Hipster Travelling Problem: Task 02

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1 Modelling Transportation Cost

1.1 Objective Function

The objective of the Hipster Tourist Problem is to minimise the transportation cost and the cost of attractions. To simplify this objective function, we assumed that the tourist will be travelling by public transport in Singapore, due to this being the most economical mode of transport. For public transport - the cost is directly proportional to the distance travelled. Thus, in order to minimize the transportation cost, we have reformulated the objective to minimise the sum of distance travelled multiplied by the average cost and the total cost of attractions visited.

1.2 Methodology

1. To find the distances of each attraction, dining location and hotel, from each other, we found the geographical location of each place in term of its latitude and longitude.
2. Using Python and the Haversine formula, we calculated all the straight path distances between any two possible locations using the latitude and longitude, assuming the earth is a perfect sphere. (Appendix 5.3 in Task 3)
3. By inputting the matrix of distance, we were able to formulate the model to minimise the distance travelled. This in turn would minimise the transportation cost, due to the linear relationship between distance and cost described above.

1.3 Assumptions

1. We assume each time instance is 10 minutes. Thus the corresponding table looks like below:

Time (a.m/p.m)	9	10	11	12	13	14	15	16	17	18	19	20	21
t (unit)	0	6	12	18	24	30	36	42	48	54	60	66	72

Table 1: Time Instances

2. A location (attraction or dining location) can only be visited once during the day.
3. All the travelling will be done by MRT at the rate of 10 km per hour estimated based on google map navigation.
4. There is a linear relation between the distance travelled and the transportation cost.
5. There is a linear relation between the travelling time and the transportation cost.

6. Lunch and Dinner must start in the time interval 12:00pm to 2:00pm and 6:00pm and 8:00pm respectively, but may end after that time interval.

2 Mathematical Modelling

The time-indexed Mixed Integer Programming (MIP) model of the Hipster Tourist Problem was formulated as shown below.

2.1 Parameters

2.1.1 Constants

$T = (21 - 9) \times (60 \div 10) = 72$
$t_a = 3$
$t_d = 6$
D : distance matrix where $D(i, j)$ represents distance (km) between location i and j. This matrix is symmetric.
T_m : time matrix where $T_m(i, j)$ represents travelling time between location i and j. This matrix is symmetric. As the assumption stated, this is in proportion to distance matrix, except that it is rounded to integer values.
$\vec{c}$: cost vector where where $\vec{c}_j$ represents entry fee for attraction with index j.
C : cost matrix where where $C(i, j)$ represents transportation fee from location i to location j. ($C = 0.1D$)

Notes: the minimum time unit regarding this optimization task is 10 minutes. T is the maximum time index for the model.

2.1.2 Sets

$J = \{0, 1, 2, \dots, 33\}$ is the set of indices for all locations
$H = \{33\}$ is the set of indices for hotel
$A = \{10,11,12, \dots,32\}$ is the set of indices for all attractions
$D = \{0,1,2, \dots,9\}$ is the set of indices for all dining places
$\Gamma = J \times J$ is the Cartesian product of J denoting indices of edges in the graph
Outdoor is the set of indices for outdoor attractions
Indoor is the set of indices for indoor attractions

$$Region\ sets \begin{cases} N & \text{set of indices for attractions \& dining places in region N} \\ E & \text{set of indices for attractions \& dining places in region E} \\ W & \text{set of indices for attractions \& dining places in region W} \\ O & \text{set of indices for attractions \& dining places in region O} \end{cases}$$

$$Activity\ time\ index\ sets \begin{cases} Lunchtime = \{18, 19, 20, \dots, 30\} \\ Dinnertime = \{54, 55, 56, \dots, 66\} \\ Nondiningtime = \{0, 1, 2, \dots, T\} - Lunchtime - Dinnertime \\ Indoortime = \{12, 13, 14, \dots, 42\} \end{cases}$$

$\Omega = \{N, E, W, O\}$ is the set of region sets

2.2 Decision Variables

$$X_{j,t} = \begin{cases} 1 & \text{arrives at location j at time t} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$Y_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$U_\omega = \begin{cases} 1 & \text{region } \omega \text{ is visited} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

2.3 Objective Function

$$\min \sum_{(l,m)\in\Gamma} \sum_{t=0}^{T} Y_{(l,m),t} C_{(l,m)} + \sum_{j\in A} \sum_{t=0}^{T} X_{j,t} \vec{c}_j \tag{1}$$

