

# Vv557 Methods of Applied Mathematics II

## Green Functions for Partial Differential Equations

### Assignment 2

Date Due: 1:00 PM, Thursday, the 14<sup>th</sup> of March 2018



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This assignment has a total of (10 Marks).

#### Exercise 2.1

Show that

$$g \in \mathcal{D}'(\mathbb{R}^2), \quad g(x) = -\frac{1}{2\pi} \log|x|$$

satisfies  $-\Delta g = \delta(x)$  in the distributional sense.

(3 Marks)

#### Exercise 2.2

For  $u: \mathbb{R}^2 \rightarrow \mathbb{R}$  given by<sup>1</sup>

$$u(x, t) = \begin{cases} 1/2 & t - |x| > 0, \\ 0 & \text{otherwise} \end{cases}$$

calculate  $u_{tt} - u_{xx}$ .

(3 Marks)

#### Exercise 2.3

Show that

$$\frac{d}{dx} \mathcal{P} \left( \frac{1}{x} \right) = -\mathcal{P} \left( \frac{1}{x^2} \right),$$

where

$$\mathcal{P} \left( \frac{1}{x^2} \right) (\varphi) := \lim_{\varepsilon \searrow 0} \int_{|x| > \varepsilon} \frac{1}{x^2} (\varphi(x) - \varphi(0)) dx.$$

(4 Marks)

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<sup>1</sup>Zuily, C., *Problems in Distributions and Partial Differential Equations*, Exercise 28