ECO208 Textbook Notes

Macroeconomic Theory Summer 2018

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July 24, 2018

Contents

1 Chapter 9 A Two-Period Model: The Consumption-Savings Decision and Credit Markets

1.1 A Two-Period Model of the Economy

1.1.1 Consumers

There are m consumers receiving exogenous income y. Incomes can be different for different consumers but all consumers pay the same lump sum tax t. Let s denote saving and the **current period income constraint** for a typical consumer is

$$c + s = y - t \tag{1}$$

Assumption: perfect credit market

Assumption 1.1. All bonds in the credit market are indistinguishable and there is no risk associated with holding a bond.

Assumption 1.2. Bonds are traded directly in the credit market without financial intermediaries.

Assumption 1.3. The real rate of interest at which a consumer can lend is the same as the real rate of interest at which a consumer can borrow.

There is no saving in the *future period* (i.e. $s' \equiv 0$). The **future period income constraint** is

$$c' = y' - t' + (1 + r)s \tag{2}$$

By combining the income constraints in two period and the **lifetime budget constraint** can be characterized as

$$c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r} \tag{3}$$

where the left hand side is the **present discounted value** of life time consumption and the right hand side captures the present discounted value of life time disposable income.

Definition 1.1. Define **lifetime wealth** we as the present value of life time disposable income

$$we := y - t + \frac{y' - t'}{1+r}$$

The life time income constraint can be rewrite as

$$c' = -(1+r)c + (1+r)we \tag{4}$$

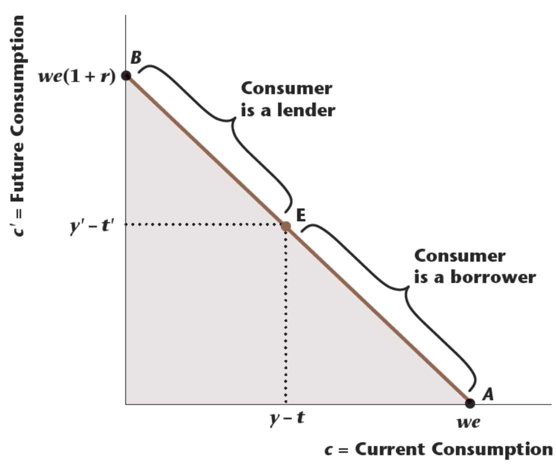


FIGURE 9.1
Consumer's Lifetime Budget Constraint
The lifetime budget constraint defines the quantities of current and future consumption the consumer can acquire, given current and future income and taxes, through borrowing and lending on the credit market. To the northwest of the endowment point *E*, the consumer is a lender with positive savings; to the southeast of *E*, he or she is a borrower with negative savings.

Definition 1.2. Endowment point is the **consumption bundle** (c, c') the consumer gets if he or she simply consumes disposable income in the current period and in the future period.

Assumption 1.4. We assume the consumers' preferences have three properties:

1. **Monotonicity** more is always preferred to less.
2. **Convexity** the consumer likes diversity in his or her consumption bundle.
3. Current consumption and future consumption are normal goods.

As figures below show, at the optimal point $A = (c^*, c'^*)$,

$$MRS_{c,c'} = \frac{dc'}{dc} = -\frac{MU_c}{MU_{c'}} = 1 + r$$

as it is on the tangency point.

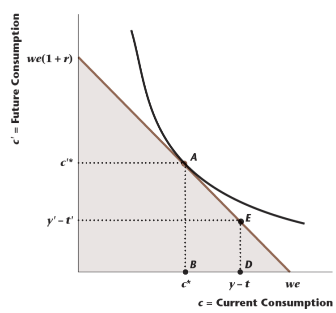


FIGURE 9.3
A Consumer Who Is a Lender
The optimal consumption bundle for the consumer is at point *A*, where the marginal rate of substitution (minus the slope of an indifference curve) is equal to 1 + *r* (minus the slope of the lifetime budget constraint). The consumer is a lender, as the consumption bundle chosen implies positive savings, with *E* being the endowment point.

