ECO206 Microeconomic Theory

Final Preparation Notes <u>First Half</u>
<u>no further update</u>

Tianyu Du

August 31, 2018

Contents

1 Lecture 1 Budget Constraints

1.1 Notation-Bundle

Definition 1.1. If we have n goods, then vector $(x_1^A, x_2^A, \dots, x_n^A) \in \mathbb{R}_+^n$ represents a **bundle**, where x_1^A represents a quantity(x) of good 1 in bundle A.

1.2 Types of Income

Exogenous independent to price of goods.

Endogenous bundle of goods endowed, value dependents on price of goods.

1.3 Budget Set

Definition 1.2. Budget Set is the set of all affordable consumption *bundles*. Let $\vec{e} = (\omega_1, \omega_2, \dots, \omega_n) \in \mathbb{R}_+^n$ denote the endowment of consumer and I denote the exogenous income, and consumers are price-takers facing price vector $\vec{p}$. Then

$$\mathcal{B} = \left\{ \vec{x} \in \mathbb{R}_+^n | \vec{x} \cdot \vec{p} \leq \vec{e} \cdot \vec{p} + I \right\}$$

1.4 Opportunity Cost

Opportunity cost is the rate at which one good can be traded for another though the *market*. It captures what the consumer is **able** to trade at market price, not what the consumer is *willing* to trade.

Consider budget constraint $\vec{x} \cdot \vec{p} = I$. Take the total differential,

$$\sum_i \frac{\partial \vec{x} \cdot \vec{p}}{\partial x_i} dx_i = 0$$

And the opportunity cost for one unit of good x is shown to be

$$\frac{dy}{dx} = -\frac{p_x}{p_y}$$

1.5 Income and Price changes

Pure Income Changes: *parallel shift* of budget line.

Price changes: *rotation* of budget line around the *invariant point* (i.e. the consumption bundle that is not affected by the price change).

1. Exogenous income: Invariant point $\Longleftarrow$ $\{x_1, \ldots, x_i = 0, \ldots, x_n\}$ with p_i changes.

2. Endogenous income: Invariant point $\Longleftarrow$ endowment point $\vec{e}$.

2 Lecture 2 Preferences and Utility

2.1 Tastes as Binary Relations

Definition 2.1.

1. Bundle A (x_1^A, x_2^A) is **strictly preferred to** bundle B (x_1^B, x_2^B).

$$(x_1^A, x_2^A) \succ (x_1^B, x_2^B)$$

2. Bundle A is **at least as good as** bundle B

$$(x_1^A, x_2^A) \succcurlyeq (x_1^B, x_2^B)$$

3. The consumer is indifferent between bundle A and bundle B

$$(x_1^A, x_2^A) \sim (x_1^B, x_2^B)$$

2.2 Rationality Assumptions

Definition 2.2. Complete: For all bundles $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$.

$$A \succcurlyeq B \vee B \succcurlyeq A$$

Definition 2.3. Transitive: for any three bundles, A, B, C

$$A \succcurlyeq B \wedge B \succcurlyeq C \implies A \succcurlyeq C$$

2.3 Convenience Assumptions

Definition 2.4. Monotonic:

$$A \succcurlyeq B \iff x_i^A \geq x_i^B \ \forall \ i$$

$$A \succ B \iff x_i^A > x_i^B \ \forall \ i$$

Definition 2.5. Convexity: Suppose $A = (x_1^A, x_2^A) \sim B = (x_1^B, x_2^B)$

$$\alpha A + (1-\alpha)B \succcurlyeq A, \ \forall \ \alpha \in [0,1]$$

Definition 2.6. Continuity: (no sudden jumps)

2.4 Utility Function and Indifference Curve

Definition 2.7. Let $\mathcal{X}$ denote the consumption set and a **utility function** is a real-valued function $u : \mathcal{X} \to \mathbb{R}_+$ so that

$$A \succcurlyeq B \iff u(A) \geq u(B)$$

and

$$A \succ B \iff u(A) > u(B)$$

Remark 2.1. The positive monotone transformations of utility function of a preference also captures the same preference.

