Vv557 Methods of Applied Mathematics II
Green Functions for Partial Differential Equations

Assignment 3

Date Due: 1:00 PM, Thursday, the 21st of March 2018

This assignment has a total of **(14 Marks)**.

Exercise 3.1
Calculate the Fourier transforms of the following elements in $L^1(\mathbb{R})$ (the theory of distributions is not needed):

i) $\Pi_{a,b}(x) = \begin{cases} 1 & a < x < b, \\ 0 & \text{otherwise}, \end{cases}$ $\qquad a, b \in \mathbb{R}$.

ii) $e^{-a|x|}$, $a > 0$.

iii) e^{-ax^2}, $a > 0$.

iv) $\cos(x)e^{-x^2}$.

v) $\cos(2x)/(4 + x^2)$.

vi) the convolution of xe^{-x^2} and e^{-x^2}.

(5 Marks)

Exercise 3.2
Calculate the Fourier transforms of the following elements in $\mathcal{S}'(\mathbb{R})$:

i) $\begin{cases} e^{-\varepsilon x} & x \geq 1, \\ 0 & x < 1, \end{cases}$ $\qquad \varepsilon > 0$,

ii) $\sin(3x - 2)$,

iii) $x^2 \cos(x)$,

iv) $xH(x - 2)$,

v) $x^2 \delta(x - 1)$.

(5 Marks)

Exercise 3.3
Consider the wave equation problem for a function $u \colon \mathbb{R}^2 \to \mathbb{R}$,

$$u_{tt} - u_{xx} = 0, \qquad u(x, 0) = f(x), \qquad u_t(x, 0) = g(x)$$

where $f, g \in \mathcal{S}'(\mathbb{R})$ are given. Take the Fourier transform of the equation with respect to the x-variable to obtain an ODE in the t-variable and solve the ODE to obtain

$$\widehat{u}(\xi, t) = \hat{f}(\xi)\cos(\xi t) + \frac{\hat{g}(\xi)}{\xi}\sin(\xi t).$$

Then calculate the inverse Fourier transform (in the distributional sense) to obtain a solution formula for $u(x, t)$.
(4 Marks)