ECO206 Microeconomic Theory Summary I

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1 Lecture 1 Introduction

Notation Assuming there are n goods, then a **bundle** of goods, $\mathcal{A}$ can be denoted as

$$\{x^A{}_1, \ldots, x^A{}_n\} \in \mathbb{R}^n_+$$

1.1 Types of Income and Budget Set

Let $\vec{p} \in \mathbb{R}^n$ denote the **price vector**.

Exogenous income Let $I \in \mathbb{R}_+$ denote the **exogenous income**, then the budget set can be expressed as

$$\mathcal{B} = \{\vec{x} \in \mathbb{R}^n_+ \mid \vec{x} \cdot \vec{p} \leq I\}$$

Endogenous income Let $\vec{\omega} \in \mathbb{R}^n_+$ denote the **endowment** and the budget set can be expressed as

$$\mathcal{B} = \{\vec{x} \in \mathbb{R}^n_+ \mid \vec{x} \cdot \vec{p} \leq \vec{\omega} \cdot \vec{p}\}$$

1.2 Opportunity Cost

MRT Marginal Rate of Transformation (MRT) measures, given budget constraint, the unit of a good need to be given up in order to consume one additional unit of the other good. *OC/MRT is expressed in units of a good, instead of dollar.*

Mathemtically,

$$\begin{gathered}
\text{Two-goods example,} \\
E = x_1 p_1 + x_2 p_2 = y \\
\text{Take total differential,} \\
dy = \frac{\partial E}{\partial x_1} dx_1 + \frac{\partial E}{\partial x_2} dx_2 = 0 \\
\implies \frac{dx_2}{dx_1} = -\frac{p_1}{p_2}
\end{gathered}$$

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Interpretation Units of good x_2 to given up (negative sign) in exchange for one unit of x_1.

1.3 Comparative Statics

1.3.1 Pure Income Effect

Exogenous Inc.	Endogenous Inc.
I	$p_M\omega_M + p_V\omega_V$

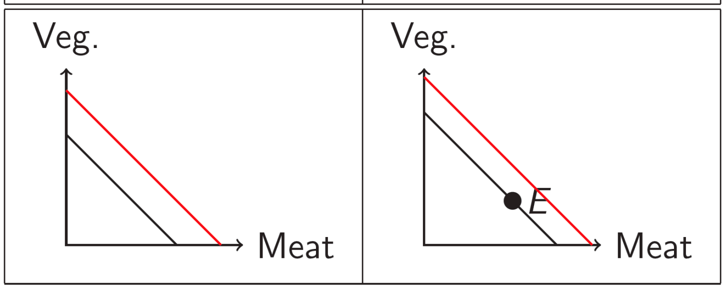

Figure 1: pure income effect from increase in income

For both types of income, pure income effect shifts the budget line parallel.

1.3.2 Price Changes

Exogenous income Consider a price increase in meat, in this case, the *invariant bundle* (i.e. the bundle that is not affected by the price change at all) is on y-intercept.

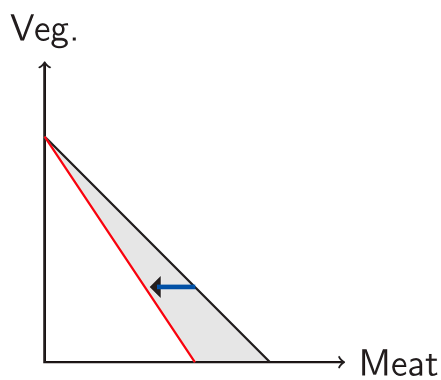

Figure 2: increase in price of meat on exogenous income budget line

Endogenous income in this case, the *invariant bundle* is the endowment bundle.

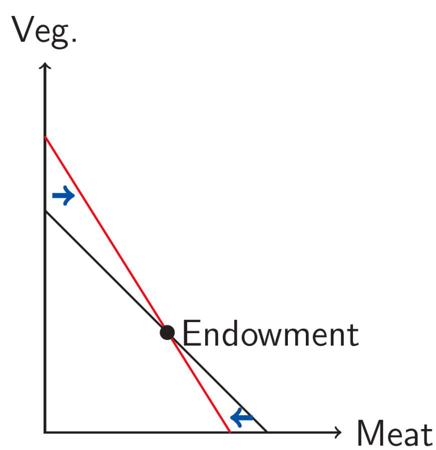

Figure 3: increase in price of meat on endogenous income budeget line

Note In both cases, price change causes the budget line rotates around the invariant bundle.

