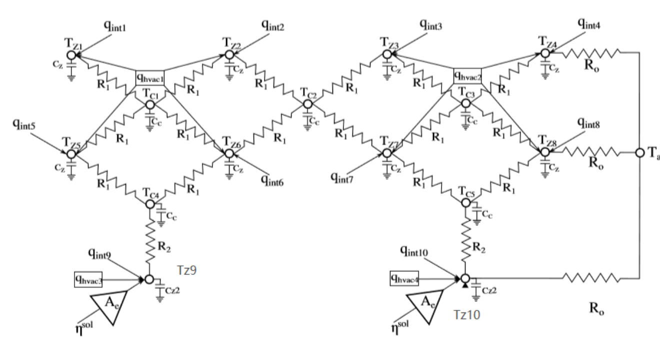

Figure 2 RC network model for multi-zone simulation data.

In Figure 2 $T_{z1}\ to\ \ T_{z10}$ stands for the indoor temperature of Zone 1 to Zone 10. $T_{c1}\ to\ \ T_{c5}$ represents the indoor temperature of the walls between different zones. The resistances(R0, R1 and R2) in the model illustrate that there is resistance to heat flow between different zones and the capacitances stand for the thermal capacitance of the building's structure (Cc>Cz because the thermal capacitance of the wall is larger than the other objects in the building); Ta is the outdoor environment temperature; q_{hvac1} to q_{hvac4} are HVAC heat gain; A_e is the effective area of the building and $A_e\eta_{sol}$ is solar radiation gain; q_{int1} to q_{int10} is the disturbance term contains all other dynamics that cannot be measured or modeled for each room.

B.Disturbance term

As shown above the disturbance term is labeled as q_{int1} to q_{int10} . We assume the disturbance as piecewise constant. So $\frac{d}{dt}q_{int}(t) = 0$ for all t(time). So those terms can be put into the state of the system in the form of state space model

C.Parameter estimation

All unknown parameters in Figure 2 need to be estimated. Among all those parameters, Ta, q_{hvac1} to q_{hvac4} and η_{sol} can be measured by sensors. That makes them to be the known input into the system; T_{z1} to T_{z10} and T_{c1} to T_{c5} will be the state of the system and also, as we illustrated above, q_{int1} to q_{int10} can be included in the state of the system. As a result, R0, R1, R2, Cz, Cz2, Cc and A_e are the parameters that need to be estimated.

We use a Kalman filter to find the best estimation of those parameters and the state of the system which contains all of the disturbance terms. The estimation process has 4 steps:

1. Rewrite the model in discreet state space form and set initial conditions for the unknown parameters.
2. Use a Kalman filter to estimate the states and outputs.
3. Find a cost function and solve for the best estimation to minimize the cost function.
4. Update the unknown parameters by the estimation and repeat step 3&4 until the estimation becomes stable.

According to Figure 2 if we can see there are ten Tzs, ten q_{int} and five T_c. We need to use the RC network model for multi-zone given in the paper to calculate them respectively.

$$\dot{T_{z1}} = \frac{1}{C_z}(\frac{T_{c1}-T_{z1}}{R_1} + q_{hvac1} + q_{int1})$$

$$\dot{T_{z2}} = \frac{1}{C_z}(\frac{T_{c1}-T_{z2}}{R_1} + \frac{T_{c2}-T_{z2}}{R_1} + q_{hvac1} + q_{int2})$$

$$\dot{T_{z3}} = \frac{1}{C_z}(\frac{T_{c2}-T_{z3}}{R_1} + \frac{T_{c3}-T_{z3}}{R_1} + q_{hvac2} + q_{int3})$$

$$\dot{T_{z4}} = \frac{1}{C_z}(\frac{T_{c3}-T_{z4}}{R_1} + \frac{T_a-T_{z4}}{R_0} + q_{hvac2} + q_{int4})$$

$$\dot{T_{z5}} = \frac{1}{C_z}(\frac{T_{c1}-T_{z5}}{R_1} + \frac{T_{c4}-T_{z5}}{R_1} + q_{hvac1} + q_{int5})$$

