ECO220: Quantitative Methods in Economics Lecture Notes (0201)

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Contents

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1.1 Notations

Variable	Population	Sample
Size	N	n
Mean	μ	$\overline{x}$
Std	σ	s

1.2 From Sample to Population

Let p denote the percentage of qualified people in population and let $\hat{p}$ denote the percentage of qualified people in sample. Then, p has an unknown value and the value $\hat{p}$ can be calculated from sample data. We say $\hat{p}$ *is an* ***estimator*** *for* p, and the value of p is still unknown and can only be estimated.

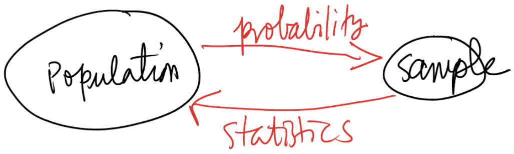
p is a **fixed value** (i.e. p is fixed once population is fixed, we can measure the exact and certain value of p if we traverse the whole population). But $\hat{p}$ will change from sample to sample. We call $\hat{p}$ an **estimator** (or **sample statistic**). The value of sample statistic will change from sample to sample. And, therefore, we call $\hat{p}$ a **random value**.

2 Lecture 2 May. 09 2018

2.1 What is Statistics?

$$\text{Statistics}\begin{cases}\text{Descriptive Statistics}\begin{cases}\text{Graphs}\\ \text{Numerical measures}\end{cases}\\ \text{Inferential Statistics}\quad \textit{Draw conclusions in a population based on sample data.}\end{cases}$$

Inferential Statistics involves uncertainties. To deal with the uncertainties, we need ***probability***



2.2 Data

$$\text{Data}\begin{cases}\text{Quantitative data}\begin{cases}\text{Discrete}\\ \text{Continuous}\end{cases}\\ \text{Qualitative data (Categorical data)}\end{cases}$$

2.3 Descriptive Statistics - Graphs

2.4 Descriptive Statistics - Numerical measures

2.4.1 Measures of centre

Mean Let $\{x_1, \ldots, x_N\}$ be measurements for the population with size N. The population mean is denoted by μ and defined as

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Let $\{x_1, \ldots, x_n\}$ be measurements for the sample of size n. The sample mean is denoted by $\overline{x}$ and defined as

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Note *The mean is sensitive to extreme values.*

Median is the value in the middle when all data are put in order of magnitude. (For data with even size, median is defined as the average of the two values in the middle.)

Mode is value(s) with highest frequency.

Percentiles the k^{th} percentile is a number such that $k\%$ of data fall below this number.

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3.1 Measures of Variation(Spread)

Variance and Standard Derivation Let $\{x_1, \ldots, x_N\}$ denote the population with size N and let $\{x_1, \ldots, x_n\}$ denote the sample with size n. Then

Measures	Population	Sample
Size	N	n
Mean	$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$	$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Variance	$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$
Std	$\sigma = +\sqrt{\sigma^2}$	$s = +\sqrt{s^2}$

Note When calculate the sample variance, use $n-1$ as denominator.

Note mathematically,

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2]$$

Range is defined as the difference between the largest value and the smallest value.

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4.1 Covariance and Correlation: on Populations

Consider two sets of data (population) with size N, denoted as $\{x_1, \dots, x_N\}$ and $\{y_1, \dots, y_N\}$, where x and y measure the age and income of observation, respectively.

Denote $\mu_x :=$ mean of x, $\mu_y :=$ mean of y
$\sigma_x :=$ std dev of x and $\sigma_y :=$ std dev of y. *When x changes, does y change?*

Covariance defined covariance between two datasets, x and y as,

$$Cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)$$

Correlation coefficient the correlation coefficient ρ between datasets x and y is defined as

$$\rho = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

4.2 Covariance and Correlation: on Samples

When N is too large, we select a sample of size n.

Let $\{x_1, \dots, x_n\}$ and $\{y_1, \dots, y_n\}$ denote the selected samples with size n, $\overline{x}, \overline{y}$ denote the sample means, and s_x, s_y denote the sample std dev.

