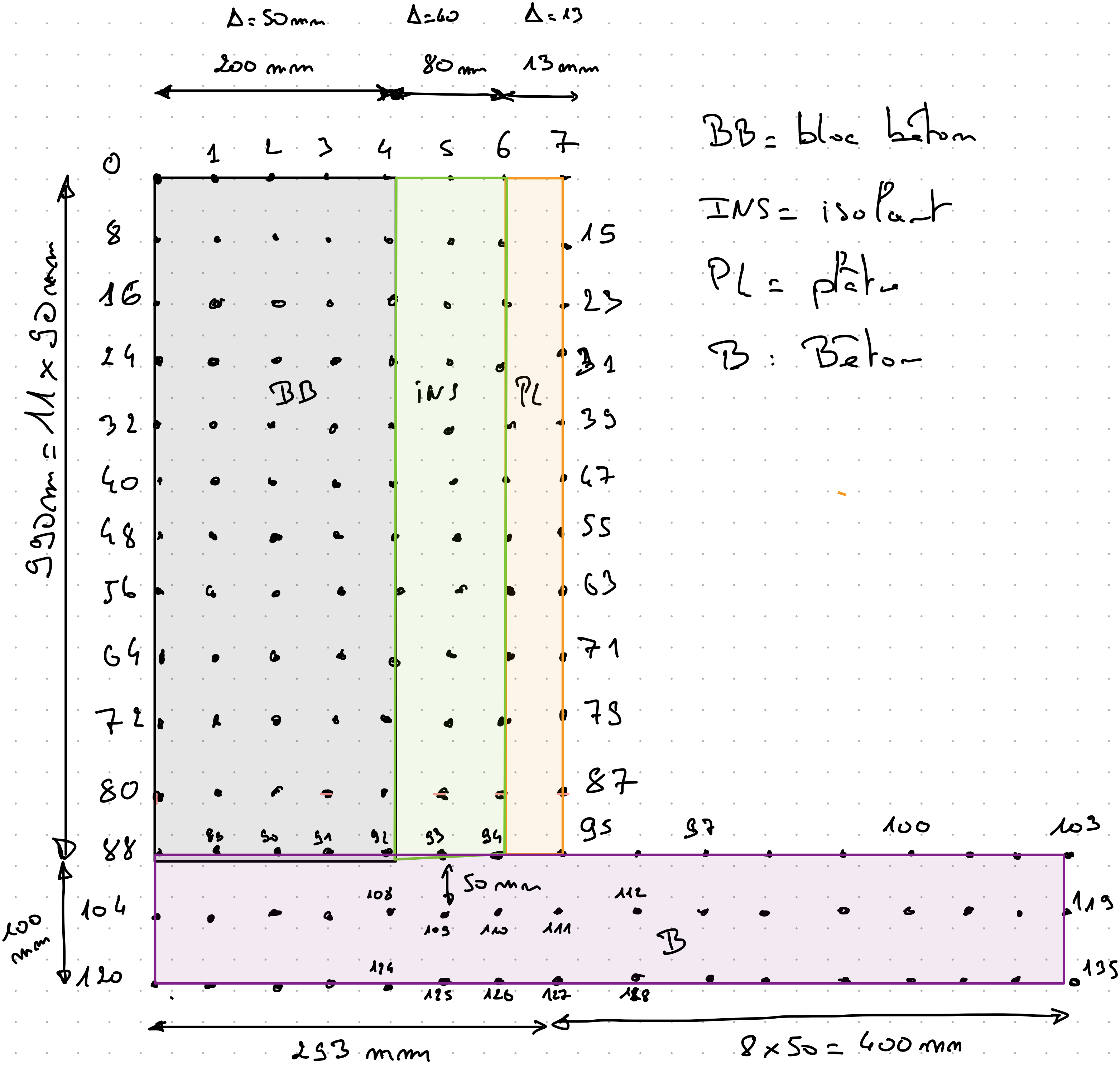
Numerical Methods

2024 - 2025



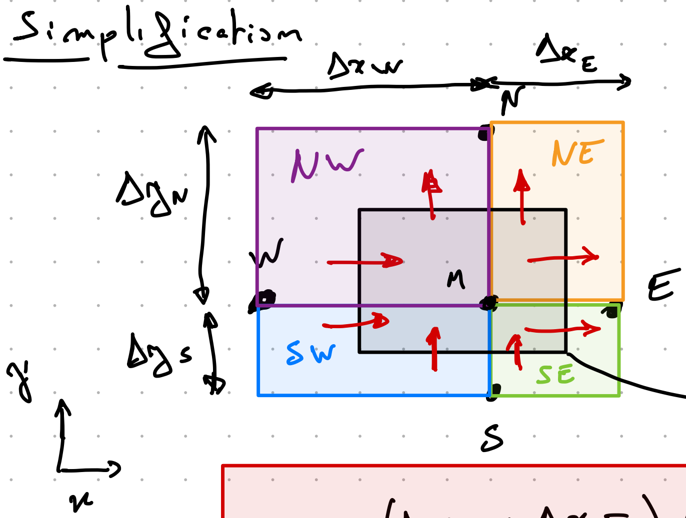
BB = bloc béton

INS = isolant

PL = plâtre

B : Béton

Simplification



each point M has
Δx_W, Δx_E
Δy_N, Δy_S

ρ_M, c_{p_M}, S_M

$$\boxed{S_M = \left(\frac{\Delta x_W + \Delta x_E}{2}\right)\left(\frac{\Delta y_S + \Delta y_N}{2}\right)}$$

