Forecasting and Time Series Econometrics
ECO374 Winter 2019

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1 Introduction and Statistics Review

Definition 1.1. Given random variable X, the k^{th} **non-central moment** is defined as

$$\mathbb{E}[X^k] \tag{1.1}$$

Definition 1.2. Given random variable X, the k^{th} **central moment** is defined as

$$\mathbb{E}[(X - \mathbb{E}[X])^k] \tag{1.2}$$

Remark 1.1. Moments of order higher than a certain k may not exist for certain distribution.

Definition 1.3. Given the **joint density** $f(X, Y)$ of two *continuous* random variables, the **conditional density** of random Y conditioned on X is

$$f_{Y|X}(y|x) = \frac{f_{Y,X}(y, x)}{f_X(x)} \tag{1.3}$$

Definition 1.4. Given discrete variables X and Y, the **conditional density** of Y conditioned on X is defined as

$$P(Y = y|X = x) = \frac{P(Y = y \wedge X = x)}{P(X = x)} \tag{1.4}$$

Assumption 1.1. Assumptions on linear regression on time series data:

(i) **Linearity**

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_k X_k + u \tag{1.5}$$

(ii) **Zero Conditional Mean**

$$\mathbb{E}[u|X_1, X_2, \ldots, X_k] = 0 \tag{1.6}$$

(iii) **Homoscedasitcity**

$$\mathbb{V}[u|X_1, X_2, \ldots, X_k] = \sigma_u^2 \tag{1.7}$$

(iv) **No Serial Correlation**

$$Cov(u_t, u_s) = 0 \ \forall t \neq s \in \mathbb{Z} \tag{1.8}$$

(v) **No Perfect Collinearity**

(vi) **Sample Variation in Regressors**

$$\mathbb{V}[X_j] > 0 \ \forall j \tag{1.9}$$

Theorem 1.1 (Gauss-Markov Theorem)**.** Under assumptions 1.1, the OLS estimators $\hat{\beta}_j$ are *best linear unbiased estimators* of the unknown population regression coefficients β_j.

Remark 1.2. The *no serial correlation* assumption is typically not satisfied for time series data. And the *linearity* assumption is also too restrictive for time series featuring complex dynamics. Hence, for time series data we typically use other models than linear regression with OLS.

2 Statistics and Time Series

Definition 2.1.