ECO206 Microeconomic Theory
Lecture Notes

Tianyu Du

May 17, 2018

Contents

1 Lecture 1 May. 8 2018 **2**

1.1 Budget Constraint 2

1.1.1 Types of Income 2

1.1.2 Exogenous Income 2

1.1.3 Endogenous Income 2

1.2 Opportunity Cost 3

1.3 Changes that affect the budget constraint 3

1.3.1 Pure income change 3

1.3.2 Price change 4

1.3.3 Endogenous income price change 4

2 Lecture 2 May. 9 2018 **4**

2.1 Tastes as Binary Relations 5

2.2 Rationality Assumptions on Preference Relation 5

2.3 Convenience Assumptions 6

2.4 Indifference Curve 6

2.5 Utility Function 6

3 Lecture 3 May. 15 2018 **7**

3.1 Different Types of Tastes 7

3.1.1 Shape and Substitutability Along IC 7

3.1.2 Diminishing MRS 7

4 Tutorial 2 May. 17 2018 **7**

4.1 Different types of utility functions 7

1 Lecture 1 May. 8 2018

1.1 Budget Constraint

- Exogenous income
- Endogenous income:

Bundle Combination of goods. If we have n goods, then x_1^A represents a quantity (x) of good 1 in bundle A.

$$A = (x_1^A, x_2^A, \ldots, x_n^A)$$

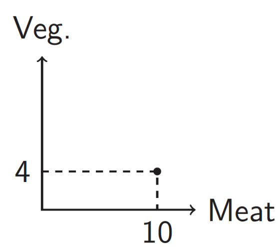


Figure 1: Consumption bundle

1.1.1 Types of Income

Exogenous income Cash(i.e. \$) in your pocket to spend.

Endogenous income Bundle fo goods you can sell to get money. e.g. *Assets*, *Skills*, *Time*, etc.

1.1.2 Exogenous Income

Consumer walk into market with a fixed amount of **cash**, budget constraint.

$$\vec{x} \cdot \vec{p} \leq I$$

1.1.3 Endogenous Income

Framework Consumer walks into a market **without cash**, but with **endowment** (ω_M, ω_V). And consumer can sell the endowment at market prices, the value of the endowment is

$$p_M \omega_M + p_V \omega_V$$

Hypothetical income Income/Cash from selling the *entire* bundle endowed.

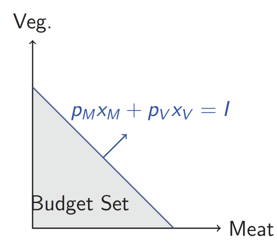


Figure 2: Budget constraint

Budget Constraint equation

$$p_M x_M + p_V x_V \leq I_{hypothetical} = p_M \omega_M + p_V \omega_V$$

Intercepts: if spend all income on one good.

- x-axis(meat) $= \frac{p_M \omega_M + p_V \omega_V}{p_M} = \omega_M + \frac{p_V}{p_M} \omega_V$
- y-axis(veg) $= \frac{p_M \omega_M + p_V \omega_V}{p_V} = \omega_V + \frac{p_M}{p_V} \omega_M$

Assumption consumers are price takers.

Affordable means *spending* $\leq$ *income* and $\vec{x} \in \mathbb{R}^n_+$

1.2 Opportunity Cost

OC/MRT Rate at which one good can be traded for another though the market, expressed in units of a good.

To get another unit of good 1 home many unit of good 2 do I need to give up?

$$\frac{dy}{dx} = -\frac{p_x}{p_y}$$

1.3 Changes that affect the budget constraint

1.3.1 Pure income change

keeping relative prices constant. i.e. $\frac{p_1}{p_2} = \overline{p}$

Note Changes in prices(relative price holds) will change the budget constraint in exogenous income budget, but will *not* affect the endogenous income constraint.

Conclusion To change budget constraint defined with endogenous income, we need **endowment changes**.

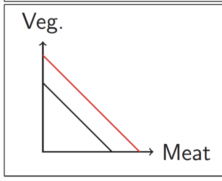
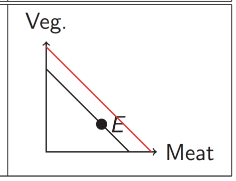
Exogenous Inc.	Endogenous Inc.
I	$p_M\omega_M + p_V\omega_V$
	

Figure 3: Pure income change

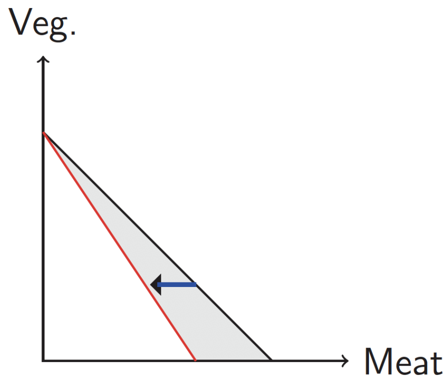


Figure 4: Relative price change

1.3.2 Price change

1.3.3 Endogenous income price change

Intuition **Rotation** about the endowment.

