SkyCoop: Promoting Cooperation in Competitive Sky Computing Marketplace

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Appendix A
Proof of Theorem 1

Since the payoff function of users is concave, when d_k satisfies the first order condition, users gain the maximum payoff. When

$$R'_{user}(d_k^*) = \frac{vq_k}{1+q_kd_k^*} - p_k = 0,$$

$$d_k^* = \frac{v}{p_k} - \frac{1}{q_k}.$$

Users would only choose one SP, other SPs with lower φ would receive zero demand.

Appendix B
Proof of Theorem 2

We first prove that $p_k^* = c_k$ is Nash equilibrium when $\eta_{max} = \eta_{sec}$, If $p_k^* = c_k, \forall k \in \mathcal{M}$, all SPs obtain zero profit. If SP $k, k \in \mathcal{M}$ sets its price $p_k > c_k$, it can not gain a positive profit. This is because another SP with the same quality-cost ratio could slightly decrease its price to gain the entire profit. Next, we prove that there is no other price vector is an equilibrium. If SP k sets its price over cost, there is always at least one SP that could set a lower price than SP k in set $\mathcal{M}$ to gain all users, and SP k's profit would be zero.

When $\eta_{max} > \eta_{sec}$, if $p_{max} < \frac{q_{max}}{q_{sec}}c_{sec}$, the SP with the second-highest quality-cost ratio has no chance to attract users from SP k_{max}. To maximize its profit, SP k_{max} will set its price $p_{max} = \frac{q_{max}}{q_{sec}}c_{sec} - \varepsilon$. In this way, other SPs would set their price as cost, since no SP could get profit with lower or higher price. If SP k_{max} try to increase its price $p_{max} > \frac{q_{max}}{q_{sec}}c_{sec}$, SP k_{sec} could set a relatively low price to make its quality-price ratio larger than that of k_{max}'s, in this case, the profit of k_{max} will be zero. When there is only one SP in set $\mathcal{M}$, the optimal price of k_{max} is $\sqrt{q_kc_kv}$. To prove the optimal price, we first substitute $d_k^*(\boldsymbol{p})$ into $R_k(p_k, \boldsymbol{p}_{-k})$,

$$R_k(p_k, \boldsymbol{p}_{-k}) = v - \frac{p_k}{q_k} - \frac{c_kv}{p_k} + \frac{c_k}{q_k},$$

$$\frac{dR_k}{dp_k} = \frac{c_kv}{p_k^2} - \frac{1}{q_k},$$

$$\frac{d^2R_k}{dp_k^2} = -\frac{c_kv}{p_k^3} \leq 0.$$

It means the optimization problem of SP is strictly concave, and there is only one optimal solution. We can calculate the optimal solution p_k^* by the zero point of the first derivative of the optimization problem:

$$\frac{dR_k}{dp_k} = \frac{c_kv}{p_k^{*2}} - \frac{1}{q_k} = 0,$$

$$p_k^* = \sqrt{q_kc_kv}.$$

Based on the above statements, we complete the proof.

Appendix C
Proof of Lemma 1

The existence of MPE has been proved in [1].

Appendix D
Proof of Theorem 3

In Stage II, we conclude the profit of the coalition $\mathcal{B}, \mathcal{B} \in \Omega$ as follows:

$$R_{\mathcal{B}}(\boldsymbol{\Omega}) = \begin{cases} \left(\sqrt{v} - \sqrt{\frac{c_{\mathcal{B}}}{q_{\mathcal{B}}}}\right)^2, & \text{if } \frac{q_{\mathcal{B}}}{q_{sec}}c_{sec} \geq \sqrt{q_{\mathcal{B}}c_{\mathcal{B}}v}, \\ & \eta_{\mathcal{B}} > \eta_{sec}, \\ v - \frac{c_{sec}}{q_{sec}} - \frac{c_{\mathcal{B}}}{q_{\mathcal{B}}}\left(\frac{vq_{sec}}{c_{sec}} - 1\right), & \text{if } \frac{q_{\mathcal{B}}}{q_{sec}}c_{sec} < \sqrt{q_{\mathcal{B}}c_{\mathcal{B}}v}, \\ & \eta_{\mathcal{B}} > \eta_{sec}, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

