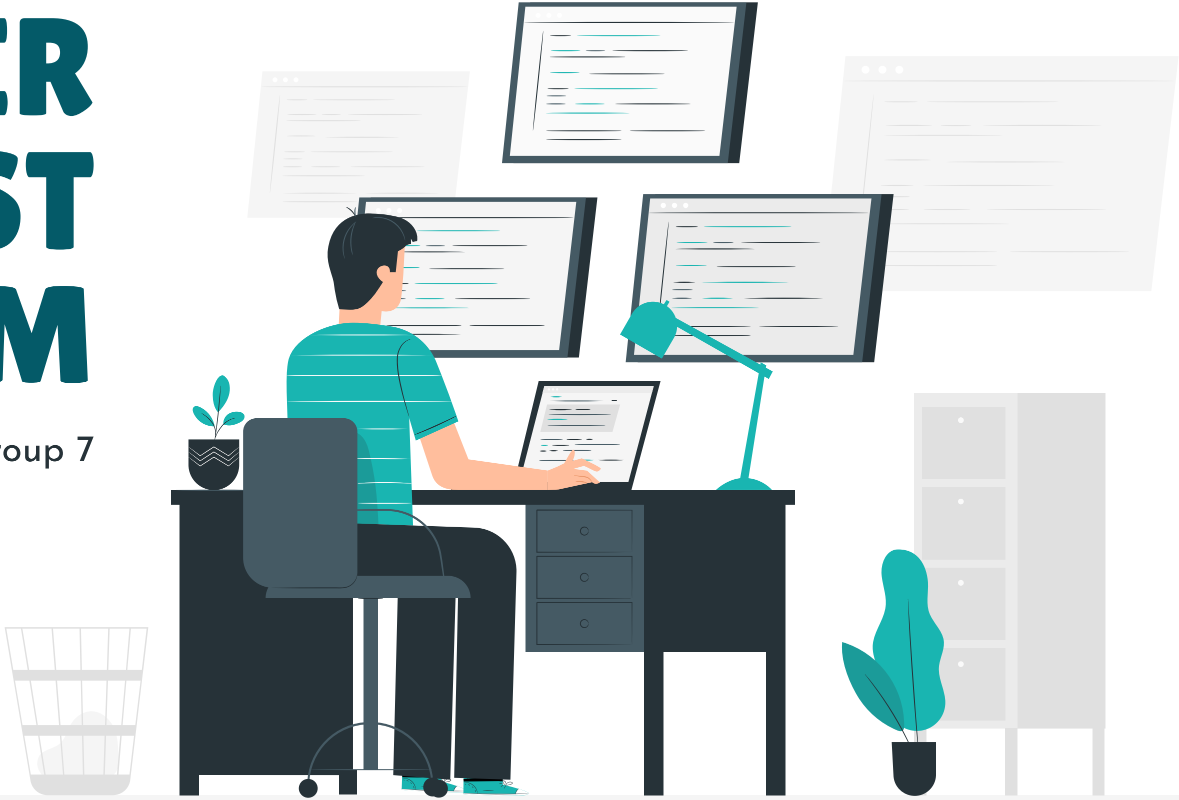
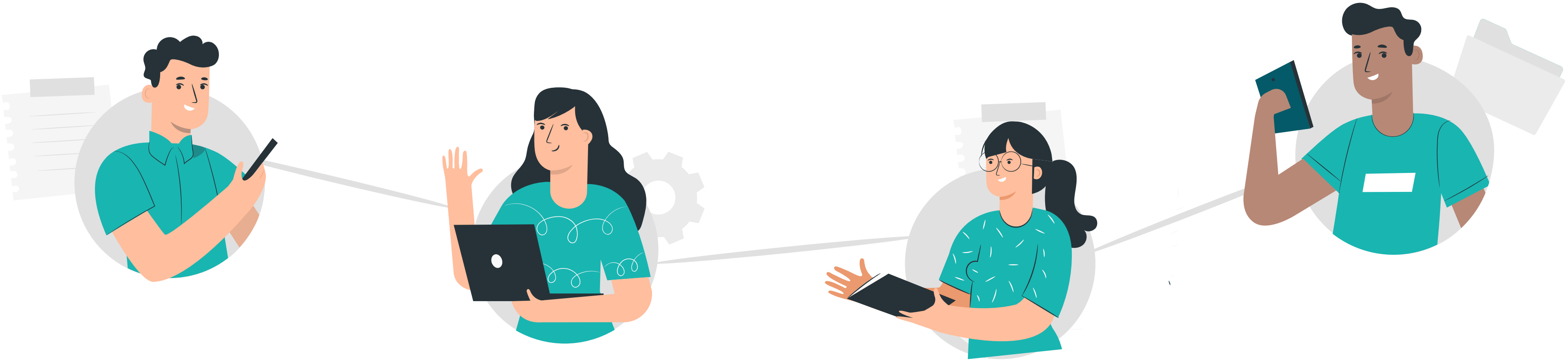
HIPSTER TOURIST PROBLEM