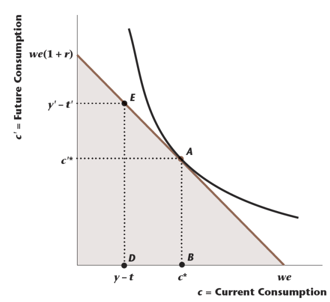


FIGURE 9.4
A Consumer Who Is a Borrower
The optimal consumption bundle is at point *A*. Since current consumption exceeds current disposable income, saving is negative, and so the consumer is a borrower.

1.2 Experiments

1.2.1 An Increase in Current-Period Income

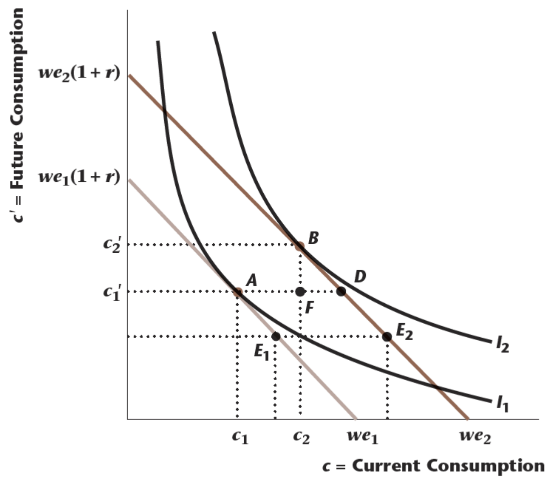
Pure income effect Increase in we and parallel shifting on budget constraint.

Consumption Smoothing As current income increases, the consumer wants to spread this additional income over both periods and not consume it all in the current period. Therefore

$$\Delta y > \Delta c > 0 \; and \; \Delta s > 0$$

FIGURE 9.5

The Effects of an Increase in Current Income for a Lender
When current income increases, lifetime wealth increases from we_1 to we_2. The lifetime budget constraint shifts out, and the slope of the constraint remains unchanged, since the real interest rate does not change. Initially, the consumer chooses A, and he or she chooses B after current income increases. Current and future consumption both increase (both goods are normal), and current consumption increases by less than the increase in current income.



Excess variability although consumption is smoother than income, as the theory predicts, consumption is not quite smooth enough to tightly match the theory. The excess variability of aggregate consumption relative to aggregate income can be explained by

1. *There are imperfections in the credit market.*
2. *When all consumers are trying to smooth consumption in the same way simultaneously, this will change the market prices/ interest rate.*

1.2.2 An Increase in Future Income

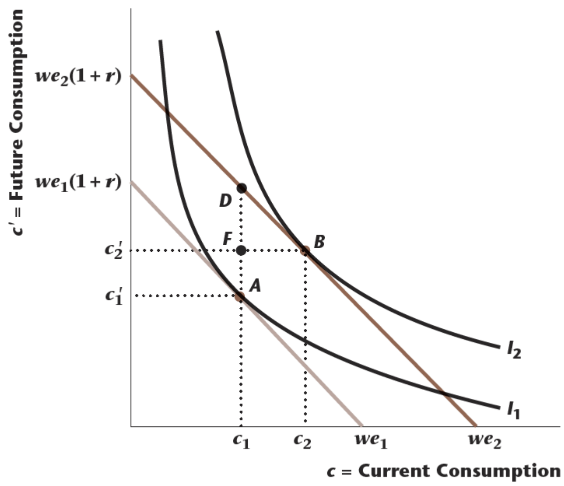
Pure income effect causes an out shifting of budget line and increases consumption in both periods. Due to backward consumption smoothing, even with $\Delta t = \Delta y = 0$,

$$\Delta c > 0 \; and \; \Delta s < 0$$

FIGURE 9.7

An Increase in Future Income

An increase in future income increases lifetime wealth from we_1 to we_2, shifting the lifetime budget constraint up and leaving its slope unchanged. The consumer initially chooses point *A*, and he or she chooses *B* after the budget constraint shifts. Future consumption increases by less than the increase in future income, saving decreases, and current consumption increases.