From the equation (D.1), we can see as the service quality-cost ratio $\eta_{\mathcal{B}} = \frac{q_{\mathcal{B}}}{c_{\mathcal{B}}}$ increases, the profit of coalition $\mathcal{B}$ increases, when $\mathcal{B}$ when the price competition game. If a CSP (or coalition) $i \in \Omega$ join the coalition $\mathcal{B}$, $\eta_{\mathcal{B}\cup i}$ is

$$\eta_{\mathcal{B}\cup i} = \frac{(q_{\mathcal{B}} + q_i)^2}{q_{\mathcal{B}}c_{\mathcal{B}} + q_ic_i},$$

when

$$\eta_{\mathcal{B}\cup i} - \eta_{\mathcal{B}} = \frac{(q_{\mathcal{B}} + q_i)^2}{q_{\mathcal{B}}c_{\mathcal{B}} + q_ic_i} - \frac{q_{\mathcal{B}}}{c_{\mathcal{B}}} > 0,$$

$$\begin{aligned} c_{\mathcal{B}}(q_{\mathcal{B}} + q_i)^2 &> q_{\mathcal{B}}(q_{\mathcal{B}}c_{\mathcal{B}} + q_ic_i), \\ c_{\mathcal{B}}q_{\mathcal{B}}^2 + 2c_{\mathcal{B}}q_iq_{\mathcal{B}} + c_{\mathcal{B}}q_i^2 &> q_{\mathcal{B}}^2c_{\mathcal{B}} + q_{\mathcal{B}}q_ic_i, \\ c_{\mathcal{B}}q_i^2 + q_{\mathcal{B}}q_i(2c_{\mathcal{B}} - c_i) &> 0, \\ c_{\mathcal{B}}q_i + q_{\mathcal{B}}(2c_{\mathcal{B}} - c_i) &> 0, \\ q_i &> q_{\mathcal{B}}(\frac{c_i}{c_{\mathcal{B}}} - 2). \end{aligned}$$

It denotes that when $c_i \leq 2c_{\mathcal{B}}$, or $c_i > 2c_{\mathcal{B}}, q_i > q_{\mathcal{B}}\left(\frac{c_i}{c_{\mathcal{B}}} - 2\right)$, the inequality always holds, which means the joined coalition i can increase the profit of coalition $\mathcal{B}$.

Assume grand coalition $\mathcal{G}$ is inefficient when $c_{max} \leq 2c_{min}$, i.e. $R_{\mathcal{G}} \leq \sum_{\mathcal{B}\in\Omega} R_{\mathcal{B}}(\Omega)$, in which $R_{\mathcal{G}}$ is the profit of grand coalition. There is only one coalition that win the price competition in Stage II can gain positive profit and represent it as $R_{win}(\Omega) = \sum_{\mathcal{B}\in\Omega} R_{\mathcal{B}}(\Omega)$, and the winner coalition might face competition from other coalition in Stage II, then we have

$$R_{\mathcal{G}} \leq R_{win}(\Omega) \leq R'_{win}(\Omega),$$

where $R'_{win}(\Omega)$ is the profit of the coalition ignoring the competition in Stage II. However, whenever the grand coalition is not formed, coalitions in Ω can always form a new coalition with any other coalition to gain more profit. Since when $c_{max} \leq 2c_{min}$, any two coalition $\mathcal{B}$ and $\mathcal{B}'$ satisfy

$$c_{\mathcal{B}} \leq c_{max} \leq 2c_{min} \leq 2c_{\mathcal{B}'},$$

which guarantees there is always a coalition would cooperate with the winner coalition in price competition to receive higher profit until the grand coalition forms, i.e.

$$R_{win'}(\Omega') \leq R_{\mathcal{G}},$$

where $R_{win'}(\Omega')$ is the profit of winner coalition in any coalition, which is contradictory to our assumptions.

Consider a coalition structure Ω that $\sum_{\mathcal{B}\in\Omega} \omega_{\mathcal{B}}(\Omega) < \omega_{\mathcal{G}}$, where $\omega_{\mathcal{G}}$ is the long-term profit of grand coalition. For any coalition structure, the proposer can receive a positive surplus if it proposes a contract to form the grand coalition, since $\omega_{\mathcal{G}} - \sum_{\mathcal{B}\in\Omega} \hat{\omega}_{\mathcal{B}}(\sigma|\Omega) > 0$, then the grand coalition would form in finite periods.

Appendix E
Proof of Theorem 4

Based on Theorem 3, CSPs that $c_i \leq 2c_{min}, \forall i \in \mathcal{S}$, would increase the η for all coalition, then it would be in the stable coalition, since as long as when they are proposer, they can propose a contract that join the only coalition that gain profit which would be the stable coalition finally.