Group 7



GROUP 07



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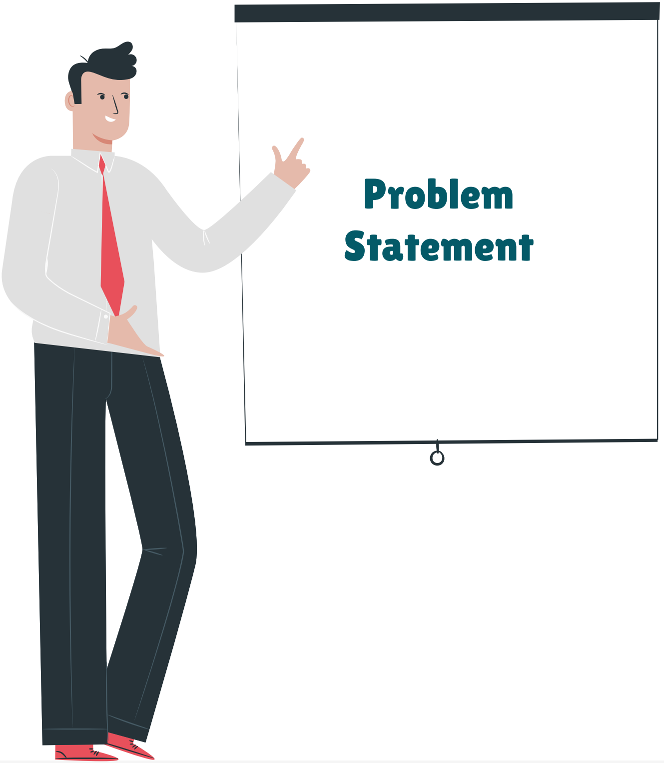


TASK 2

Schedule your friend's one-day visit in Singapore from 9 a.m to 9 p.m, given:

24 attractions,
10 dining places,

subjected to a series of constraints.

ASSUMPTIONS

01

Time Instance

Each time instance is 10 minutes.

02

Mode of Transport

All transportation is done by MRT at the rate of 10 km per hour according to google map navigation

03

Distance and Cost

Linear relationship between distance travelled and transport cost.

04

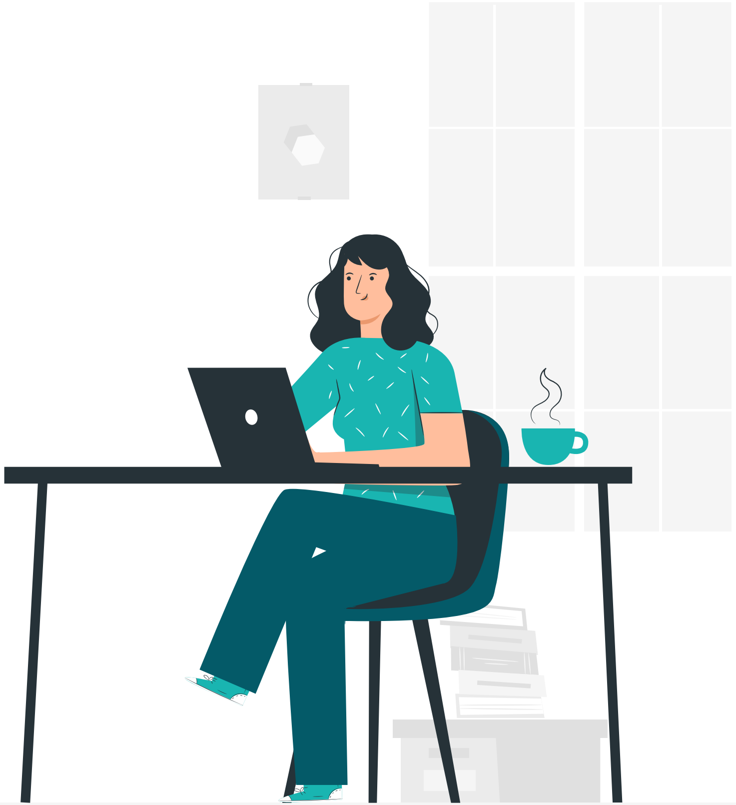
Meal Timings

Lunch and Dinner must start in the time interval 12-2 pm and 6-8 pm respectively, but may end after.

