Efficient Task Offloading in MEC via UAV-UGV Collaboration

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1 Appendix A

First, we define the quadratic Lyapunov function:

$$L(t) = \frac{1}{2}\left(\sum_{j\in\mathcal{A}} {Q_j^t}^2 + \sum_{k\in\mathcal{G}} {Q_k^t}^2\right), \tag{1}$$

and Lyapunov drift,

$$\Delta(\mathbf{Q}(t)) = \mathbb{E}\{L(t+1) - L(t)|\mathbf{Q}(t)\},$$

where $\mathbf{Q}(t) = \{Q_j^t, Q_k^t\}_{j\in\mathcal{A},k\in\mathcal{G}}$. According to [1], for any $\lambda > 0$, there exist a strategies π^t satisfied,

$$\begin{aligned}
&\mathbb{E}\{T_{sum}^t(\pi^t)|\mathbf{Q}(t)\} \leq T_{sum}^{opt} + \lambda,\\
&\mathbb{E}\{E_j^t(\pi^t) - e_j^t - \bar{E}_A\} \leq \lambda, \forall j \in \mathcal{A},\\
&\mathbb{E}\{E_k^t(\pi^t) - \bar{E}_G\} \leq \lambda, \forall k \in \mathcal{G}.
\end{aligned}$$

Based on (1), we have

$$\begin{aligned}
&\Delta(\mathbf{Q}(t)) + V\sum_{t=1}^{T} T_{sum}^t \leqslant (B_A + B_G)T + V\sum_{t=1}^{T} T_{sum}^t\\
&+ \leq \mathbb{E}\{\sum_{j\in\mathcal{A}} Q_j^t(E_j^t - e_j^t - \bar{E}_A)|\mathbf{Q}(t)\} + \mathbb{E}\{\sum_{k\in\mathcal{G}} Q_k^t(E_k^t - \bar{E}_G)|\mathbf{Q}(t)\},
\end{aligned}$$

taking $\lambda \to 0$, we have for any $\tilde{t} > 0$,

$$\frac{1}{\tilde{t}}\sum_{t=1}^{\tilde{t}} \mathbb{E}\{T_{sum}^t|\mathbf{Q}(t)\} \leq T_{sum}^{opt} + \frac{B_A + B_G}{V}.$$

2 Appendix B

We let $f^*(\mathbb{M}_G, \mathit{\Gamma})$ denote the objective function value achieved under optimal strategies, and $\hat{f}(\mathbb{M}_G, \mathit{\Gamma})$ be the approximated objective function value obtained via the SAA method. Consider a larger sample set $\mathcal{N}'$ of size $|\mathcal{N}'| \gg |\mathcal{N}|$, where the mean SAA estimate is given by:

$$\hat{f}_{\mathcal{N}'}(\mathbb{M}_G, \Gamma) = \frac{1}{|\mathcal{N}'|} \sum_{n \in \mathcal{N}'} f^n(\mathbb{M}_G, \Gamma), \tag{2}$$

with variance:

$$\sigma^2_{\mathcal{N}'} = \frac{1}{|\mathcal{N}'|(|\mathcal{N}'| - 1)} \sum_{n \in \mathcal{N}'} \left(f^n(\mathbb{M}_G, \Gamma) - \hat{f}_{\mathcal{N}'}(\mathbb{M}_G, \Gamma) \right)^2. \tag{3}$$

Additionally, consider a collection $\mathcal{C} = \{\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_C\}$ of i.i.d. SAA replications, each with sample size $|\mathcal{N}|$. The mean SAA estimate across these replications is:

$$\bar{f}_{|\mathcal{N}|,\mathcal{C}}(\mathbb{M}_G, \Gamma) = \frac{1}{|\mathcal{C}|} \sum_{c \in \mathcal{C}} \hat{f}^{(c)}_{|\mathcal{N}|}(\mathbb{M}_G, \Gamma), \tag{4}$$

with the corresponding variance:

$$\sigma^2_{|\mathcal{N}|,\mathcal{C}} = \frac{1}{|\mathcal{C}|(|\mathcal{C}| - 1)} \sum_{c \in \mathcal{C}} \left(\hat{f}^{(c)}_{|\mathcal{N}|}(\mathbb{M}_G, \Gamma) - \bar{f}_{|\mathcal{N}|,\mathcal{C}}(\mathbb{M}_G, \Gamma) \right)^2. \tag{5}$$

