Vv557 Methods of Applied Mathematics II
Green Functions for Partial Differential Equations

JOINT INSTITUTE
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Assignment 4

Date Due: 1:00 PM, Thursday, the 28th of March 2018

This assignment has a total of **(15 Marks)**.

Exercise 4.1
We want to find a fundamental solution of the stationary equation for a simply supported beam, i.e., a function $g(x, \xi)$ satisfying

$$\frac{d^4 g}{dx^4} = \delta(x - \xi), \qquad 0 < x, \xi < 1,$$

with boundary conditions

$$g(0, \xi) = g''(0, \xi) = g(1, \xi) = g''(1, \xi) = 0.$$

i) Find a causal fundamental solution, i.e., a function E satisfying

$$\frac{d^4 E}{dx^4} = \delta(x - \xi), \qquad 0 < x, \xi < 1,$$

and $E(x) = 0$ for $x < \xi$.
(3 Marks)

ii) Add a solution of the homogeneous equation $\frac{d^4 u}{dx^4} = 0$ to E to obtain a function that satisfies the boundary conditions.
(2 Marks)

Exercise 4.2
We want to find a fundamental solution of the stationary equation for a travelling wave with wavenumber k, i.e., a function $g(x, \xi)$ satisfying

$$-\frac{d^2 g}{dx^2} - k^2 g = \delta(x - \xi), \qquad 0 < x, \xi < 1,$$

with boundary conditions

$$g(0, \xi) = g(1, \xi) = 0.$$

i) Find a causal fundamental solution, i.e., a function E satisfying

$$-\frac{d^2 E}{dx^2} - k^2 E = \delta(x - \xi), \qquad 0 < x, \xi < 1,$$

and $E(x) = 0$ for $x < \xi$.
(3 Marks)

ii) Add a solution of the homogeneous equation $-\frac{d^2 u}{dx^2} - k^2 u = 0$ to E to obtain a function that satisfies the boundary conditions.
(2 Marks)

iii) Another approach to the same problem: Use the Fourier transform to find a fundamental solution on $\mathbb{R}$, i.e., a function E satisfying

$$-\frac{d^2 E}{dx^2} - k^2 E = \delta(x - \xi), \qquad x, \xi \in \mathbb{R}.$$

(3 Marks)

iv) Add a solution of the homogeneous equation $-\frac{d^2 u}{dx^2} - k^2 u = 0$ to E to obtain a function that satisfies the boundary conditions.
(2 Marks)