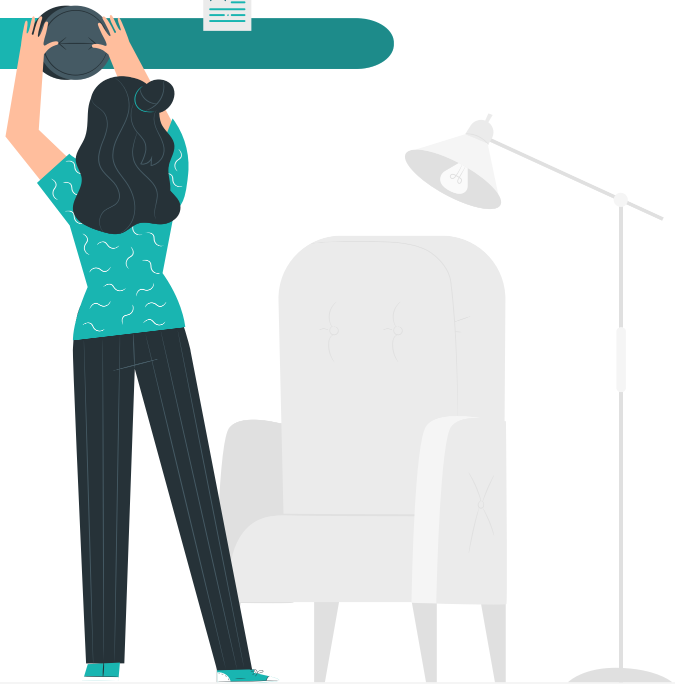
05

Visiting Time

Each attraction is visited for exactly 30 minutes.



METHODOLOGY



01 To find the distances of each attraction, dining location and hotel, from each other, we found the **geographical location** of each place in term of its **latitude and longitude.**

02 Using Python and the **Haversine** formula, we calculated all the **straight path distances** between any two possible locations using the latitude and longitude, assuming the earth is a perfect sphere. (code in the Appendix)

03 By inputting the matrix of distance, we were able to formulate the model to minimise the distance travelled. This in turn would minimise the transportation cost, due to the **linear relationship between distance and cost.**

Task 3

In addition to the constraints in Task 2:

- There are 2 days to travel
- Option to change hotels out of 4 alternatives
- Maximize diversity instead of minimising cost
- Three different budget travel plans of different total costs

ASSUMPTIONS

01 Time Instance

02 Mode of Transport

03 Distance and Cost

04 Meal Timings

Lunch and Dinner must **start and finish** in the time interval 12-2 pm and 6-8 pm respectively.

05 Visiting Time

Each attraction is visited for **at least 30** minutes, with **no upper bound**.

05 Definition of the Objective Function

In order to maximise the diversity of visited places, we have set a minimum bar for our MIP model:

1. Visit at least 3 outdoor attractions each day.
2. Visit at least 4 regions in two days.

METHODOLOGY

01 , 02, 03 **Keep as the same.**

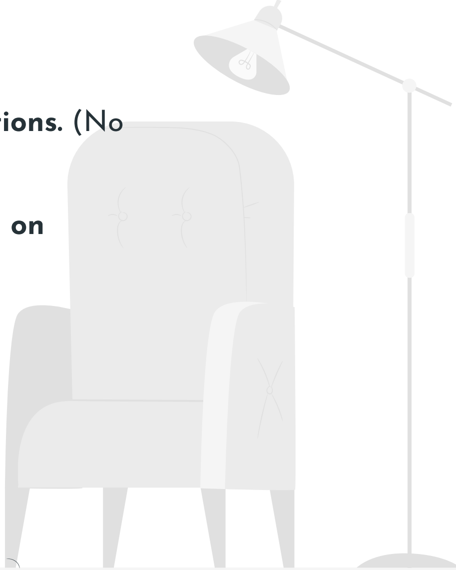
04 Non-heuristic approach: pure Mixed Integer Program to **consider all feasible solutions.** (No shrinkage of solution space)

05 Add one more subscript to decision variable Xi,j. So that Xi,j,p has **no upper bound on processing time** on each task.

06 **Two sets of variables** of the same definitions for each day.

07 Use **Big M** to decide whether to move hotel.

08 Use **Regularization** to obtain 3 budget plans by tuning parameter Lambda.



MIP Model : Task 02

Our MIP Model

Objective Function
& Decision Variables

Decision Variables

$$X_{j,t} = \begin{cases} 1 & \text{arrives at location j at time t} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t} \\ 0 & \text{otherwise} \end{cases}$$

$$U_\omega = \begin{cases} 1 & \text{region } \omega \text{ is visited} \\ 0 & \text{otherwise} \end{cases}$$

Objective Function

$$\min \sum_{(l,m)\in\Gamma} \sum_{t=0}^{T} Y_{(l,m),t} C_{(l,m)} + \sum_{j\in A} \sum_{t=0}^{T} X_{j,t} \vec{c}_j$$

Our MIP Model

Parameters

Parameters

Constants

$T = (21 - 9) \times (60 \div 10) - 1 = 71$
$t_a = 3$
$t_d = 6$

D : distance matrix where $D(i, j)$ represents distance between location i and j. This matrix is symmetric.

