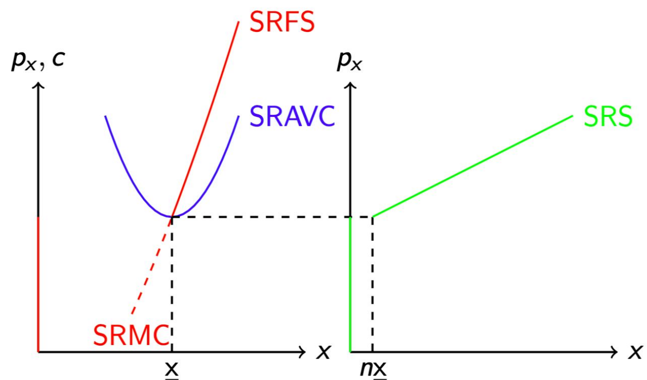
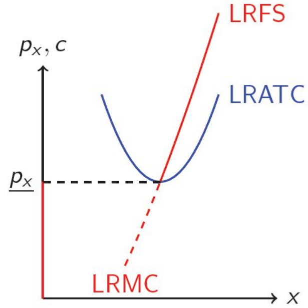
Lecture 13 Perfect Competition

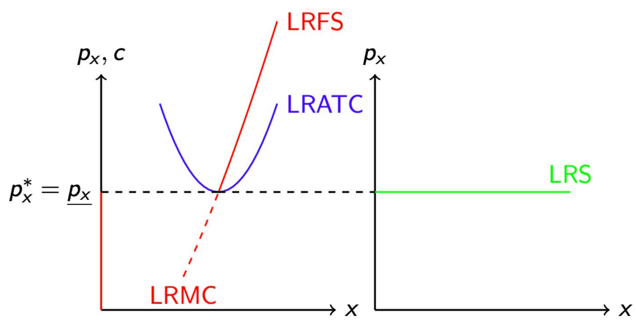
1. Characteristics of Perfect Competition.
    a. Price-taker
    b. Perfect information
2. Difference between SR and LR.
    a. Short Run
        i. Number of producers is exogenous.
        ii. Market supply: summation of individual firm supply (from firm's profit optimization)
        iii. Some input factors are fixed. (Fixed cost incurred)
        iv. Finitely many(n) producers in the market.
        v. Infinitely potential producers outside the market.
    b. Long Run
        i. Number of producers is endogenous.
        ii. Market supply is determined by zero-profit condition for the marginal producer (The last producer joint the market earn zero profit.)
            1. (*See below*) If all firms are identical then the long run market supply is perfectly elastic at $\underline{p}$
3. Find short run firm supply.
    a. Given production function $x_i = f(L, \overline{K})$
    b. Check if $p \geq min\{AVC\}$ to decide produce or not.
    c. Producer maximize profit $\pi = x_i * p - w * l - F$ (Producers are price-takers for p, l, F) and find the optimal quantity to produce.
    d. FOC gives $MC(x) = MR(x) = p$
4. Find short-run market supply.
    a. Summation of short run supplies from individual firm.



5. Find long-run firm supply.
    a. Fixed cost $F = 0$ in the long run
    b. Firm maximize $\pi = x_i(p_x - ATC(x_i))$
    c. Firm's supply curve is the portion of LRMC no less than $min\{LRATC\}$



6. Find long-run market supply.
    a. Long run market price $p^* = min\{LRATC\}$ (endogenous price)
        i. Proven by contradiction.
    b. (Assuming firms are identical)LRS perfect elastic at $p^* = min\{LRATC\}$
        i. If $p > \underline{p}$ then infinitely many producers enter the market.
        ii. If $p < \underline{p}$ then all producers exit the market.
        iii. If firms are not identical, LRS might be upward sloping: as price increases, producers with higher $min\{LRATC\}$ enter the market.



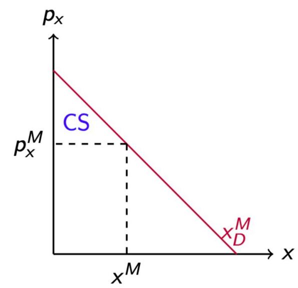
7. Find short-run equilibrium price and quantity.
    a. Solving $x_s(p) = x_d(p)$
8. Find long-run equilibrium price and quantity.
    a. LR equilibrium price is determined by $min\{LRATC\}$
    b. (If all firms identical) Quantity found by $x_d(\underline{p})$
9. Find number of producers.
    a. Short-run: number of producers is fixed.
    b. Long-run:
        i. Find x^M using $p_D(x) = min\{LRAC\} = \underline{p}$
        ii. Find individual firm's supply $x_f = argmin_x\{LRATC\}$
        iii. Number of firms $n = \frac{x_M}{x_f}$

