ECO421: Topics in Economic of Information

Games with Incomplete Information

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1 Knowledge

Notation 1.1. Let Ω denote the all possible states of world, and let A denote the set of all agents.

Definition 1.1. Every $E \subseteq \Omega$ is called an **event** or a type/piece of **information**.

Definition 1.2. Let $E \subseteq \Omega$ be a piece of information and $i \in A$, and we say agent i **knows** E in state $\omega \in \Omega$ if this agent knows

(i) The true state of world $\omega^* \in E$;

(ii) but, all $\omega \in E$ are considered to be possible.

Remark: the agent is certain about the true state ω^* if and only if $E = \{\omega^*\}$.

Definition 1.3. For an agent $i \in A$, the **information structure** of this agent, $\mathcal{T}_i$, is a *partition* of Ω.

Notation 1.2. Each element of the partition corresponds to one piece of information.
One may define a mapping $T_i(\cdot) : \Omega \to \mathcal{T}_i$, such that for every $\omega \in \Omega$, let $T_i(\omega) \in \mathcal{T}_i$ denote the piece of information known by this agent in state ω.
Equivalently, $T_i(\omega) \subseteq \Omega$ denotes the set of all states that i considers possible given her information in state ω.

Definition 1.4. A **knowledge-based type space** is defined to be a tuple

$$(\Omega, (\mathcal{T}_i)_{i \in A}) \tag{1.1}$$

Definition 1.5 (Equivalent Definition)**.** An agent i is said to **know** information E in state ω if and only if

$$T_i(\omega) \subseteq E \tag{1.2}$$

That is, the true state $\omega^* \in T_i(\omega) \subseteq E$, so that i believes E to be certain.

Definition 1.6. Define $K_i(E)$ to be the set of states in which agent i knows E, that is,

$$K_i(E) := \{\omega \in \Omega : T_i(\omega) \subseteq E\} \tag{1.3}$$

Definition 1.7. Given a piece of information $F \subseteq \Omega$ characterizing that $\omega^* \in F$, the new **information structure updated by** F is defined to be

$$(\Omega^F, (T_i^F(\cdot))) \tag{1.4}$$

where

$$\Omega^F = \Omega \cap F \tag{1.5}$$
$$T_i^F(\omega) = T_i(\omega) \cap F \quad \forall \omega \in \Omega^F \tag{1.6}$$

Definition 1.8. The set of states where E is known given F is

$$K_i(E|F) := \left\{\omega \in \Omega^F : T_i^F(\omega) \subseteq E\right\} \tag{1.7}$$

Notation 1.3. For $i, j \in A$, the following are equivalent:

1. states in which i knows that player j knows E;
2. states in which i knows $K_j(E)$.

This set of states can be denoted as

$$K_i(K_j(E)) \tag{1.8}$$

Definition 1.9. An event $E \subseteq \Omega$ is a **common knowledge** in state ω if

$$\omega \in \bigcap_{i \in A} K_i(E) \tag{1.9}$$

$$\omega \in \bigcap_{j \in A} K_j \left(\bigcap_{i \in A} K_i(E) \right) \tag{1.10}$$

$$\omega \in \bigcap_{k \in A} K_k \left[\bigcap_{j \in A} K_j \left(\bigcap_{i \in A} K_i(E) \right) \right] \tag{1.11}$$