T_m : time matrix where $T_m(i, j)$ represents travelling time between location i and j. This matrix is symmetric. As the assumption stated, this is in proportion to distance matrix, except that it is rounded to integer values.

$\vec{c}$: cost vector where where $\vec{c}_j$ represents entry fee for attraction with index j.

C : cost matrix where where $C(i, j)$ represents transportation fee from location i to location j. According to MRT charging standard, $C = 0.1D$

Notes: the minimum time unit regarding this optimization task is 10 minutes. T is the maximum time index for the model.

Our MIP Model

Defining Sets

Sets

$J = \{0, 1, 2, \ldots, 33\}$ is the set of indices for all locations
$H = \{33\}$ is the set of indices for hotel
$A = \{10{,}11{,}12, \ldots, 32\}$ is the set of indices for all attractions
$D = \{0{,}1{,}2, \ldots, 9\}$ is the set of indices for all dining places
$\Gamma = J \times J$ is the Cartesian product of J denoting indices of edges in the graph
$Outdoor$ is the set of indices for outdoor attractions
$Indoor$ is the set of indices for indoor attractions

$$Region\ sets \begin{cases} N & \text{set of indices for attractions \& dining places in region N} \\ E & \text{set of indices for attractions \& dining places in region E} \\ W & \text{set of indices for attractions \& dining places in region W} \\ O & \text{set of indices for attractions \& dining places in region O} \end{cases}$$

$$Activity\ time\ index\ sets \begin{cases} Lunchtime = \{18, 19, 20, \ldots, 30\} \\ Dinnertime = \{54, 55, 56, \ldots, 66\} \\ Nondiningtime = \{0, 1, 2, \ldots, T\} - Lunchtime - Dinnertime \\ Indoortime = \{12, 13, 14, \ldots, 42\} \end{cases}$$

$\Omega = \{N, E, W, O\}$ is the set of region sets

Our MIP Model

Constraints

Constraints

$$\begin{cases} \sum_{t=0}^{T} X_{j,t} \leqslant 1, & \forall j \in A \cup D \\ \sum_{t=0}^{T} Y_{(l,m),t} \leqslant 1, & \forall (l,m) \in \Gamma \end{cases} \tag{1}$$

Visits each attraction/dining place at most once
Travels between two locations at most once

$$\sum_{s=max(t+1-P_j,0)}^{t} \sum_{j\in J} X_{j,s} + \sum_{s=max(t+1-P_{(l,m)},0)}^{t} \sum_{(l,m)\in\Gamma} Y_{(l,m),t} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T+1 \tag{2}$$

At most one activity is taken at each time instant.

$$X_{j,t} = 0, \text{ if } t \in Nondiningtime, \quad \forall j \in D \tag{3}$$

At non-dining hours, no dining is allowed.

$$\sum_{j\in A} \sum_{t=0}^{T} X_{j,t} \geqslant 8 \tag{4}$$

At least 8 attractions must be visited.

$$\sum_{j\in Outdoor} \sum_{t=0}^{T} X_{j,t} \geqslant 3 \tag{5}$$

At least 3 attractions must be outdoor.

$$X_{j,t} = 0, \quad \forall j \in Outdoor, \quad t \in Indoortime \tag{6}$$

From 11am to 4pm, no outdoor attraction is allowed to be visited.

Our MIP Model

More Constraints

$$\begin{cases} \sum_{j\in\omega}\sum_{t=0}^{T} X_{j,t} \geqslant U_\Omega \\ \sum_{j\in\omega}\sum_{t=0}^{T} X_{j,t} \leqslant MU_\Omega, \quad \forall\omega\in\Omega \end{cases} \tag{7}$$

At least 2 regions other than the region of the hotel must be visited.

$$\left.\begin{array}{l} \sum_{k\in A\cup D\setminus j} Y_{(k,j),max(t-Tm_{(k,j)},0)} = X_{j,t}, \quad \forall j\in J \quad , \quad \forall t = 1,2,3\ldots,T+1 \\ \sum_{k\in A\cup D\setminus j} Y_{(j,k),min(t+P_a,0)} = X_{j,t}, \quad \forall j\in A, \quad \forall t = 0,1,2,\ldots,T \\ \sum_{k\in A\cup D\setminus j} Y_{(j,k),min(t+P_d,0)} = X_{j,t}, \quad \forall j\in D, \quad \forall t = 0,1,2,\ldots,T \\ \sum_{k\in A\cup D\setminus j} Y_{(33,k),0} = 1 \end{array}\right\} \tag{8}$$