2.4 Constraints

s.t

$$\begin{cases} \sum_{t=0}^{T} X_{j,t} \leqslant 1, \quad \forall j \in A \cup D \\ \sum_{t=0}^{T} Y_{(l,m),t} \leqslant 1, \quad \forall (l,m) \in \Gamma \end{cases} \tag{1}$$

Visits each attraction/dining place at most once
Travels between two locations at most once
Notes: The second constraint is redundant for most of the cases, because no edge will be taken twice if the location is not allowed to visit more than once. This constraint is added to prevent traveler from returning back to hotel during the day, which seems a bit weird.

$$\sum_{s=max(t+1-P_j,0)}^{t} \sum_{j\in J} X_{j,s} + \sum_{s=max(t+1-Tm_{(l,m)},0)}^{t} \sum_{(l,m)\in\Gamma} Y_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T \tag{2}$$

At most one activity is taken at each time instant.

$$X_{j,t} = 0, \text{ if } t \in Nondiningtime, \quad \forall j \in D \tag{3}$$

At non-dining hours, no dining is allowed.

$$\sum_{j\in A} \sum_{t=0}^{T} X_{j,t} \geqslant 8 \tag{4}$$

At least 8 attractions must be visited.

$$\sum_{j\in Outdoor} \sum_{t=0}^{T} X_{j,t} \geqslant 3 \tag{5}$$

At least 3 attractions must be outdoor.

$$X_{j,t} = 0, \quad \forall j \in Outdoor, \quad t \in Indoortime \tag{6}$$

From 11am to 4pm, no outdoor attraction is allowed to be visited.

$$\begin{cases} \sum_{j\in\omega}\sum_{t=0}^{T} X_{j,t} \geqslant U_\omega \\ \sum_{j\in\omega}\sum_{t=0}^{T} X_{j,t} \leqslant MU_\omega, \quad \forall\omega\in\Omega \\ \sum_{\omega\in\Omega} U_\omega \geqslant 2 \end{cases} \tag{7}$$

At least 2 regions other than the region of the hotel must be visited.

$$\left.\begin{array}{l} \sum_{k\in A\cup D\setminus j} Y_{(k,j),max(t-Tm_{(k,j)},0)} = X_{j,t}, \quad \forall j\in J \quad , \quad \forall t=1,2,3\ldots,T \\ \sum_{k\in A\cup D\setminus j} Y_{(j,k),min(t+P_a,T)} = X_{j,t}, \quad \forall j\in A, \quad \forall t=0,1,2,\ldots,T-1 \\ \sum_{k\in A\cup D\setminus j} Y_{(j,k),min(t+P_d,T)} = X_{j,t}, \quad \forall j\in D, \quad \forall t=0,1,2,\ldots,T-1 \\ \sum_{k\in A\cup D\setminus j} Y_{(33,k),0} = 1 \end{array}\right\} \tag{8}$$

Inspired by Network Flow Problem. These constraints combined together guarantees that the scheduled forms an end-to-end loop. First constraint guarantees that there is an edge pointing towards each node while the last three guarantee that there is an edge out of each node. Assuming each attraction is visited for exactly 30 minutes.

$$X_{H,0} = 1, \quad X_{H,T} = 1 \tag{9}$$

The hotel must be the starting point at time 0 and ending point at time T.

$$Y_{(j,j),t} = 0, \quad \forall j\in J, \quad \forall t=0,1,2\ldots,T \tag{10}$$

Travelling between same places is prohibited.

$$\left.\begin{array}{l} \sum_{t\in lunchtime}\sum_{j\in D} X_{j,t} = 1 \\ \sum_{t\in dinnertime}\sum_{j\in D} X_{j,t} = 1 \end{array}\right\} \tag{11}$$

Lunch and dinner are compulsory.

