PHL245 Modern Symbolic Logic

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`Github Page: https://github.com/TianyuDu/Spikey_UofT_Notes`
`Note Page: TianyuDu.com/notes`

1 Lecture 1

2 Lecture 2

Definition 2.1. An **argument** consists two parts, set of **premises** and **conclusions**.

$$\texttt{premise} \implies \texttt{conclusion} \tag{2.1}$$

Where a *premise* is a reason given to believe something, and the *conclusion* is the thing the argument is supposed to cause one to believe.

Remark 2.1. Two types of arguments

1. Inductive Arguments.
2. Deductive Arguments.

Remark 2.2. Deductive arguments are **truth-preserving**.

Definition 2.2. An argument is **valid** if and only if there's *no logically possible situation* in which all of its premises are true and its conclusion is false.

Remark 2.3 (Vacuous)**.** A valid argument could have false premise, or false conclusion.

Definition 2.3. An argument is **sound** if it's valid and its premises are all true.

Definition 2.4. An argument is **trivially valid** either

1. it has false premise,
2. <u>or</u>, its conclusion is unconditionally true (does not depend on the premises).

3 Lecture 3

Definition 3.1. An **atomic sentence**(**proposition**) is the basic element of sentential logic, which has a **truth-value**.

Definition 3.2. A **molecular sentence** is an object with **truth-value** built up out of atomic sentences using connectives.

Definition 3.3. The **main connective** is the object determines the overall truth-value of the sentence.

Definition 3.4. **Parsing** is the process to figure out which symbol is the **main connective**.

Definition 3.5. A sentence is in **official notation** if and only if it is an atomic sentence, or can be constructed from atomic sentences using (i) negation and (ii) conditionals.

Definition 3.6. A sentence is in **informal notation** if you could turn it into official notation just by adding parentheses around it.

Definition 3.7. A sentence is **not well-formed** if it is not in official notation or informal notation.

Remark 3.1. Equivalences to Negation

(i) Not X.

(ii) It's not the case X.

(iii) It's failed to X.

Definition 3.8. A **conditional** connective takes the form

$$\texttt{antecedent} \implies \texttt{consequent} \tag{3.1}$$

where `antecedent` is *sufficient* for `consequent`, and, `consequent` is *necessary* for `antecedent`.

Remark 3.2. Equivalences to Conditionals $P \implies Q$

(i) If P then Q.

(ii) Provided P, Q.

(iii) Assuming that P, Q.

(iv) Given that P, Q.

(v) In case P, Q.

(vi) On the condition that P, Q¿

```
Show P -> Q:
        P ass cd
        ...
        Q cd
```

Figure 5.1: conditional derivation usage

```
Show ...:
        ...
        11. ~Q
        12. Q
        11 12 id
```

Figure 5.2: indirect derivation usage

4 Lecture 4

Definition 4.1. A **derivation** is a method for proving that an argument is valid. It starts from some given *premises*, leas by applications of *rules*, to some *conclusions*.

Rule 4.1 (Repetition (`R`)).

$$P \therefore P \tag{4.1}$$

Rule 4.2 (Double Negation (`DN`)).

$$P \therefore \sim\sim P \tag{4.2}$$

$$\sim\sim P \therefore P \tag{4.3}$$

Rule 4.3 (Modus Ponens (`MP`)).

$$P \implies Q.\ P \therefore Q \tag{4.4}$$

Rule 4.4 (Modus Tollens (`MT`)).

$$P \implies Q.\ \sim Q \therefore \sim P \tag{4.5}$$

5 Lecture 5

Remark 5.1. You must provide an exact match as input to a rule that involves negations. If a rule that involves negations produces a double negation *as the major connective*, you may give yourself the unnegated version.

Definition 5.1. A **conditional derivation** is a proof that if we assume the antecedent, (`ass cd`) then the consequent will follow.

Definition 5.2. An **indirect derivation** is a proof that if we assume the opposite of the conclusion (`ass id`), and show it leads to a contradiction.

Remark 5.2. When you close off a sub-derivation, you can't use its contents in further derivations, except for the cancelled show line.

Remark 5.3. If we wish to use a line *outside* the sub-derivation, we need to use the *repetition rule* to bring the available line into the sub-derivation.

6 Lecture 6

Definition 6.1. Connective **conjunction**

$$(P \land Q) \tag{6.1}$$

Definition 6.2. Connective **disjunction**

$$(P \lor Q) \tag{6.2}$$

Definition 6.3. Connective **bi-conditional**

$$(P \iff Q) \tag{6.3}$$

Remark 6.1. In general, for *informal notations*, we assume the conditional or bi-conditional is the major connective, unless the brackets tell you otherwise

Example 6.1.

$$P \land Q \implies R \equiv (P \land Q) \implies R \tag{6.4}$$
$$P \implies Q \lor R \equiv P \implies (Q \lor R) \tag{6.5}$$

Remark 6.2. For informal notations for mixed disjunction and conjunction, the sentence is evaluated *from left to right*.