2 Lecture 2 Preference and Utility

2.1 Preference Relation

Preference Relation is Binary. Let $\mathcal{X}$ be the consumption set, and let $\mathcal{A}, \mathcal{B} \in \mathcal{X}$.

Definition If a bundle $\mathcal{A}$ is no worse than (i.e. **at least as good as**) another bundle $\mathcal{B}$, then we denote this as

$$\mathcal{A} \succcurlyeq \mathcal{B}$$

Definition A consumer **strictly prefers** bundle $\mathcal{A}$ than bundle $\mathcal{B}$ if and only if

$$\mathcal{A} \succcurlyeq \mathcal{B} \wedge \neg \mathcal{B} \succcurlyeq \mathcal{A}$$

and denoted as

$$\mathcal{A} \succ \mathcal{B}$$

Definition A consumer is **indifferent** between two bundles $\mathcal{A}$ and $\mathcal{B}$ if and only if

$$\mathcal{A} \succcurlyeq \mathcal{B} \wedge \mathcal{B} \succcurlyeq \mathcal{A}$$

2.2 Rationality Assumptions

Let $\mathcal{X}$ denote the consumption set.

A1.Completeness A preference relation $\succcurlyeq$ is **complete** if and only if

$$\mathcal{A} \succcurlyeq \mathcal{B} \vee \mathcal{B} \succcurlyeq \mathcal{A}, \ \forall \mathcal{A}, \mathcal{B} \in \mathcal{X}$$

A2.Transitivity A preference relation $\succcurlyeq$ is **transitive** if and only if

$$\mathcal{A} \succcurlyeq \mathcal{B} \wedge \mathcal{B} \succcurlyeq \mathcal{C} \implies \mathcal{A} \succcurlyeq \mathcal{C}, \ \forall \mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathcal{X}$$

Definition a preference relation is **rational** if and only if it satisfies assumptions A1 and A2 above.

2.3 Convenience Assumptions

A3.Monotonicity Let $\mathcal{A} = \{x^A{}_1, \dots, x^A{}_n\}$ and $\mathcal{B} = \{x^B{}_1, \dots, x^B{}_n\} \in \mathcal{X}$, then

$$x_i^A \geq x_i^B, \ \forall i \in \{1, \dots, n\} \implies \mathcal{A} \succcurlyeq \mathcal{B}$$

and

$$x_i^A > x_i^B, \ \forall i \in \{1, \dots, n\} \implies \mathcal{A} \succ \mathcal{B}$$

Example In figure below, region 3 (including boundary) represents the *no worse than set* of $\mathcal{A}$, i.e. $R_3 = \succcurlyeq (\mathcal{A}) := \{\vec{x} \in \mathcal{X} \mid \vec{x} \succcurlyeq \mathcal{A}\}$ and region 2 (including boundary) represents the *no better than set* of $\mathcal{A}$, i.e. $R_2 = \succcurlyeq (\mathcal{A}) := \{\vec{x} \in \mathcal{X} \mid \mathcal{A} \succcurlyeq \vec{x}\}$

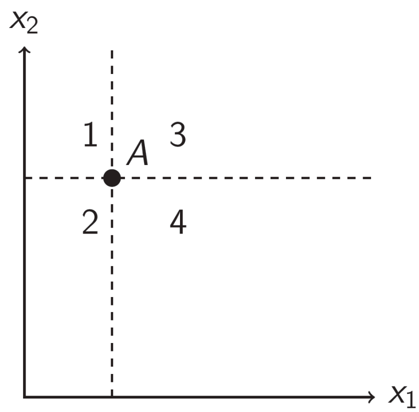

Figure 4: monotonic preference

A4.(Weak) Convexity If a preference relation is **convex**, then, for any $\mathcal{A}, \mathcal{B} \in \mathcal{X}$, suppose $\mathcal{A} \sim \mathcal{B}$,

$$\alpha\mathcal{A} + (1-\alpha)\mathcal{B} \succcurlyeq \mathcal{A}, \ \forall \alpha \in [0,1]$$

Meaning the *no worse than set* for any given bundle $\mathcal{A}$ over preference relation $\succcurlyeq$ is a convex set.
Implication the utility function has to be quasi-concave.
Lemma the upper level contour for a quasi-concave function is convex.

A5.Continuity Loosely speaking, no sudden switch over preference. Formally, $\succcurlyeq(\mathcal{A})$ and $\preccurlyeq(\mathcal{A})$ sets are closed.