Definition 2.8. The **indifference curve** corresponding to utility level $\overline{u}$ is a set of consumption bundles

$$\{\vec{x} \in \mathbb{R}_+^n \mid u(\vec{x}) = \overline{u}\}$$

2.5 Marginal Rate of Substitution

Definition 2.9. The **MRS** is the number of good y that a consumer is **willing** to given up in order to get one more unit of good x.
Take total differential of equation $u(\vec{x}) = \overline{u}$,

$$\sum_i \frac{\partial u(\vec{x})}{\partial x_i} dx_i = 0 \implies \frac{dy}{dx} = -\frac{MU_x}{MU_y}$$

And MRS is a function of bundle $\vec{x}$.

Diminishing MRS is when the **absolute MRS** decreases as more good x is consumed along the indifference curve.

2.6 Different Types of Tastes

- **Perfect Substitutes** MRS constant at every bundle.
- **Perfect Complements** Unwilling to substitute.
- **Homothetic** MRS constant on a ray ($\beta\vec{x_o},\ \beta > 0$) through origin.
- **Quasi-linear** MRS depends on $\vec{x_{-i}}$ where x_i is excluded.

3 Lecture 3 Choice

Diminishing MRS in words Comparing two bundles on the *same* indifferent curve. At each bundle consider how much of y we are willing to given up for an additional unit of x. *If we have relatively more x in one bundle, then we are less willing to give up y at that bundle compared to the other bundle.*

Remark 3.1. Perfect Substitutes (linear in every commodity) $\implies$ quasi-linear.

3.1 Lagrangian Multiplier

Sufficiency FOCs from Lagrangian multiplier is both necessary and sufficient if and only if

1. There are no "flat spots" on indifference curves.
2. All goods are essential (interior solution)
3. Both choice set (characterized by budget constraint) and tastes (characterized by preference and utility function, quasi-concave).

Uniqueness Assuming **convexity** of preferences and choice set guarantees *uniqueness* of optimal choice. (Reducing problem to a convex optimization problem).

Remark 3.2. If convexity is not satisfied, Lagrangian multiplier could give *minimal* instead.

3.2 Problem Solving

1. Determine preference type.
2. Check expectation of interior solution.
   - If interior solution (e.g. Cobb-Douglas): setup and solve Lagrangian multiplier, check sufficiency.
   - If not:
     (a) Perfect Substitute: Corner Solution if $MRS \neq rel.Price$.

(b) Perfect Complement: Solution on path $x_1 = \gamma x_2$.

(c) Quasi-linear: interior solutions found for some parameter values only. *Use LM to find expression of solution and check the condition for positiveness.*

3. Check for *multiple optimal solutions*.

4 Lecture 4 Demand and Income Effects

4.1 Demand

Definition 4.1. A **demand function** is an equation that expresses bundle choices as a function of prices and incomes.

4.2 Income Effects

Definition 4.2. Income effect is the change in behavior arising from just a change in income and leads to a parallel shift in the budget constraint.

Remark 4.1. With quasi-linear preference with numeraire x_1, then the demand for all other commodities other than x_1 have no income effect. We often refer to quasi-linear goods as the borderline goods between normal and inferior.

Remark 4.2. With pure income effect and homothetic preference, the ratios of commodities before and after are the same.

Definition 4.3. A **Engel curve** $x_i^D(I, \vec{p})$ captures the relationship between demand and income. Note: put I on y-axis.

- **Normal goods** $\frac{\partial x}{\partial I} > 0$
- **Inferior goods** $\frac{\partial x}{\partial I} < 0$

4.3 Decomposition of Substitution Effect and Income Effect

- **Substitution Effect** changing relative prices holding the utility level constant. (Moving along Hicksian demand)
- **Income Effect** changing purchasing power of the consumer's income holding relative prices constant.

