PHY566 - Percolation

Tianyu Dai, Xiaoqing Li, Yuheng Liao, Xuejian Ma

4/12/2017

Percolation theory describes the behavior of connected cluster in a random graph. It usually concerns the movement and filtering of fluids through porous materials. In this project, we simulate the percolation transition and try to analyze its properties.

1 Determine the critical probability p_c

The basic idea to simulate the percolation transition is to take a $N \times N$ lattice, and then occupy the lattice sites at random according to a certain probability p. If a cluster of occupied sites spans the entire lattice from edge to edge, it's called a spanning cluster. The probability of the appearance of a spanning cluster rises with the rise of p. When p is relatively small, it is highly unlikely to get a spanning cluster. However, for p large enough, the existence of spanning cluster is guaranteed. The transition from one regime to the other is rather sharp and occurs at a critical concentration, which we call p_c. For $N \times N$ lattice, our basic task is to simulate the percolation transitions with different probability p, and figure out the p_c. Doing those for different N, and then plot $p_c - N^{-1}$ graph. From $p_c(N^{-1})$ graph, we can extrapolate to the infinite size limit $p_c(0)$.

1.1 Basic Algorithm

The program details we should follow for $N \times N$ lattice is as below:

1. Begin with an empty $N \times N$ lattice and initialize all sites to zero, i.e. unoccupied.
2. Select and occupy a random site, labeling it with a cluster number.
3. Select another site at random and check neighboring sites to see if any belongs to a cluster created before. If not, the new site should be labeled with another cluster number. If so, we need to discuss among the following two situations:
   (a) If there's only one cluster number among all neighboring sites, give this number to the new site.

(b) If there are several cluster number among all occupied neighbors, the new site is considered as a site bridge which connects multiple clusters into one. We should choose a common unique cluster number for resulting cluster, and then also give this number to the new site and relabel all sites in merged cluster to that number.

4. Repeat step 3 until a common cluster number appears on all edges of the lattice.

5. The common cluster number is the number of the spanning cluster, and the fraction of all occupied sites among all lattice sites is p_c

1.2 Revised Algorithm

To simplify the boundary condition, we revised the original algorithm.
The original $N \times N$ lattice is expanded to $(N+1) \times (N+1)$ first. Then, each site on the boundary should be labeled a unique number, as shown in Fig. (1.2)

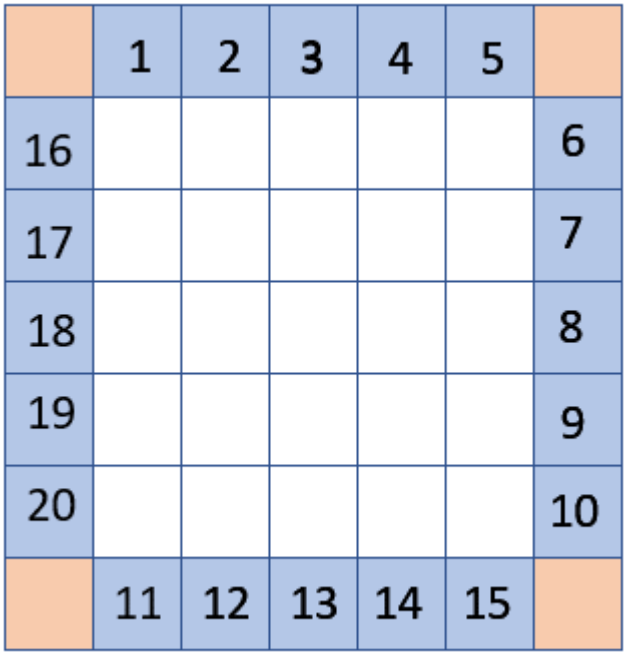
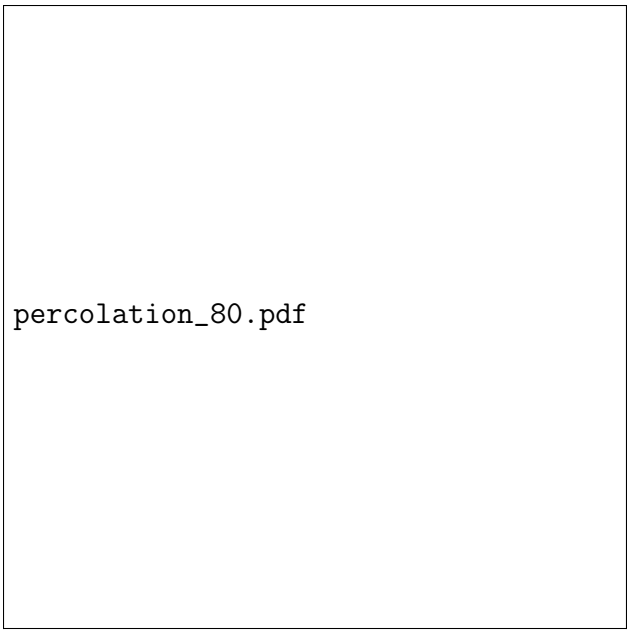


Figure 1: Boundary Setting

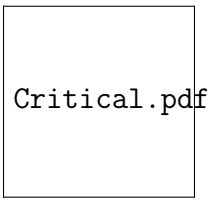
Following the steps above, we can get the spanning clusters for different $N \in \{5, 10, 15, 20, 30, 50, 80\}$ as below:

percolation_5.pdf	percolation_10.pdf
percolation_15.pdf	percolation_20.pdf
percolation_30.pdf	percolation_50.pdf



In the pictures above, black sites represent sites of spanning clusters, gray ones represent sites occupied while not belonging to spanning clusters, and white ones represent sites unoccupied.

By repeating the procedures over 50 times for each N, we can calculate the average $p_c(N^{-1})$. After doing so for all $N \in \{5, 10, 15, 20, 30, 50, 80\}$, we could pplot the graph of $p_c(N^{-1})$ as below:



According to the graph, we could extrapolate to the infinite size limit $p_c(0) \approx 0.602$.

2 (b)