As long as a CSP can increase the profit of stable coalition in the price competition would be in stable coalition. We use E to denote the set of CSPs that form the coalition with the highest η, for any coalition $\mathcal{B}$, we have

$$\eta_{E\backslash i} \geq \eta_{\mathcal{B}\backslash i}, \forall i \in E.$$

We need to prove

$$q_i \geq q_{\mathcal{B}\backslash i}(\frac{c_i}{c_{\mathcal{B}\backslash i}} - 2). \tag{2}$$

which can guarantee coalition B could gain higher profit if CSP i join in based on (D.1). When $q_{\mathcal{B}\backslash i} < q_{E\backslash i}, \forall i \in E$, we have

$$\begin{aligned} q_i \geq & q_{E\backslash i}(\frac{c_i}{c_{E\backslash i}} - 2), \\ \eta_{E\backslash i} > & \eta_{\mathcal{B}\backslash i}, \\ \frac{q_{E\backslash i}}{q_{\mathcal{B}\backslash i}} > & \frac{c_{E\backslash i}}{c_{\mathcal{B}\backslash i}}, \\ \frac{q_{E\backslash i}}{q_{\mathcal{B}\backslash i}} \geq & 1, \\ \frac{c_{E\backslash i}}{c_{\mathcal{B}\backslash i}} \geq & 1, \\ c_{E\backslash i} \geq & c_{\mathcal{B}\backslash i}. \end{aligned}$$

Then

$$\frac{q_{E\backslash i}}{q_{\mathcal{B}\backslash i}} \geq 1 \geq \frac{\frac{c_i}{c_{E\backslash i}} - 2}{\frac{c_i}{c_{\mathcal{B}\backslash i}} - 2}$$

$$\Leftrightarrow q_{E\backslash i}(\frac{c_i}{c_{E\backslash i}} - 2) - q_{\mathcal{B}\backslash i}(\frac{c_i}{c_{\mathcal{B}\backslash i}} - 2) > 0.$$

We have

$$q_i \geq q_{E\backslash i}(\frac{c_i}{c_{E\backslash i}} - 2) \geq q_{\mathcal{B}\backslash i}(\frac{c_i}{c_{\mathcal{B}\backslash i}} - 2) \tag{3}$$

If $q_{\mathcal{B}\backslash i} \geq q_{E\backslash i}$, (D.2) is true

$$\eta_i \geq \eta_{E\backslash i} - \frac{2q_{E\backslash i}}{c_i} \geq \eta_{\mathcal{B}\backslash i} - \frac{2q_{E\backslash i}}{c_i} > \eta_{\mathcal{B}\backslash i} - \frac{2q_{\mathcal{B}\backslash i}}{c_i}.$$

Then (D.2) is always true, and we finish the proof.

Appendix F
Proof of Theorem 5

If $\delta = 0$, for winner coalition in price competition game, its long-term discount profit is $\hat{\omega}_{win}(\Omega) = R_{win}(\Omega)$, a newly formed coalition $\mathcal{D}$ satisfies $\eta_{\mathcal{D}} > \eta_{win}$ would be enough since $R_{\mathcal{D}} > R_{win}$, then

$$e_{\mathcal{D}}(\Omega) = \hat{\omega}_{\mathcal{D}}(\Omega_{\mathcal{D}}) - \sum_{\mathcal{B}\in\mathcal{D}} \hat{\omega}_{\mathcal{B}}(\Omega) = R_{\mathcal{D}} - R_{win} > 0,$$

which means the contract that form coalition $\mathcal{D}$ can generate positive surplus.

When $\delta \to 1$, and coalition $\mathcal{D}$ is stable coalition, we have $\sum_{i\in\mathcal{S}} \hat{\omega}_i(\Omega) = \hat{\omega}_{\mathcal{D}}(\Omega_{\mathcal{D}})$, and $\mathcal{D}$ is the coalition that gain the most profit, otherwise, it would not be stable coalition. Since grand coalition is not efficient coalition, we have $\omega_{\mathcal{D}}(\Omega_{\mathcal{D}}) > \omega_{\mathcal{G}}$, and for any coalition structure forming the grand coalition, the surplus is

$$e_{\mathcal{G}}(\Omega) = \hat{\omega}_{\mathcal{G}} - \sum_{\mathcal{B}\in\Omega} \hat{\omega}_{\mathcal{B}}(\Omega) = \hat{\omega}_{\mathcal{G}} - \hat{\omega}_{\mathcal{D}}(\Omega_{\mathcal{D}}) < 0,$$

then the grand coalition would not form.

