# PHY566 - Diffusion Equation

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# 1 Problem 1

# 2 Problem 2

## 2.1 Part A

To calculate $\left\langle x(t)^2 \right\rangle$, we can integrate it by part:

$$\begin{aligned}
\rho(x,t) &= \frac{1}{\sqrt{2\pi\sigma(t)^2}} exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \\
\left\langle x(t)^2 \right\rangle &= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma(t)^2}} exp\left(-\frac{x^2}{2\sigma(t)^2}\right) dx \\
\left\langle x(t)^2 \right\rangle &= \int_{-\infty}^{\infty} \sigma(t)^2 x \frac{1}{\sqrt{2\pi\sigma(t)^2}} d\left(\exp\frac{-x^2}{2\sigma(t)^2}\right) \\
\left\langle x(t)^2 \right\rangle &= \sigma(t)^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma(t)^2}} exp\left(-\frac{x^2}{2\sigma(t)^2}\right) dx \\
\left\langle x(t)^2 \right\rangle &= \sigma(t)^2
\end{aligned}$$

Therefore,the spatial expectation value $\left\langle x(t)^2 \right\rangle$ of the 1D Normal Distribution equals $\sigma(t)^2$

## 2.2 Part B

The diffusion equation is:

$$\frac{\partial\phi(\vec{r},t)}{\partial t} = D\nabla^2\phi(\vec{r},t)$$

where $D = 2$ is a diffusion constant. For a linear diffusion equation:

$$\frac{\partial\phi(\vec{r},t)}{\partial t} = D\frac{d^2\phi(\vec{r},t)}{dx^2}$$

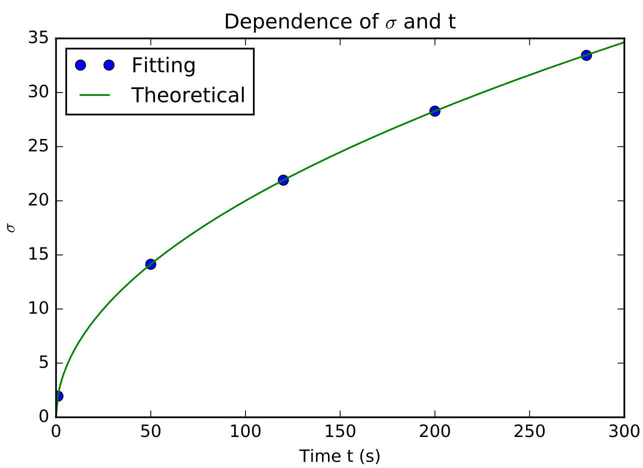
The numerical solution is:

$$\begin{aligned}\frac{y_{j+1,i}-y_{j,i}}{\Delta t} &= D\frac{y_{j,i+1}+y_{j,i-1}-2y_{j,i}}{\Delta x^2}\\ y_{j+1,i} &= y_{i,j}+\frac{D\Delta t}{\Delta x^2}(y_{j,i+1}+y_{j,i-1}-2y_{j,i})\end{aligned}$$

After getting y in each x for each time snapshot, we obtained $\sigma(t)$ by fitting it into equation:

$$\rho(x,t) = \frac{1}{\sqrt{2\pi\sigma(t)^2}}exp\left(-\frac{x^2}{2\sigma(t)^2}\right)$$

We choose 5 time snapshots [1,50,120,200,280]($\Delta t = 0.01$) to get $\sigma(t)$ and plot them in following figure.




The fitting points match the analytical function perfectly.