ECO208 Macroeconomic Theory
Test 1 Review

Tianyu Du

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Contents

1 Chapter 2: Measurements

1.1 Measuring GDP

Three approaches

1. Expenditure Approach.
2. Income Approach.
3. Production (Value-added) Approach.

Net Factor Payment(NFP) Income paid towards domestic factors aboard minus income paid to foreign factors in board.

$$GNP = GDP + NFP$$

Problem with GDP

1. Inequality.
2. Non-market/ home production.
3. Underground economy.
4. Value added for government services. Workaround: estimate with expenditure.

1.2 Measuring Changes over Time

1. Constant price: take prices in **base year**.
2. Chain-weighted method: calculate the growth rate with prices in difference year, then take the **geometric average**.

Calculation Let p_n^i be the price of good n in year i, and q_n^i be the quantity of good n produced in year i. Then

$$NGDP_t = \sum_{n=1}^{N} p_n^t q_n^t \text{ (nominal GDP of year } t\text{)}$$

$$RGDP_t^b = \sum_{n=1}^{N} p_n^b q_n^t \text{ (real GDP of year } t \text{ with base year } b)$$

For chain-weighted method, to calculate the growth rate between year t and $t+1$, let

$$1 + g_t = \frac{RGDP_{t+1}^t}{RGDP_t^t}$$

$$1 + g_{t+1} = \frac{RGDP_{t+1}^{t+1}}{RGDP_t^{t+1}}$$

Then, the chain-weighted growth rate, g_c is

$$1 + g_c = \sqrt{(1 + g_t)(1 + g_{t+1})}$$

GDP Price Deflator

$$Deflator = \frac{NGDP}{RGDP} \times 100$$

1.3 Saving and Investment

Private Disposable Income

$$Y^d = Y + NFP + TR + INT - T$$

Private Saving

$$S^{private} = Y^d - C = Y + NFP + TR + INT - T - C$$

Public(Government Saving)

$$S^{public} = T - TR - INT - G$$

Total Saving

$$S = S^{private} + S^{public} = Y - C - G + NFP = I + NX + NFP = I + CA$$

Where CA stands for **current account**, and $CA = NFP + NX$. Current account measures the net cash *inflow* (from factor payment and product payment) into the country.

1.4 Labor Market Measurement

Measurements

$$\text{Unemployment Rate} = \frac{\text{Unemployment}}{\text{Labor Force}}$$

$$\text{Participation Rate} = \frac{\text{Labor Force}}{\text{Total Working Age Population}}$$

$$\text{Employment-Population Ratio} = \frac{\text{Employment}}{\text{Total Working Age Population}}$$

2 Chapter 4: Consumer and Firm Behaviour in a One-period Model

2.1 Representative Agent: Consumer

2.1.1 Controls

Physical Goods (C) Assumed to be normal good, and abstracted to be a composite good, denoted as **aggregate consumption** (C).

Leisure (ℓ) Measured in hours (or other unit of time.)

2.1.2 Assumptions

1. **Monotonicity** More is better.
2. **Convexity** Diversity preferred.

2.1.3 Constraints

Budget Constraint Let w denote the real wage rate, π denote the dividend payment from. firms owned by household and T be the lump sum tax collected by government. By Walras' Law, the constraint would holds as equality.

$$C = wN^s + \pi - T \tag{1}$$

Time Constraint Let h denote the total hours available in a single time period, let N^s denote the time devoted to work and ℓ denote the time of leisure enjoyed.

$$N^s + \ell = h \tag{2}$$

Other (implicit) Constraints Those constraints make sure that values of variable makes sense.

$$C \geq 0 \tag{3}$$
$$0 \leq \ell \leq h \tag{4}$$
$$0 \leq N^s \leq h \tag{5}$$

2.1.4 Experiments

1. Change in <u>non-labor income</u> $\implies$ pure income effect.
2. Change in <u>real wage rate</u> $\implies$ both income effect and substitution effect.

2.1.5 Constructing Labor Supply

Labor Supply Let $\ell^*(w, \cdot)$ denote the optimal level of leisure chosen by the consumer at real wage rate w and other parameter given. Then by equation (2), the supply of labor could be constructed as $N^{s*}(w, \cdot) = h - \ell^*(w, \cdot)$.

