Vv557 Methods of Applied Mathematics II
Green Functions for Partial Differential Equations

Assignment 4

Date Due: 1:00 PM, Thursday, the 11th of April 2018

This assignment has a total of **(21 Marks)**.

Exercise 4.1

Consider the boundary value problem operator given by

$$L = \frac{d^2}{dx^2}, \quad 0 < x < 1, \qquad\qquad B_1 u = u(0).$$

Characterize M^* by three boundary functionals.
(2 Marks)

Exercise 4.2

Consider the boundary value operator given by

$$L = \frac{d^4}{dx^4}, \quad 0 < x < 1, \qquad B_1 u = u(0), \qquad B_2 u = u'''(0), \qquad B_3 = u(1), \qquad B_4 = u''(1)$$

i) Find $g(x, \xi)$.
(2 Marks)

ii) It is obvious that $L = L^*$. Find the adjoint boundary conditions and calculate $g^*(x, \xi)$.
(3 Marks)

iii) Show that $g(x, \xi) \neq g(\xi, x)$.
(1 Mark)

Exercise 4.3

Find the solvability condition for the forced harmonic oscillator

$$-u'' - u = f, \quad -\pi < x < \pi, \qquad u(\pi) - u(-\pi) = \gamma_1, \qquad u'(\pi) - u'(-\pi) = \gamma_2.$$

Suppose that $\gamma_1 = \gamma_2 = 0$. Interpret the result in terms of the type of forcing function that can give a periodic solution.
(3 Marks)

Exercise 4.4

Find the modified Green function for

$$L = \frac{d^2}{dx^2} + \pi^2, \quad 0 < x < 1, \qquad B_1 u = u(0) + u(1), \qquad B_2 u = u'(0) + u'(1)$$

(3 Marks)

Exercise 4.5

i) Find the nontrivial solutions of

$$u^{(4)} = 0, \quad 0 < x < 1, \qquad u''(0) = u'''(0) = u''(1) = u'''(1) = 0$$

(1 Mark)

ii) Show that the problem is self-adjoint. **(1 Mark)**

iii) Define and construct the modified Green's function. **(3 Marks)**

iv) Solve $u^{(4)} = f$ with the homogeneous boundary conditions above when f satisfies the solvability conditions.
(2 Marks)