$$\dot{T_{z6}} = \frac{1}{C_z}(\frac{T_{c1}-T_{z6}}{R_1} + \frac{T_{c2}-T_{z6}}{R_1} + \frac{T_{c4}-T_{z6}}{R_1} + q_{hvac1} + q_{int6})$$

$$\dot{T_{z7}} = \frac{1}{C_z}(\frac{T_{c2}-T_{z7}}{R_1} + \frac{T_{c3}-T_{z7}}{R_1} + \frac{T_{c5}-T_{z7}}{R_1} + q_{hvac2} + q_{int7})$$

$$\dot{T_{z8}} = \frac{1}{C_z}(\frac{T_{c3}-T_{z8}}{R_1} + \frac{T_{c5}-T_{z8}}{R_1} + \frac{T_a-T_{z8}}{R_0} + q_{hvac8} + q_{int8})$$

$$\dot{T_{z9}} = \frac{1}{C_{z2}}(\frac{T_{c4}-T_{z9}}{R_2} + q_{hvac3} + q_{int9} + A_e\eta_{sol})$$

$$\dot{T_{z10}} = \frac{1}{C_{z2}}(\frac{T_{c5}-T_{z10}}{R_2} + \frac{T_a-T_{z10}}{R_0} + q_{hvac4} + q_{int10} + A_e\eta_{sol})$$

$$\dot{T_{c1}} = \frac{1}{C_c}(\frac{T_{z1}-T_{c1}}{R_1} + \frac{T_{z2}-T_{c1}}{R_1} + \frac{T_{z5}-T_{c1}}{R_1} + \frac{T_{z6}-T_{c1}}{R_1})$$

$$\dot{T_{c2}} = \frac{1}{C_c}(\frac{T_{z2}-T_{c2}}{R_1} + \frac{T_{z3}-T_{c2}}{R_1} + \frac{T_{z6}-T_{c2}}{R_1} + \frac{T_{z7}-T_{c2}}{R_1})$$

$$\dot{T_{c3}} = \frac{1}{C_c}(\frac{T_{z3}-T_{c3}}{R_1} + \frac{T_{z4}-T_{c3}}{R_1} + \frac{T_{z7}-T_{c3}}{R_1} + \frac{T_{z8}-T_{c3}}{R_1})$$

$$\dot{T_{c4}} = \frac{1}{C_c}(\frac{T_{z5}-T_{c4}}{R_1} + \frac{T_{z6}-T_{c4}}{R_1} + \frac{T_{z9}-T_{c4}}{R_2})$$

$$\dot{T_{c5}} = \frac{1}{C_c}(\frac{T_{z7}-T_{c5}}{R_1} + \frac{T_{z8}-T_{c5}}{R_1} + \frac{T_{z10}-T_{c5}}{R_2})$$

Then we can rewrite those equations into state space function (notice that q_{int} have been put into the state of the system as q_{dist} in order to match the parameters' name listed in the proposal)

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$\begin{bmatrix} \dot{T_{z1}} \\ \vdots \\ \dot{T_{z10}} \\ \dot{T_{c1}} \\ \vdots \\ \dot{T_{c5}} \\ \dot{q_{dist1}} \\ \vdots \\ \dot{q_{dist10}} \end{bmatrix}_{25\times1} = \begin{bmatrix} \frac{-1}{C_z R_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{25\times25} \begin{bmatrix} T_{z1} \\ \vdots \\ T_{z10} \\ T_{c1} \\ \vdots \\ T_{c5} \\ q_{dist1} \\ \vdots \\ q_{dist10} \end{bmatrix}_{25\times1} + \begin{bmatrix} \frac{1}{C_z} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{25\times6} \begin{bmatrix} q_{hvac1} \\ q_{hvac2} \\ q_{hvac3} \\ q_{hvac4} \\ T_a \\ \eta_{sol} \end{bmatrix}_{6\times1}$$

$$\begin{bmatrix} \dot{T_{z1}} \\ \vdots \\ \dot{T_{z10}} \end{bmatrix}_{10\times1} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{10\times25} \begin{bmatrix} T_{z1} \\ \vdots \\ T_{z10} \\ T_{c1} \\ \vdots \\ T_{c5} \\ q_{dist1} \\ \vdots \\ q_{dist10} \end{bmatrix}_{25\times1}$$

Where A is a 25*25 matrix, B is a 25*6 matrix; C is a 10*25 matrix.