Covariance between two sample is defined as

$$Cov(x,y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$$

Correlation coefficient the sample correlation r is defined as

$$r = \frac{Cov(x,y)}{s_x s_y}$$

4.3 Interpretations

4.3.1 Interpreting covariance

Example Consider samples x and y with

$$Cov(x, y) = -25.31$$

The negative sign means x and y have a negative linear relationship. As x increases, y tends to decrease. The magnitude 25.31 has **no** meaning.

4.3.2 Interpreting correlation coefficient

Example Consider samples x and y with

$$r = -0.94$$

The negative sign means x and y have negative linear relationship. As x increases, y decreases. The magnitude 0.94 means the linear relationship is strong. When r is close to 1 or -1, the string line relation is strong, when r is close to 0, the relation is weak.

Note $\rho \in [-1, 1]$ and $r \in [-1, 1]$

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5.1 Introduction to Simple Regression

Let the linear estimator to be $\hat{y} = b_0 + b_1 x$ and let y_i denote the actual value at x_i, $\hat{y}$ is the estimated y value at x_i. Then, $e_i := y_i - \hat{y_i}$ is the error of y value at x_i (a.k.a. **residual**).

Note notice that $\sum_{i=1}^{n} e_i \equiv 0$.

SSE **Sum of Squared Error**(SSE) as

$$SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

OLS Minimize SSE with respect to b_0 and b_1, we have the FOC as

$$\begin{cases} \frac{\partial SSE}{\partial b_0} = 0 \\ \frac{\partial SSE}{\partial b_1} = 0 \end{cases}$$

By solving the first order conditions, we have

$$\begin{cases} b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2} \\ b_0 = \overline{y} - b_1 \overline{x} \end{cases}$$

The above method to find b_0 and b_1 is called the method of least square, or method of Ordinary Least Square (OLS).

5.2 Relationship between b_1 and r

$$b_1 = \frac{Cov(x,y)}{Var(x)} = \frac{Cov(x,y)}{std(x)std(y)}\frac{std(y)}{std(x)} = r\frac{s_y}{s_x}$$

5.3 Analysis of Variance (ANOVA)

Let y_i denote the actual y value at x_i and $\hat{y}_i$ denote the estimated y value at x_i.

Definition

$$SST = \sum_{i=1}^{n}(y_i - \overline{y})^2$$

$$SSR = \sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2$$

$$SSE = \sum_{i=1}^{n}(y_i - \hat{y}_1)^2$$

Notice $SST = SSR + SSE$

Anova Table

	SS	df	MS	F
Regression	SSR	1	MSR	MSR/MSE
Error(Residual)	SSE	MSE	$n-2$	
Total	SST	$n-1$		

where MS stands for **mean square** and is defined as

$$MS = \frac{SS}{df}$$

$$MSR = \frac{SSR}{1}$$

$$MSE = \frac{SSE}{n-2}$$

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6.1 OLS, continued.

R-square **coefficient of determination** is defined as

$$R^2 = \frac{SSR}{SST}$$

and notice that $R^2 \in [0, 1]$ and can be interpreted as **% of variation in** y **explained by** x **(via the linear model)**

Note in ECO220, we use R^2 or r^2 to represent the same thing.

6.2 Sample space, Event and Probability

Experiment an experiment is a process that creates two or more outcomes.

Random Experiment a random experiment is an experiment such that the outputs *cannot* be determined with certainty before the end of the experiment.

Sample Space a sample space is the set of all possible outcomes in a random experiment.

Event an event is a subset of a sample space.

Prob Let S be the sample space, let E be an event, then the **probability of** E, $P(E)$ is defined as

$$P(E) = \text{probability of } E = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } S}$$

assuming that each outcome in S has equal likelihood to be chosen into E.

6.3 Some Rules of Probability

Let E be an event in sample space S, then

- $P(E) \in [0, 1]$.
- $P(S) = 1$.
- Let E^c denote the **complementary** of E, then $P(E^c) = 1 - P(E)$.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Addition Rule)

7 Lecture 7 May. 29 2018

Mutually Exclusive Event If $A \cap B = \emptyset$, we say events A and B are **mutually exclusive/disjoint**. Then, if A, B are disjoint, we have

$$P(A \cup B) = P(A) + P(B)$$

7.1 Conditional Probability

Conditional Prob In general, if A and B are events in sample space S, the conditional probability of A given B is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication rule

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

7.2 Independent Event

Independent Event We say two events, A and B are **independent** if any of the following is true. (those definitions below are equivalent.)

