PHY781 - Project 2

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3.

Assume that $d = 2$, $g = 0$, $m_0 a = 0.5$, $L_X = 8$. Then, $\kappa = \frac{1}{(2d+m_0^2 a^2)} = 1/4.25$. The values of correlation function C_0^l is shown in the following table.

d	L_X	L_T	κ	C_0^l
2	8	16	0.235294118	0.1573400844
2	8	24	0.235294118	0.0217219005
2	8	32	0.235294118	0.0029999106
2	8	48	0.235294118	5.72E-05

Data in the table are fitted to the equation $C_0^l = A * \exp(-BL_T/2) + C$. The fitted parameters are $A = 8.25528324$, $B = 0.495025299$ and $C = 1.08084156e - 08$.
According to the function in problem, $a = B/M_{phys}$. In natural units, $1 = \hbar c = 0.197327 GeV * fm$. Assume $M_{phys} = 100GeV$, we have $a = 9.76819 \times 10^{-4} fm$. $L = L_X a = 7.81455 \times 10^{-3} fm$, so $C = \frac{L_x |\langle 0|\phi|0\rangle|^2}{\kappa}$. We have $\langle 0|\phi|0\rangle = 1.22800 \times 10^{-4}$. $A = \frac{L_x |\langle \pi|\phi|0\rangle|^2}{\kappa}$, we have $\langle \pi|\phi|0\rangle = 0.492750$.

4.

To keep M_{phys} and L the same, we should double L_X and find κ to make the fitted $M_{phys}a$ half of the original value. Since we can calculate the exact value in free case, we can easily find the proper κ to be 0.246212599. The code is attached in Appendix B. To make the result more accurate, I fit the data for $L_T = 32, 48, 64, 96, 128, 256$.

d	L_X	L_T	κ	C_0^l
2	16	16	0.246212599	2.28858
2	16	24	0.246212599	0.310258
2	16	32	0.246212599	0.0428486
2	16	48	0.246212599	0.00591972
2	16	64	0.246212599	8.17841e-04
2	16	96	0.246212599	1.12989e-04
2	16	128	0.246212599	2.15662e-06
2	16	256	0.246212599	2.86231e-13

The fitted parameters are $A = 16.2666$, $B = 0.247467$ and $C = 5.29679886e - 04$. Then, $\langle 0|\phi|0\rangle = 2.8550 \times 10^{-3}$, $\langle \pi|\phi|0\rangle = 0.500315$.
When the lattice spacing is reduced to quater of original one. We should choose $\kappa = 0.249075732$ to keep M_{phys} and L the same. We have the data as the following table.

d	L_X	L_T	κ	C_0^l
2	32	16	0.249075732	14.4831
2	32	24	0.249075732	4.78586
2	32	32	0.249075732	1.77532
2	32	48	0.249075732	0.668601
2	32	64	0.246212599	0.252340
2	32	96	0.246212599	0.0952655
2	32	128	0.246212599	0.0135792
2	32	256	0.246212599	5.60596e-06

The fitted parameters are $A = 34.6075$, $B = 0.123705$ and $C = 3.09979 \times 10^{-3}$. Then, $\langle 0|\phi|0\rangle = 4.91198 \times 10^{-3}$, $\langle \pi|\phi|0\rangle = 0.519010$.
From the calculated result, we can observe that $\langle 0|\phi|0\rangle$ is much smaller than $\langle \pi|\phi|0\rangle$, but both $\langle 0|\phi|0\rangle$ and $\langle \pi|\phi|0\rangle$ increases slightly when the lattice spacing decreases. Since the calculation has some error, the increasement might just be a coincidence. We can conculude that when the field is observed, it is nearly impossible to be in the ground state, while the probability for the field to be in the one particle state is around 0.5.

5.

Define $m_0^2 = M_{phys}^2 + \delta m_0^2$. Since $\kappa = \frac{1}{(2d+m_0^2 a^2)}$, $m_0^2 = \frac{1}{a^2}(\frac{1}{\kappa} - 2d)$. Thus, for $a = 9.76819 \times 10^{-4} fm$, $m_0 = 101.005 GeV$, $\delta m_0^2 = 202.010 GeV^2$. When we halve the lattice spacing, $a = 4.88319 \times 10^{-4} fm$, $m_0 = 100.237 GeV$, $\delta m_0^2 = 47.4717 GeV^2$. When we make the lattice spacing quater of the original value, $a = 2.44103 \times 10^{-4} fm$, $m_0 = 98.4865 GeV$, $\delta m_0^2 = -300.412 GeV^2$. Since there are only four points to be fitted and the result are invalid when L_T is too small, the fitting result might be inaccurate.