2.1.6 Formalizing Consumer's Optimization Problem

Utility Consider the log form of **Cobb-Douglas Utility Function**

$$u(c, \ell) = \log c + \eta \log \ell, \ \eta > 0$$

Optimization

$$\begin{gathered} \max_{c,\ \ell} u(c, \ell) = \log c + \eta \log \ell \\ s.t. \\ c = (h - \ell)w + \pi - T \\ c \geq 0 \\ 0 \leq \ell \leq h \end{gathered} \tag{6}$$

Solution Set up the Lagrangian function and solve for the first order condition, we have

$$\mathcal{L}(c, \ell, \lambda) = \log c + \eta \log \ell + \lambda((h - \ell)w + \pi - T - c)$$

$$c^*(\cdot) = \frac{hw + \pi - T}{1 + \eta} \tag{7}$$

$$\ell^*(\cdot) = \frac{hw + \pi - T}{w(1 + \frac{1}{\eta})} \tag{8}$$

2.1.7 Comparative Statistics

Lump Sum Tax (T)

$$\frac{\partial c^*(\cdot)}{\partial T} = -\frac{1}{1 + \eta} < 0 \tag{9}$$

Therefore <u>negative</u> correlation.

$$\frac{\partial \ell^*(\cdot)}{\partial T} = -\frac{1}{w(1 + \frac{1}{\eta})} < 0 \tag{10}$$

Therefore <u>negative</u> correlation.

Real Wage Rate (w)

$$\frac{\partial c^*(\cdot)}{\partial w} = \frac{h}{1 + \eta} > 0 \tag{11}$$

Therefore <u>positive</u> correlation.

$$\frac{\partial \ell^*(\cdot)}{\partial w} = \frac{hw(1 + \frac{1}{\eta}) - (1 + \frac{1}{\eta})(hw + \pi - T)}{w^2(1 + \frac{1}{\eta})^2} = -\frac{\pi - T}{w^2(1 + \frac{1}{\eta})} \tag{12}$$

The correlation is up to the sign of non-labor income $(\pi - T)$.

2.2 Representative Agent: Firm

2.2.1 Production Function

Production Function maps the inputs (K, N) to output (Y).

$$Y = zF(L, N^d),\ z > 0$$

Where z is the **total factor productivity (TFP)**

2.2.2 Assumptions

Constant Return to Scale (CRS)

$$F(tK, tN^d) = tF(K, N^d),\ \forall t > 0 \tag{13}$$

Increasing Return in Input

$$\frac{\partial F(K, N^d)}{\partial K} > 0 \wedge \frac{\partial F(K, N^d)}{\partial N^d} > 0 \tag{14}$$

Diminishing in Marginal Return

$$\frac{\partial^2 F(K, N^d)}{\partial K^2} < 0 \wedge \frac{\partial^2 F(K, N^d)}{\partial N^{d2}} < 0 \tag{15}$$

Marginal Product Increases in other Inputs

$$\frac{\partial^2 F(K, N^d)}{\partial K \partial N^d} > 0 \wedge \frac{\partial^2 F(K, N^d)}{\partial N^d \partial K} > 0 \tag{16}$$

2.2.3 Optimization Problem

$$\max_{N^d}\{zF(K, N^d) - wN^d\} \tag{17}$$

Intuition The optimal choice would be where $MP_N = w$, this means an extra unit of labor hired leads to negative profit change.

2.2.4 Formalizing Firm's Optimization Problem

Production Function Take the **Cobb-Douglas Production Function** so that an interior solution for this optimization problem is guaranteed to be existing.

$$Y = zF(K, N^d) = zK^\alpha {N^d}^{1-\alpha},\ \alpha \in (0,\ 1) \tag{18}$$

Solution

$$N^{d*}(\cdot) = (\frac{(1-\alpha)zK^\alpha}{w})^{\frac{1}{\alpha}} \tag{19}$$

2.2.5 Comparative Statistics

Total Factor Productivity (z)

$$\frac{\partial N^{d*}(\cdot)}{\partial z} = \frac{1}{\alpha} z^{\frac{1}{\alpha}-1} (\frac{(1-\alpha)K^{\alpha}}{w})^{\frac{1}{\alpha}} > 0 \tag{20}$$

Capital K

$$\frac{\partial N^{d*}(\cdot)}{\partial K} = (\frac{(1-\alpha)z}{w})^{\frac{1}{\alpha}} > 0 \tag{21}$$

Wage (w)

$$\frac{\partial N^{d*}(\cdot)}{\partial w} = -\frac{1}{\alpha} w^{-\frac{1+\alpha}{\alpha}} [(1-\alpha)zK^{\alpha}]^{\frac{1}{\alpha}} < 0 \tag{22}$$

2.2.6 Adding Taxes

Tax on output (τ)

$$N^{d^*}(\cdot) = (\frac{(1-\alpha)(1-\tau)zK^{\alpha}}{w})^{\frac{1}{\alpha}} \tag{23}$$

Tax on labor hired (τ_N)

$$N^{d^*}(\cdot) = (\frac{(1-\alpha)zK^{\alpha}}{(1+\tau_N)w})^{\frac{1}{\alpha}} \tag{24}$$