Lecture 14 Welfare and Distortions

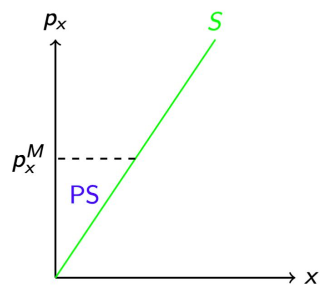
1. Measuring consumer surplus
    a. Use compensated demand to capture MWTP.
    b. Aggregation of MWTP is total utility gained.
    c. Minus the cost of consumption $p_x * x$

d. Individual $CS_i = \int_{p_x}^{\infty} x_i(p)\, dp$

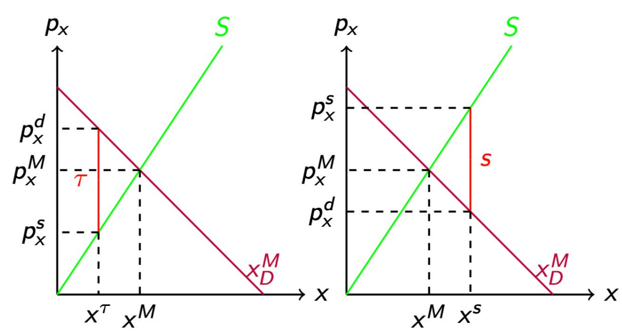
e. Aggregate $CS \ = \ \sum_{i=1}^{N} CS_i = \sum_{i=1}^{N} \int_{p_x}^{\infty} x_i(p)\, dp \ = \ \int_{p_x}^{\infty} \sum_{i=1}^{N} x_i(p)\, dp = \int_{p_x}^{\infty} x_D{}^M(p)\, dp$



2. Measuring producer surplus
   a. $PS = \int_{0}^{p_x} x_s{}^M(p)\, dp$



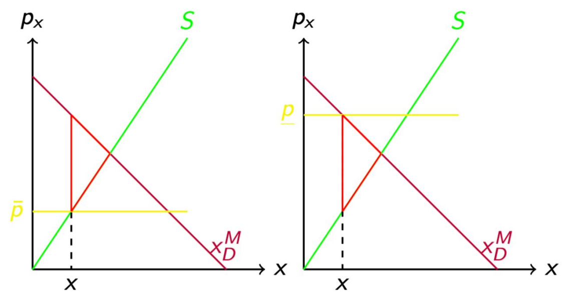
3. Measuring welfare
   a. CS + PS + Government Revenue (if applicable and could be negative)
4. Per-unit taxes and subsidies.



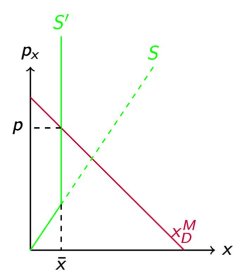
   a. (Generally, also applicable for percentage tax/sub) Market price (after) is the price faced by the party unaffected.
      i. Statutory incidence on producer ⇒ Market price is the price paid by consumers.
      ii. Statutory incidence on consumer ⇒ Market price is the price received by producers.
   b. Expressions
      i. Tax on producer $p^d \ = \ p_x,\ p^s = p^d - \tau$
      ii. Tax on consumer $p^s = p_x,\ p^d = p^s + \tau$
      iii. Subsidy on producer $p^d = p_x,\ p^s = p^d + s$
      iv. Subsidy on consumer $p^s = p_x,\ p_d = p_s - s$
5. Percentage tax/subsidy.

a. Expressions
   i. Tax on producer $p^d = p_x,\ p^s = (1-t)p^d$
   ii. Tax on consumer $p^s = p_x,\ p^d = (1+t)p^s$
   iii. Subsidy on producer $p^d = p_x,\ p^s = (1+s)p^d$
   iv. Subsidy on consumer $p^s = p_x,\ p^d = (1-s)p^s$

6. Price manipulation and quantity manipulation.
   a. Price ceiling and price floor.
      i. Notice, at a given price p, the actual quantity sold is $min\{S(p),\ D(p)\}$
      ii. With price ceiling, only most efficient producers produce.
      iii. With price floor, only consumers willing to pay the most purchase.



   b. Quantity manipulation. (quota)



7. Statutory and Economic incidence
   a. Statutory incidence does not affect the new equilibrium quantity.
   b. Economic incidence depends on relative elasticities $\varepsilon,\ \eta$ of consumers and producers.