1.2.3 Temporary and Permanent Changes in Income

Definition 1.3. Permanent income hypothesis by Milton Friedman argued that a primary determinant of a consumer's current consumption is his or her **permanent income**.

Temporary changes in income yield small changes in permanent income (lifetime wealth) and therefore leads to a small effects on current consumption. Whereas *permanent* changes in income have large effects on permanent income and current consumption.

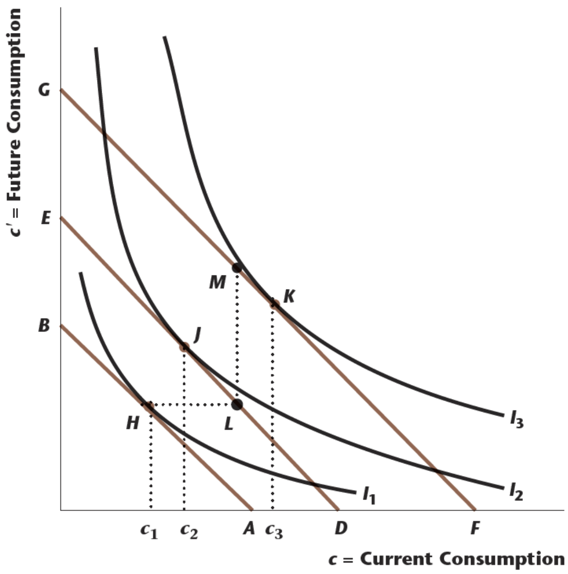


FIGURE 9.8
Temporary versus Permanent Increases in Income
A temporary increase in income is an increase in current income, with the budget constraint shifting from *AB* to *DE*, and the optimal consumption bundle changing from *H* to *J*. When there is a permanent increase in income, current and future income both increase, and the budget constraint shifts from *AB* to *FG*, with the optimal consumption bundle changing from *H* to *K*.

1.2.4 An Increase in the Real Interest Rate

Effect combined effect of both income effect and substitute effect.

Remark 1.1. the relative price of consumption goods in terms of current consumption goods is

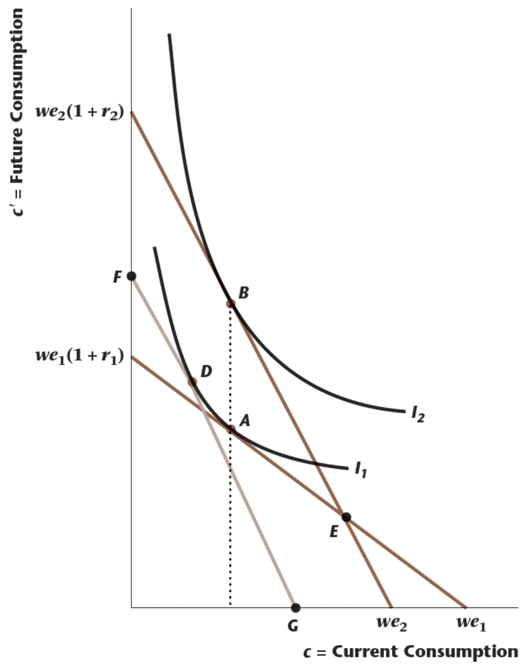
$$\frac{1}{1+r}$$

Remark 1.2. the effect on optimal (c, c', s) would be different for lenders and borrowers.

Lender Consider an increase in real interest rate. For lenders, they intend to lend out current period income in exchange for future period consumption. Therefore, increase in return on saving would have a positive income effect on them.

Var	SE	IE	Total
c	-	+	?
c'	+	+	+
s	+	-	?

FIGURE 9.12
An Increase in the Real Interest Rate for a Lender
When the real interest rate increases for a lender, the substitution effect is the movement from *A* to *D*, and the income effect is the movement from *D* to *B*. Current consumption and saving may rise or fall, while future consumption increases.