Inspired by Travelling Salesman Problem (TSP). These constraints combined together guarantee that the scheduled plan forms a perfect loop. First constraint guarantees that there is an edge pointing towards each node while the last three guarantee that there is an edge out of each node. Assumption that each attraction is visited exactly 30 minutes is made.

$$X_{H,0} = 1, \quad X_{H,T+1} = 1 \tag{9}$$

The hotel must be the starting point at time 0 and ending point at time T+1.

$$Y_{(j,j),t} = 0, \quad \forall j\in J, \quad \forall t = 0,1,2\ldots,T+1 \tag{10}$$

Travelling between same places is prohibited.

$$\left.\begin{array}{l} \sum_{t\in lunchtime}\sum_{j\in D} X_{j,t} = 1 \\ \sum_{t\in dinnertime}\sum_{j\in D} X_{j,t} = 1 \end{array}\right\} \tag{11}$$

Lunch and dinner are compulsory.

FINAL SOLUTION

SGD $ 75.09

Minimised Cost of Transport and Attractions for Task 2

SCHEDULE

9:00 Mandarin Orchard

9:30 Haw Par Villa

10:30 Chinese Garden

11:30 Singapore Discovery Centre

12:30 Science Centre Singapore

13:30 Timbre+

14:50 Mandarin Orchard

15:20 Garden by the Bay

16:20 East Coast Park

17:40 Punggol Waterway Park

18:40 Chomp Chomp Food Centre

20:10 Botanic Garden

20:50 Mandarin Orchard

MIP Model : Task 03

MIP Model

Objective Function

Objective Function

$$\max \sum_{j\in A}\sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p} - \lambda\Big(\sum_{(l,m)\in\Gamma}\sum_{t=0}^{T} Y_{(l,m),t}C_{(l,m)} + \sum_{j\in J}\sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p}c_j\Big)$$

Notes: λ is the regularization parameter used to indicate the importance of cost. A high lambda means the budget limit is low. A low lambda means the budget limit is high.

MIP Model

Decision Variables

Decision Variables

$$X_{j,t,p} = \begin{cases} 1 & \text{arrives at location j at time t and stays for p units of time in the first day} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t in the first day} \\ 0 & \text{otherwise} \end{cases}$$

$$W_{j,t,p} = \begin{cases} 1 & \text{arrives at location j at time t and stays for p units of time in the second day} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{(l,m),t} = \begin{cases} 1 & \text{leaves location l for location m at time t in the second day} \\ 0 & \text{otherwise} \end{cases}$$

$$H_{h_i,h_j} = \begin{cases} 1 & \text{hotel changes from hotel i to hotel j} \\ 0 & \text{otherwise} \end{cases}$$

$$U_{\omega} = \begin{cases} 1 & \text{region } \omega \text{ is visited in two days} \\ 0 & \text{otherwise} \end{cases}$$

Notes: $\forall t = 0, 1, 2, \ldots T, \quad \forall p = 0, 1, 2, \ldots T, \quad \forall (h_i, h_j) \in H \times H, \quad \forall \omega \in \Omega$

MIP Model

Constraints

Constraints

$$\left.\begin{array}{l} \sum_{t=0}^{T}\sum_{p=0}^{T} X_{j,t,p} + W_{j,t,p} \leqslant 1, \quad \forall j \in A \cup D \\ \sum_{t=0}^{T} Y_{(l,m),t} + Z_{(l,m),t} \leqslant 1, \quad \forall (l,m) \in \Gamma \end{array}\right\}$$

Visits each attraction/dining place at most once
Travels between two locations at most once
Notes: The second constraint is probably redundant.

$$\left.\begin{array}{l} \sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} X_{j,s,p} + \sum_{s=max(t+1-P_{(l,m)},0)}^{t}\sum_{(l,m)\in\Gamma} Y_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T \\ \sum_{j\in J}\sum_{s=0}^{t}\sum_{p=t-s+1}^{T} W_{j,s,p} + \sum_{s=max(t+1-P_{(l,m)},0)}^{t}\sum_{(l,m)\in\Gamma} Z_{(l,m),s} \leqslant 1, \quad \forall t = 0,1,2,\ldots,T \end{array}\right\}$$

At most one activity is taken at each time instant.