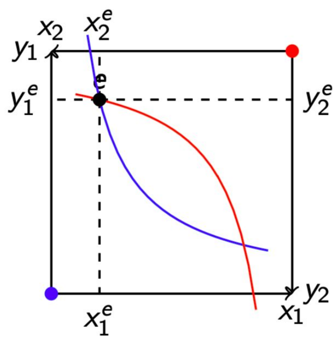
8. Compute economic incidence
   a. Calculate by definitions (*Normally calculated by definition*): comparing price before and after.
      i. EI of tax on consumer $\frac{p^d{}_x - p^M{}_x}{\tau}$
      ii. EI of tax on producer $\frac{p^s{}_x - p^M{}_x}{\tau}$
      iii. EI of subsidy on consumer $\frac{p^d{}_x - p^M{}_x}{s}$
      iv. EI of subsidy on producer $\frac{p^M{}_x - p^s{}_x}{s}$
   b. Calculate with linear approximation. (Absolute proportion)
      i. On producer $\frac{-\varepsilon}{-\varepsilon+\eta}$
      ii. On consumer $\frac{\eta}{-\varepsilon+\eta}$

Lecture 15 General Equilibrium

1. Assumptions on individual.
   a. Individuals maximize their utility given constraints (e.g. budget, non-negativity)

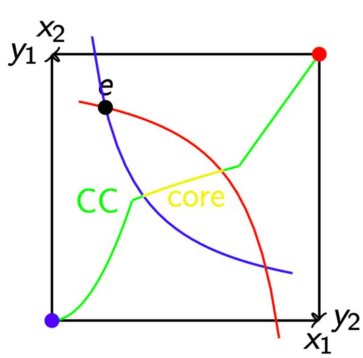
b. Convexity.
c. Monotonicity.
d. No externality.

2. Edgeworth Box
    a. Represents <u>feasible</u> (i.e. new allocations of individuals sum up to the aggregate initial endowment) <u>allocation</u> of all goods among all individuals.
    b. Let there are n goods to trade between N agents, then the Edgeworth Box is a set of N vectors from R^n. Therefore, the Edgeworth box is a subset $n*N$ dimensional space.
    c. One point in the box represents a <u>feasible allocation/distribution of goods among individuals</u>.



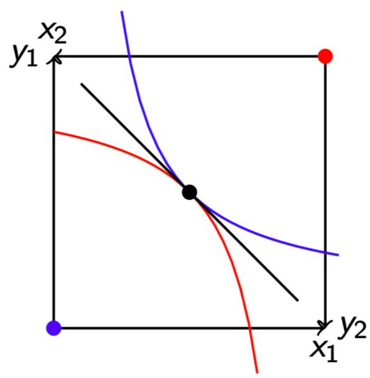
3. Contract curve (See attached images for examples)
    a. **Pareto Efficiency**: an allocation is Pareto-Efficient if it is impossible to make someone better off without making someone worse off. (Impossible to make <u>Pareto-improvement</u>)
    b. **Contract Curve**: the contract curve is the set of <u>all</u> pareto-efficient allocations.
    c. Find contract curve.
        i. <u>In general (Always consider corner solution, i.e. on the edge of box)</u> it is where indifference curves are tangent.
        ii. Find expression $MRS_1(x_1, y_1)$, $MRS_2(x_2, y_2)$
        iii. Use <u>aggregate</u> initial endowment to substitute $x_2 = e_x - x_1$, $y_2 = e_y - y_1$
        iv. Solve equation $MRS_1(x_1, y_1) = MRS_2(e_x - x_1, e_y - y_1)$ for y_1
        v. Solution $y_1(x_1)$ plots the contract curve.

4. Core
    a. **Core** the core is the set of points which are Pareto-efficient and are mutually beneficial for each individual to trade. (Mutually beneficial portion of contract curve)