Borrowers Consider an increase in real interest rate

Var	SE	IE	Total
c	-	-	-
c'	+	-	?
s	+	+	+

Remark 1.3. For both borrower and lender, *the substitution effects of change in real interest rate towards the same direction.* SE simply captures the change in relative price of current and future consumption ($\frac{1}{1+r}$). *The income effects for borrower and lender work towards the opposite directions.*

1.3 Government

Assumption 1.5. Government bonds and private bonds are indistinguishable.

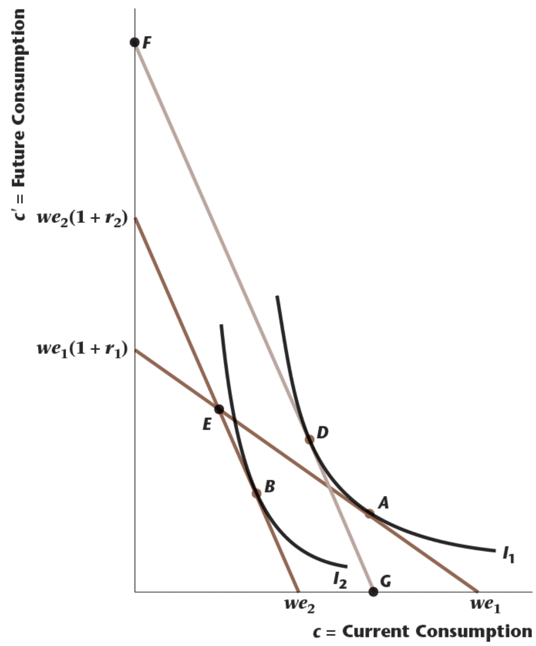
Let $T = mt$ denote the total tax collected by government. The **current period budget** for government is

$$G = T + B, \ B \in \mathbb{R} \tag{5}$$

FIGURE 9.13

An Increase in the Real Interest Rate for a Borrower

When the real interest rate increases for a borrower, the substitution effect is the movement from *A* to *D*, and the income effect is the movement from *D* to *B*. Current consumption decreases, saving increases, and future consumption may rise or fall.



and the **future period budget** for government is

$$G' + (1+r)B = T' \tag{6}$$

The **government present-value budget constraint** is

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \tag{7}$$

1.4 Competitive Equilibrium

Consumers and the government interact in the **credit market**. And in a competitive equilibrium, three conditions must hold:

1. Each consumer chooses (c, c', s) optimally given the real interest rate r.

2. Government present-value budget constraint, equation (7), holds.

3. The credit market clears.

Equivalently, let $\mathcal{I}$ be the collection of consumer indices,

$$\begin{cases} MRS^i_{c,c'} = 1 + r \ \forall i \in \mathcal{I} \\ G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \\ S^p = \sum_{i \in \mathcal{I}} s_i = B \end{cases}$$

On aggregate for consumers,

$$S^p = \sum_{i \in \mathcal{I}} s_i = \sum_{i \in \mathcal{I}} \{y_i - c_i - t\} = Y - C - T \tag{8}$$

and by the second condition for competitive equilibrium and government's first period budget constraint, we have

$$B = Y - C - T = G - T \implies Y = C + G \tag{9}$$

1.5 The Ricardian Equivalence Theorem

1.5.1 Assumptions

Assumption 1.6.

1. When taxes change in the experiment, they change by the same amount for all consumers, both in the present and in the future.
2. Any debt issued by the government is paid off during the lifetimes of the people alive when the debt was issued.
3. Taxes are lump-sum.
4. There are perfect credit markets.

1.5.2 The Theorem

Definition 1.4. The **Ricardian Equivalence Theorem** states that if *current and future government spending are held constant*, a change in current taxes with an equal and opposite change in the present value of future taxes [1] leaves the equilibrium interest rate and consumption of individuals unchanged.