2.4 Indifference Curve

Definition Let $\mathcal{A} \in \mathcal{X}$, then indifference set of $\mathcal{A}$ over preference relation $\succcurlyeq$ is defined as

$$\sim(\mathcal{A}) = \{\vec{x} \in \mathcal{X} \mid \vec{x} \sim \mathcal{A}\}$$

2.5 Utility Function

Definition a real-valued function $u : \mathbb{R}_+^n \to \mathbb{R}$ represents a preference relation if and only if, let $\vec{x_1}, \vec{x_2} \in \mathbb{R}_+^n$ denote the quantities of goods in bundles $\mathcal{A}_1$ and $\mathcal{A}_2 \in \mathcal{X}$,

$$\mathcal{A}_1 \succcurlyeq \mathcal{A}_2 \iff u(\vec{x_1}) \geq u(\vec{x_2})$$

Theorem an utility function is invariant to positive-monotonic transformations. That's, let g denote a positive-monotonic transformation, and $u : \mathbb{R}_+^n \to \mathbb{R}$ be a utility function representing preference relation $\succcurlyeq$, then $g \circ u$ also is an utility function representing $\succcurlyeq$.

Definition we say two consumers have the **same tastes** if and only if (1) they have same willingness to trade (MRS) at the same bundle and (2) same direction of increasing preference.
Mathematically,

$$MRS_1|_{\vec{x}} = MRS_2|_{\vec{x}},\ \forall \vec{x} \in \mathcal{X}$$

and, let u_1 and u_2 denote utility functions, then

$$\nabla u_1 \cdot \vec{d} \geq 0 \iff \nabla u_2 \cdot \vec{d} \geq 0,\ \forall\ \vec{d} \in \mathbb{R}^n$$

2.6 Marginal Rate of Substitution

Definition MRS represents the willingness to trade. In two-goods situations, it measures the number of goods 2 the consumer is willing to give up for one unit of good 1, keeping his/her utility level constant.

Mathematically,

$$du(\cdot) = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0$$

$$\implies \frac{dx_2}{dx_1} = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{MU_1}{MU_2}$$

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2.7 Types of Preference Relations

Definition A preference relation is homothetic if and only if there exists a homogeneous utility function to represent it. That's

$$u(\alpha\vec{x}) = \alpha^k u(\vec{x}) \text{ for some } k \in \mathbb{Z}^+\ \forall \alpha \in \mathbb{R}_+,\ \vec{x} \in \mathbb{R}^n_+$$

Note that, utility function is invariant to positive monotonic transformation.

Proposition MRS of a homothetic preference only depends on the ratio of consumption. i.e. $MRS_{homothetic} = f(\frac{x_2}{x_1})$.

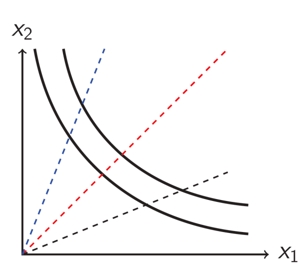


Figure 5: homothetic preference and MRS of it

Definition A preference relation is **quasi-linear** in good i if and only if, for all $\mathcal{A}$ and $\mathcal{B} \in \mathcal{X}$,

$$\mathcal{A} \sim \mathcal{B} \implies (\mathcal{A} + \alpha \vec{e_i}) \sim (\mathcal{B} + \alpha \vec{e_i}), \ \forall a \in \mathbb{R}$$

where $\vec{e_i}$ is the i^{th} standard basis vector of $\mathbb{R}^n$.

Proposition MRS of a quasi-linear utility depends only on one goods.

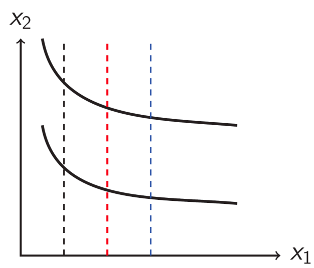


Figure 6: quasi-linear preference and MRS of it

3 Lecture 3 Choice

3.1 Different Types of Tastes

Examine

1. How MRS changes **along** an IC.
2. How MRS changes as we move **across** ICs.

3.2 Shape and Substitutability Along a given IC

Type	MRS (Trade-offs)
Perfect Substitutes	Constant at every bundle
Perfect Complements	Unwilling to substitute
In Between	Changes as we move along IC

Figure 7: different types of preference and substituability

3.3 Diminishing MRS

Definition When we move down (more x and less y) along an indifference curve.

