ECO2020 Microeconomic Theory I (PhD)

Individual Decision Making, Market Equilibrium, Market Failure, and Other Topics.

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- GitHub: `https://github.com/TianyuDu/Spikey_UofT_Notes`
- Website: `TianyuDu.com/notes`

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1 Chapter 1. Preference and Choice

1.1 Preference Relations

Definition 1.1.

(i) The **strict preference** relation, $\succ$, is defined by

$$x \succ y \iff x \succsim y \wedge \neg(y \succsim x) \tag{1.1}$$

(ii) The **indifference** relation, $\sim$, is defined by

$$x \sim y \iff x \succsim y \wedge y \succsim x \tag{1.2}$$

Definition 1.2 (1.B.1)**.** The preference relation $\succsim$ is **rational** if it possesses the following two properties

(i) *Completeness*

$$\forall x, y \in X,\ x \succsim y \vee y \succsim x \tag{1.3}$$

(ii) *Transitivity*

$$\forall x, y, z \in X,\ x \succsim y \wedge y \succsim z \implies x \succsim z \tag{1.4}$$

Proposition 1.1 (1.B.1)**.** If $\succsim$ is rational, then

(i) $\succ$ is both **reflexive** ($\neg\ x \succ x$) and **transitive** ($x \succ y \wedge y \succ z \implies x \succ z$);

(ii) $\sim$ is both **reflexive** and **transitive**;

(iii) $x \succ y \succsim z \implies x \succ z$.

Example 1.1. Typical scenarios when transitivity of preference is violated:

(i) *Just perceptible differences*;

(ii) *Framing problem*;

(iii) *Observed preference might from the result of the interaction of several more primitive rational preferences (Condorcet paradox)*;

(iv) *Change of tastes.*

Definition 1.3 (1.B.2)**.** A function $u : X \to \mathbb{R}$ is a **utility function representing preference relation** $\succsim$ if

$$\forall x, y \in X,\ x \succsim y \iff u(x) \geq u(y) \tag{1.5}$$

Proposition 1.2 (1.B.2)**.** If a preference relation $\succsim$ can be represented by a utility function, then $\succsim$ is rational.

1.2 Choice Rules

Definition 1.4. A **choice structure**, $(\mathscr{B}, C(\cdot))$, is a tuple consists of

(i) The collection of **budget sets** $\mathscr{B}$, which is a set of nonempty subsets of X.

(ii) The **choice rule**, $C(B) \subset B$, is a *correspondence* for every $B \subset \mathscr{B}$ denotes the individual's choice from among the alternatives in B. If $C(B)$ is not a singleton, it can be interpreted as the *acceptable alternatives* in B, which the individual would actually chosen if the decision-making process is run repeatedly.

Definition 1.5 (1.C.1)**.** The choice structure $(\mathscr{B}, C(\cdot))$ satisfies the **weak axiom of revealed preference** if

$$\Big(\underbrace{\exists B \in \mathscr{B} \text{ s.t. } x, y \in B \wedge x \in C(B)}_{x \succsim y \text{ revealed.}}\Big) \implies \Big(\forall B' \in \mathscr{B} \text{ s.t. } x, y \in B',\ y \in C(B') \implies x \in C(B')\Big) \tag{1.6}$$

Definition 1.6. Given a choice structure $(\mathscr{B}, C(\cdot))$, the **revealed preference relation** $\succsim^*$ is defined as

$$x \succsim^* y \iff \exists B \in \mathscr{B} \text{ s.t. } x, y \in B \wedge x \in C(B) \tag{1.7}$$

Remark 1.1 (Interpretation on the definition of WARP)**.** If x is *revealed* at least as good as y, then y cannot be revealed preferred to x.

1.3 The Relationship between Preference Relations and Choice Rules

Definition 1.7. Given rational preference relation $\succsim$ on X, the **preference-maximizing choice rule** is defined as

$$C^*(B, \succsim) := \{x \in B : x \succsim y\ \forall y \in B\}\ \forall B \in \mathscr{B} \tag{1.8}$$

We say the rational preference relation **generates** the choice structure $(\mathscr{B}, C^*(\cdot, \succsim))$.

Assumption 1.1. Assume $C^*(B, \succsim) \neq \varnothing$ for all $B \in \mathscr{B}$.

Proposition 1.3 (1.D.1 (Rational → WARP))**.** Suppose that $\succsim$ is a rational preference relation. Then the choice structure generated by $\succsim$, $(\mathscr{B}, C^*(\cdot, \succsim))$, satisfies the weak axiom.

Definition 1.8 (1.D.1)**.** Given choice structure $(\mathscr{B}, C(\cdot))$, we say that the rational preference relation $\succsim$ **rationalizes** $C(\cdot)$ relative to $\mathscr{B}$ if

$$C(B) = C^*(B, \succsim)\ \forall B \in \mathscr{B} \tag{1.9}$$

That is, $\succsim$ *generates the choice structure* $(\mathscr{B}, C(\cdot))$.

Remark 1.2. In general, for a given choice structure $(\mathscr{B}, C(\cdot))$, there may be more than one rational preference relation $\succsim$ rationalizing it.

Proposition 1.4 (1.D.2 (WARP → Rational))**.** If $(\mathscr{B}, C(\cdot))$ is a choice structure such that

(i) The weak axiom is satisfied;

(ii) $\mathscr{B}$ includes all subsets of X up to three elements.

Then there is a rational preference relation $\succsim$ that rationalizes $C(\cdot)$ relative to $\mathscr{B}$.

2 Chapter 2. Consumer Choice

2.1 Commodities

Definition 2.1. Assume the number of **commodities** is finite and equal to L. In general, a **commodity vector** or **commodity bundle** is an element in a **commodity space**, typically $\mathbb{R}^L$.