Then we can use "c2d" function in MATLAB to convert this continuous form into a discreet form of state space function:

$$x_{k+1} = Ax_k + Bu_k + \xi_k$$
$$z_k = Cx_k + n_k$$

The cost function will be the sum of absolute value of the error: f (p) = $\sum_{k=1}^{kmax}|(zk - C\hat{x}k|k)|$ where zk is the measurement and $\hat{x}k|k$ is the state estimation.

Now we have the discreet version of the state space and the cost function, we can use the SPDI algorithm described in [1] and solve the best estimation of the parameters vector p = [R0, R1, R2, Cc, Cz, Cz2, Ae]. The initial condition of p and covariance of the model noise (Q) and sensor noise (R) are listed in [1], this initial conditions will be adjusted if the result is not good enough. Function fmincon in MATLAB is used in this process.

After the best estimation of p being computed, a new state estimation ($\hat{x}k|k$) and output (zk) can be calculated using the Kalman filter used in SPDI algorithm. $\hat{x}k|k$ contains the estimation of Tz's, Tc's and $q_{int}'s$ and zk is the estimation of Tz.

2. Implementation

We realize the algorithm using MATLAB.

Load and preprocess data

:

First, we should load the data and change the Celsius unit to Kelvin temperature unit. Which means add 273.15 to Ta and Tz. Because all the units of operation in this system are Kelvin, we need to unify the units first. Get $T = 300s$ for one data and $T_s = \frac{1}{12} hour$ for one data.

Initialize parameters

Then we need to set the initial condition to calculate A, B and C. The initial system parameters we need to set are $R_0, R_1, R_2, C_c, C_z, C_{z2}$ and A_e. Because of the data input, the initial input parameters need ten T_{zInit}, five T_{wInit} and ten $Q_{distInit}$.

Build Kalman Filter

In order to get the best predictions, we need to get the best $R_0, R_1, R_2, C_c, C_z, C_{z2}$ and A_e to minimize the error of the error function, we call them collectively p.

$$p = [R_0, R_1, R_2, C_c, C_z, C_{z2}, A_e]$$

Frist, we need to build a Kalman Filter and create an equation of this Kalman Filter with p as an unknown number.

```
error_fun = @(p) kalmanfilter(p,Ta,etaSol,Qhvac,Ts,Tz',TzInit,TwInit,QdistInit,N);
```

In order to build the Kalman Filter, we use the model we build before. Then we can calculate A, B and C. We use 'ss' and 'c2d' function in MATLAB to calculate the estimation value of A, B, C, and D.

After that initialize the process and noise covariance:

$$Q_k = Q = diag(\alpha_1, \alpha_1, \alpha_2)$$

Where $\alpha_2 \approx 10^{-3}$ and $\alpha_1 \approx 10^{-7}$.

$$R = \frac{\alpha_2}{3}$$

Using the state estimator build in class:

$$\widehat{x_{k|k}} = A_{k-1}\widehat{x_{k-1|k-1}} + B_{k-1}U_{k-1} + M_k[Z_k - C_k(A_{k-1}\widehat{x_{k-1|k-1}} + B_{k-1}U_{k-1})]$$

Where M_k is Kalman Gain,

$$M_k = \Sigma_k C_k^T (C_k \Sigma_k C_k^T + \Theta_k)^{-1}$$

Using 'for' loop in MATLAB to implement the above equation.

```
for k=2:1:N
    Y_k = Y(:,k);
    U_k = uKal(:,k-1);
    P_k = A_est*P_k_fin*A_est' + Q_k;
    K_k = P_k*C_est'/(C_est*P_k*C_est'+R_k);
    x_hat_fin = A_est*x_hat_fin + B_est*U_k + K_k*(Y_k - C_est*(A_est*x_hat_fin + B_est*U_k));
    P_k_fin = (I - K_k*C_est)*P_k;
    xHatHist(:,k) = x_hat_fin;
end
```