- $P(A|B) = P(A)$ or
- $P(B|A) = P(B)$ or
- $P(A \cap B) = P(A)P(B)$

8 Lecture 8 May. 30 2018

8.1 Bayes Theorem

Let A and B be two events. Then,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Proven by definition of conditional probability.

8.2 Random Variable and Prob. Distributions

Prob. distribution

Cumulative Prob. distribution

8.3 Expected Values

Expected Value Let X be a random variable with probability distribution $P(X)$. Then defined the expected value of X, $\mathbb{E}(X)$ as

$$\mu = \mathbb{E}(X) = \sum_x xP(X = x)$$

Variance of Random Variable For random variable X, we have

$$\sigma^2 = Var(X) = \sum_x (x - \mu)^2 P(X = x) = \mathbb{E}(X - \mu)^2$$

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9.1 Expected Value of a Random Variable

Mean $\mu = \mathbb{E}(X) = \sum_x xP(X = x)$.

Variance $\sigma^2 = \mathbb{E}(x - \mu)^2 = \mathbb{E}(X^2) - \mu^2$.

9.2 Laws of Expectation

In general, let X be a random variable, and let $a, c \in \mathbb{R}$, then

$$\mathbb{E}(aX + c) = a\mathbb{E}(X) + c$$

$$Var(aX + c) = Var(aX) = a^2 Var(X)$$

Let X and Y be random variables, and let $a, b, c \in \mathbb{R}$, then

$$\mathbb{E}(aX + bY + c) = a\mathbb{E}(X) + b\mathbb{E}(Y) + c$$

$$Var(aX + bY + c) = Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab\ Cov(X, Y)$$

Note if X and Y are independent, then $\rho = Cov(X, Y) = 0$ and

$$Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y)$$

9.3 Binomial Distribution

In general, let n be the number of independent trails and $p = P(\#success)$. Let X be a random variable which is the number of successes in n trails, we have

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},\ \text{for } x = \{0, 1, 2, \dots, n\}$$

$$\mu = \mathbb{E}(X) = np$$

$$\sigma^2 = Var(X) = npq,\ q = 1 - p$$

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10.1 Uniform Distribution

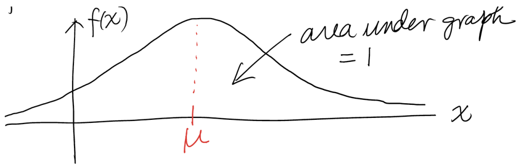
Let X be uniform from a to b. $f(x) = \frac{1}{b-a}, a \leq x \leq b$

$$\mu = \mathbb{E}(X) = \int_a^b xf(x)dx = \frac{a+b}{2}$$

$$\sigma^2 = Var(X) = \mathbb{E}(X^2) - \mu^2$$

10.2 Normal Distribution

Let X be a continuous random variable, satisfying $-\infty < x < \infty$. The mean of X is μ and the variance of X is σ^2. The graph of X is The graph is



symmetric at μ and the variance σ^2 determines the shape(spread) of X. We say X follows a normal distribution with mean μ and variance σ^2. And denote as

$$X \sim N(\mu, \sigma^2)$$

Standard Normal Distribution A standard normal distribution is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. Denote the standard normal distribution as

$$Z \sim N(0, 1)$$

11 Lecture 11 June. 12 2018

11.1 Applying normal distribution

Theorem Let $X \sim N(\mu, \sigma^2)$, then

$$z = \frac{x - \mu}{\sigma} \sim N(0, 1)$$

11.2 Normal Approximation to Binomial

Consider a random variable $X \sim B(n, p)$, then we can approximate the binomial with a normal distribution $X \approx N(np, npq)$.

12 Lecture 12 June. 13 2018

12.1 Sampling Distributions

Consider population with size N has p as percentage of *success* (qualified) and sample with size n with $\hat{p} = \frac{x}{n}$ as percentage of *success.*

p is a parameter which has a fixed value. In real life, the value of p is usually unknown. $\hat{p}$ is a sample statistic, which does not have fixed value (random variable, value of $\hat{p}$ vary from sample to sample). Also, μ and σ are parameters, which are fixed but usually unknown. $\overline{x}$ is a sample statistic, and is random.