5. Competitive Equilibrium

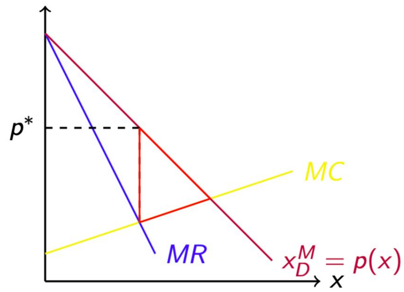
a. **Competitive Equilibrium**: A Competitive Equilibrium is an allocation and a set of prices such that all individuals, given prices, choose to consume their allocation (allocation satisfies the solution of their constrained optimization problem).
b. We have to determine the (relative) price of the trade (competitive equilibrium price).
c. Compute: budget line must be tangent to IC of each individual and ICs of each individual are tangent at Pareto efficiencies.
    i. (notes: for corner cases, the tangency equation might not hold.)
    ii. Let p be the placeholder for relative price (normalize the price of one produce to 1, as the numeraire).
    iii. (Optimization) Solve demand for each good for each individual as function of p and initial endowments as their endogenous income (no exogenous income).
    iv. Substitute $x_2(p) = e_x - x_1(p)$ and $y_2(p) = e_y - y_1(p)$
    v. Solve $MRS_1(x_1(p),\ x_2(p)) = MRS_2(e_x - x_1(p),\ e_y - y_1(p))$ for p^* as the competitive equilibrium price.



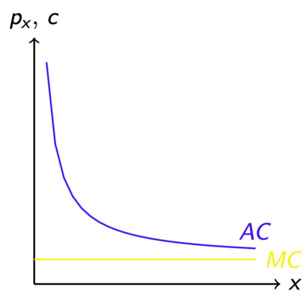
6. Welfare Theorems
    a. First welfare theorem(CE ⇒ PE): Given our assumptions, the allocation of a competitive equilibrium is Pareto-efficient.
    b. Second welfare theorem(PE⇒ CE): Given our assumptions, any Pareto-efficient allocation can be implemented as a competitive equilibrium with an appropriate choice of initial allocation.
7. Walras' Law
    a. With n markets in total, if $n-1$ markets are clear, the last market also clears.
    b. Only have to consider $n-1$ markets in our analysis and the other market left is automatically clear.

Lecture 16 Monopoly and Monopsony

1. Monopoly



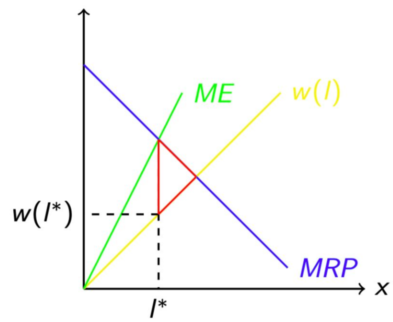
a. Possible causes of monopoly
   i. Barriers to entry
   ii. Patents
   iii. Ownership of scarce resource
   iv. Technology
   v. Natural Monopoly (large minimum efficient scale)



2. Monopoly's optimization problem
   a. Monopoly faces market demand $x(p)$
   b. $max_x\ x * p(x) - c(x)$
   c. Or, equivalently $max_p\ x(p) * p - c(x(p))$
   d. Check if $\pi > 0$ otherwise, exist the market.
3. Marginal revenue below demand
   a. Without implementing price discrimination, have to lower price for all consumers to get one additional consumer.
4. Monopoly and distortions (Adding to price or subtracting from per-unit cost are equivalent)
   a. With per-unit tax τ on product.
      i. Optimization $max_x\ x * (p(x) - \tau) - c(x)$
   b. With per-unit subsidy s
      i. Optimization $max_x\ x * (p(x) + s) - c(x)$
5. Monopsony
   a. Definition: they have market power in their <u>input markets</u>. (Output market might be perfectly competitive or not)
   b. (No input price discrimination) all inputs have to be paid same price.
6. Monopsony optimization problem
   a. Optimization: $max_l\ p * x(l) - w(l) * l$ (assuming monopsony only in input market)

b. FOC Marginal Revenue Product (of labor) = Marginal Expenditure. $MRP = ME$

c. Assert $\pi(\cdot) \geq 0$



7. Price distortion
   a. With per-worker subsidy s to monopsony. $max_x\ p * x(l) - l * (w(l) - s)$
   b. With per-worker tax τ. $max_x\ p * x(l) - l * (w(l) + \tau)$

Lecture 17 Price Discrimination

1. Different types of price discrimination

Type	Market Segments	Required Information	Implementation
First Degree	Each individual	i) Identifiable individual ii) know Individual demand/MWTP	Charge individual exactly their MWTP
Second Degree	Each individual	i) Know demand for each type	Customer self-selection Monopoly selects (Quantity, Price) bundles
Third Degree	Each groups	i) Identifiable group ii) Know demand for each group	Charge different per-unit price for each group
Two-Part Tariff	Each individual	i) Know demand for each type	Charge per-unit price and a fixed fee. Sell to high type only or sell to both types