5 Lecture 5 Income and Substitution Effects

Definition 5.1. Hicksian substitution effect derived by compensating consumer enough income so that the consumer can reach the original utility level after the price changes.

Definition 5.2. Slutsky substitution effect derived by compensating consumer enough income so that the consumer can afford the original optimal bundle after the price changes.

5.1 Expenditure Minimization

Definition 5.3. With new prices, given consumer enough (hypothetical) income to just reach the original indifference curve. Mathematically,

$$min_{x_1,x_2} p_1^{final} x_1 + p_2^{final} x_2 \ s.t. \ u(x_1, x_2) = U^{initial} = u(x_1^{initial}, x_2^{initial})$$

Solving the expenditure using Lagrangian multiplier method

$$\mathcal{L}(\vec{x}, \lambda) = \vec{x} \cdot \vec{p} + \lambda\{\overline{u} - u(\vec{x})\}$$

Let $\vec{x}^{SE} = (x_1^{SE}, x_2^{SE}, \ldots, x_N^{SE}) = argmin_{\vec{x}}\{\vec{p} \cdot \vec{x} \ s.t. \ u(\vec{x}) = \overline{u}\}$ and $\vec{p}^{new} \cdot \vec{x}$ denote the **compensated income**.

Definition 5.4. From expenditure minimization, we define the **expenditure function** $e(\vec{p}, \overline{u}) : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ as a function of price vector $\vec{p}$ and utility level $\overline{u}$.

$$e(\vec{p}, \overline{u}) := min_{\vec{x}}\{\vec{p} \cdot \vec{x} \ s.t. \ u(\vec{x}) = \overline{u}\}$$

5.2 Income Effect

Remark 5.1. Decomposing substitution and income effects.

1. The utility value achieved is $\overline{u} = u(\vec{x}_1^*)$.
2. Given prices $\vec{p}_1, \vec{p}_2$, use utility maximization to find optimal bundles $\vec{x}_1^*, \vec{x}_2^*$.
3. Use expenditure minimization with respect to new price and original utility level to find $\vec{x}_{SE}(\vec{p}_2, \overline{u})$.
4. Substitution effect is $\vec{x}_{SE} - \vec{x}_1^*$.
5. Income effect is $\vec{x}_2^* - \vec{x}_{SE}$.

Figure 1: Decomposition of two effects

5.3 Compensated Demand Curve

Remark 5.2. Regular demand (Marshallian) changes in x_1 as p_1 changes when income held fixed. (On graph: initial to final bundles)

Definition 5.5. **Compensated demand (Hicksian)** captures the changes in x_1 as p_1 changes, when utility level is held fixed(hitting the same IC) by compensating consumer additional income. (On graph: initial to SE)
Notation:
Given prices p_i and prices of other goods, $\vec{p}_{-i}$ and original utility $\overline{u}$, compensated demand for x_i is denoted as

$$h_i(p_i, \vec{p}_{-i}, \overline{u}) : \mathbb{R}^n_+ \to \mathbb{R}_+$$

Remark 5.3.
- Change in prices of other goods / changes in original utility level Movements of the compensated demand curve.
- Own-price changes Movement *along* the curve.

5.4 Slutsky Equation (Optional)

Theorem 5.1.

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - h_j \frac{\partial x_i}{\partial y}$$

Proof. Notice that at the optimal, $x_i(\vec{p}, e(\vec{p}, u)) = h_i(\vec{p}, u)$, take derivative with respect to p_j we have

$$\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial e}\frac{\partial e}{\partial p_j} = \frac{\partial h_i}{\partial p_j}$$

By Sheppard's Lemma $\frac{\partial e(\vec{p},u)}{\partial p_j} = h_j(\cdot)$,

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - h_j \frac{\partial x_i}{\partial y}$$

■

6 Lecture 6 Labor Supply and Elasticities

6.1 Labor Supply

- Endogenous income: Leisure endowment (L) sold at real wage rate w.
- Exogenous income: Non-labor income (M)