Last, use the initialize value $T_{zInit}, T_{wInit}, q_{distInit}$ to estimate the prior value. We put $T_{zInit}, T_{wInit}, q_{distInit}$ in uKal(:,1), and use it to calculate the error between X_{k-1} and the true value. Then use the error to estimate the X_k value. Then use X_k value to estimate X_{k+1}. At last, we can get the finial value. The x_hat_fin list includes T_z, T_w and q_{dist} value.

Then we can get an error function of parameter p from Kalman Filter.

```
error = Y-C*xHatHist;
Fval = sum(abs(error(:)));
```

Fval is the finial error of the system. We need to use it to calculate the best p.

Get p (unknown parameters)

Calculate parameter p to get the min value of this error function error_fun:

```
[X,FVAL] = fmincon(error_fun,X0,[],[],[],[],LB,UB,[],options);
```

We should set initial value of p. Set $R_0 = 7, R_1 = 48, R_2 = 48, C_c = 48, C_z = 25, C_{z2} = 25$ and $A_e = 25$. We try the initial condition for several times and find out that the initial value should not be too small. Because that will influence the estimation of itself and the parameter behind it become too small, even approaching to the lower bounded. And the upper bounded of the parameter is 50, so set the parameter not so small and less than 50 is all right.

X is the new value of p. We will put the new p into the RC model to get new A, B, C and D, and use them to establish a new Kalman Filter to adapt our data.

In 'fmincon' function LB and UB are lower bounded and upper bounded respectively. We can get the parameter from the paper.

$$0.01 \le C_w,\ C_z\left(\frac{\mathrm{K}}{\mathrm{kWh}}\right),\ R_w,\ R_z\left(\frac{\mathrm{kW}}{\mathrm{K}}\right),\ A_e\ (\mathrm{m}^2) \le 50$$

The lower bounded is 0.01 and the upper bounded is 50. Initialize it with the right from.

```
LB = 0.01*ones(nParams,1);
UB = 50*ones(nParams,1);
```

Calculate the T_z and q_{dist}:

Using the new p we calculated in the previous step, we can build a new Kalman Filter.

```
[Fval,x_hat_fin] = kalmanfilter(p,Ta,etaSol,Qhvac,Ts,Tz',TzInit,TwInit,QdistInit,N);
```

x_hat_fin is the system state we want to estimate. It contains ten T_{z_hat}, five T_{w_hat} and ten q_{dist_hat}. We can compute y using x_hat_fin and then compare y to with their true values respectively to verify the accuracy of our model.

3. Result Analysis

Since the full result will show the comparison between 20 estimation (10 T_z s and 10 $q_{dist}s$) and their true values. We only choose several typical results for analysis.

Result for T_z s

Several results of T_z s are shown below:

