CSC165 Lecture notes

Tianyu Du

January 15, 2018

Info.
Created: January 15, 2018.
Last modified: January 15, 2018

Contents

1 Lecture 2 Jan.15 2018

1.1 Predicate Logic

Allow you to have a domain of objects that you want to talk about, we want to be express reason about this domain.

Predicate Simplest kind of Predicate Logical Formula is called a **predicate**. A predicate is a function with range $\{0, 1\}$.
Examples

1. *Less-than-or-equal-to:* $\leq : \mathbb{Z} \times \mathbb{Z} \to \{0, 1\}$
2. *Equality:* $=: \mathbb{Z} \times \mathbb{Z} \to \{0, 1\}$
3. *Prime:* $Prime : \mathbb{N} \to \{0, 1\}$
4. Define: $R : \{a, b\} \times \{1, 2, 3\} \to \{0, 1\}$ as $R(a, 1) = R(b, 1) = R(c, 1) = 1, R = 0$ elsewise.

When you specify/define a predicate you have to specify the domain.

Quantifiers Introduce two quantifiers, **exists**: $\exists$ and **for all**: $\forall$, that let us express,

$\exists x \in \mathbb{DOMAIN}$: There is at least one element in domain of predicate that is true.

equivalently, represent as $\vee$,

$$"\exists" \equiv p(x_0) \vee p(x_1) \vee p(x_2) \dots$$

$\forall x \in \mathbb{DOMAIN}$: All element in domain of predicate satisfy the predicate.

equivalently, represented as $\wedge$,

$$"\forall" \equiv p(x_0) \wedge p(x_1) \wedge p(x_2) \dots$$

Negation of quantifier statements

$$\neg(\exists x \in \mathbb{D},\ s.t.\ p(x)) \equiv \forall x \in \mathbb{D}, \neg p(x)$$

$$\neg(\forall x \in \mathbb{D},\ s.t.\ p(x)) \equiv \exists x \in \mathbb{D}, \neg p(x)$$

Nested quantifier more than one variable quantified.
Example

For every natural number x, if x is a power of 2 then $2x$ is a power of 2.

$$\forall x \in \mathbb{N}, (\exists k \in \mathbb{N}\ s.t.\ x = 2^k) \implies (\exists k' \in \mathbb{N}\ s.t.\ 2^{k'} = 2x)$$