Setup
Budget constraint:

$$c = w(L - \ell) + M$$

Optimization problem:

$$\max_{c,\ell} u(c, \ell)$$
$$s.t.\ c \leq w(L - \ell) + M$$

Let $c^*(w), \ell^*(w)$ denote the maximizers and **labor supply** is

$$h(w) = L - \ell(w)$$

Example 6.1. For wage rate w increases:

- Substitution effect: $\ell \downarrow$ and $h \uparrow$
- Income effect (positive):
  - (Inferior leisure) $\ell \downarrow$ and $h \uparrow$
  - (Normal leisure) $\ell \uparrow$ and $h \downarrow$

Remark 6.1. Labor supply is upwards sloping if substitution effect dominates income effect or leisure is considered as inferior.

6.2 Taxes and Labor Supply

Remark 6.2. *Proportional income tax* is effective a change in real wage w faced by consumers. Consider both income effect (and notice leisure could be inferior or normal) and substitution effect.

6.2.1 Laffer Curve (Optional)

Definition 6.1. Laffer curve captures the relation between tax revenue and marginal tax rate. For proportional income tax, tax revenue $T(\tau)$ is

$$T(\tau) = \tau * w * h(\tau)$$

and marginal tax revenue depends on the elasticity of labor supply.

6.3 Elasticity

Definition 6.2. The **own price elasticity of demand** for commodity i at coordinate (x_i, p_i) on its demand curve is

$$\epsilon_i = \frac{\frac{dx_i}{x_i}}{\frac{dp_i}{p_i}} = \frac{dx_i}{dp_i}\frac{p_i}{x_i}$$

Definition 6.3. Cross price elasticity of demand for commodity i with respect to price of commodity j is

$$\epsilon_{i,j} = \frac{dx_i}{dp_j}\frac{p_j}{x_i}$$

Definition 6.4. The **income elasticity of demand** for commodity i is

$$\eta_i = \frac{dx_i}{dI}\frac{I}{x_i}$$

Definition 6.5. Elasticity ϵ is classified as

- **Elastic** if $|\epsilon| > 1$.
- **Inelastic** if $|\epsilon| < 1$.
- **Unit elastic** if $|\epsilon| = 1$.
- **Perfectly inelastic** if $|\epsilon| = 0$.
- **Perfectly elastic** if $|\epsilon| = \infty$.

6.4 Change in Revenue

Total revenue $TR(p) = p * x(p)$, take derivative with respect to p on both sides

$$\frac{\partial TR(p)}{\partial p} = x(p) + p\frac{\partial x(p)}{\partial p}$$
$$\implies \frac{\partial TR(p)}{\partial p} = x(p)\{1 + \frac{\partial x(p)}{\partial p}\frac{p}{x(p)}\} = x(p)\{1 - |\epsilon|\}$$
$$\implies \frac{\partial TR(p)}{\partial p} > 0 \iff |\epsilon| < 1$$

7 Lecture 7 Elasticity and Consumer Surplus

7.1 Consumer Surplus

Relating the regular demand, choice diagram and compensated demand.

Definition 7.1. Consumer surplus is the difference between what you are willing to pay (MRS) and what you have to pay for each unit in your chosen bundle quantified in dollars.

Calculating Consumer Surplus

1. Identify the original bundle chosen and utility level $\overline{u}$.
2. Calculate $h_1(p_1, p_2 = 1, \overline{u})$.
3. $CS = \int_{p^*}^{\infty} h_1(p_1, p_2 = 1, \overline{u})\ dp_1$

Remark 7.1. If the preference is **quasi-linear**, then for non-numeraire commodities, consumer surplus calculated from compensated and uncompensated demand curves are the same.