[1] so that government present-value constraint holds

Proof. By assumption of our model, all consumers pay the same lump sum tax t, t' in two periods. Therefore the total tax collected is $T = mt$ and $T' = mt'$. Rewrite the government budget as

$$G + \frac{G'}{1+r} = mt + \frac{mt'}{1+r}$$

For each consumer,

$$t + \frac{t'}{1+r} = \frac{1}{m}(G + \frac{G'}{1+r})$$

The life-time budget constraint for individual consumer becomes

$$C + \frac{C'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{m}(G + \frac{G'}{1+r})$$

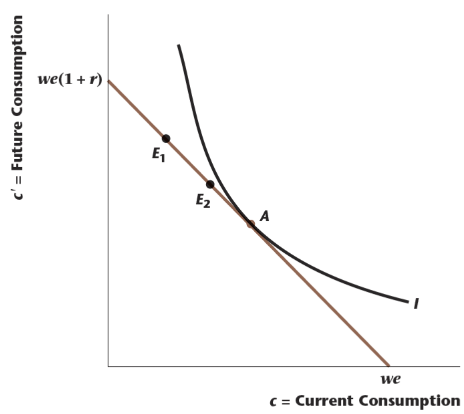
If (G, G') is left unchanged, the life time constraint for consumer is also unchanged. Therefore consumers' optimal (c, c', s) is unchanged. ■

Under conditions stated in above definition, such a change in tax scheme is equivalent to a **movement of endowment point** along the original inter-temporal budget line.

FIGURE 9.15

Ricardian Equivalence with a Cut in Current Taxes for a Lender

A current tax cut with a future increase in taxes leaves the consumer's lifetime budget constraint unchanged, and so the consumer's optimal consumption bundle remains at *A*. The endowment point shifts from E_1 to E_2, so that there is an increase in saving by the amount of the current tax cut.



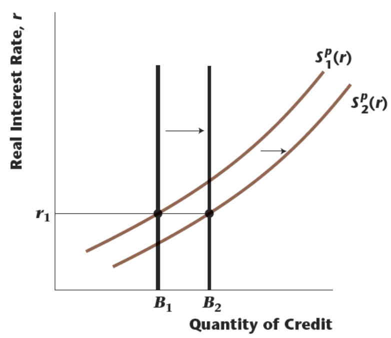


FIGURE 9.16
Ricardian Equivalence and Credit Market Equilibrium
With a decrease in current taxes, government debt increases from B_1 to B_2, and the credit supply curve shifts to the right by the same amount. The equilibrium real interest rate is unchanged, and private saving increases by an amount equal to the reduction in government saving.

1.5.3 Credit Market

Assumption 1.7. For $S^p(r)$, when r changes, substitution effects outweights the income effects when we add these effects across all consumers. (*upward sloping private saving*)

Ricardian Equivalence Theorem states that the change in tax scheme will not affect the real interest rate r. This means when there is a tax cut associated with an increase in B, consumers anticipates the future increase in tax. Then consumers increase s and $S^p(r)$ for consumption smoothing. The supply in credit market increases so that the the equilibrium interest rate (i.e. r^* $s.t.$ $S^p(r^*) = B$) is unchanged.

2 Chapter 11 A Real Inter-temporal Model with Investment

2.1 The Representative Consumer

2.1.1 Setup

Consumers are endowed with h hours per period to allocate between working and leisure and non-labor income $\pi - T$ in each period.

The representative consumer has **current period budget**

$$C + S^p = (h - l)w + \pi - T \tag{10}$$

and **future period budget constraint** is

$$C' = (h - l')w' + (1 + r)S^p + \pi' - T' \tag{11}$$

The **life time present value constraint** is

$$C + \frac{C'}{1+r} = (h-l)w + \pi - T + \frac{(h-l')w' + \pi' - T'}{1+r} \tag{12}$$

2.1.2 Optimization problem

Given $w, w', T, T', \pi, \pi', r$, the representative consumer maximize his or her life time utility by choosing C, C', l, l'.

$$\begin{aligned} &max_{C,C',l,l'}\big\{\ln C + \eta \ln l + \beta\big[\ln C' + \eta \ln l'\big]\big\} \\ s.t.\ &C + \frac{C'}{1+r} = (h-l)w + \pi - T + \frac{(h-l')w' + \pi' - T'}{1+r} \end{aligned} \tag{13}$$

