UM–SJTU Joint Institute

VE477 Intro to Algorithms

Homework 4

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Question 1 Time vs. Space

In this problem we suppose one clock cycle is enough for executing an operation.

(1) NUDT

For 2^{64}

$$\frac{2^{64}}{33.86 \times 10^{15}} = 544.8s$$

For 2^{80}

$$\frac{2^{80}}{33.86 \times 10^{15}} = 3.6 \cdot 10^7 s$$

(2) PC

For each computer, one day it can calculate

$$24 \cdot 60 \cdot 60 \cdot 4 \text{ (cores)} \cdot 3.8 \cdot 10^9 = 1.31 \cdot 10^{15}$$

The total number of computers needed is

$$\frac{2^{64}}{1.31 \cdot 10^{15}} = 14047$$

For 2^{80} operations,

$$\frac{2^{80}}{30 \cdot 1.31 \cdot 10^{15}} = 3.07 \cdot 10^7$$

(3) Storage

To store 2^{64} with the hard drive (16TB),

$$\frac{2^{64}}{8 \cdot 2^{12} \cdot 16} = 3.5 \cdot 10^1 3$$

for 2^{80}

$$\frac{2^{80}}{8 \cdot 2^{12} \cdot 16} = 2.3 \cdot 10^1 8$$

Question 2 Critical Thinking

The critical part in this problem is that every element must be visited. So in one pass, S' could be obtained as

1. First fill in an array $A[k]$ of size k with the first k element in S. *Obviously these k elements are visited.*
2. For the rest elements, each time get a random number $t = rand(1, \text{element index})$, and if the rand number t is smaller or equal to k, replace $A[t]$ with the current element.

This method is effective in that each time when visiting a new element, it has $1/k \cdot k/index = 1/index$ of chance to be selected. And the former elements are derived similarly, so each element has same probability of being selected.

Question 3   Algorithm and Complexity

(1)   Algorithm

The algorithm is given as

Input : i
Output: Sum over the i-th line

Function sum(i)**:**
    $A[0:i][0:i] \leftarrow$ an $(i+1) \times (i+1)$ matrix;
    $A[1:i][1] \leftarrow 1$;
    $A[1][1:i] \leftarrow 1$;
    $A[0][1:i] = A[0:i][0] = 0$;
    for $0 < k \leq i$ *and* $0 < j \leq i$ **do**
        if $k < j$ **then**
            $A[k][j] = A[k-1][j-1] + A[k-2][j-1] + A[k-3][j-1]$
        else if $k > j$ **then**
            $A[k][j] = A[k-1][j-1] + A[k-1][j-2] + A[k-1][j-3]$
        else
            $A[k][j] = A[k-1][j-1] + A[k-2][j-1] + A[k-1][j-2]$
        end if
    end for
    return $\sum A[i][1:i] + A[1:i][i] - A[i][i]$
end

(2)   Complexity

The complexity of this algorithm is $O(i^2)$ since we need to calculate each element in the matrix.
This method is correct in that it exactly follows the definition of the triangle. Each layer i is defined as

$$A[i][1:i] \text{ and } A[1:i][i]$$

Question 4   SAT to 3-SAT

Not done yet.

Question 5   Clique Problem

(1)   Definition

A clique problem is to find the maximum clique(Complete graph) in a given graph.

(2) $\mathcal{NP}$

It is easy to verify that, given a simple certificate, namely given the clique, we are able to check the clique is in the graph and whether it is a clique in polynomial time. ($\mathcal{O}(n^2)$)

(3) 3-SAT to Clique

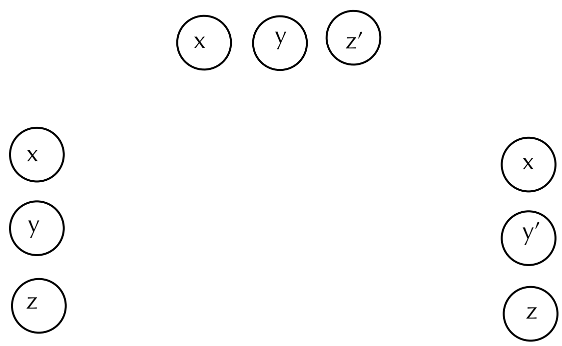
A 3-SAT problem can be transferred to a Clique problem by: setting the 3 literals as from the same group, and connect each literal to all literals satisfying:

1. In other groups
2. Different literal from this literal

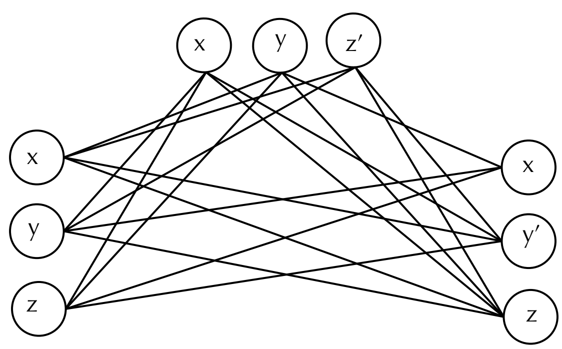
Do this step for all groups. And the clique in this graph evaluates true will lead to a true for the 3-SAT problem. Since this operation can be done in polynomial time, we can conclude the 3-SAT problem can be reduced to a clique problem.

An example is given as

$$(x \vee y \vee z) \wedge (x \vee y \vee z') \wedge (x \vee y' \vee z)$$



The clique is then



(4) Complexity Class

We know that 3-SAT problem is NP-Complete. And since it can reduce to a clique problem in polynomial time, the clique problem is also $\mathcal{NP}$-complete.

Question 6 IND-SET problem

(1) Definition

A set of vertices from a graph that no two are adjacent.

(2) Definition II

Given a graph G and an integer k, does there exist a set of vertex whose size is larger or equal to k s.t. all elements inside are non-adjacent.

(3) $\mathcal{NP}$

To prove it is in $\mathcal{NP}$, we let the clue to be the set of vertex. Through linear search, we can check whether all vertices in the set are non-adjacent, this will lead to $\mathcal{O}(n^2)$ operations, which is polynomial time.

(4) Graph construction

The Graph is constructed as:

Given the graph G with (V, E), let G_1 be the complete graph with V, and denote all edges from G_1 as E_1, the G' is then defined as

$$(V, E_1 \backslash E)$$

(5) Complexity Class

Since we can reduce a 'k-clique' problem into an 'independent set problem' within polynomial operations, the independent set problem is also in $\mathcal{NP}$-complete complexity class.