Since $\mathbb{E}[\hat{f}_{\mathcal{N}}] = \mathbb{E}[\bar{f}_{|\mathcal{N}|,\mathcal{C}}]$ and $\mathbb{E}[\hat{f}_{\mathcal{N}}] \leq f^*(\mathbb{M}_G, \Gamma)$ (indicating a negative bias) as established in [2], we analyze the optimality gap $\hat{f}_{\mathcal{N}'}(\mathbb{M}_G, \Gamma) - f^*(\mathbb{M}_G, \Gamma)$. Given that

$$\mathbb{E}\left[\hat{f}_{\mathcal{N}'}(\mathbb{M}_G, \Gamma) - \bar{f}_{|\mathcal{N}|,\mathcal{C}}(\mathbb{M}_G, \Gamma) \right] \geq \hat{f}(\mathbb{M}_G, \Gamma) - f^*(\mathbb{M}_G, \Gamma), \tag{6}$$

and invoking the Central Limit Theorem as in [2], the optimality gap with confidence level ρ for a sample size $|\mathcal{N}|$ is:

$$\Delta_{|\mathcal{N}|,\rho} = \left(\bar{f}_{\mathcal{N}'}(\mathbb{M}_G, \Gamma) - \bar{f}_{|\mathcal{N}|,\mathcal{C}}(\mathbb{M}_G, \Gamma) \right) + z_\rho \left(\sigma^2_{\mathcal{N}'} + \sigma^2_{|\mathcal{N}|,\mathcal{C}} \right)^{\frac{1}{2}}, \tag{7}$$

where $z_\rho = \phi^{-1}(1 - \rho)$. This result establishes an upper bound on the deviation from optimality under the given confidence level.

3 Appendix C

We introduce auxiliary variables $\boldsymbol{\eta} = \{\eta^t_{i,j}, \forall i \in \mathcal{I}, \forall j \in \mathcal{A}\}$

$$\eta^t_{i,j} \geq \frac{1}{\beta \log_2 \left(1 + \frac{\delta p_A}{\left\| c^t_j - c^t_i \right\|^2 + H^2}\right)}. \tag{8}$$

Then the problem becomes

$$\begin{aligned} \min_{\mathbb{C}_A, \boldsymbol{\eta}} \quad & V \sum_{i \in \mathcal{I}} \sum_{j} \alpha^t_{i,j} \varphi_i \eta^t_{i,j} + \sum_{j \in \mathcal{A}} Q^t_j E^{pro,t}_j(c^t_j) \\ \text{s.t.} \quad & \left\| c^t_j - c^{t-1}_j \right\| \leq C_{max}, \forall j \in \mathcal{A}, \ \beta \log_2 \left(1 + \frac{\delta p_A}{\left\| c^t_j - c^t_i \right\|^2 + H^2}\right) \geq \frac{1}{\eta^t_{i,j}}, \forall i \in \mathcal{I}, j \in \mathcal{A}. \end{aligned}$$

We apply the first-order Taylor expansion at point $c_j^t(m)$ to approximate the constraint,

$$\beta \log_2(1 + \frac{\delta p_A}{\left\|c_j^t - c_i^t\right\|^2 + H^2}) \geq \Omega_{i,j}^{t,1} + \Omega_{i,j}^{t,2}(||c_j^t - c_i^t||^2 - ||c_j^t(m) - c_i^t||^2) \geq \frac{1}{\eta_{i,j}^t},$$

where

$$\begin{aligned}
\Omega_{i,j}^{t,1} &= \beta \log_2(1 + \frac{\delta p_A}{\left\|c_j^t(m) - c_i^t\right\|^2 + H^2}), \\
\Omega_{i,j}^{t,2} &= \frac{-\beta \delta p_A}{(\left\|c_j^t(m) - c_i^t\right\|^2 + H^2)(\left\|c_j^t(m) - c_i^t\right\|^2 + H^2 + \delta p_A)\ln 2}.
\end{aligned}$$

According to the above statements, we turn to solve the approximate convex problem of problem $\mathbf{P}_{2,1}^t$ at iteration m as follows:

$$\begin{aligned}
\mathbf{P}_{2,1}^t(m) \min_{\mathbb{C}_A, \boldsymbol{\eta}} \quad & V \sum_{i \in \mathcal{I}} \sum_{j} \alpha_{i,j}^t \varphi_i \eta_{i,j}^t + \sum_{j \in \mathcal{A}} Q_j^t E_j^{pro,t}(c_j^t) \\
\text{s.t.} \quad & \left\|c_j^t - c_j^{t-1}\right\| \leq C_{max}, \forall j \in \mathcal{A},\ \Omega_{i,j}^{t,1} + \Omega_{i,j}^{t,2}(||c_j^t - c_i^t||^2 - ||c_j^t(m) - c_i^t||^2) \geq \frac{1}{\eta_{i,j}^t}.
\end{aligned}$$

References

1. Neely, M.: Stochastic Network Optimization with Application to Communication and Queueing Systems, vol. 3 (01 2010)
2. Kleywegt, A.J., Shapiro, A., Homem-de Mello, T.: The sample average approximation method for stochastic discrete optimization. SIAM Journal on Optimization **12**(2), 479–502 (2002)