3 Solution

The mathematical model was able to find an optimal integer solution which is shown below:

Time	Location	Type of Location
9:00 a.m	Mandarin Orchard	Hotel
10:00 a.m	East Coast Park	Attraction
10:50 a.m	Punggol Waterway Park	Attraction
12:00 a.m	Mandarin Orchard Singapore	Attraction
1:20 p.m	Miam Miam French Japanese Cafe Kitchen	Dining
2:30 p.m	Science Centre Singapore	Attraction
3:30 p.m	Singapore Discovery Centre	Attraction
4:30 p.m	Sungei Buloh Wetland Reserve	Attraction
5:20 p.m	Chinese Garden	Attraction
6:20 p.m	Haw Par Villa	Attraction
7:10 p.m	Botanic Gardens	Attraction
7:50 p.m	Jamie's Italian	Dining
9:00 p.m	Mandarin Orchard	Hotel

Table 2: Schedule for the Day

Thus, the total cost of the day amounts to SGD 53.12 (rounded off to 2 decimal place).

Hipster Travelling Problem: Task 03

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1 Problem Statement

The objective of the problem is to suggest three schedules with low, medium, and high budget limits that maximizes the diversity of activities during the two-day stay. The model should include the scenario where there might be a hotel change on the first day.

1.1 Definition of the Objective Function

We define the objective to be maximising the number of attractions visited in two days by covering at least 3 attractions each day and 4 regions in two days.

1.2 Methodology

Methodologies 1-3 from task 02 still hold.

4. One more subscript to introduce more flexibility
5. Two sets of variables for two days
6. Big M method to model hotel moving at any time in the first day
7. Regularization parameter to obtain different plans under different budget limits

1.3 Assumptions

Assumptions 1-5 from task 02 still hold.

6. At least 3 outdoor attractions are visited each day to guarantee diversity
7. At least 4 regions are visited in two days to guarantee diversity
8. Zero entry fee for hotels

2 Mathematical Modelling

In this Mixed Integer Programming formulation, all feasible paths/schedules are considered (there is no shrinkage of solution space)

2.1 Parameters

2.1.1 Constants: Same as the constants from task 02

2.1.2 Sets

Except for the following changes, the rest of the sets remain the same as task 02.

$H = \{33,34,35,36,37\}$ is the set of indices for hotel

$$\textit{Region sets} \begin{cases} N & \text{set of indices for attractions \& dining places \& hotels in region N} \\ E & \text{set of indices for attractions \& dining places \& hotels in region E} \\ W & \text{set of indices for attractions \& dining places \& hotels in region W} \\ O & \text{set of indices for attractions \& dining places \& hotels in region O} \\ C & \text{set of indices for attractions \& dining places \& hotels in region C} \end{cases}$$

$\Omega = \{N, E, W, O, C\}$ is the set of region sets

2.2 Decision Variables

$$X_{j,t,p} = \begin{cases} 1 & \text{arrives at location j at time t and stays for p units of time in the first day} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$Y_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t in the first day} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$W_{j,t,p} = \begin{cases} 1 & \text{arrives at location j at time t and stays for p units of time in the second day} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$Z_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t in the second day} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$H_{h_i,h_j} = \begin{cases} 1 & \text{hotel changes from hotel i to hotel j} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$U_\omega = \begin{cases} 1 & \text{region } \omega \text{ is visited in two days} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Notes: $\forall t = 0, 1, 2, \ldots T, \quad \forall p = 0, 1, 2, \ldots T, \quad \forall (h_i, h_j) \in H \times H, \quad \forall \omega \in \Omega$

2.3 Objective Function

$$\max \sum_{j \in A} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} - \lambda\Big(\sum_{(l,m) \in \Gamma} \sum_{t=0}^{T} Y_{(l,m),t} C_{(l,m)} + \sum_{j \in J} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} c_j \Big)$$

Notes: λ is the regularization parameter used to indicate the importance of cost. A high lambda means the budget limit is low. A low lambda means the budget limit is high.

2.4 Constraints

s.t

$$\left.\begin{array}{l}\sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p} + W_{j,t,p} \leqslant 1, \quad \forall j \in A \cup D \\ \sum_{t=0}^{T} Y_{(l,m),t} + Z_{(l,m),t} \leqslant 1, \quad \forall (l,m) \in \Gamma\end{array}\right\} \tag{1}$$

Visits each attraction/dining place at most once
Travels between two locations at most once (might be redundant)

$$\left.\begin{array}{l}\sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} X_{j,s,p} + \sum_{(l,m)\in\Gamma}\sum_{s=max(t+1-Tm_{(l,m)},0)}^{t} Y_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T \\ \sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} W_{j,s,p} + \sum_{(l,m)\in\Gamma}\sum_{s=max(t+1-Tm_{(l,m)},0)}^{t} Z_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T\end{array}\right\} \tag{2}$$