Consider an efficient coalition $\mathcal{D}$, for all coalition structures Ω and $\Omega_{\mathcal{D}}$, satisfy $\omega_{\mathcal{D}}(\Omega_{\mathcal{D}}) \geq \sum_{\mathcal{B}\in\Omega} \omega_{\mathcal{B}}(\Omega)$, then for any $\mathcal{D}_1 \cup \mathcal{D}_2 \cup \cdots \cup \mathcal{D}_n = \mathcal{D}$,

$$\omega_{\mathcal{D}}(\Omega_{\mathcal{D}}) \geq \sum_{\mathcal{B}\in\Omega} \hat{\omega}_{\mathcal{B}}(\Omega),$$

$$\hat{\omega}_{\mathcal{D}}(\Omega_{\mathcal{D}}) - (\hat{\omega}_{D_1} + \cdots \hat{\omega}_{D_n}) \geq 0,$$

and for any coalition $\mathcal{C} \subset \mathcal{D}$,

$$e_{\mathcal{D}} - e_{\mathcal{C}} = \omega_{\mathcal{D}} - \omega_{\mathcal{C}} - \sum_{\mathcal{B} \in \mathcal{D} \setminus \mathcal{C}} \omega_{\mathcal{B}} > 0 \tag{4}$$

which means for coalitions in $\mathcal{D}$, they would immediately form $\mathcal{D}$, since this contract with the largest surplus.

Appendix G
Proof of Lemma 2

For CSP $i \in \mathcal{S}$ that $\eta_i < \eta_{max}$, unilaterally join intercloud broker would not gain positive profit since $\eta_{\{i\}} \leq \eta_i$, where $\{i\}$ is intercloud broker with only CSP i. For CSP i with η_{max}, if it joins intercloud broker, and it has $\eta_{\{i\}} > \eta_{sec}$, i.e. CSP i still win in the price competition, however, it has to share its profit with intercloud broker which leads to profit decreasing. If $\eta_{\{i\}} \leq \eta_{sec}$, CPS i would gain zero profit. Other CSPs that do not have η_{max} would still gain zero profit due to competition.

Appendix H
Proof of Lemma 3

In intercloud broker model, CSP i's profit is

$$R_i(x_i, p_i) = \begin{cases} p_i d_i - c_i d_i, & x_i = 0, \\ (p_i^s(q_i) - c_i)\, \dfrac{q_i}{q_s} d_s. & x_i = 1. \end{cases}, \forall i \in \mathcal{S} \tag{5}$$

To maximize the number of CSPs that gain profit, we aim to let CSPs cooperate through intercloud broker, since there is only one SP that could gain positive profit due to the price competition. For CSPs that $p_i^s(q_i) > c_i, \forall i \in \mathcal{S}$, their cooperation strategies are joining intercloud broker, the rest of CSPs have $\eta \leq \dfrac{1}{K}$ would not join intercloud broker, since they would receive negative profit due to the high cost compare to purchase price from intercloud broker.

Appendix I
Proof of Theorem 6

When

$$R_i^{coop} \sum_{j=1}^{\infty} \delta_i^j > R_i^{coop} \sum_{j=1}^{t-1} \delta^j + R_i^{dev} \delta_i^{t-1} + R_i^{pun} \sum_{j=t+1}^{\infty} \delta_i^{j-1},$$

CSP i would not deviate from the cooperation at time slot t. Then for CSPs $\forall i \in \mathcal{S}$ with $\delta_i > \dfrac{R_i^{dev} - R_i^{coop}}{R_i^{dev} - R_i^{pun}}$, they would play cooperation strategies $\boldsymbol{x}^{coop}$ at each stage, and would not deviate from cooperation in the long run. If $R_i^{dev} \leq R_i^{coop}$, it means CSP i has no incentive to deviate in any step. For restriction $R_i^{dev} \leq R_i^{pun}$, only CSP with η_{max} could probably satisfy since the punishment does not affect its deviation, then the cooperation strategy would not happen.

