

Notes on MAT 137 study guide

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Contents

1	Part 1. Sets and functions	1
2	Part 2. Limits & Continuity	2

1 Part 1. Sets and functions

Functions

$$f : A \rightarrow B.$$

- **A: domain** of f .
- **B: codomain** of f .
- $f(A)$: **range** of f .
- **Onto/surjective** if and only if codomain = range.
- **One-to-one/injective** if and only if $f(x) = f(y) \implies x = y$.
- **Bijective** if and only if a function is both surjective and injective.
- **Even**: $f(x) = f(-x) \forall x \in A$.
- **Odd**: $f(x) = -f(-x) \forall x \in A$.
- **Increasing[Decreasing]**: $x < y \implies f(x) < f(y) [f(x) > f(y)]$.

Operations on functions

- $(f \pm g)(x) = f(x) \pm g(x)$.
- $(f \cdot g)(x) = f(x) \cdot g(x)$.
- $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$.

Composite of functions for functions

$$f : A \rightarrow B$$

$$g : C \rightarrow D$$

$f(g(x))$ is defined when $D \subset A$.

Exponential and logarithms for $x, y, n \in \mathbb{R}, x, y > 0$

- $b^{x+y} = b^x + b^y$.
- $b^{x-y} = \frac{b^x}{b^y}$.
- $\log_b(xy) = \log_b(x) + \log_b(y)$.
- $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$.
- $\log_b(x^n) = n \log_b(x)$.
- $\log_y(b) \log_b(x) = \log_y(x)$.

The supremum and infimum A set S of real numbers is said to be **bounded above**[below] if there is a $s \in \mathbb{R}$ such that $s \geq x$ [$s \leq x$] for all $s \in A$.

Definition A number is a **least upper bound**(supremum) of S if both:

- s is an upper bound of S .
- $\forall x$: upper bound of S , then $s \leq x$.

2 Part 2. Limits & Continuity

Definition We say that the limit of a function $f(x)$ as x approaches a is L ($\lim_{x \rightarrow a} f(x) = L$) if:

$$\forall \epsilon > 0, \exists \delta > 0, s.t. 0 < |x - a| \implies |f(x) - L| < \epsilon$$

$\lim_{x \rightarrow a} f(x) = L$ exists if and only if the left-hand and right-hand limits both exist and are equal.