2.1.3 Optimal Solution

Solve Solve above optimization problem with Lagrangian Multiplier method,

$$\mathcal{L} = \ln C + \eta \ln l + \beta[\ln C' + \eta \ln l'] + \lambda\big\{w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r} - C - \frac{C'}{1+r}\big\} \tag{14}$$

the first order condition for optimal choice (C^*, C'^*, l^*, l'^*) is

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial C} = \frac{1}{C} - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial C'} = \frac{\beta}{C'} - \frac{\lambda}{1+r} = 0 \\ \frac{\partial \mathcal{L}}{\partial l} = \frac{\eta}{l} - w\lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial l'} = \frac{\beta\eta}{l'} - \lambda\frac{w'}{1+r} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r} - C - \frac{C'}{1+r} = 0 \end{cases} \tag{15}$$

Inter-temporal consumption

$$\frac{1}{C} = (1+r)\frac{\beta}{C'} \tag{16}$$

Inter-temporal leisure

$$\frac{\beta\eta}{l'} = \frac{w'}{1+r}\frac{\eta}{wl} \tag{17}$$

Intra-temporal leisure consumption For current period

$$\frac{\eta}{l} = w\frac{1}{C} \tag{18}$$

and for future period

$$\frac{\beta\eta}{l'} = \frac{\beta}{C'}w' \tag{19}$$

2.1.4 Current Labor Supply

Assumption 2.1. The current labor supply satisfies the following assumptions.

1. The quantity of current labor supplied increases when the current real wage increases.
2. Current labor supply increases when the real interest rate increases.
3. Current labor supply decreases when lifetime wealth increases.

2.1.5 Current Demand for Consumption Goods

Primary factors affecting current consumption are lifetime wealth and the real interest rate.

Definition 2.1. Marginal propensity to consume measures the amount that current consumption increases when there is a unit increase in aggregate real income.

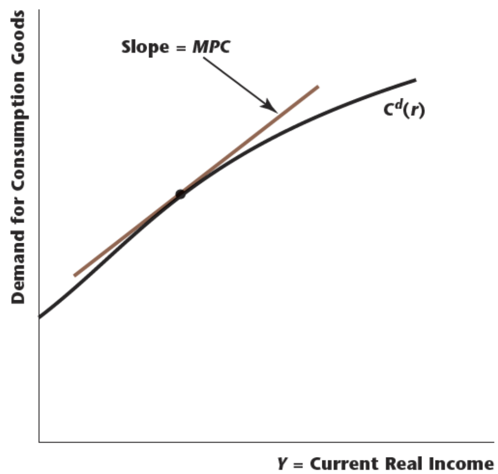


FIGURE 11.4
The Representative Consumer's Current Demand for Consumption Goods Increases with Income
The slope of the curve is the marginal propensity to consume, *MPC*. We have *MPC* < 1, since part of an increase in current income is saved.

Assumption 2.2. when interest rate change, for consumption scheme, the substitute effect dominates income effect.

Therefore, an increase in r leads to a downward shifting of $C^d(r)$ as current period consumption becomes relatively more expensive.

2.2 The Representative Firm

2.2.1 Optimization Problem

Setup the firm want to maximize the **present value of lifetime profit** by choosing numbers of labor hired in each period and the investment, (N, N', I) (equivalently, choosing K').

Production function

$$Y = zF(K, N),\ Y' = z'F(K', N') \tag{20}$$

Current period profit

$$\pi = Y - wN - I \tag{21}$$

Capital accumualtion

$$K' = (1 - d)K + I \tag{22}$$

Future period profit in future period, firm sales all its capital stock altogether with its output.

$$\pi' = Y' + (1-d)K' - wN' \tag{23}$$

Present value for life time profit

$$V = Y - wN + (1-d)K - K' + \frac{Y' + (1-d)K' - wN'}{1+r} \tag{24}$$

2.2.2 Optimal Solution

Labor choice In both periods, the representative firm chooses N, N' by equating the marginal revenue product(since price of output is normalized to 1, $MRP_N = MP_N$) and marginal cost (w) of labor. That's

$$\frac{\partial zF(K,N)}{\partial N} = MP_N = w \tag{25}$$

$$\frac{\partial z'FK',N'}{\partial N'} = MP'_N = w' \tag{26}$$

Investment schedule Solving $\frac{\partial V}{\partial K'} = 0$ gives

$$1 = \frac{MP'_K + (1-d)}{1+r} \tag{27}$$

rearrange the above equation,

$$r = MP'_K - d \tag{28}$$

where the left hand side is the opportunity cost of investing one additional unit of K' and the right hand side is the **net (of depreciation) marginal product of capital**.

