ECO206 Microeconomic Theory
Second Degree Price Discrimination

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1 Setup

Consider two groups of customers with different marginal willingness to pay. Their demands are captured by demand function

$$D_i = f_i(p) \; i \in \{L, H\} \tag{1}$$

Assuming H group has higher MWTP at any quantity of x, that's

$$f_H^{-1}(x) > f_L^{-1}(x) \; \forall x \in \mathbb{R}_+ \tag{2}$$

Let function $C_L(x)$ and $C_H(x)$ denote the cost to provide goods/services to each group of customer.
Define welfare function for customers

$$u_i(x_i) = \int_0^{x_i} f_i^{-1}(x) \; dx \; i \in \{L, H\} \tag{3}$$

By equation (2), we have

$$u_H(x) > u_L(x) \; \forall x \in \mathbb{R}_+ \tag{4}$$

2 Mathematical Form

The profit optimization problem of producer can be formalized as

$$\max_{x_L, p_L, x_H, p_H} \{\pi(\cdot) = P_L - C_L(x_L) + P_H - C_H(x_H)\} \tag{5}$$

subject to individual rationality and intensive compatibility constraints

$$\begin{cases} u_L(x_L) - p_L \geq 0 \\ u_H(x_H) - p_H \geq 0 \\ u_L(x_L) - p_L \geq u_L(x_H) - p_H \\ u_H(x_H) - p_H \geq u_H(x_L) - p_L \end{cases} \tag{6}$$

3 Reduce the Constraint Region

The constraint can be reduced to

$$\begin{cases} u_L(x_L) - p_L = 0 \\ u_L(x_H) - p_H \leq 0 \\ u_H(x_H) - p_H = u_H(x_L) - p_L \end{cases} \tag{7}$$

4 Solving the Problem

We want to write $\pi(\cdot)$ as a function of x_L and solve the optimal x_L.

4.1 Step 1: Directly solve x_H^*

We wish to sell <u>efficient amount</u> to the high types, that's where MC(supply) equals demand. Find x_H^* by solving

$$\left.\frac{\partial C_H(x_H)}{\partial x_H}\right|_{x_H = x_H^*} = f_H^{-1}(x_H^*) \tag{8}$$

4.2 Step 2: Rewrite P_L

Using equation (7.1), we can map x_L to p_L by

$$p_L(x_L) = u_L(x_L) = \int_0^{x_L} f_L^{-1}(x)\ dx \tag{9}$$

4.3 Step 3: Rewrite p_H

Using equation (7.3) to map x_L to p_H as

$$p_H(x_L) = \overline{u_H(x_H^*)} - \int_0^{x_L} f_H^{-1}(x)\ dx + p_L(x_L) \tag{10}$$

4.4 Step 4: Solving Problem

By equation (7.2)

$$p_H \geq u_L(x_H^*) \iff x_L \leq x_H^* \tag{11}$$

Proof.

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\begin{aligned}
7.2 &\implies u_L(x_H) \leq p_H \\
\text{(with 7.3)} &\implies u_L(x_H) \leq u_H(x_H) - u_H(x_L) + p_L \\
&\implies u_L(x_H) \leq u_H(x_H) - u_H(x_L) + u_L(x_L) \\
&\implies u_L(x_H) - u_L(x_L) \leq u_H(x_H) - u_H(x_L) \\
&\implies \int_{x_L}^{x_H} f_L^{-1}(x)\ dx \leq \int_{x_L}^{x_H} f_H^{-1}(x)\ dx \\
&\text{as } 0 < f_L^{-1}(x) < f_H^{-1}(x)\ \forall x \in \mathbb{R}_+ \\
&\iff x_L \leq x_H
\end{aligned}
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Rewrite the objective function as

$$\pi(x_L) = P_L(x_L) + P_H(x_L) - C_L(x_L) - \overline{C_H(x_H^*)} \tag{12}$$

and maximize $\pi(x_L)$ by solving $\frac{\partial \pi(x_L)}{\partial x_L} = 0$ and $x_L \in [0, x_H^*)$.