2. First Degree Price Discrimination
   a. Requirements
      i. Know Demand/MWTP for each individual.
      ii. Could identify consumers.
      iii. Prevent arbitrage.

b. Implementation
    i. Produce at the efficient quantity by solving $MC(x) = MR(x) = p$
    ii. Then charge each individual at their MWTP.
    iii. Monopoly captures all the consumer surplus.
    iv. $\pi = CS + PS = Social\ Welfare$
c. Alternative implementation
    i. Sell the efficient quantity x^*.
    ii. Charge each individual the same price using demand $p^* = P_D(x^*)$
    iii. Charge each individual different cost for the right of purchasing $F_i = CS_i$

3. Second Degree Price Discrimination
    a. Requirements
        i. Known demands for different types.
        ii. Prevent arbitrage
    b. No need to identify different types ⇒ Customer self-selection process.
    c. Optimization problem
        i. Objective $max_{\{x_L, x_H, p_L, p_H\}} \pi = p_L + p_H - C_L(x_L) - C_H(x_H)$
        ii. Constraints (Original form)
            1. Individual Rationality
                a. $u_H(x_H) - p_H \geq 0$
                b. $u_L(x_L) - p_L \geq 0$
            2. Incentive Compatibility
                a. $u_H(x_H) - p_H \geq u_H(x_L) - p_L$
                b. $u_L(x_L) - p_L \geq u_L(x_H) - p_H$
        iii. Constraints (Reduced form)
            1. $u_L(x_L) = p_L$ ⇐ Low type is indifferent between buying or not.
            2. $p_H = u_H(x_H) - u_H(x_L) + p_L$ ⇐ High type is indifferent between buying two bundles.
            3. $x_L < x_H$
        iv. Problem solving procedure
            1. Find x_H by solving $MC_H(x_H) = MR_H(x_H)$ ⇐ Efficient quantity for high type.
            2. Find expression of $p_L(x_L) = u_L(x_L)$
            3. Find expression of $p_H(x_L) = u_H(x_H) - u_H(x_L) - u_L(x_L)$
            4. Write objective function $\pi(x_L) = \theta_L * p_L(x_L) + \theta_H * p_H(x_L) - \theta_L * C_L(x_L) - \theta_H * C_H(x_H)$
            5. $max_{x_L} \pi(x_L)$ and find $x_L = argmax_x \pi$

4. Third Degree Price Discrimination
    a. Requirements
        i. Identifiable groups, Known group demands.
        ii. Prevent arbitrage.
    b. Optimization problem: $max_{x_A, x_B}\ p_A(x_A) * x_A + p_B(x_B) * x_B - C(x_A + x_B)$
    c. Consider corner solutions. (sell to one group only)
    d. Third degree price discrimination do weakly better than single price monopoly.

5. Two-part tariff (As alternative to second degree price discrimination)

a. Requirements
    i. Known demand for each type.
    ii. Not identifiable (requires self-selection).
    iii. Need to prevent arbitrage.
b. Operations: same unit price p + fixed fee F .
c. Options: comparing the profit from two options.
    i. Sell to both types.
        1. Set $F(p) = \int_0^{x_L(p)} p_L{}^D(x)\,dx - p * x_L(p)$ as the consumer surplus for **low type**.

    Let $\theta_L \in (0,1)$ denote the percentage of low type customer.
        2. $max_p\ \theta_L * \pi_L(p) + (1-\theta_L) * \pi_H(p)$
        3. $max_x\ \theta_L * x_L(p) * p + (1-\theta_L) * x_H(p) * p + 1 * F(p) - C(\theta_L * x_L(p) + (1-\theta_L) * x_H(p))$
    ii. Sell to high type only.
        1. Set p and $x_H(p)$ such that $MC(p) = MR_H(p)$ the efficient (maximize consumer surplus for high type) quantity and price.
        2. Charge F as the amount of individual consumer surplus.
        3. $\pi = x * p + CS$