7.2 Expenditure Function

Definition 7.2. Expenditure function is the value function for consumer expenditure minimization problem.

$$e(\vec{p}, \overline{u}) = \min_{\vec{x}}\{\vec{x} \cdot \vec{p} \; s.t. \; u(\vec{x}) \geq \overline{u}\} = \vec{p} \cdot \vec{h}^*(\vec{p}, \overline{u})$$

7.3 Compensating Variation

Definition 7.3. Compensating Variation(CV) is the amount of compensation consumer needs to achieve it's original utility level with the new price.

$$|CV| = e(\vec{p}_{final}, \overline{u}_{initial}) - I$$

Remark 7.2. Consider a change on p_i from p_i^1 to p_i^2, the prices for other commodities $\overline{p}_{-i}$ are unchanged. By Sheppard's lemma, $\frac{\partial e(\cdot)}{\partial p_i} = h_i$.

$$|CV| = e(p_i^2, \overline{p}_{-i}, \overline{u}_{initial}) - e(p_i^1, \overline{p}_{-i}, \overline{u}_{initial}) = \int_{p_i^1}^{p_i^2} h_i(p_i, \overline{u}_{initial}) \; dp_i$$

7.4 Equivalent Variation

Definition 7.4. Equivalent Variation (EV) is the amount of compensation consumer needs at initial price to make him as well off as after the price change.

$$|EV| = e(\vec{p}_{initial}, \overline{u}_{final}) - I$$

Remark 7.3. Consider a change on p_i from p_i^1 to p_i^2, the prices for other commodities $\overline{p}_{-i}$ are unchanged.

$$|EV| = e(p_i^1, \overline{p}_{-i}, \overline{u}_{final}) - e(p_i^2, \overline{p}_{-i}, \overline{u}_{final}) = \int_{p_i^2}^{p_i^1} h_i(p_i, \overline{u}_{final}) \; dp_i$$

Exit Question CV and EV are derived from integral of compensated demand h_i at difference utility levels. For quasi-linear preferences, $|CV| = |EV|$, since there is no income effect, compensated demand curves at any utility level coincides the uncompensated demand curve. Therefore both CV and EV are derived from integral compensated demand and they have the same absolute value *but opposite sign*.

8 Lecture 8 Dead Weight Loss and Uncertainty

Remark 8.1. In general, the compensated and uncompensated demand curves intersect at one point (where the compensated demand curve is constructed). As we move away from that point by changing prices, the quantity on CD and Reg.D will differ because the CD captures only SE but regular demand captures both IE and SE.

8.1 Finding ΔCS on Graph

Figure 2: CV, EV and CS on graph for normal goods

8.2 Dead Weight Loss(DWL)

Definition 8.1. **Dead weight loss** is the loss in surplus that could have been captured by someone in the economy but isn't.

Remark 8.2. DWL is due to a **substitution effect** - changing relative prices causes people to move away from relatively more expensive good, lowering tax revenue.

Remark 8.3. Lump-sum tax causes pure income effect and therefore no dead weight loss. We calculate the DWL from taxation using

$$DWL = |T - L|$$

where T is the actual tax revenue.

Figure 3: Lump-sum revenue, tax revenue and dead weight loss on graph

8.2.1 Calculating DWL from taxation

1. Put composite good on y-axis of the choice diagram.
2. Find post tax choice $\vec{x}_t$ and utility level $\overline{u}_t$.
3. Calculate $EV = I - e(\vec{p}_{initial}, \overline{u}_t)$. And notice EV is the maximum amount of lump sum tax could be charged leaving consumer at least as well as charging them proportional tax t on commodity.
4. Calculate $T = t * p_x * x^t$.
5. $DWL = |T - L|$. (focus on the absolute value)

8.3 Duality

Remark 8.4. Notice at the optimal,

$$v(\vec{p}, e(\vec{p}, u)) = u$$

and

$$e(\vec{p}, v(\vec{p}, I)) = I$$

Indirect utility function $v(\vec{p}, I)$ and expenditure function $e(\vec{p}, u)$ are invert of each other.

8.4 Risk and Uncertainty

8.4.1 Setup

Axes Accident state (x_A) consumption on x-axis and safe state consumption (x_S) on y-axis.

