UM–SJTU Joint Institute

VV557 Methods of Applied Math II

Assignment 3

Group 22

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Exercise 2. 1 Fourier Transform

The Fourier Transform is defined as

$$\mathcal{F}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

i).

Plug in the definition of $f(x)$

$$f(x) = \Pi_{a,b}(x) = \begin{cases} 1 & a < x < b \\ 0 & \text{otherwise} \end{cases} \quad , \quad a, b \in \mathbb{R}$$

The Fourier transform is then calculated as

$$\begin{aligned}
\mathcal{F}(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \\
&= \int_{-\infty}^{a} 0 \cdot e^{-i\omega t}dt + \int_{a}^{b} e^{-i\omega t}dt + \int_{b}^{\infty} 0 \cdot e^{-i\omega t}dt \\
&= \int_{a}^{b} e^{-i\omega t}dt \\
&= \left.\frac{e^{-i\omega t}}{-i\omega}\right|_{a}^{b} = \frac{e^{-i\omega b} - e^{-i\omega a}}{-i\omega}
\end{aligned}$$

ii).

$$f(x) = e^{-a|x|}$$

Plug it in Fourier transform, which yields to

$$\begin{aligned}
\mathcal{F}(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \\
&= \int_{-\infty}^{0} e^{ax}e^{-i\omega x}dx \cdot \int_{0}^{+\infty} e^{-ax}e^{-i\omega x}dx \\
&= \left.\frac{e^{x(a-i\omega)}}{a - i\omega}\right|_{-\infty}^{0} \cdot \left.\frac{e^{-x(a+i\omega)}}{-a - i\omega}\right|_{0}^{+\infty} \\
&= -\frac{e^{a-i\omega}}{a - i\omega} \cdot \frac{e^{a+i\omega}}{-a - i\omega} \\
&= \frac{e^{a-i\omega}}{a - i\omega} \cdot \frac{e^{a+i\omega}}{a + i\omega}
\end{aligned}$$