At most one activity is taken at each time instant.

$$\left.\begin{array}{l}\sum_{j\in D}\sum_{t,p:[t,t+p]\subseteq Lunchtime} X_{j,t,p} = 1, \\ \sum_{j\in D}\sum_{t,p:[t,t+p]\subseteq Dinnertime} X_{j,t,p} = 1, \\ \sum_{j\in D}\sum_{t,p:[t,t+p]\subseteq Lunchtime} W_{j,t,p} = 1, \\ \sum_{j\in D}\sum_{t,p:[t,t+p]\subseteq Dinnertime} W_{j,t,p} = 1, \\ \sum_{j\in D}\sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p} = 2 \\ \sum_{j\in D}\sum_{t=0}^{T}\sum_{p=0}^{T} W_{j,t,p} = 2\end{array}\right\} \tag{3}$$

Lunch and dinner are the only meals and compulsory for each day, they must begin and end inside the specified periods.

$$\left.\begin{array}{l}X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in A, \quad \forall t = 0,1,2,\ldots,T, \quad \forall p < t_a \\ X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in D, \quad \forall t = 0,1,2,\ldots,T, \quad \forall p < t_d \\ X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in J, \quad \forall p + t > T \\ Y_{i,j,t} = Z_{i,j,t} = 0 \quad \forall (i,j) \in \Gamma, \quad \forall t > T - T_{m(i,j)}\end{array}\right\} \tag{4}$$

Each attraction needs to be visited for at least 30 minutes. Each restaurant needs to be given 60 minutes. No activity is allowed to end after 9pm.

$$\left.\begin{array}{l}\sum_{j\in Outdoor}\sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p} \geqslant 3 \\ \sum_{j\in Outdoor}\sum_{t=0}^{T}\sum_{p=0}^{T} W_{j,t,p} \geqslant 3\end{array}\right\} \tag{5}$$

At least 3 attractions must be outdoor for each day.

$$\left.\begin{array}{l}X_{j,t,p} = 0, \quad \forall j \in Outdoor, \quad \forall (t,p) : (t \in Indoortime \vee t + p \in Indoortime \vee Indoortime \subseteq [t, t+p]) \\ W_{j,t,p} = 0, \quad \forall j \in Outdoor, \quad \forall (t,p) : (t \in Indoortime \vee t + p \in Indoortime \vee Indoortime \subseteq [t, t+p])\end{array}\right\} \tag{6}$$

From 11am to 4pm, no outdoor attraction is allowed to be visited during the period.

$$\left.\begin{array}{l}\sum_{j\in\omega}\sum_{t=0}^{T}\sum_{p=0}^{T}X_{j,t,p}+W_{j,t,p}\geqslant U_\omega,\quad\forall\omega\in\Omega\\ \sum_{j\in\omega}\sum_{t=0}^{T}\sum_{p=0}^{T}X_{j,t,p}+W_{j,t,p}\leqslant MU_\omega,\quad\forall\omega\in\Omega\\ \sum_{\omega\in\Omega}U_\omega\geqslant 4\end{array}\right\}\tag{7}$$

At least 4 regions must be visited.

$$\left.\begin{array}{l}\sum_{k\in J\setminus j}Y_{(k,j),max(t-Tm_{(k,j)},0)}=\sum_{p=0}^{T}X_{j,t,p},\quad\forall j\in J,\quad\forall t=1,2,3\dots,T\\ \sum_{k\in J\setminus j}Y_{(j,k),t}=\sum_{s,p:s+p=t}X_{j,s,p},\quad\forall j\in J,\quad\forall t=0,1,2,\dots,T-1\\ \sum_{k\in J\setminus j}Z_{(k,j),max(t-Tm_{(k,j)},0)}=\sum_{p=0}^{T}W_{j,t,p},\quad\forall j\in J,\quad\forall t=1,2,3\dots,T\\ \sum_{k\in J\setminus j}Z_{(j,k),t}=\sum_{s,p:s+p=t}W_{j,s,p},\quad\forall j\in J,\quad\forall t=0,1,2,\dots,T-1\end{array}\right\}\tag{8}$$

Inspired by Network Flow Problem. These constraints combined together guarantees that the scheduled forms a end-to-end loop.