Figure 4: Coordinate used to analyze uncertainty

Definition 8.2. A **gamble** is a bundle giving payoff in different states, and each state has its probability of being realized.

$$(x_A, x_S) = \begin{cases} x_A \text{ with probability } \delta \\ x_S \text{ with probability} 1-\delta \end{cases}$$

8.4.2 Budget Constraint

Remark 8.5. Endowed income here is the consumption in each state when there are no interventions.

Definition 8.3. A general form of insurance is paying **premium** p in both states and get **benefit** b only in a certain state.

Example 8.1. With endowed income (e_A, e_S), an insurance costs premium p in both states and pays benefit b in accident state only transforms the consumption bundle

$$\begin{cases} x_A = e_A \\ x_S = e_S \end{cases} \rightarrow \begin{cases} x_A = e_A - p + b \\ x_S = e_S - p \end{cases}$$

Remark 8.6. We usually consider two types of insurance contracts:

1. **Buy or not:** Comparing the expected utility level with and without insurance.
2. **How much to buy:** negative amount allowed.

   Consider the insurance contract

$$b = \gamma p$$

Then the budget constraint between x_A and x_S becomes

$$x_S = e_S + \frac{\gamma}{1-\gamma}(e_A - x_A)$$

Therefore we can find the opportunity cost for each unit of x_A is

$$\frac{dx_S}{dx_A} = -\frac{\gamma}{1-\gamma}$$

Exit Question When insurance gets more expensive ($\gamma \uparrow$), how does the budget constraint change? The budget constraint **rotates** around the endowment point. It's getting steeper (with accident consumption on x-axis and safe consumption on y-axis) as more safe state consumption are needed to be given up (p) for one more unit of accident state consumption ($b - p = (1-\gamma) * b$).

9 Lecture 9 Uncertainty

9.1 Budget Constraint

Remark 9.1. Given insurance in $p = \gamma b$ form, budget constraint captures the trade off between safe state disposable income and accident state disposable income.

Given disposable incomes in both states

$$\begin{cases} x_A = e_A + \frac{1-\gamma}{\gamma} p \\ x_S = e_S - p \end{cases}$$

and combining above equations gives the **budget constraint**,

$$x_A = e_A + (e_S - x_S)(\frac{1-\gamma}{\gamma})$$

where the **opportunity cost** is

$$\frac{dx_S}{dx_A} = -\frac{\gamma}{1-\gamma}$$

interpreted as the quantity of safe state income *given up* for one additional unit of accident state income.

9.2 Expected Value of Gamble

Without insurance,

$$EV = \delta e_A + (1-\delta)e_S$$

and with insurance

$$EV = \delta e_A + (1-\delta)e_S + \delta b - p$$

Definition 9.1. An insurance plan in the form of $p = \gamma b$ paying b when state with probability δ happens is a **fair insurance** if the expected payoff from this insurance is zero. That's

$$\begin{gathered} \mathbb{E}(insurance) = \\ \delta(b - \gamma b) + (1-\delta)(-\gamma b) = 0 \\ \implies \delta = \gamma \end{gathered}$$

Definition 9.2. Given $u_i(\cdot)$ represents the utility function for state i and the **VNM(Expected) utility** from our simple gamble is

$$U(x_A, x_S) = \delta u_A(x_A) + (1 - \delta)u_S(x_S)$$

Theorem 9.1. (von Neumann-Morgenstern) Expected Utility Theorem: if tastes over gambles satisfy the independence Axiom, then there exists a function $u(x)$ such that the tastes over gambles can be represented by a function U that takes the form

$$U(x_A, x_S) = \delta u(x_A) + (1 - \delta)u(x_S)$$

9.2.1 MRS

Definition 9.3. MRS: holding **expected utility** constant, how much safe state consumption is consumer **willing** to give up for an additional unit of consumption in accident state.

$$\begin{aligned} MRS = \frac{dx_S}{dx_A} &= -\frac{MU_A}{MU_S} \\ &= -\frac{\frac{\partial U}{\partial x_A}}{\frac{\partial U}{\partial x_S}} \\ &= -\frac{\delta}{1-\delta}\frac{\frac{\partial u_A}{\partial x_A}}{\frac{\partial u_S}{\partial x_S}} \end{aligned}$$

Definition 9.4. Preference over wealth is **state independent** if the utility functions in different states are taking the same functional form

$$u_A(\cdot) = u_S(\cdot) = u(\cdot)$$

otherwise, the preference is **state dependent**.