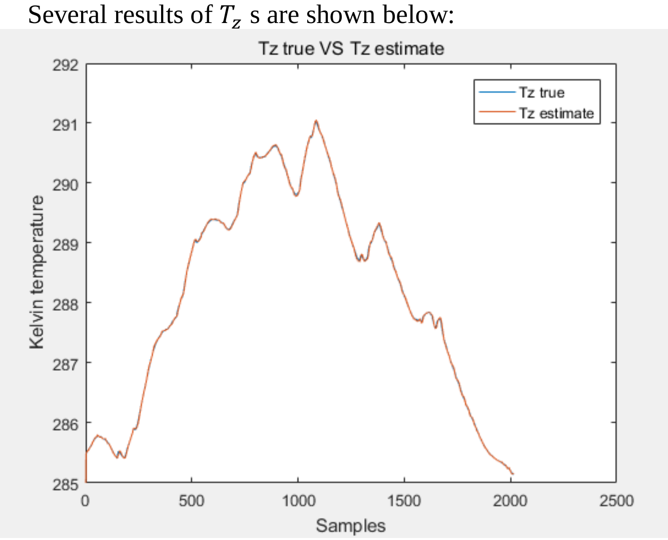

Figure 3 the estimation of T_{z6} VS its true value

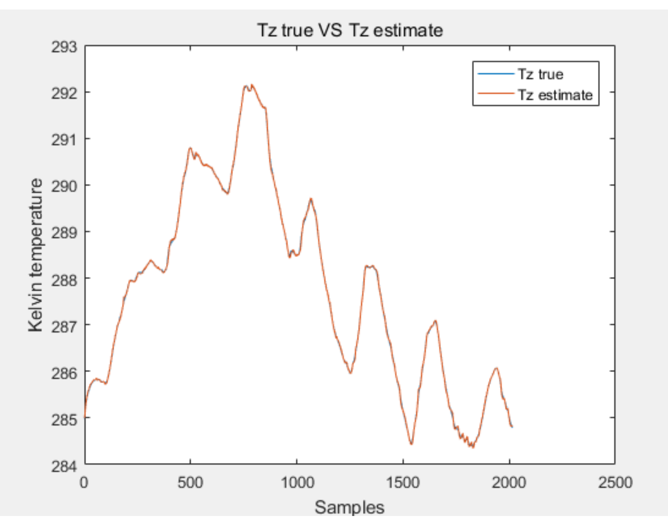

Figure 4 the estimation of T_{z8} VS its true value

T_{z6} and T_{z8} are only two representatives, in fact , all estimations of T_zs match their true value very well. That is probably because we used the true value to form our cost function, since the estimation of the model will minimize the cost function, the error between T_zs and their true value will be relatively small.

Result for q_{dist} s

Most of the estimation of q_{dist} s have good results, but some of them are not good enough, I will discuss them as follow.

Several results of q_{dist} s are shown below:

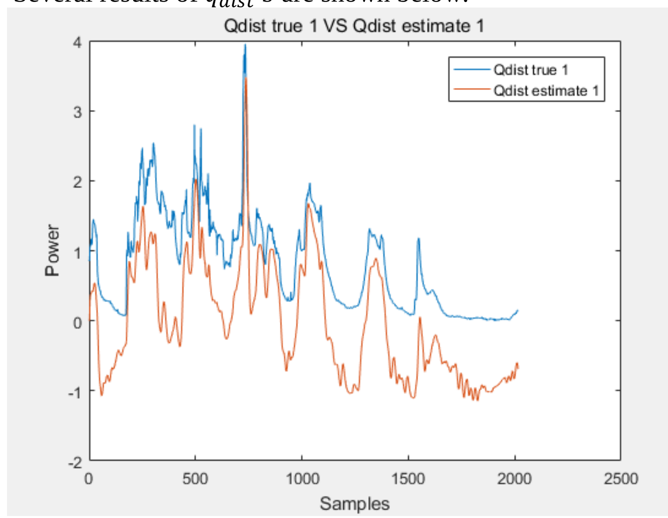

Figure 5 the estimation of q_{dist1} VS its true value

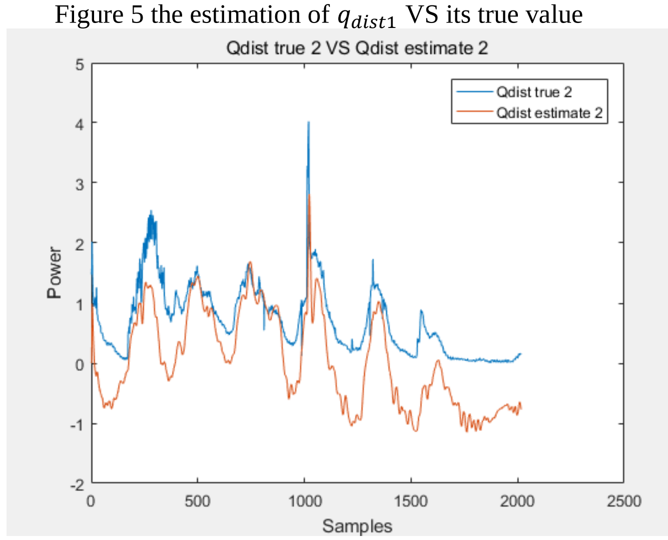

Figure 6 the estimation of q_{dist2} VS its true value

From these images above we can see that although q_{dist_est} is not as same as q_{dist}, but the trend of two lines are approximately the same. So, the result of q_{dist_est1} and q_{dist_est2} performs well. Also the estimation of q_{dist3}, q_{dist5} and q_{dist6} show good performances.