6. Firm's choice if arbitrage was a concern.
7. For more than two market segments.

Lecture 18 Game Theory

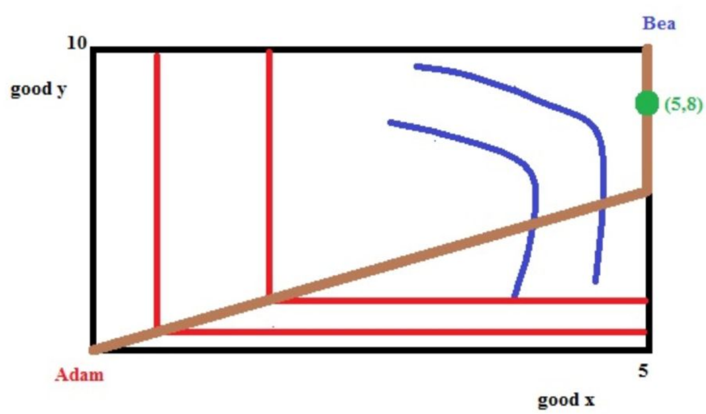
1. Normal form game.
    a. Players $i \in \{1, ..., N\}$
    b. Strategy picked $s_i \in S_i$ from individual's strategy set S_i
    c. Strategy profiles: collection of individuals' strategies. $\{s_1, ..., s_N\}$
    d. Payoff functions: transformation strategy profile $\to R^N$.
    e. Competitor's strategies s_{-i}
    f. Assumptions
        i. Complete information: common knowledge on payoff function.
        ii. Pure strategies: no distribution among strategies.
2. Strictly dominated/dominates
    a. $u(s_i, s_{-i}) > u(s_i', s_{-i})\ \forall s_{-i}$
        i. then s_i strictly dominates s_i'
        ii. and s_i' is strictly dominated by s_i
    b. $u(s_i, s_{-i}) \geq u(s_i', s_{-i})\ \forall s_{-i}$
        i. then s_i weakly dominates s_i'
        ii. and s_i' is weakly dominated by s_i
3. Best Response

a. The best response for one individual corresponding to competitor's strategy s_{-i} is a set of strategies(could be multiple) such that any s_i in this set satisfies $u(s_i, s_{-i}) \geq u(s_i', s_{-i})\ \forall s_i'$. (maximizing payoffs)

4. Nash Equilibrium
   a. Definition: a Nash equilibrium is a strategy profile such that each player's strategy is one of their best responses to their competitor's strategy.
5. Finding Nash Equilibrium
   a. Method 1: find overlapping of best responses.
   b. Method 2: iterative elimination of strictly dominated strategies.
6. Relations.
   a. Strictly dominated strategies are never chosen in a NE.
   b. NE must be constructed from best responses.

Past Test Questions.

1. [Fall 2015 2(c)] Difference between MR for perfectly competitive firm and monopolist.
   a. PC ⇒ Horizontal demand. (Price taker) ⇒ Horizontal MR
   b. Monopolist
      i. Downward sloping (To sell extra unit, need to lower the price).
      ii. Lies below the demand curve (Have to reduce price for all previous units).
2. [Fall 2016 1(b)] Explaining Nash Equilibrium
   a. By definition, no incentives to change strategy.
   b. Consider scenario and show that neither agent has incentives to deviate.
   c. Remember to state any possible strictly/weakly dominated/dominant strategies.
3. [Fall 2016 4(b)]



4. Competitive equilibrium and competitive equilibrium questions (G.E.)

    a. Consider the supply and demand side of each good. (Comparing the equilibrium consumption and their endowment.)
        i. (Total) Consuming more than endowed ⇒ Demand
        ii. Consuming less than endowed ⇒ Supply
    b. (After changes) Adjust relative price consider the change in relative demand and supply
        i. Excess demand ⇒ relative price increases.
        ii. Excess supply ⇒ relative price falls
5. [Fall 2017 2(b)] (Game Theory) Communication without changing on the payoff matrix will **not** affect the resulting equilibrium(NE). (Since players still have incentives to cheat/deviate after communication)

Contract Curve with different preferences.

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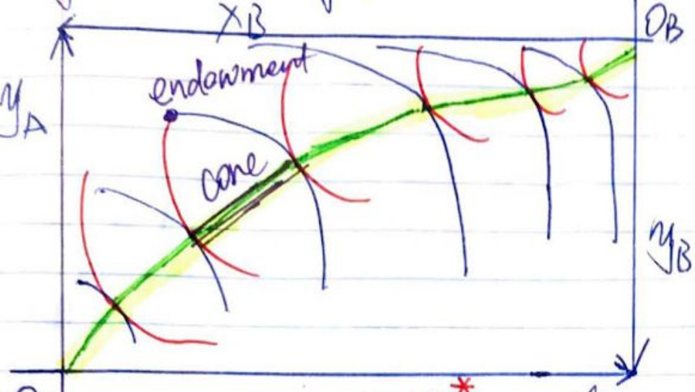
Contract-curves over different ~~utility~~ preferences