$$\left.\begin{array}{l}X_{h_i,0,0}+\sum_{t=0}^{T}Y_{(h_i,h_j),t}+X_{h_j,T,0}+W_{h_j,0,0}+W_{h_j,T,0}\geqslant 5-5(1-H_{h_i,h_j}),\quad\forall(h_i,h_j)\in H\times H\\ \sum_{(h_i,h_j)}H_{h_i,h_j}=1,\quad\forall(h_i,h_j)\in H\times H\\ \sum_{h\in H}X_{h,0,0}=1,\quad\sum_{h\in H}X_{h,T,0}=1,\quad\sum_{h\in H}W_{h,0,0}=1,\quad\sum_{h\in H}W_{h,T,0}=1,\end{array}\right\}\tag{9}$$

These constraints together models the moving from any hotel to another hotel (including the case where hotel remain unchanged) at any time on the first day.

$$Y_{(j,j),t}=0,\quad\forall j\in A\cup D,\quad\forall t=0,1,2\dots,T\tag{10}$$

Travelling between same places is prohibited.
This constraint is redundant given constraint 8, but is included because it reduces the number of variables. It is important to note that moving between same hotels is allowed. This is crucial for constraint 9 when there is no hotel changing.

$$\left.\begin{array}{l}X_{h,0,p}=0,\quad\forall h\in H,\quad\forall p\geqslant 1\\ W_{h,0,p}=0,\quad\forall h\in H,\quad\forall p\geqslant 1\end{array}\right\}\tag{11}$$

Leaves hotel at exactly 9 am. This reduces number of variables. The beginning and ending of each day is fixed. The flexibility of schedule is given by other X's/W's.

3 Solutions

3.1 Low Budget Plan

The mathematical model was able to find an optimal integer solution which is shown below:

Time	Location	Type of Location
9:00 a.m	Mandarin Orchard	Hotel
9:10 a.m	Botanic Gardens	Attraction
10:30 a.m	Mandarin Orchard Singapore	Hotel
12:00 a.m	Odette	Dining
1:20 p.m	Hilton Garden Inn Singapore Serangoon	Hotel
4:00 p.m	East Coast Park	Attraction
5:20 p.m	Punggol Waterway Park	Attraction
6:20 p.m	Chomp Chomp Food Centre	Dining
8:10 p.m	Mandarin Orchard Singapore	Hotel
9:00 p.m	Oasia Residence Singapore	Hotel

Table 1: Schedule for Day 01

Time	Location	Type of Location
9:00 a.m	Oasia Residence Singapore	Hotel
9:10 a.m	Haw Par Villa	Attraction
10:50 a.m	Oasia Residence Singapore	Hotel
12:20 a.m	Timbre+	Dining
2:20 p.m	Oasia Residence Singapore	Hotel
6:00 p.m	Buey Tahan See-Food CCK	Dining
7:20 p.m	Sungei Buloh Wetland Reserve	Attraction
8:10 p.m	Chinese Garden	Attraction
9:00 p.m	Oasia Residence Singapore	Hotel

Table 2: Schedule for Day 02

Thus, the total cost of the day amounts to SGD 25.60 (rounded off to 2 decimal place).

3.2 Medium Budget Plan

The mathematical model was able to find an optimal integer solution which is shown below:

Time	Location	Type of Location
9:00 a.m	Hilton Garden Inn Singapore Serangoon	Hotel
9:10 a.m	Mandarin Orchard Singapore	Hotel
09:30 a.m	Botanic Gardens	Attraction
10:20 a.m	Haw Par Villa	Attraction
11:00 p.m	Oasia Residence Singapore	Hotel
12:00 p.m	Miam Miam French and Japanese Cafe Kitchen	Dining
1:10 p.m	The Rink	Attraction
2:00 p.m	Science Centre Singapore	Attraction
3:10 p.m	Singapore Discovery Centre	Attraction
4:00 p.m	Jurong Bird Park	Attraction
4:50 p.m	Chinese Garden	Attraction
5:40 p.m	Sungei Buloh Wetland Reserve	Attraction
7:00 p.m	Jamie's Italian	Dining
8:10 p.m	Fort Canning Art Centre	Attraction
9:00 p.m	Hilton Garden Inn Singapore Serangoon	Hotel