9.2.2 Certainty Equivalent(CE)

Definition 9.5. Certainty Equivalent (CE)(x_{CE}) the amount of money the consumer gets for sure that gives them the same utility as gamble

$$u(x_{CE}) = EU(e_A, e_S) = \delta u_A(x_A) + (1 - \delta)u_S(x_S)$$

9.2.3 Risk Preference

Definition 9.6. Consider gamble with possible outcomes associated with probabilities $g = (\vec{x} \circ \vec{p})$ and therefore $\mathbb{E}(x)$ represents the expected payoff from gamble, then a preference is **risk averse**(*prefer the expected payoff with certainty*) if

$$u(\mathbb{E}(x)) > EU(g)$$

and is **risk neutral** if

$$u(\mathbb{E}(x)) = EU(g)$$

and is **risk-loving/affine** if

$$u(\mathbb{E}(x)) < EU(g)$$

where

$$EU(g) = \sum_{i \in \mathcal{I}} p_i u_i(x_i)$$

9.2.4 Optimal Insurance Choice

$$\max_{(p,b)\in\mathbb{R}^2} \delta u(e_A + b - p) + (1-\delta)U(e_S - p)$$

$$s.t.\ p = \gamma b$$

Remark 9.2. Negative quantity of b and p are allowed.

Definition 9.7. When **full** insurance is chosen, consumption in both states is the same. That's

$$e_A + b - p = e_S - p$$

Theorem 9.2. The individual will choose to be fully insured if the following three conditions are satisfied,

1. Fair insurance.
2. State independent tastes.
3. Risk-aversion.

10 Lecture 10 Production

10.1 Production Function

Definition 10.1. Production function $f : \mathbb{R}^n_+ \to \mathbb{R}_+$ maps input bundle to the maximum possible output.

Remark 10.1. Given short run production function $f(l)$, $f^{-1}(x)$ gives the minimum quantity of labor required for output quantity x.

Definition 10.2. The **isoquant** curve captures the input bundles that can be used to produce the quantity $\overline{q}$.

$$\{\vec{x} \in \mathcal{X} \mid f(\vec{x}) = \overline{q}\}$$

Definition 10.3. Marginal Rate of Technical Substitution(MRTS) captures at what rate can a firm substitute one input for another without changing its output level.

$$\frac{dk}{dl} = -\frac{MPL}{MPK}$$

10.2 Technologies

Definition 10.4. Monotonicity let $\vec{x}$ denote input bundle,

$$\frac{\partial f(\cdot)}{\partial x_i} > 0 \ \forall \ i$$

Definition 10.5. Convex Isoquants $\iff$ *quasi-concave* production function.

Definition 10.6.

1. **CRS** $f(\lambda\vec{x}) = \lambda f(\vec{x}), \ \forall\lambda > 0$
2. **IRS** $f(\lambda\vec{x}) > \lambda f(\vec{x}), \ \forall\lambda > 0$
3. **DRS** $f(\lambda\vec{x}) < \lambda f(\vec{x}), \ \forall\lambda > 0$

10.2.1 Elasticity of Substitution

Definition 10.7. Elasticity of substitution between two input factors l, k is defined as

$$\begin{aligned}\sigma &= \frac{\frac{d(k/l)}{k/l}}{\frac{dMRTS}{MRTS}} \\ &= \frac{d\ln k/l}{d\ln |MRTS|}\end{aligned}$$

11 Lecture 11 Cost Minimization and Cost Curve

11.1 Firm's Problem

$$\max \pi(x) = p(x)x - C(x, w, r)$$

11.2 Types of Costs

Classification of costs:

