UM–SJTU Joint Institute
VE477 Intro to Algorithms

Homework 1

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Question 1 Hash Tables

(a)

The condition *exactly k keys hash to a same slot* means *exactly k keys hash to a same slot and the rest* $(n-k)$ *keys hash to the other slots*. Then the probability is calculated as

$$P = \frac{\text{Combs satisfying the condition}}{\text{All Combs}} = \frac{\binom{n}{k} \cdot 1^{n-k} \cdot (n-1)^{n-k}}{n^n} = \binom{1}{n}^k \left(1 - \frac{1}{n}\right)^{n-k} \binom{n}{k}$$

The numerator means *choosing k numbers from n which only belongs to one slot, and the rest has totally* $(n-1)$ *spaces to go*. And the denominator means *All combinations for n numbers*.

(b)

The strongest requirement for *most keys to have exactly 1 key hash to a place*. And in this case, all entries are filled by one element. So the probability is then nP_k.

(c)