2 Lecture 2 May. 9 2018

Tophat Assume endogenous income and 2 goods - meat and vegetables. When the price of meat goes up, can you afford more bundles? Explain your reasoning briefly.

Key points

1. Relative price → Exchange rate.
2. Holding fixed amount of meat, consider the quantity of veg could be consumed.
3. Endowment point.

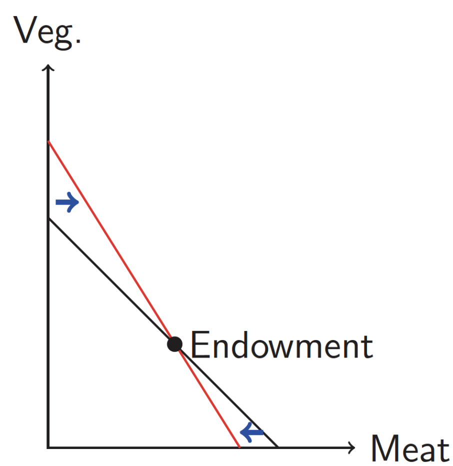


Figure 5: Endogenous income price change

2.1 Tastes as Binary Relations

Strictly preferred Consider bundles $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$, we denote Bundle A is strictly preferred as B as,

$$(x_1^A, x_2^A) \succ (x_1^B, x_2^B)$$

At least as good as Consider bundles $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$, we denote Bundle A is at least as good as B as,

$$(x_1^A, x_2^A) \succcurlyeq (x_1^B, x_2^B)$$

Indifference Consider bundles $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$, we denote Bundle A is indifferent to B as,

$$(x_1^A, x_2^A) \sim (x_1^B, x_2^B)$$

2.2 Rationality Assumptions on Preference Relation

Completeness Let X denote the consumption set, then we say a preference relation $\succcurlyeq$ satisfies completeness if and only if

$$\forall \vec{x_1}, \vec{x_2} \in X,\ \vec{x_1} \succcurlyeq \vec{x_2} \vee \vec{x_1} \preccurlyeq \vec{x_2}$$

Transitivity Let X denote the consumption set, then we say a preference relation $\succcurlyeq$ satisfies transitivity if and only if

$$\forall \vec{x_1}, \vec{x_2}, \vec{x_3} \in X,\ (\vec{x_1} \succcurlyeq \vec{x_2} \wedge \vec{x_2} \succcurlyeq \vec{x_3}) \implies \vec{x_1} \succcurlyeq \vec{x_3}$$

Rational tastes If those two assumption hold, we say the individual has rational tastes.

2.3 Convenience Assumptions

(Strict) Monotonicity Let X denote the consumption set, a preference relation $\succcurlyeq$ satisfies monotonicity if and only if

$$\forall \vec{x_1}, \vec{x_2} \in X,\ (\vec{x_1} \geq \vec{x_2} \implies \vec{x_1} \succcurlyeq \vec{x_2}) \wedge (\vec{x_1} \gg \vec{x_2} \implies \vec{x_1} \succ \vec{x_2})$$

(Weak) Convexity *Intuitively, Averages are better than extremes, or at least no worse.* Let X denote the consumption set, a preference relation $\succcurlyeq$ satisfies (weak) convexity if and only if

$$\forall \vec{x_1}, \vec{x_2} \in X,\ \vec{x_1} \sim \vec{x_2} \implies \lambda \vec{x_1} + (1-\lambda)\vec{x_2} \succcurlyeq \vec{x_1},\ \forall \lambda \in [0,1]$$

Continuity *Intuitively, no sudden preference switching*, mathematically, let consumption set $X \subseteq \mathbb{R}^n_+$, $\forall \vec{x} \in X$, the "no better than" set, $\preccurlyeq(\vec{x})$ and the "no worse than" set, $\succcurlyeq(\vec{x})$ are closed in $\mathbb{R}^n_+$. [1]

2.4 Indifference Curve

Definition Let consumption bundle $A \in X \subseteq \mathbb{R}^n_+$, the indifference curve corresponding to consumption bundle A is defined as

$$\sim(A) = \{\vec{x} \in X \mid \vec{x} \sim A\}$$

Properties

1. IC slopes downwards $\impliedby$ monotonicity.
2. ICs does not cross (for individual preference).
3. Direction of increasing preference.

2.5 Utility Function

Definition A real-valued function $u : \mathbb{R}^n_+ \to \mathbb{R}$ is called a **utility function** representing the preference relation $\succcurlyeq$, if for all $\vec{x_0}, \vec{x_1} \in \mathbb{R}^n_+$, $u(\vec{x_0}) \geq u(\vec{x_1}) \iff \vec{x_0} \succcurlyeq \vec{x_1}$.

[1] The mathematical definition is not required in ECO206.

