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CS454 - Theory of Computation

Homework 5

1) a) L(A) = { (a^m)(b^n)(c^n) | m, n >= 0 }

L(B) = { (a^n)(b^n)(c^m) | m, n >= 0 }

in L(A), #b = #c

in L(B), #a = #b

in the intersection of L(A) and L(B),

-> #a = #b

#b = #c

so #a = #b = #c

this means that

-> L(A) n L(B) = { (a^n)(b^n)(c^n) | n >= 0 }

therefore, by Ex2.36:

context-free grammars are not close under complement

b) Let L(A), L(B) be CFGs

Let L(C) = L(A) U L(B)

-> L'(C) = L'(A) n L'(B) : by DeMorgan's law applied to sets

which would imply that CFGs are closed for intersection

intersection is shown above to not be closed in CFGs

therefore:

CFGs are not closed for complement

2) Let G = (V,Σ,R,<STMT>) be defined by:

<STMT> -> <ASSIGN> | <IF-THEN> | <IF-THEN-ELSE>

<IF-THEN> -> if condition then <STMT>

<IF-THEN-ELSE> -> if condition then <STMT> else <STMT>

<ASSIGN> -> a:=1

Σ = {if, condition, else, a:=1}

V = {<STMT>, <IF-THEN>, <IF-THEN-ELSE>, <ASSIGN>}

a) this is an ambiguous grammar since the else in the string

-> "if condition then if condition then a:=1 else a:=1"

can be either

-> "if condition then (if condition then a:=1 else a:=1)"

or

-> "if condition then (if condition then a:=1) else a:=1"

b) an unambiguous grammar for the language is:

Let H = (V,Σ,R,<STMT>)

<STMT> -> <ASSIGN> | <IF-THEN> | <IF-THEN-ELSE>

<IF-THEN> -> ( if condition then <STMT> )

<IF-THEN-ELSE> -> ( if condition then <STMT> else <STMT> )

<ASSIGN> -> a:=1

Σ = { if, condition, else, a:=1, (, ) }

V = { <STMT>, <IF-THEN>, <IF-THEN-ELSE>, <ASSIGN> }

3) Let G a context-free grammar defined as:

S -> ABS | AB

A -> aA | a

B -> bB | b

Determine if the following strings are in L(G):

a) aabaab: rejected

b) aaaaba: accepted

S

/ \

A B

/ \ / \

a A b A

/ \ \

a A a

/ \

a A

/

a

c) aabbaa: rejected

d) abaaba: accepted

S

/ | \

/ | \

A B S

| / \ / \

a b A / B

| / / \

a A b A

| |

a a

4) L = { w <- {a,b}\* | #a(w) <= #b(w) <= 2#a(w) }

let G = (V,Σ,R,<STMT>) be the grammar for L and defined as:

S -> aSB | BSa | ε

B -> bb | b

Σ = { a, b }

V = { S, B }

then the PDA for L is defined as:

Q = { q0, q1, q2 }

Σ = { a, b }

Γ = { 1, 2, S, B }

S = q0

F = { q2 }

δ(q0,ε,Z) -> (q1,SZ)

δ(q1,ε,S) -> (q1,1SB)

δ(q1,ε,S) -> (q1,BS1)

δ(q1,ε,B) -> (q1,22)

δ(q1,ε,B) -> (q1,2)

δ(q1,a,1) -> (q1,ε)

δ(q1,b,2) -> (q1,ε)

δ(q1,ε,Z) -> (q2,ε)