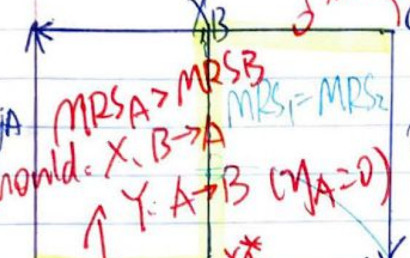
① (Normal)
Cobb Douglas - Cobb-Douglas

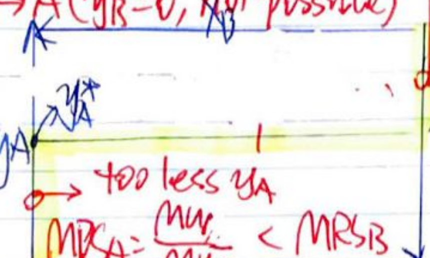


② Quasilinear - Quasilinear

$X_A > X_A^*$
$MRS_A < MRS_B$
should $\begin{cases} X: A \to B \\ y: B \to A \ (y_B = 0, \text{ not possible}) \end{cases}$

too less y_B
MU_y too large
$MRS_A > MRS_B$
should $(X_B = 0)$ $\begin{cases} X: B \to A \\ y: A \to B \end{cases}$





$\ln(y) + X$
y_B (w.r.t. X)

$\ln(x) + y$
form
(quasilinear
w.r.t. y)

O_A should $\begin{cases} X: A \to B \ (X_A = 0) \\ y: B \to A \end{cases}$

$X_1 = \frac{ad\, e_x}{ad + cb}$

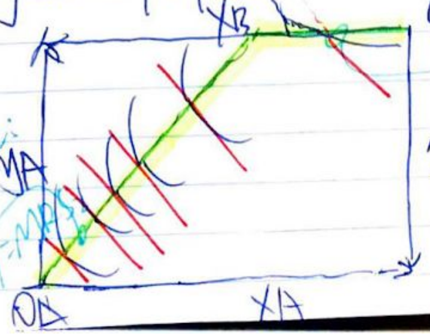
e.g. $u = \ln(x) + y$
constant MU_y
decreasing MU_x
decreasing $MRS_i = \frac{MU_x}{MU_y}$

for interior soln: $u_1 = a\ln(x_1) + by_1$, $u_2 = c\ln(x_2) + dy_2$

$MRS_1 = \frac{a}{bx_1}$ $MRS_2 = \frac{c}{dx_2} = \frac{c}{d(e_x - x_1)}$

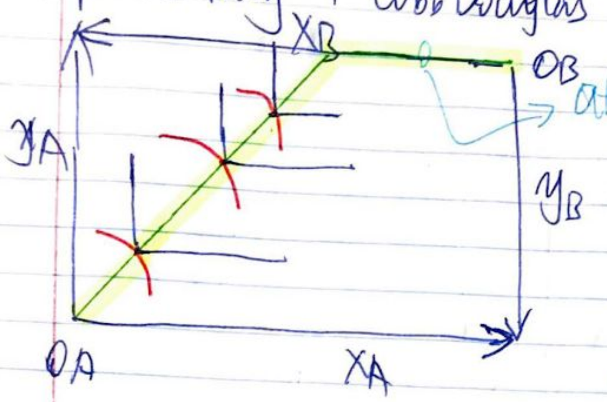
solve $ad(e_x - x_1) = cbx_1$
$ad\, e_x = (ad + cb)x_1$
$x_1^* = \frac{ad\, e_x}{ad + cb}$

③ Cobb Douglas + perfect substitute



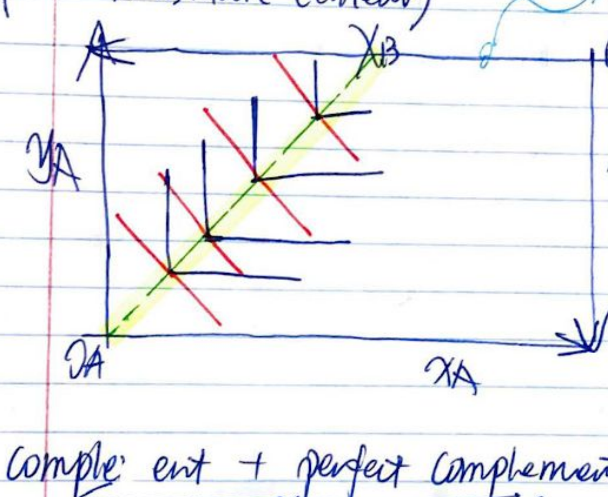
$|MRS_1| \downarrow$ as $X_A \uparrow$
→ at this portion $|MRS_1| < |MRS_2|$
Should transfer $X: A \to B$ for pareto improvement $y: B \to A$
But $y_B = 0$
therefore no pareto improvement!