6.

When we set $d = 2$, $g = 0.01$, $m_0 a = 0.1$, $L_X = 8$, $\kappa = 1/4.01$, data is shown in the following table.

d	L_X	L_T	κ	C_l^0	
				Monte Carlo	Error
2	8	16	0.249376559	0.0153983	4.81E-05
2	8	24	0.249376559	0.00363053	1.67E-05
2	8	32	0.249376559	0.000848118	6.04E-06
2	8	48	0.249376559	1.6178E-07	5.87E-08

The fitted parameters are $A = 5.08059117$, $B = 0.724869785$, $C = 4.98659168e - 07$. We can calculate $a = 1.42903 \times 10^{-3} fm$. $\langle 0|\phi|0\rangle = 1.17199 \times 10^{-4}$, $\langle \pi|\phi|0\rangle = 0.397960$.
When we make the lattice spacing to be half of the original value by setting $\kappa = 0.2796$, we have the data in the following table.

d	L_X	L_T	κ	C_l^0	
				Monte Carlo	Error
2	16	16	0.2796	0.472222	0.0010133
2	16	24	0.2796	0.118301	0.000366868
2	16	32	0.2796	0.0272178	0.000131877
2	16	48	0.2796	0.00143473	9.45866E-06

The fitted parameters are $A = 9.47422169$, $B = 0.365324486$, $C = 4.53728078e - 08$. From these parameters, we can see that B is nearly half of the original fitted data, which means that the choice of κ is resonable. We can calculate $a = 7.20885 \times 10^{-4} fm$. $\langle 0|\phi|0\rangle = 2.81583 \times 10^{-5}$, $\langle \pi|\phi|0\rangle = 0.406893$.
When we make the lattice spacing to be quater of the original value by setting $\kappa = 0.2905$, we have the data in the following table.

d	L_X	L_T	κ	C_l^0	
				Monte Carlo	Error
2	32	16	0.2905	3.74834	0.00487458
2	32	24	0.2905	1.77304	0.00525049
2	32	32	0.2905	0.832098	0.00304017
2	32	48	0.2905	0.183132	0.000312508

The fitted parameters are $A = 16.8381679$, $B = 0.187757525$, $C = 5.64099863e - 16$. From these parameters, we can see that B is nearly quater of the original fitted data, which means that the choice of κ is resonable. We can calculate $a = 3.70497 \times 10^{-4} fm$. $\langle 0|\phi|0\rangle = 2.26296 \times 10^{-9}$, $\langle \pi|\phi|0\rangle = 0.390972$.
When $\kappa = 0.249376559$, we have $(m_0 a_0)^2 = \frac{1}{\kappa} - 2d = 0.1$, $\delta(m_0 a)^2 = -0.624192$. When $\kappa = 0.2796$, $(m_0 a_0)^2 = \frac{1}{\kappa} - 2d = -0.423462$, $\delta(m_0 a)^2 = -0.788786$. When $\kappa = 0.2905$, $m_0 = \frac{1}{a}\sqrt{\frac{1}{\kappa} - 2d} = -0.557659$, $\delta(m_0 a)^2 = -0.745416$.

7.

In the free case, $\langle \pi|\phi|0\rangle$ increases slightly when the lattice spacing decreases. However, in the interacting case, $\langle \pi|\phi|0\rangle$ fluctuates around 0.4 when the lattice spacing decreases. In addition, in the free case, $\langle\pi|\phi|0\rangle$ is around 0.5, while in the interacting case, $\langle\pi|\phi|0\rangle$ is around 0.4.

8.

In cutoff renormalization, we put a cutoff on the momentum to avoid diverging caused by infinite momentum. In lattice renormalization, we constrain the momentum in 1st Brillouin zone ($p \leq \frac{2\pi}{a}$). When the lattice spacing decreases, the range of the momentum increases. Thus, decreasing the lattice spacing is equivalent to increasing the cutoff. If eventually $\delta m_0 a^2$ reaches a non-zero constant instead of continuing to decrease as the lattice spacing decreases, δm_0 diverges. Thus, this statement is equivalent to "The additive renormalization δm_0^2 diverges quadratically with the cutoff in an interacting theory. "