1. Fixed cost.
2. Sunk cost.
3. Variable cost.

11.3 Cost Minimization

Definition 11.1. Conditional input demands capture the demands for input factors to produce output quantity $\overline{x}$.

$$(l, k) = argmin_{(l,k)} w * l + r * k \ s.t. \ f(k, l) = \overline{x}$$

11.3.1 Short-run Cost Minimization

$$\min_{l} wl + r\overline{k} \; s.t. \; f(l, \overline{k}) = \overline{x}$$
$$\min_{k} w\bar{l} + rk \; s.t. \; f(\bar{l}, k) = \overline{x}$$

and **conditional input demand**

$$l(\overline{x}, w)$$
$$k(\overline{x}, r)$$

and **SR Cost function**

$$C(\overline{x}, w, r) = wl(\overline{x}, w) + r\overline{k}$$
$$C(\overline{x}, w, r) = w\bar{l} + rk(\overline{x}, w)$$

11.3.2 Long-run Cost Minimization

$$\min_{k,l} wl + rk$$
$$s.t. f(l, k) = \overline{x}$$

and get **conditional input demands**

$$l(\overline{x}, w, r) and k(\overline{x}, w, r)$$

and **long run cost** as a function of output quantity, which captures the minimum cost required to produce certain among of output $\overline{x}$:

$$C(\overline{x}, l, r) = w * l(\overline{x}, w, r) + k * k(\overline{x}, w, r)$$

Remark 11.1. For some types of production function (e.g. *(quasi)linear*), solution of cost minimization could be corner solution. Use isoquant and isocost curve to solve the problem.

11.4 LR vs SR on Choice Graph

Remark 11.2. LR total cost curve is the lower envelope of the short-run total cost curves. And The LR total cost curve always has one point in common with any particular short-run total cost curve (*not always the lowest point*). Since the short run conditional input demand is always available in the long run. Therefore, long run cost minimization problem has larger choice set and therefore the value function is weakly lower than the short run minimization problem.

Remark 11.3. Returns to scale determine how average (variable) production costs change with output level. If scaling up output by a a factor λ average cost becomes

$$\frac{f(\lambda\vec{x})}{\lambda\vec{p}\cdot\vec{x}} \sim \frac{\lambda f(\vec{x})}{\lambda\vec{p}\cdot\vec{x}}$$

then AVC increases if DRS and decreases if IRS.

12 Lecture 12 Profit Maximization and Factor Demand

12.1 Two step method: profit maximization

$$\max_x \pi(x) = p(x)x - C(x, w, r)$$

Step 1 Minimize cost and construct **conditional** input demand and cost function $C(x, \cdot)$

Step 2 Profit maximization and determine the optimal quantity $x(p)$ using $MR = \frac{\partial C(x,w,r)}{\partial x}$, assert non-negative profit.

12.2 Short Run Firm Supply

$$x_{SR}(p, w, r) = \begin{cases} MC_{SR}^{-1}(p) \text{ if } p \geq \min\{AVC_{SR}\} \\ 0 \text{ (shutdown) otherwise} \end{cases}$$

12.3 Long Run Firm Supply

$$x_{LR}(p, w, r) = \begin{cases} MC_{LR}^{-1}(p) \text{ if } p \geq \min\{AVC_{LR}\} \\ 0 \text{ (exit market) otherwise} \end{cases}$$

12.4 One step method: profit maximization

$$\max_{l,k} p * f(l, k) - w * l - r * k$$

Remark 12.1. *argmax* for above optimization is the **unconditional factor demands**.

Remark 12.2. Profit maximization $\implies$ Cost minimization. But Cost minimization $\not\Longrightarrow$ Profit maximization.

12.5 Edge Cases

12.5.1 Corner Solutions

- $x = 0 \Longleftarrow \max\{\pi\} < 0$ shutdown.
- $x = \infty \Longleftarrow$ increasing return to scale. $(AVC(x) < p \ \forall x \geq x^*)$

12.5.2 Multiple Solutions

Solution Choose all candidates and pick the highest valued one.