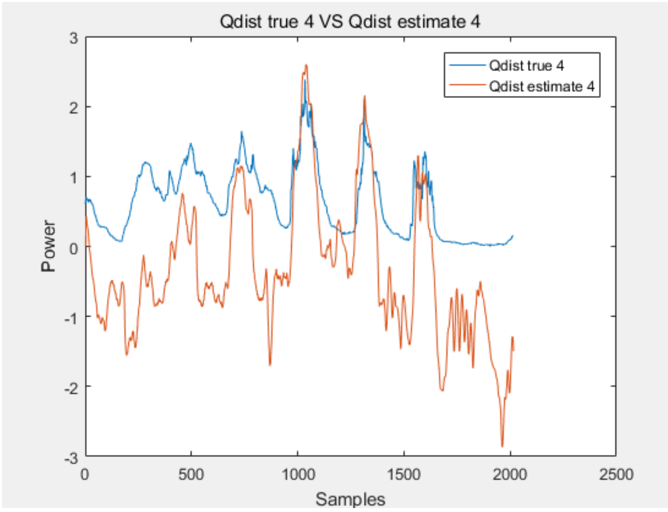

Figure 6 the estimation of q_{dist4} VS its true value

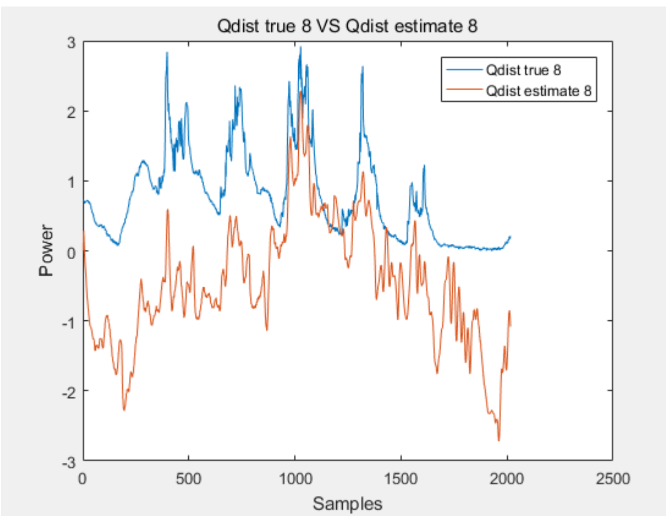

Figure 7 the estimation of q_{dist8} VS its true value

In Figure 6&7 we can see that there are some errors of the estimations of q_{dist4} and q_{dist8}, I think the reason is that R_0 and T_a have some influence on the result.

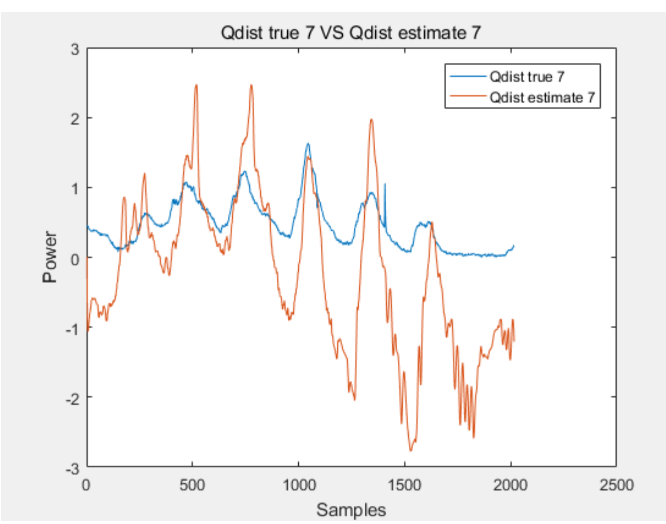

Figure 8 the estimation of q_{dist7} VS its true value

The image above shows, the predicted value of q_{dist7} fluctuates greatly. I am not sure what cause the result like that, but according to the building layout, T_{z4} and T_{z8} influence T_{c3}, T_{z10} will affect T_{c5}, and finally T_{c3} and T_{c5} will influence zone 7. So the error of estimation of q_{dist4} , q_{dist8} and q_{dist10} may somehow affect the result of q_{dist7}.