Suppose we know p for population, then we can conclude about random variables from a random sample,

1. $\mathbb{E}(\hat{p}) = p$.
2. $Var(\hat{p}) = \frac{pq}{n},\ q = 1 - p$
3. When sample size n is large, the distribution of $\hat{p}$ is approximately normal (**Central Limit Theorem in proportion**) [1]

That's

$$\hat{p} \approx\sim N(p, \frac{pq}{n}),\ \text{when } n \text{ is large.}$$

Example Given $p_{success} = 0.3$ for the whole population and find the probability that at least 320 *success* found in a sample of size $n = 1000$. i.e. Let X denote the number of success in sample with $n = 1000$, find $P(X \geq 320)$.

Method 1 Use Central Limit Theorem, check $np = 300 \geq 10 \wedge nq = 700 \geq 10$, thus n is *large*. And approximate $\hat{p}$ of sample as

$$\hat{p} \sim N(p, \frac{pq}{n})$$

Soln.

$$P(X \geq 320) = P(\hat{p} \geq 0.32)$$
$$= P(\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \geq \frac{0.32 - 0.3}{\sqrt{\frac{0.3*0.7}{1000}}}) = P(z \geq \frac{0.02}{\sqrt{\frac{0.21}{1000}}})$$

Find z in z-table

■

[1] As a rule of thumb, n is considered to be large when $np \geq 10 \wedge nq \geq 10$.

Method 2 Use Normal Approximation to Binomial. $p = 0.3$ and $n = 1000$.

$$X \approx Y \sim N(300, 210)$$

Soln.

$$\begin{aligned} P(X \geq 320) &= P(Y > 319.5) \\ &= P(\frac{Y-\mu}{\sigma} > \frac{319.5 - 300}{\sqrt{210}}) \\ &= P(z > 1.35) \text{ find in z table} \end{aligned}$$

■

Note *methods 1 and 2 do* ***not*** *give exactly same answer, but the answers should be close.*

12.2 Sampling distribution of $\overline{X}$, the sample mean

1. $\mathbb{E}(\overline{X}) = \mu$.
2. $Var(\overline{X}) = \frac{\sigma^2}{n}$.
3. When n is large, the distribution of $\overline{X}$ is approximately normal. (**Central Limit Theorem in Mean**).
4. When population is normal, the distribution of $\overline{X}$ is exactly normal, regardless of the sample size n.

Putting together,

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n}), \text{ when } n \text{ is large.}$$

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13.1 Confidence Interval

To find $100(1-\alpha)\%$ confidence interval for p estimated from $\hat{p}$ is

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

13.2 Sample Size Required

When we specify the confidence level $1-\alpha$, and the margin of error, the required sample size is

$$n = \frac{z^2_{\frac{\alpha}{2}}}{(ME)^2} pq$$

If p can be estimated from previous surveys, use it to find n. Else, use $p = 0.5$ to find n.

14 Lecture 14 Jul. 3 2018

14.1 Confidence Interval for Population Proportion

Point estimator for p is $\hat{p}$, confidence interval for p is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}},\ \textit{with large } n$$

n is considered as *large* iff $np \geq 10 \wedge nq \geq 10$. $\sqrt{\frac{\hat{p}\hat{q}}{n}}$ is the **standard error/deviation** of estimation. And $z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ is the **margin of error**.

14.2 Two populations

p_1, p_2 denote the qualification percentages for population 1 and 2. And $\hat{p_1}, \hat{p_2}$ denote the qualification percentages in samples with sample sizes n_1, n_2 from population 1 and 2.

Point estimator To estimate p_1 to p_2, we estimate $p_1 - p_2$. The point estimator for $p_1 - p_2$ is $\hat{p_1} - \hat{p_2}$.