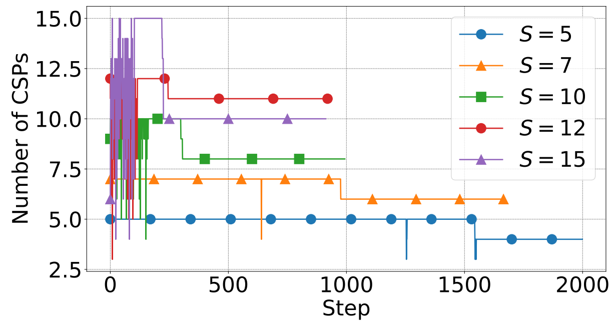

Fig. 1: Number of CSP in coalition under different number of CSP S in market

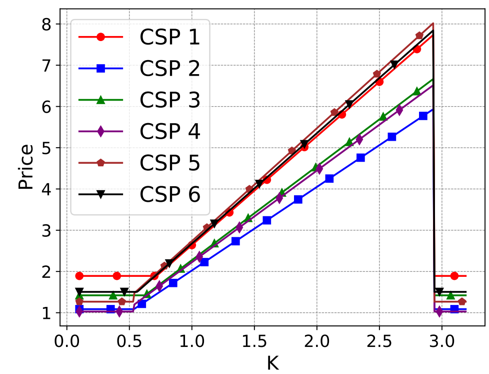

Fig. 2: Price of each CSP under different K

Appendix J
Additional Simulation Analysis

Fig. 1 shows that in different numbers of CSPs in the coalition model. There is a regularity that the grand coalition would form in the early training steps. This is because we set the k of k-nearest-neighbor as 10, and we stop choosing a proposer to form a new coalition only when the grand coalition is formed, or we have chosen a proposer for over 50 times. When there are 5 CSPs, the action space for the proposer is 2^5, and the k-nearest-neighbor would be more likely to form a coalition with more CSPs since the larger coalition tends to be more efficient than smaller ones.

Fig. 2 shows the purchase prices of different CSPs in intercloud broker. Due to price competition among CSPs, the pricing is often set at its cost. Even for the winning CSP, its price will not exceed the cost by much. However, after joining the broker, CSPs' prices are determined by intercloud broker and quality of their own. As the broker's pricing to CSPs increases, the profit per unit obtained by CSPs also continues to increase.

Fig. 3 shows the price, demand from users and price per unit of quality of winner SP in price competition in intercloud broker model. Within $0.5 < K < 3.0$, the

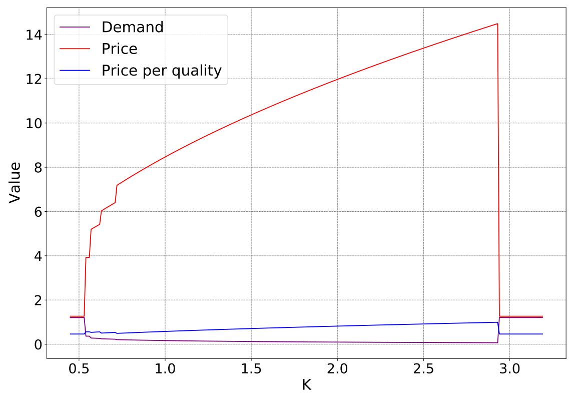

Fig. 3: Demand under different K

winner is intercloud broker, the addition of CSPs improves service quality and intercloud sets higher prices, however, as CSPs leave the market, competition eases, leading to a significant increase in service prices. The price per unit of quality also slightly increases. Due to the increase in prices, user demand decreases as more CSPs join the broker.

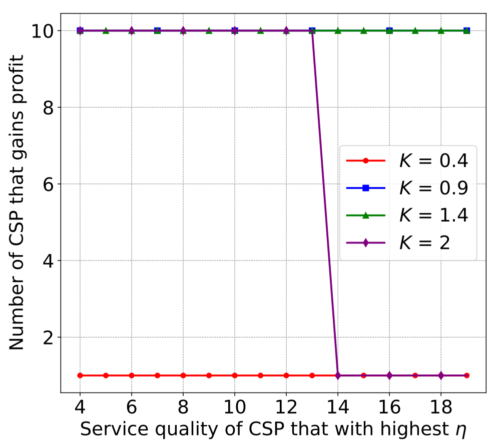

Fig. 4: Number of CSP that receives positive profit under different service quality of CSP with the highest η

Fig. 4 shows the number of CSPs that gain profit, as the service quality of CSP with the maximum η increases. When there is a large difference among CSPs, i.e. the service quality of CSP with the maximum η is high, the remaining CSPs would not join the broker, and the broker can not attract enough CSPs to generate positive profit when intercloud broker sets a low price (K is small) for CSPs. However, when $K = 2$, as the service quality of CSP with the maximum η increases, intercloud broker still can not attract CSPs, since CSP with the maximum η can gain more profit to be stand-alone SP even though other CSPs all join in intercloud broker than all CSPs join in intercloud broker.

References

[1] A. Gomes, "Multilateral contracting with externalities," Econometrica, vol. 73, no. 4, pp. 1329–1350, 2005.