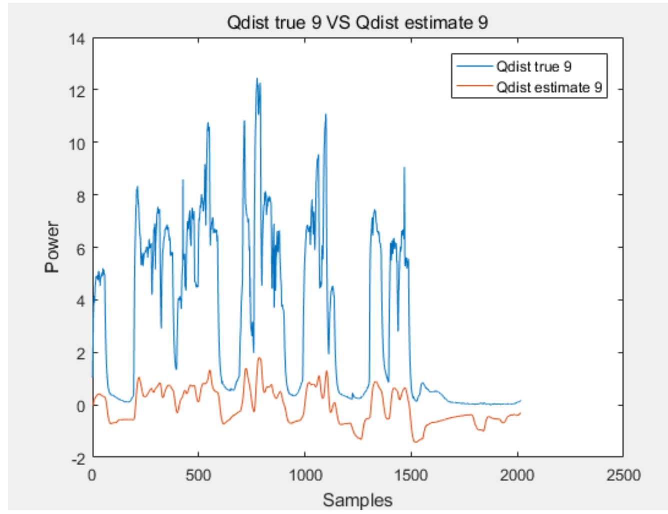

Figure 9 the estimation of q_{dist9} VS its true value

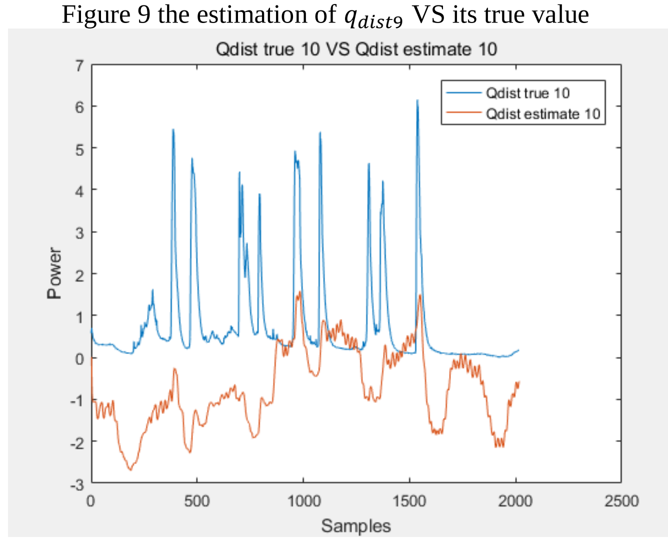

Figure 10 the estimation of q_{dist10} VS its true value

The estimation of q_{dist9} and q_{dist10} get the worst result among all 10 estimation of q_{dist}. These two results are shown above. Maybe the reason is the complexity of these two zones makes it difficult to estimate their parameters. We can see in Figure 2 that zone 9&10 are not only affected by the indoor situations but also influenced by the solar radiation (both zone 9&10) and the outside temperature (zone 10).

Conclusion and future work

Our model provides good estimation of T_z s and relatively bad estimation of the disturbance terms. There are some improvements can be made in the future:

1. A new error function that contains both error of T_z

and q_{dist} may improve the estimation of q_{dist}, but it may cause a worse performance on T_z s.

2. The initial condition of those parameters can influence the result greatly, so a better guess of the initial conditions may improve the performances.
3. We use Kalman filter and fmincon function in MATLAB to find the best estimation of the model parameters. Maybe a different algorithm can give us a better result.
4. The model we use is a linear RC model. A different model may provide a better result.

4. REFERENCES

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