MIP Model

More Constraints

$$\left.\begin{array}{l}
\sum_{t,p:[t,t+p]\subseteq Lunchtime} X_{j,t,p} = 1, \quad \forall j \in D \\
\sum_{t,p:[t,t+p]\subseteq Dinnertime} X_{j,t,p} = 1, \quad \forall j \in D \\
\sum_{t,p:[t,t+p]\subseteq Lunchtime} W_{j,t,p} = 1, \quad \forall j \in D \\
\sum_{t,p:[t,t+p]\subseteq Dinnertime} W_{j,t,p} = 1, \quad \forall j \in D \\
\sum_{j\in D} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} = 2 \\
\sum_{j\in D} \sum_{t=0}^{T} \sum_{p=0}^{T} W_{j,t,p} = 2
\end{array}\right\}$$

Lunch and dinner are compulsory for each day. The duration of lunch/dinner must be inside the specified lunch/dinner period. There are only two meals each day.

$$\left.\begin{array}{l}
X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in A, \quad \forall t = 0,1,2,\ldots,T, \quad \forall p < t_a \\
X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in D, \quad \forall t = 0,1,2,\ldots,T, \quad \forall p < t_d \\
X_{j,t,p} = W_{j,t,p} = 0 \quad \forall j \in J, \quad \forall p + t > T \\
Y_{i,j,t} = Z_{i,j,t} = 0 \quad \forall (i,j) \in Gamma, \quad \forall t > T - T_{m(i,j)}
\end{array}\right\} \tag{4}$$

Each attraction needs to be visited for at least 30 minutes. Each restaurant needs to be given 60 minutes. No activity is allowed to end after 9pm.

$$\left.\begin{array}{l}
\sum_{j\in Outdoor} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} \geqslant 3 \\
\sum_{j\in Outdoor} \sum_{t=0}^{T} \sum_{p=0}^{T} W_{j,t,p} \geqslant 3
\end{array}\right\} \tag{5}$$

At least 3 attractions must be outdoor for each day.

MIP Model

Even More Constraints

$$\left.\begin{array}{l} X_{j,t,p} = 0, \quad \forall j \in Outdoor, \quad (t,p): t \in Indoor \vee t+p \in Indoor \vee Indoor \subseteq [t, t+p] \\ W_{j,t,p} = 0, \quad \forall j \in Outdoor, \quad (t,p): t \in Indoor \vee t+p \in Indoor \vee Indoor \subseteq [t, t+p] \end{array}\right\} \quad (6)$$

From 11am to 4pm, no outdoor attraction is allowed to be visited during the period.

$$\left.\begin{array}{l} \sum_{j\in\omega} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} + W_{j,t,p} \geqslant U_\omega, \quad \forall \omega \in \Omega \\ \sum_{j\in\omega} \sum_{t=0}^{T} \sum_{p=0}^{T} X_{j,t,p} + W_{j,t,p} \leqslant MU_\omega, \quad \forall \omega \in \Omega \\ \sum_{\omega\in\Omega} U_\omega \geqslant 4 \end{array}\right\} \quad (7)$$

At least 4 regions must be visited.

$$\left.\begin{array}{l} \sum_{k\in J\setminus j} Y_{(k,j),max(t-Tm_{(k,j)},0)} = \sum_{p=0}^{T} X_{j,t,p}, \quad \forall j \in J, \quad \forall t = 1,2,3\ldots,T \\ \sum_{k\in J\setminus j} Y_{(j,k),t} = \sum_{s,p:s+p=t} X_{j,s,p}, \quad \forall j \in J, \quad \forall t = 0,1,2,\ldots,T-1 \\ \sum_{k\in J\setminus j} Z_{(k,j),max(t-Tm_{(k,j)},0)} = \sum_{p=0}^{T} W_{j,t,p}, \quad \forall j \in J, \quad \forall t = 1,2,3\ldots,T \\ \sum_{k\in J\setminus j} Z_{(j,k),t} = \sum_{s,p:s+p=t} W_{j,s,p}, \quad \forall j \in J, \quad \forall t = 0,1,2,\ldots,T-1 \end{array}\right\} \quad (8)$$

Inspired by Travelling Salesman Problem (TSP). These constraints combined together guarantee that the scheduled plan forms a perfect loop.

MIP Model

Even Even More Constraints

$$\left.\begin{array}{l} X_{h_i,0,0} + \sum_{t=0}^{T} + X_{h_j,T,0} + W_{h_j,0,0} + W_{h_j,T,0} \geqslant 5 - 5(1 - H_{h_i,h_j}), \quad \forall (h_i, h_j) \in H \times H \\ \sum_{(h_i,h_j)} H_{h_i,h_j} = 1, \quad \forall (h_i, h_j) \in H \times H \\ \sum_h X_{h,0,0} = 1, \quad \sum_h X_{h,T,0} = 1, \quad \sum_h W_{h,0,0} = 1, \quad \sum_h W_{h,T,0} = 1, \end{array}\right\} \tag{9}$$

These constraints together models the moving from any hotel to another hotel (including the case the hotel remain unchanged) at any time in the first day.