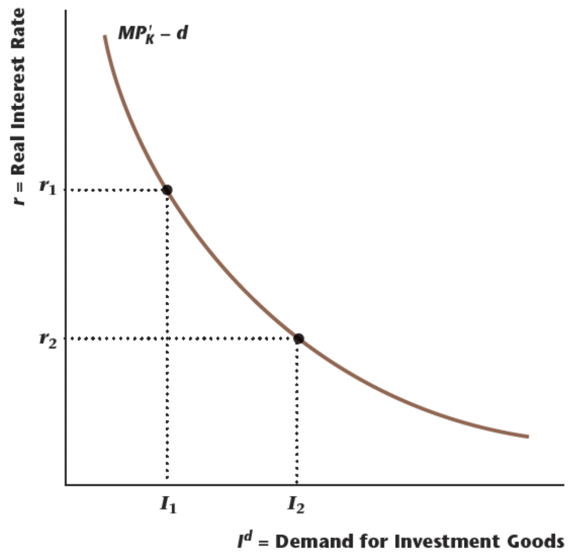
Change in optimal investment schedule notice that we infer the optimal investment level by finding the optimal capital stock in the future period first.

1. The optimal investment schedule shifts to the right if future total factor productivity, z', increases.
2. The optimal investment schedule shifts to the left if initial capital endowment, K, increases.

FIGURE 11.9

Optimal Investment Schedule for the Representative Firm

The optimal investment rule states that the firm invests until $MP'_K - d = r$. The future net marginal product schedule, $MP'_K - d$, is the representative firm's optimal investment schedule, since this describes how much investment is required for the net marginal product of future capital to equal the real interest rate.



2.3 Government

Government sets government purchases of consumption goods exogenously in each period and have to satisfy its budget in terms of present discounted value.

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \tag{29}$$

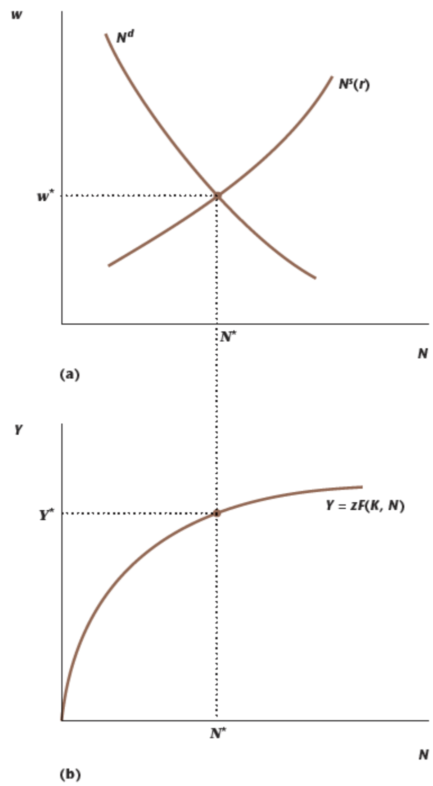
2.4 Competitive Equilibrium

2.4.1 The Current Labor Market and the Output Supply Curve

Steps

1. Find labor market equilibrium N^* by solving $N^s(w, r, \cdot) = N^d(w, \cdot)$
2. Find equilibrium consumption good output using aggregate production function. $Y^* = zF(K, N^*)$

Then we can construct the output supply curve $Y^s(r)$ as in figure 11.13.