Table 3: Schedule for Day 01

Time	Location	Type of Location
9:00 a.m	Hilton Garden Inn Singapore Serangoon	Hotel
9:10 a.m	Gardens by the bay	Attraction
10:20 a.m	East Coast Park	Attraction
11:20 a.m	Crowne Plaza Hotel @Changi	Hotel
12:00 p.m	Simpang Bedok	Dining
1:50 p.m	Jewel Changi Airport	Attraction
4:00 p.m	Wild Wild Wet	Attraction
4:40 p.m	Pulau Ubin	Attraction
5:30 p.m	Coney Island	Attraction
6:10 p.m	Punggol Waterway Park	Attraction
6:50 p.m	Punggol Nasi lemak	Dining
8:10 p.m	ArtScience Museum	Attraction
9:00 p.m	Hilton Garden Inn Singapore Serangoon	Hotel

Table 4: Schedule for Day 02

Thus, the total cost of the day amounts to SGD 269.50 (rounded off to 2 decimal place).

3.3 High Budget Plan

The mathematical model was able to find an optimal integer solution which is shown below:

Time	Location	Type of Location
9:00 a.m	Hilton Garden Inn Singapore Serangoon	Hotel
9:10 a.m	Gardens by the bay	Attraction
10:10 a.m	East Coast Park	Attraction
11:20 a.m	1-Altitude	Attraction
12:00 p.m	Fort Canning Art Centre	Attraction
12:40 p.m	Odette	Dining
1:50 p.m	ArtScience Museum	Attraction
3:20 p.m	Jewel Changi Airport	Attraction
4:10 p.m	Wild Wild Wet	Attraction
4:50 p.m	Pulau Ubin	Attraction
5:40 p.m	Coney Island	Attraction
6:20 p.m	Punggol Waterway Park	Attraction
7:00 p.m	Punggol Nasi lemak	Dining
8:20 p.m	Hilton Garden Inn Singapore Serangoon	Hotel
9:00 p.m	Oasia Residence Singapore	Hotel

Table 5: Schedule for Day 01

Time	Location	Type of Location
9:00 a.m	Oasia Residence Singapore	Hotel
9:10 a.m	Haw Par Villa	Attraction
10:10 a.m	Botanic Gardens	Attraction
11:00 a.m	River Safari	Attraction
12:00 p.m	Miam Miam French and Japanese Cafe Kitchen	Dining
1:20 p.m	The Rink	Attraction
2:00 p.m	Science Centre Singapore	Attraction
3:00 p.m	Singapore Discovery Centre	Attraction
4:00 p.m	Jurong Bird Park	Attraction
4:50 p.m	Sungei Buloh Wetland Reserve	Attraction
5:40 p.m	Chinese Garden	Attraction
6:20 p.m	Buey Tahan See-Food CCK	Dining
7:40 p.m	Night Safari	Attraction
8:10 p.m	Singapore Zoo	Attraction
9:00 p.m	Oasia Residence Singapore	Hotel

Table 6: Schedule for Day 02

Thus, the total cost of the day amounts to SGD 442.32 (rounded off to 2 decimal place).

4 Limitations and Reflections

1. In order to increase the solver efficiency, attractions visited might not be distributed uniformly in different regions due to non-linear representation of entropy.
2. One more dimension for decision variables to introduce more flexibility
3. When there is no feasible solution returned by solver: firstly, eliminate constraints that do not relate to fundamental framework of the formulation, such as the requirement of 3

outdoor activities and 4 regions. Secondly, eliminate constraints one by one to identify the problematic constraint. Thirdly, assign value manually to decision variables such that it represents a simple feasible solution that should satisfy the constraints. Finally, run the model and analyse the error message.

4. Edge case construction
5. Be careful about equality constraint
6. Finding: when lambda becomes larger, the running time tends to be shorter.

5 Appendix

5.1 explanation of some Constraints in Task 3

$$\left.\begin{array}{l} \sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} X_{j,s,p} + \sum_{(l,m)\in\Gamma}\sum_{s=max(t+1-Tm_{(l,m)},0)}^{t} Y_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\dots,T \\ \sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} W_{j,s,p} + \sum_{(l,m)\in\Gamma}\sum_{s=max(t+1-Tm_{(l,m)},0)}^{t} Z_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\dots,T \end{array}\right\}$$

This is the second constraint in the model. The first term on the left-hand-side means whether there is an activities in any attraction or restaurant at time instance **t** (If there is an activity at time **t**, it must start between time **0** and time **t**, let's call it **s**, also this activity should lasts longer than **t - s + 1**). The second term means whether at time **t**, the traveller is taking transportation. The summation of this two terms should less or equal than **1** to guarantee at most one activity is taken at each time instant.