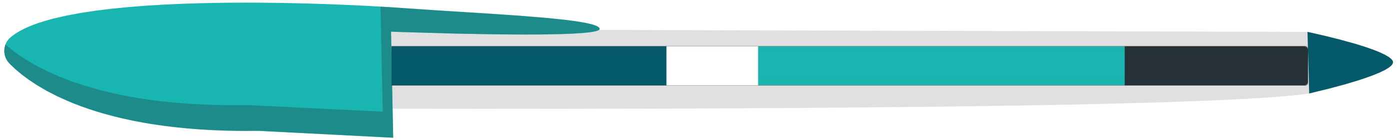
$$Y_{(j,j),t} = 0, \quad \forall j \in A \cup D, \quad \forall t = 0, 1, 2 \ldots, T \tag{10}$$

Travelling between same places is prohibited. This constraint is redundant given constraint 8. This constraint is included because it reduces number of variables for the problem. It is important to note that moving between same hotels is allowed. This is crucial for constraint 9 when there is no hotel changing.

$$\left.\begin{array}{l} X_{h,0,p} = 0, \quad \forall h \in H, \quad \forall p \geqslant 1 \\ W_{h,0,p} = 0, \quad \forall h \in H, \quad \forall p \geqslant 1 \end{array}\right\} \tag{11}$$

Leaves hotel at exactly 9am. This reduces number of variables. The beginning and ending of each day is fixed. The flexibility of schedule is given by other X's/W's.

TASK 03 : FINAL SOLUTIONS



SGD $ 25.60

Low Cost & **Low** Diversity

SGD $ 269.50

Medium Cost & **Average** Diversity

SGD $ 442.32

High Cost & **High** Diversity

Schedule of LOW Budget : Day 1

9:00 Mandarin Orchard Singapore

9:10 Botanic Gardens

10:30 Mandarin Orchard Singapore

12:00 Odette

13:20 Hilton Garden Inn SIngapore Serangoon

16:00 East Coast Park

17:20 Punggol Waterway Park

18:20 Chomp Chomp Food Centre

20:10 Mandarin Orchard Singapore

21:00 Oasia Residence Singapore

Schedule of LOW Budget : Day 2

9:00
Oasia Residence Singapore

9:10
Haw Par Villa

10:50
Oasia Residence Singapore

12:20
Timbre+

14:20
Oasia Residence Singapore

18:00
Buey Tahan See-Food CCK

19:20
Sungei Buloh Wetland Reserve

20:10
Chinese Garden

21:00
Oasia Residence Singapore

Schedule of MEDIUM Budget : Day 1

- **9:00** Hilton Garden Inn Singapore Serangoon
- **9:10** Mandarin Orchard Singapore
- **9:30** Botanic Gardens
- **10:20** Haw Par Villa
- **11:00** Oasia Residence Singapore
- **12:00** Miam Miam French and Japanese Cafe Kitchen
- **13:00** The Rink
- **14:00** Science Centre Singapore
- **15:10** Singapore Discovery Centre
- **16:00** Jurong Bird Park
- **16:50** Chinese Garden
- **17:40** Sungei Buloh Wetland Reserve
- **19:00** Jamie's Italian
- **20:10** Fort Canning Art Centre
- **21:00** Hilton Garden Inn Singapore Serangoon

Schedule of MEDIUM Budget : Day 2

- **9:00** Hilton Garden Inn Singapore Serangoon
- **9:10** Gardens by the Bay
- **10:20** East Coast Park
- **11:20** Crowne Plaza Hotel @Changi
- **12:00** Simpang Bedok
- **13:50** Jewel Changi Airport
- **16:00** Wild Wild Wet
- **16:40** Pulau Ubin
- **17:30** Coney Island
- **18:10** Punggol Waterway Park
- **18:50** Punggol Nasi Lemak
- **20:10** Art Science Museum
- **21:00** Hilton Garden Inn Singapore Serangoon