Interval estimator The interval estimation for $p_1 - p_2$ is

$$PointEstimator \pm z_{\alpha/2} \times Std(PointEstimator)$$

$$\hat{p_1} - \hat{p_2} \pm z_{\alpha/2} \times Std(\hat{p_1} - \hat{p_2})$$

To find $Std(\hat{p_1} - \hat{p_2})$, by *law of expectation*

$$Var(aX + bY) = Var(aX) + Var(bY) + 2abCov(X, Y)$$

We select two independent samples of size n_1 and n_2 from populations, therefore

$$V(\hat{p_1} - \hat{p_2}) = V(\hat{p_1}) + V(\hat{p_2})$$

When n_1 and n_2 are large, by *central limit theorem*,

$$\hat{p_1} \sim N(p_1, \frac{p_1 q_1}{n_1}), \quad \hat{p_2} \sim N(p_2, \frac{p_2 q_2}{n_2})$$

Then, for two *independent* samples, $\hat{p_1}$ and $\hat{p_2}$ are independent,

$$V(\hat{p_1} - \hat{p_2}) = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$$

But we do not know p_1 and p_2, we cannot calculate $V(\hat{p_1} - \hat{p_2})$ directly from above equation. We estimate p_1 and p_2 by $\hat{p_1}$ and $\hat{p_2}$
Therefore the **estimated variance** is

$$(Estimated)V(\hat{p_1} - \hat{p_2}) = \frac{\hat{p_1}\hat{q_1}}{n_1} + \frac{\hat{p_2}\hat{q_2}}{n_2}$$

$$(Estimated)Std(\hat{p_1} - \hat{p_2}) = \sqrt{\frac{\hat{p_1}\hat{q_1}}{n_1} + \frac{\hat{p_2}\hat{q_2}}{n_2}}$$

Result confidence interval for $\hat{p_1} - \hat{p_2}$

$$C.I._\alpha = \hat{p_1} - \hat{p_2} \pm z_{\alpha/2}\sqrt{\frac{\hat{p_1}\hat{q_1}}{n_1} + \frac{\hat{p_2}\hat{q_2}}{n_2}}$$

Example Compare the percentage of people going to casino in Ontario and Manitoba.

Var	Ontario	Manitoba
Actual	p_1	p_2
Sample size	$n_1 = 4151$	$n_2 = 389$
Point estimator	$\hat{p_1} = 66.5\%$	$\hat{p_2} = 75.2\%$

Point estimator for $p_1 - p_2$ is $\hat{p_1} - \hat{p_2} = -0.087$
the 95% C.I. for $p_1 - p_2$ is

$$-.0087 \pm (z_{.025} = 1.96) * \sqrt{\frac{.665 * .335}{4151} + \frac{.752 * .248}{389}} = (-.132, -.042)$$

Interpretation 95% of the times, $p_1 - p_2$ falls between -.132 and -.042. We are 95% confident that $p_1 - p_2$ is between -.132 and -.042.
Remark Is there a significant difference between the % going to casinos between Ontario and Manitoba? Since the 95% C.I. for $p_1 - p_2$ does not contain 0, we conclude that there **is** a significant difference between % in two population.
Remark You can also use estimate the $p_2 - p_1$ and the 95% C.I. would be $(.042, .132)$.

14.3 Chapter 12. Hypothesis Testing in Population Proportion

$$\text{Statistical Inference}\begin{cases}\text{Estimation}\begin{cases}\text{Point Estimation}\\ \text{Interval Estimation}\end{cases}\\ \text{Hypothesis Testing}\end{cases}$$

H_0: the **null hypothesis**. H_1: the **alternative hypothesis**.

Reality

$$\begin{cases}H_0 \text{ Person not murder}\begin{cases}Guilty \mid H_0 \ \textbf{Type I Error}\\ NotGuilty \mid H_0 \ \textbf{No error}\end{cases}\\ H_1 \text{ Person is murderer}\begin{cases}Guilty \mid H_1 \ \textbf{No error}\\ NotGuilty \mid H_1 \ \textbf{Type II Error}\end{cases}\end{cases}$$

When the court concludes "Guilty" and H_0 is true, type I error occurs and denote the probability of type I error as α

$$\alpha = P(\text{Reject } H_0|H_0)P(\text{Type I Error})$$

And in this case, Type II Error will not occur.

Let β denote the probability for type II error to occur.

$$\beta = P(\text{Reject } H_1/\text{Fail to reject } H_0|H_1) = P(\text{Type II Error})$$

Remark β becomes large as α becomes small.

