MAT 344 Problem Set 1
(due 11:59pm, January 14)

Part A

These questions relate to the learning outcome *analyze a counting problem by proving an exact or approximate enumeration*. Three of these questions will be marked.

1. The Greek alphabet consists of 24 letters. How many seven-character strings can be made using the Greek alphabet?

2. Let n be an even number. How many ternary strings (i.e. strings over the alphabet $\{0, 1, 2\}$) of length n are there in which the only places that zeroes can appear are in the odd-numbered positions?

3. How many license plates are there of the form $L_1L_2L_3D_1D_2D_3$ where L_1, L_2, L_3 are each capital letters in the English alphabet (of which there are 26), and D_1, D_2, D_3 are each decimal digits (elements of the set
$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$), subject to the restriction that at least one digit is nonzero and at least one letter is K?

4. Pizza Hut has a special for a large 2 topping pizza. There are 6 choices of sauce (including none) and 27 choices of toppings (including fries). How many distinct pizzas with 2 toppings can be made?

Part B

These questions relate to the learning outcomes *select and justify appropriate tools to analyze a counting problem* and *analyze a counting problem by proving an exact or approximate enumeration*. Two of these questions will be marked.

5. Tim Horton's sells 15 types of donuts. A manager wants to buy 7 donuts, one for herself and one for each of her six employees.

   (a) Suppose that she does this by selecting a specific type of donut for each person. She can select the same type of donut for more than one person. In how many ways can she do this?

   (b) How many ways could she select the donuts if she wants to choose a different type of donut for each person?

   (c) Suppose instead that she wishes choose a collection of seven donuts, each of different types, and place them in the breakroom. How many ways can she do this?

6. Twenty students compete in a programming competition in which the top four students are recognized with trophies for first, second, third, and fourth places.

   (a) How many different outcomes are there for the top four places?

(b) The judges decide that they will award honourable mention certificates to four individuals who did not receive trophies. In how many ways can the honourable mention recipients be selected (after the top four places have been determined)? How many total outcomes (trophies and certificates) are there then?

7. Jorge Luis Borges's "Library of Babel" is made up of hexagonal rooms. Each room has 5 bookshelves on 4 walls, and each shelf contains 32 books. Every book is 410 pages, with 40 lines on each page, and each line contains 80 characters. The characters are spaces, commas, periods, and 22 letters. If every book is unique, and the library contains every possible book, which is larger: the number of rooms in the library, or the largest known prime?

Part C

This question relates to the learning outcomes *construct counting problems which show the usefulness or limitations of combinatorial tools.* It will be marked for completeness only.

8. Give an example of a counting problem that can be solved using strings, permutations, or combinations.