Schedule of HIGH Budget : Day 1

9:00
Hilton Garden Inn Singapore Serangoon

9:10
Gardens by the Bay

10:10
East Coast Park

11:20
1 Altitude

12:00
Fort Canning Art Centre

12:40
Odette

13:50
Art Science Museum

15:20
Jewel Changi Airport

16:10
Wild Wild Wet

16:50
Pulau Ubin

17:40
Coney Island

18:20
Punggol Waterway Park

19:00
Punggol Nasi Lemak

20:20
Hilton Garden Inn Singapore Serangoon

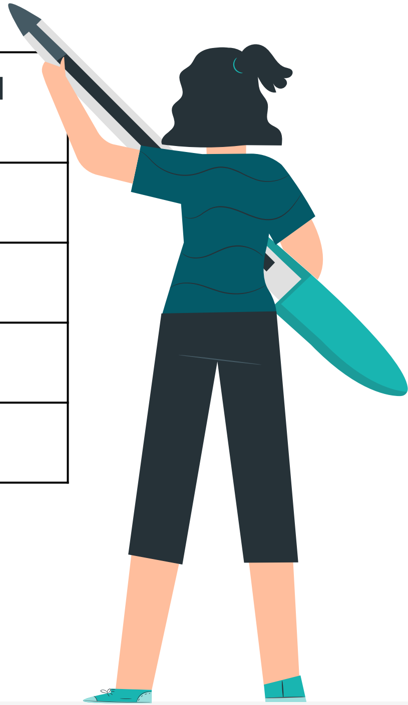
21:00
Oasia Residence Singapore

Schedule of HIGH Budget : Day 2

- **9:00** Oasia Residence Singapore
- **9:10** Haw Par Villa
- **10:10** Botanic Gardens
- **11:00** River Safari
- **12:00** Miam Miam French & Japanese
- **12:40** The Rink
- **14:00** Singapore Centre Singapore
- **15:00** Singapore Discovery Centre
- **16:00** Jurong Bird Park
- **16:50** Sungei Buloh Wetland Reserve
- **17:40** Chinese Garden
- **18:20** Buey Tahan Sea-Food CCK
- **19:40** Night Safari
- **20:10** Singapore Zoo
- **21:00** Oasia Residence Singapore

Solutions Given Under Different Lambda

Lambda	Objective Value	Cost (S$)	No. of Attractions to Visit	Time Elapsed (AMPL)
0.01	16.5768	442.32	21	6230.7
0.025	10.2623	269.51	17	2815.22
0.05	6.0682	98.64	11	323.72
0.2	0.88	25.6	6	303.02



Reflections

Decision Variables

Introduction of more dimensions for decision variables introduces flexibility.

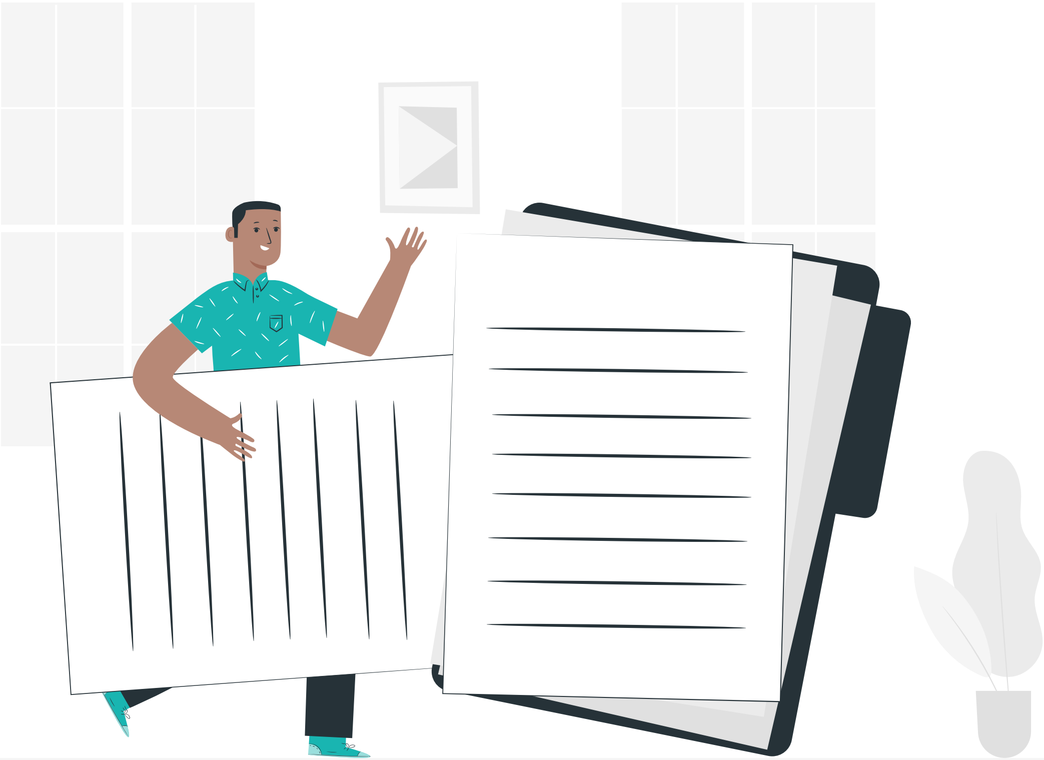
Be careful about equality constraint

Edge Case Constructions

Lambda (Regularization parameter)

When lambda becomes larger, the running time tends to be shorter.

THANK YOU!



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