Conclusion Therefore, in *hypothesis testing*, we have two hypotheses, H_0 and H_1. Based on sample results (evidence), we either *reject* H_0 or *do not reject* H_0

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15.1 Recall: Concepts of Hypothesis Testing

Concepts H_0 **null hypothesis** and H_1 **alternative hypothesis**

Case I Rejecting H_0 while H_0 is true. Failed to accept null hypothesis H_0.

$$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ True}) = \textbf{Significance Level}$$

In real life, we want the ***significance level*** *to be as low as possible.* And note that significance level is probability for type I error to occur when H_0 is true.

Case II Accepting H_0 with H_1 is true. Fail to accept alternative hypothesis. β is the probability of type II error while H_1 is true.

$$\beta = P(\text{Accept } H_0 \mid H_1 \text{ True})$$

Remark In hypothesis testing, we wish both α and β to be small. However, by setting the rule of decision making and lowering α, β goes higher.

15.2 Hypothesis Testing

Steps

1. Set up null and alternative hypotheses H_0 and H_1.
2. Setup a decision rule.
3. Test statistic.
4. Conclusion.

15.3 Hypothesis Testing in Population Proportion

Data Population proportion p. Select sample with sample size n and sample proportion $\hat{p}$.

H_0 null hypothesis on p and H_1 as the alternative hypothesis.

Example Government claims that more than 50% of Canadian are in favour of a policy.
Step 1 setup hypotheses $\underline{H_0 := p \leq 0.5 \text{ and } H_1 := p > 0.5}$
Step 2 setup a decision rule $\underline{\text{If } \hat{p} < c \text{ we accept } H_0. \text{ If } \hat{p} \geq c \text{ we reject } H_0.}$
Consider $c = 0.55$. Then in a random sample of $n = 200, \hat{p} = \%$ in favour from sample. Decision rule is

$$\begin{cases} \text{Accept } H_0 : p \leq 0.5 \text{ if } \hat{p} < 0.55 \\ \text{Accept } H_1 : p > 0.5 \text{ if } \hat{p} \geq 0.55 \end{cases}$$

Step 3 test statistic. In a sample of $n = 200, \hat{p} = 0.58$.
Step 4 Conclusion: Reject H_0.

15.3.1 Finding Critical Value c

Finding α

$$\alpha = P(\text{Type I Error}) = P(\hat{p} \geq 0.55 | p \leq 0.5)$$

By <u>central limit theorem</u>, when n is large,

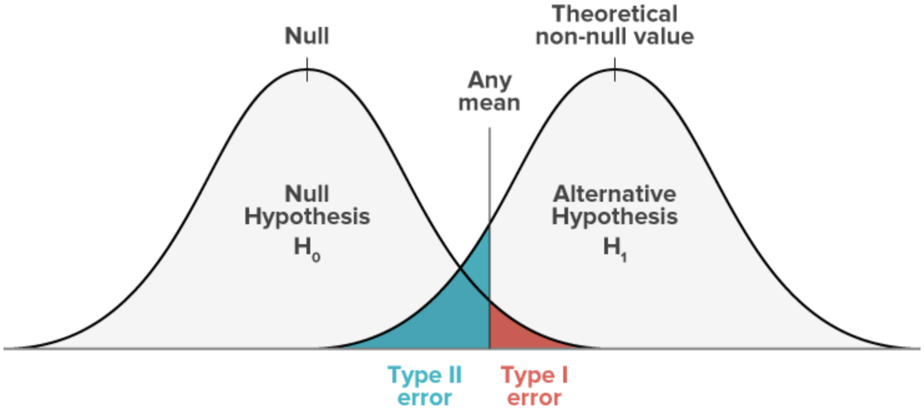
$$\hat{p} \sim N(np, \frac{pq}{n})$$

Then normalized data,

$$\alpha = p(z \geq \frac{0.05\sqrt{200}}{0.5}) = 0.0793 \approx 8\%$$

Therefore, by setting the critical value $c = 0.55$, the probability of type I error, $\alpha = 0.08$.
Finding β

$$\beta = P(\hat{p} < 0.55 | p > 0.5)$$


Null
Theoretical non-null value
Any mean
Null Hypothesis H_0
Alternative Hypothesis H_1
Type II error
Type I error