The eighth constraints:

$$\left.\begin{array}{l} \sum_{k\in J\backslash j} Y_{(k,j),max(t-Tm_{(k,j)},0)} = \sum_{p=0}^{T} X_{j,t,p}, \quad \forall j\in J, \quad \forall t = 1,2,3\dots,T \\ \sum_{k\in J\backslash j} Y_{(j,k),t} = \sum_{s,p:s+p=t} X_{j,s,p}, \quad \forall j\in J, \quad \forall t = 0,1,2,\dots,T-1 \\ \sum_{k\in J\backslash j} Z_{(k,j),max(t-Tm_{(k,j)},0)} = \sum_{p=0}^{T} W_{j,t,p}, \quad \forall j\in J, \quad \forall t = 1,2,3\dots,T \\ \sum_{k\in J\backslash j} Z_{(j,k),t} = \sum_{s,p:s+p=t} W_{j,s,p}, \quad \forall j\in J, \quad \forall t = 0,1,2,\dots,T-1 \end{array}\right\}$$

For the equation, right-hand-side means whether the traveller starts doing activity at place **j** time **t**, left-hand-side means whether there is a travelling from some place **k** to place **j** at certain time, this two events should be logically equivalent. The second equation means if there is a travelling from **j** to **k**, there should be an activity at place j time **s** and lasts p such that **s + p = t**.

The constraints about changing hotel:

$$\left.\begin{array}{l} X_{h_i,0,0} + \sum_{t=0}^{T} Y_{(h_i,h_j),t} + X_{h_j,T,0} + W_{h_j,0,0} + W_{h_j,T,0} \geqslant 5 - 5(1 - H_{h_i,h_j}), \quad \forall (h_i,h_j)\in H\times H \\ \sum_{(h_i,h_j)} H_{h_i,h_j} = 1, \quad \forall (h_i,h_j)\in H\times H \\ \sum_{h\in H} X_{h,0,0} = 1, \quad \sum_{h\in H} X_{h,T,0} = 1, \quad \sum_{h\in H} W_{h,0,0} = 1, \quad \sum_{h\in H} W_{h,T,0} = 1, \end{array}\right\}$$

If the traveller will change the hotel from **i** to **j** at day one, the traveller must stay in hotel **i** at the start of the first day, move from **i** to **j** at a time instance (corresponds to the summation of Y), stay in hotel **j** at the end of the first day, and stay in **j** at the start and end of the second day.

5.2 Verify the Solution

Since the decision variables in task 3 have three dimensional index, it is not practical to check the decision variables directly by using the command:

display w;

So instead, we use the command:

display {j in J, t in T, p in T: w[j, t, p] == 1};

Then AMPL will console will print a set of indices (j, t, p) such that w[j, t, p] is 1:

{(3,18,9) (14,1,4) (19,46,3) (27,55,3) (37,72,0) (7,59,6) (17,8,3) (20,42,3) (35,14,2) (11,67,4) (18,29,11) (26,51,3) (37,0,0);};

Then sort the tuple by the second index (corresponding to the starting time of the activity). Extract the first index in sorted list of tuple, that is the sequence of places in chronological order:

37, 14, 17, 35, 3, 18, 20, 19, 26, 27, 7, 11, 37

For transportation decision variable z[i, j, t], using similar command, we will get:

{(3,18,27) (11,37,71) (17,35,11) (19,26,49) (26,27,54) (35,3,16) (7,11,65) (14,17,5) (18,20,40) (20,19,45) (27,7,58) (37,14,0)};

Sort the list of tuple by the third index of the tuple:

{ (37, 14, 0) (14, 17, 5) (17, 35, 11) (35, 3, 16) (3, 18, 27) (18, 20, 40) (20, 19, 45) (19, 26, 49) (26, 27, 54) (27, 7, 58) (7, 11, 65) (11, 37, 71) };

This means, travel from 37 to 14 at 0, from 14 to 17 at 5, so on and so forth. Then we can verify w and z are corresponding with each other.

5.3 Code for Distance Generation

See the attached dataPrepocess.py file

5.4 Code for Solution Parsing

See the attached recoverPath.py file