Example 6.2.

$$P \lor Q \land R \equiv (P \lor Q) \land R \tag{6.6}$$

Remark 6.3. Equivalences to AND

(i) X and Y.

(ii) X, but Y.

(iii) X, although Y.

(iv) X, even though Y.

(v) Even though X, Y.

(vi) Despite X, Y.

Remark 6.4. Equivalences to OR

(i) X or Y.

(ii) Either X or Y.

(iii) X unless Y.

Remark 6.5. Not both is equivalent to

$$\sim (P \land Q) \tag{6.7}$$

Remark 6.6. **Neither ... nor** is equivalent to

$$\sim (X \vee Y) \tag{6.8}$$

Remark 6.7. Equivalents to iff

(i) X if and only if Y.

(ii) X exactly on the condition that Y.

(iii) X *just* in case Y.

(iv) X is necessary and sufficient for Y.

Rule 6.1 (Negation Introduction (`dn`))**.**

$$P \therefore \sim\sim P \tag{6.9}$$

Rule 6.2 (Negation Elimination (`dn`))**.**

$$\sim\sim P \therefore P \tag{6.10}$$

Rule 6.3 (Simplification (`s`))**.**

$$P \wedge Q \therefore P \tag{6.11}$$
$$P \wedge Q \therefore Q \tag{6.12}$$

Rule 6.4 (Adjunction (`adj`))**.**

$$P.\ Q \therefore P \wedge Q \tag{6.13}$$
$$P.\ Q \therefore Q \wedge P \tag{6.14}$$

Remark 6.8. Order matters in conjunction,

$$P \wedge Q \not\equiv Q \wedge P \tag{6.15}$$

Rule 6.5 (Addition (`add`))**.**

$$P \therefore P \vee Z P \therefore Z \vee P \tag{6.16}$$

Rule 6.6 (Modus Tollendo Ponens (`mtp`))**.**

$$P \vee Q.\ \sim Q \therefore P \tag{6.17}$$
$$P \vee Q.\ \sim P \therefore Q \tag{6.18}$$

Rule 6.7 (Bi-conditional to Conditional (`bc`))**.**

$$P \iff Q \therefore P \implies Q P \iff Q \therefore Q \implies P \tag{6.19}$$

Rule 6.8 (Conditional to Bi-conditional (`cb`))**.**

$$P \implies Q.\ Q \implies P \therefore P \iff Q \tag{6.20}$$
$$P \implies Q.\ Q \implies P \therefore Q \iff P \tag{6.21}$$

7 Lecture 8

Definition 7.1. A **theorem** of logic is a conclusion that can be proven *using no premises.*

Rule 7.1 (Negation of Conditional (`NC`))**.**

$$\sim (P \implies Q) \therefore P \wedge \sim Q \tag{7.1}$$

$$P \wedge \sim Q \therefore \sim (P \implies Q) \tag{7.2}$$

Rule 7.2 (Conditional as Disjunction (`CDJ`))**.**

$$P \implies Q \therefore \sim P \vee Q \tag{7.3}$$

$$\sim P \vee Q \therefore P \implies Q \tag{7.4}$$

$$\sim P \implies Q \therefore P \vee Q \tag{7.5}$$

$$P \vee Q \therefore \sim P \implies Q \tag{7.6}$$

Rule 7.3 (Separation of Cases (`SC`))**.**

$$P \vee Q.\ P \implies R.\ Q \implies R \therefore R \tag{7.7}$$

$$P \implies R.\ \sim P \implies R \therefore R \tag{7.8}$$

Rule 7.4 (DeMorgans (`DM`))**.** General form

$$\sim (\wedge_i P_i) \therefore \vee_i(\sim P_i) \tag{7.9}$$

$$\sim (\vee_i P_i) \therefore \wedge_i(\sim P_i) \tag{7.10}$$

Binary case

$$\sim (P \wedge Q) \therefore \sim P \vee \sim Q \tag{7.11}$$

$$\sim (P \vee Q) \therefore \sim P \wedge \sim Q \tag{7.12}$$

converse also works.

Rule 7.5 (Negation of Bi-conditional (`NB`))**.**

$$\sim (P \iff Q) \therefore P \iff \sim Q \tag{7.13}$$

$$P \iff \sim Q \therefore \sim (P \iff Q) \tag{7.14}$$

8 Lecture 9

Definition 8.1. A **tautology** is a sentence that comes out true on truth value assignments. *Major connective comes out as True for* <u>*all rows*</u>*.*

Definition 8.2. A **contradiction** is a sentence that comes out as false on all truth value assignments. *Major connective comes out as False for* <u>*all rows*</u>*.*

Definition 8.3. A sentence is **contingent** if it is neither a tautology, nor a contradiction.

Definition 8.4. A set of sentences is **consistent** if they can all be true at once, otherwise, they are **contradictory**. *If* <u>*there is*</u> *a truth value assignment where all the sentences come out as true, then they are consistent.*

Remark 8.1 (Truth Tables for Determining Validity)**.** A proposition is valid if and only if <u>for all</u> rows where major connectives of the primes are all true, the conclusion is also true.