④ Complementary + Cobb Douglas



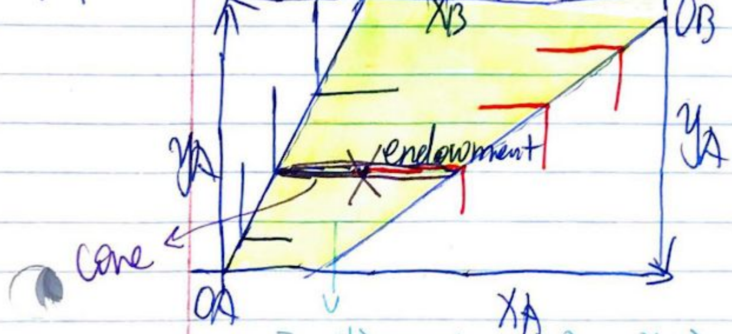
at this portion
for A: X_A redundant
so $U_A = a \cdot X_A$
and the only way to improve U_B is $Y: A \to B$
since $MU^B_X = 0$
cannot transfer without hurting A.
$\Rightarrow$ Pareto efficient.

⑤ perfect complement
\+ perfect substitute (linear)



at this portion:
for A: X_A is redunant
and Y_A is the sole determinant factor for U_A:
could transfer $X: A \to B$
to increase U_B without changing U_A.

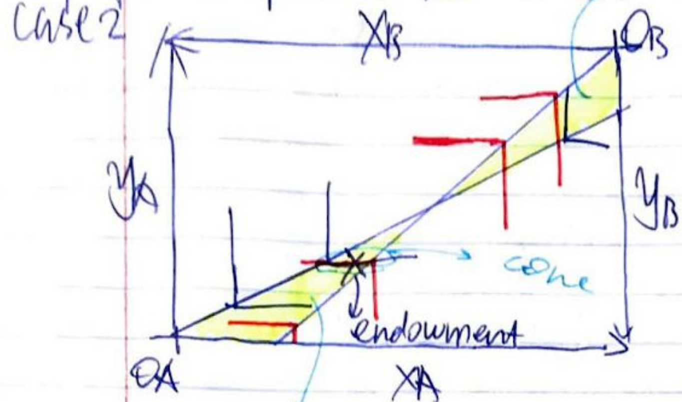
⑥ perfect complement + perfect complement



in this region A: X_A is redunant B: X_B is redunant
Both agents have to transfer y so their utilitives change
could not make pareto improvement

perfect complement X2

case 2



Both agents y redundant.

only exchange X.

in this region: both agents X redundant.
only exchanging y affects
their utilities.
No pareto improvement

at this portion
$MRS_A > MRS_B$
should $\begin{cases} X: & B \to A \ (X_B = 0) \\ Y: & A \to B \end{cases}$

Quasilinear
Additional.

A: quasilinear w.r.t. y: $u_A = a\ln(x_A) + by_A$

B: quasilinear w.r.t. x: $u_B = cx_B + d\ln(y_B)$

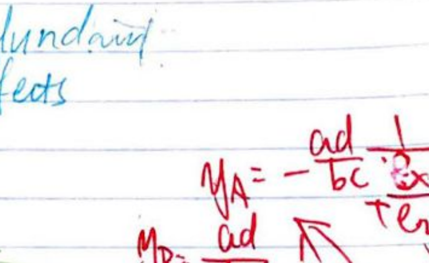
$MRS_A = \frac{a}{bx_A}$ $MRS_B = \frac{cy_B}{d} = \frac{c(\bar{e}_y - y_A)}{d}$

solve. $\frac{a}{bx_A} = \frac{c(\bar{e}_y - y_A)}{d} \Rightarrow ad = bx_A c\bar{e}_y - bx_A c y_A$

$\Rightarrow b.c\, x_A y_A = bc\bar{e}_y x_A - ad$

$\Rightarrow x_A y_A = \bar{e}_y x_A - \frac{ad}{bc}$

$\Rightarrow y_A = -\frac{ad}{bc}\frac{1}{x_A} + \bar{e}_y$



$x_A = \frac{ad}{bc \cdot \bar{e}_y}$

at this portion:
$MRS_A > MRS_B$
should $\begin{cases} X: & B \to A \\ Y: & A \to B \ (y_A = 0) \end{cases}$