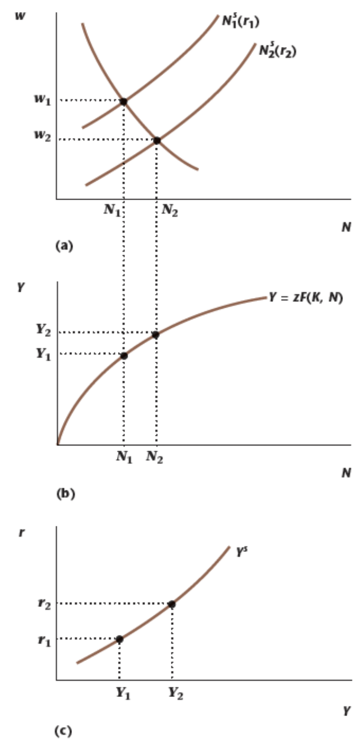


FIGURE 11.13
Construction of the Output Supply Curve
The output supply curve, Y^s, is an upward-sloping curve in panel (c) of the figure, consisting of real current output and real interest rate pairs for which the labour market is in equilibrium.

2.4.2 Current Good Market and the Output Demand Curve

Steps In a closed economy, the national income identity must hold,

$$Y = C^d(Y, r) + I^d(r) + G \tag{30}$$

And $C^d(Y, r)$ depends on Y as in figure 11.4, solving the equilibrium Y^* by solving

$$Y^* \ s.t. \ Y^* = C^d(Y^*, r) + I^d(r) + G \tag{31}$$

2.4.3 Credit Market

By Walras' Law, as labor and consumption good markets clear, the credit market clears as a result.

2.4.4 Complete Model

The complete general equilibrium model can be constructed as

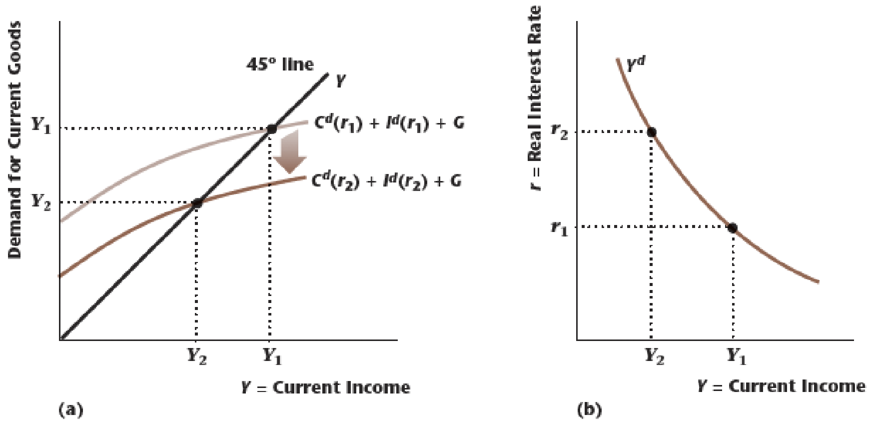


FIGURE 11.17

Construction of the Output Demand Curve

The output demand curve Y^d in (b) is a downward-sloping one describing the combinations of real output and the real interest rate for which the current goods market is in equilibrium.

2.5 Shocks to the General Equilibrium

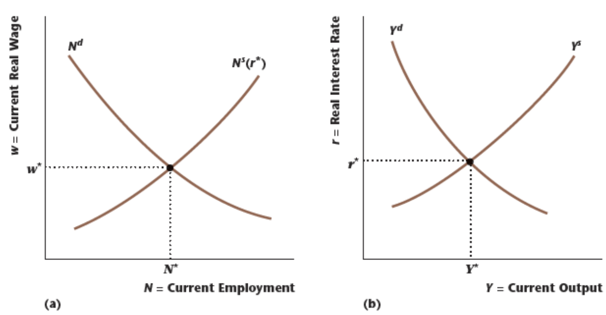


FIGURE 11.19

The Complete Real Intertemporal Model

(a) The current real wage and current employment are determined by the intersection of the current labour supply and demand curves, given the real interest rate. (b) Current aggregate output and the real interest rate are determined by the intersection of the output supply and demand curves.