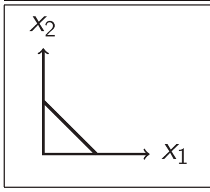
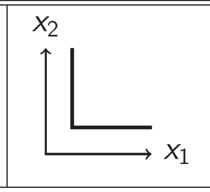
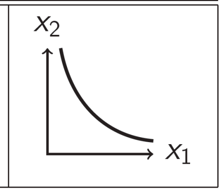
Constant MRS	Unwilling to Subs.	Changes along IC
		

Figure 8: different types of preference and graphs

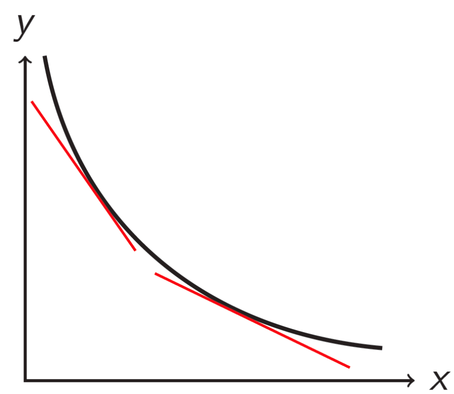


Figure 9: diminishing in MRS on graph

Diminishing MRS in words Compare two bundles on the *same* indifference curve. At each bundle consider how much of y this consumer are willing to give up for an additional unit of x. Diminishing means if we have relative *more* x in one bundle, then we are *less* willing to give up y at that bundle compared to the other bundle.

Note Perfect substitution preference is linear and therefore quasi-linear.

3.4 Choice

3.4.1 Tangency

MRS units of x_2 that we are **willing** to pay for 1 more unit of x_1.

Opp. Cost units of x_2 that we **have** to pay for 1 more x_1.

Tangency units we are willing to pay is, on the margin, equal to the amount we have to pay.

$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2} = Opp.Cost$$

3.4.2 Lagrangian Multiplier Method

Method:

$$\max_{\vec{x}} u(\vec{x})$$
$$s.t. \vec{x} \cdot \vec{p} \leq I$$

By monotonicity, income constraint holds as equality.

$$\mathcal{L}(\vec{x}, \lambda) = u(\vec{x}) + \lambda \times (I - \vec{x} \cdot \vec{p})$$

First Order Conditions.

$$\begin{cases} \frac{\partial \mathcal{L}(\cdot)}{\partial x_i} = \frac{\partial u(\cdot)}{\partial x_i} - \lambda p_i = 0, \ \forall i \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda} = I - \vec{x} \cdot \vec{p} = 0 \end{cases}$$

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Note our assumption on preference relations ensure the sufficiency of first order condition and uniqueness of solution.[1]

Shadow price Let $v(\vec{p}, I)$ denote the highest level (i.e. value function, indirect utility) of utility achievable given price $\vec{p}$ and income I. By *envelope theorem*, we can show that

$$\lambda^* = \frac{\partial v}{\partial I}$$

where λ^* measures the "value" of relaxing the constraint by a tiny bit. λ^* here is called the **shadow price** of income.

Summary the though process of solving consumer's optimization problem:

4 Lecture 4 Demand and Income Effects

4.1 Income Effects

Definition **income effect** captures the change in behaviour arising from just a change in income. A pure income effect leads to a *parallel shift in the budget constraint.*

[1] Basically, the generic optimization problem has been reduced to a convex optimization problem.

Thought Process

- Preference Type? *Check MRS*
- Can I use LM or not?
  - expect *Interior tangency* point?
  - Any nonconvexities?

Note: for Cobb-Douglas preference, interior solution is guaranteed.

- If Yes
  - Identify, choice variables, objective and constraints then setup and solve.
  - Be careful about necessary vs. sufficient conditions
- If No $\Rightarrow$ use Intuition, Graphs and Logic
  - Highest IC given constraint.
- Check for Multiple Optimal Solutions! - non-convexities, flat spots etc.
- Check to make sure answers make sense. This is when step 1 and graphs can help.

Definition let $x_i(\vec{p}, I)$ denote the demand for good i given price vector $\vec{p}$ and income I. Then good i is classified as **normal goods** if and only if

$$\frac{\partial x_i(\cdot, I)}{\partial I} > 0$$

Good i is classified as **inferior goods** if and only if

$$\frac{\partial x_i(\cdot, I)}{\partial I} < 0$$

Note if preference relation is *quasi-linear* in good i, then a change in p_i has **no** income effect.

4.2 Engel Curves

Definition **Engel curve** captures the correlation between consumer's *income* and the *quantity demanded* by the consumer.

Note Engel curve have slope $\frac{dI}{dx}$. Therefore, if good i is normal, it has upward sloping Engel curve. If good i is inferior, it has downward sloping Engel curve.

5 Lecture 5 Income and Substitution Effects

When price changes, both relative price (substitution effect) and real income (income effect) changes.