Marginal Rate of Substitution (MRS) Consider the scenario with two commodities, intuitively, MRS could be interpreted as the quantity of one commodity must be forgone for one unit increment in the other commodity, holding the utility value constant. Graphically, MRS is the (absolute value of) slope of indifference curve. Mathematically, MRS can be computed as [1]

$$\frac{dy}{dx} = -\frac{\frac{\partial u(\cdot)}{\partial x}}{\frac{\partial u(\cdot)}{\partial y}}$$

3 Lecture 3 May. 15 2018

3.1 Different Types of Tastes

Questions

- How MRS changes **along** IC. (*substitutability*)
- How MRS changes **across** ICs.

3.1.1 Shape and Substitutability Along IC

Type	MRS
Perfect Substitutes	Constant
Perfect Complements	∞ or 0 or undefined
In Between	changes along IC

3.1.2 Diminishing MRS

4 Tutorial 2 May. 17 2018

MRS In this course, defined <u>with</u> the minus sign.

$$MRS = \frac{dy}{dx}|_{u(x,y)=\overline{u}} = -\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}}$$

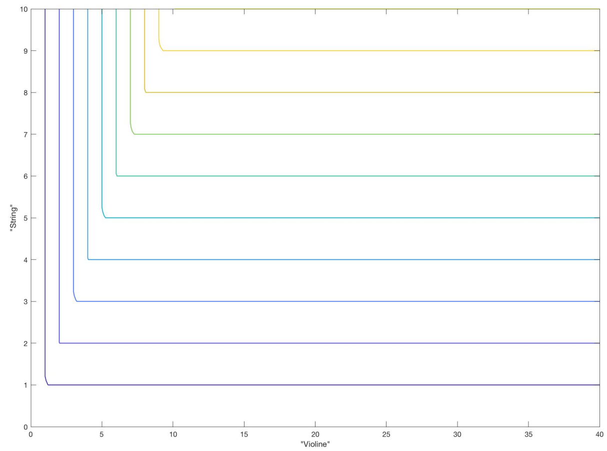
4.1 Different types of utility functions

Exercise1 (perfect complement) Preference over violins (v) and strings (s). Consume 4 strings with every violin. Any extras are discarded and cannot be used.
(a) utility function.

$$u(v, s) = min\{4v, s\}$$

(b) Graph an indifference curve
(c) What's the MRS

[1] The negative sign indicates *forgoing*.



1. 0 if $4v > s$ (flat part)
2. ∞ if $4v < s$ (vertical part)
3. *undefined* on kink point

Exercise2 (homothetic) Calculate MRS when $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$

Solution.

$$\frac{\partial u}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{\beta}$$
$$\frac{\partial u}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta-1}$$
$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{\alpha x_2}{\beta x_1}$$

Check convexity holds:

Method 1: Check convexity.

Method 2: Quasi-concave utility function.

■

Exercise 3 (quasi-linear) Calculate MRS if $u(x_1, x_2) = \sqrt{x_1} + x_2$

Solution.

$$MRS = -\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = -\frac{0.5}{\sqrt{x_1}}$$

Check convexity: check if absolute value of MRS diminish

$$|MRS| = \frac{0.5}{\sqrt{x_1}}$$

$$\frac{d|MRS|}{dx_1} < 0 \text{ for } x_1 > 0$$

■

Exercise 4 (perfect substitute) A consumer considers pepsi(p) and coca-cola(c) as perfect substitute and is willing to exchange 0.5 can of p for 1 can of c. What is a utility function for her preference?

Solution.

$$MRS = -0.5 = -\frac{dp}{dc} = -\frac{\frac{\partial u}{\partial c}}{\frac{\partial u}{\partial p}}$$

$$\implies 0.5\frac{\partial u}{\partial p} = \frac{\partial u}{\partial c}$$

$$\implies \frac{\partial u}{\partial p} = 2\frac{\partial u}{\partial c}$$

$$u(c, p) = 2p + c \text{ (not unique)}$$

■

Exercise (linear, corner solution) Consider optimization problem

$$\max_{x,y} u(x, y) = 3x + 2y$$
$$s.t.\ 2x + 5y = 100$$

Solution $(x^*, y^*) = (50, 0)$

Exercise (Concave, corner solution)

$$\max_{x,y} u(x,y) = x^2 + y^2$$
$$s.t.\ x + y = 100$$

Solution.

$$\text{Check MRS}|MRS| = \frac{dy}{dx} = \frac{x}{y}$$
Thus increasing MRS.
Lagrange provides a minimum.
$$\text{Corner solutions.}(x^*, y^*) = (0, 1000), (x^*, y^*) = (1000, 0)$$

■

Exercise Suppose $u(x_1, x_2) = x_1^2 x_2^2$ and the budget constraint is defined as

$$\begin{cases} 2x_1 + x_2 = 80 \text{ if } x_1 \leq 20 \\ x_1 + x_2 = 60 \text{ if } x_1 \geq 20 \end{cases}$$

How does the optimal x$_1 compare to 20$?