5.1 SE: Expenditure Minimization

Definition to capture substitution effect from price change, we compensate the consumer enough exogenous income so that this consumer can reach the **original indifference curve**[2] with the **new price**.

$$min_{x_1,x_2} p_1^{final} x_1 + p_2^{final} x_2 \text{ subject to } u(x_1, x_2) = U^{initial}$$

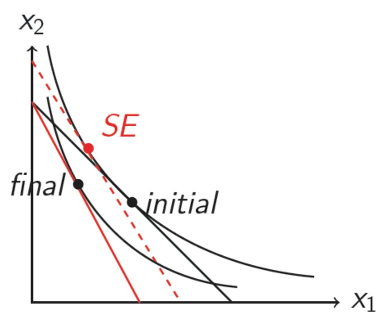

Figure 10: decomposing total effect into income and substitution effects in graph

5.1.1 Calculating a Substitution Effect

Intuition to find SE, we need to find the demand of goods with new price level and the origin utility level achieved.

Method:

This can be done via expenditure minimization.

Let $\overline{U}$ denote the origin utility level.

$$\min_{\vec{x}} \vec{p}^{new} \vec{x} + \lambda \times (\overline{U} - u(\vec{x}))$$

Extracting the first order conditions:

$$\begin{cases} \frac{\partial \mathcal{L}(\cdot)}{\partial x_i} = p_i^{new} - \lambda \frac{\partial u}{\partial x_i} = 0, \ \forall i \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda} = \overline{U} - u(\vec{x}) = 0 \end{cases}$$

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and by solving these first order conditions above, we have $h_i(\vec{p}, \overline{U})$ as **compensated demand curve** (aka Hicksian demand).

[2] In ECO206, we analyze *Hicksian* substitution effect. If we compensate the consumer enough to reach the *origin bundle*, we are capturing the *Slutsky* substitution effect.

5.2 Income Effect

Definition Income Effect can be captured by proportion of total effect unexplained by substitution effect.

$$\text{Total Effect} = x_i^{final} - x_i^{initial}$$
$$\text{Substitution Effect} = x_i^{SE} - x_i^{initial}$$
$$\text{Income Effect} = x_i^{final} - x_i^{SE}$$

5.3 Compensated Demand Curve

Definition **compensated demand curve** captures the changes in quantity demanded for good i when p_i changes, while holding the utility level fixed. And compensated demanded is denoted as

$$h_i(\vec{p}, \overline{U})$$

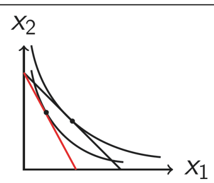
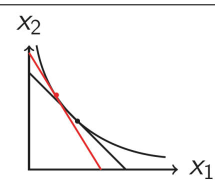
Regular Demand	**Compensated Demand**
Holds fixed income	Holds fixed IC
moves across IC	Moves along an IC
	

Figure 11: regular demand and compensated demand on graph: framework 1

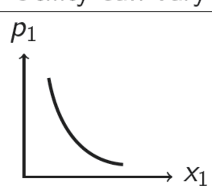
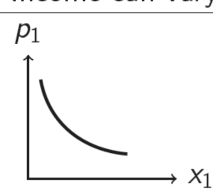
Regular Demand	**Compensated Demand**
Income held fixed	Utility held fixed
Utility can vary	Income can vary
	

Figure 12: regular demand and compensated demand on graph: framework 2

5.4 Slutsky Equation

Proof.

$$
\begin{aligned}
& h_i(\vec{p}, \overline{U}) = x_i(\vec{p}, I) \\
\implies & h_i(\vec{p}, \overline{U}) = x_i(\vec{p}, e(\vec{p}, \overline{U})) \\
\implies & \frac{\partial h_i}{\partial p_j} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial I}\frac{\partial E}{\partial p_j} \\
\implies & \frac{\partial h_i}{\partial p_j} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial I} h_j \\
\implies & \frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - \frac{\partial x_i}{\partial I} h_j
\end{aligned}
$$

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6 Lecture 6 Labor Supply and Elasticities

6.1 Model Setup

Goods c **consumption** and ℓ **leisure**.

Preference $u(c, \ell)$.

Income endogenous (L **time endowment**) and exogenous incomes (M as **non-labor income**).

6.2 Deriving Labor Supply

$$\max_{c,\ell} u(c, \ell) \ s.t. \ c + w\ell \leq wL + M$$

By solving the above optimization, we have (c^*, ℓ^*). And hours of working h is given by $h = L - \ell^*$.

6.2.1 Shape of Labor supply

Note notice the assumption on leisure, inferior or normal.