$$\rho_M = \frac{\rho_{NW} V_{NW} + \rho_{NE} V_{NE} + \rho_{SW} V_{SW} + \rho_{SE} V_{SE}}{V_M}$$

$$\rho_M = \boxed{\rho_{NW} \frac{S_{NW}}{S_M}} + \rho_{NE} \frac{S_{NE}}{S_M} + \rho_{SW} \frac{S_{SW}}{S_M} + \rho_{SE} \frac{S_{SE}}{S_M}$$

$$S_{NW} = \frac{\Delta x_W \Delta y_N}{4} \; ; \quad S_{NE} = \frac{\Delta x_E \Delta y_N}{4} \; ;$$

$$S_{SW} = \frac{\Delta x_W \Delta y_S}{4} \; ; \quad S_{SE} = \frac{\Delta x_E \Delta y_S}{4}$$

$$\frac{\Delta x_E}{r_E} = \Delta x_W + \Delta x_E$$

$$\frac{\Delta x_W}{r_W} = \Delta x_W + \Delta x_E$$

$$\rho_{NW} \frac{\Delta x_W \Delta y_N}{(\Delta x_W + \Delta x_E)(\Delta y_S + \Delta y_W)} = \rho_{NW} \underbrace{\frac{\Delta x_W}{\Delta x_W + \Delta x_E}}_{r_W} \times \underbrace{\frac{\Delta y_N}{\Delta y_S + \Delta y_N}}_{r_N}$$

$$\boxed{\rho_{NW} \, r_W \, r_N}$$

$$\boxed{\rho_M = \rho_{NW} r_N r_W + \rho_{NE} r_N r_E + \rho_{SW} r_S r_W + \rho_{SE} r_S r_E}$$

with

$$r_N = \frac{\Delta y_N}{\Delta y_N + \Delta y_S}, \quad r_W = \frac{\Delta x_W}{\Delta x_W + \Delta x_E}, \quad r_S = \frac{\Delta y_S}{\Delta y_N + \Delta y_S}$$

$$r_E = \frac{\Delta x_E}{\Delta x_W + \Delta x_W}$$

and

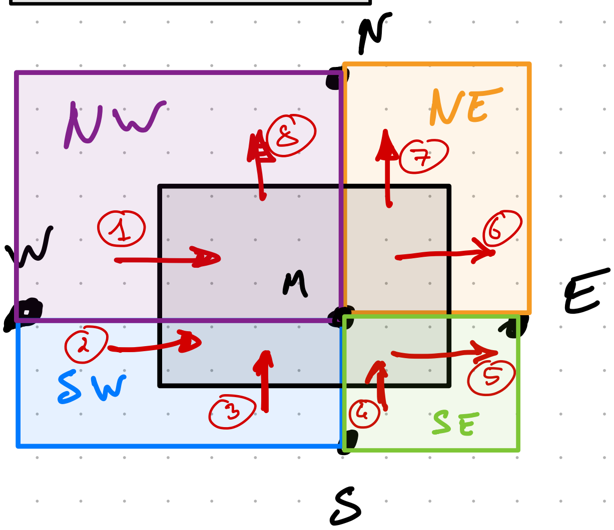
$$\boxed{\begin{aligned} r_N + r_S &= 1 \\ r_E + r_W &= 1 \end{aligned}}$$

$$\rho_M C_{p_M} S_M = \rho_{NW} C_{p_{NW}} S_{NW} + \rho_{NE} C_{p_{NE}} S_{NE} + \rho_{SW} C_{p_{SW}} S_{SW} + \rho_{SE} C_{p_{SE}} S_{SE}$$

$$\rho_M C_{p_M} = \rho_{NW} r_N r_W C_{p_{NW}} + \rho_{NE} r_N r_E C_{p_{NE}} + \rho_{SW} r_S r_W C_{p_{SW}} + \rho_{SE} r_S r_E C_{p_{SE}}$$

$$\boxed{C_{p_M} = \frac{\rho_{NW} r_N r_W C_{p_{NW}} + \rho_{NE} r_N r_E C_{p_{NE}} + \rho_{SW} r_S r_W C_{p_{SW}} + \rho_{SE} r_S r_E C_{p_{SE}}}{\rho_M}}$$

I CONDUCTION

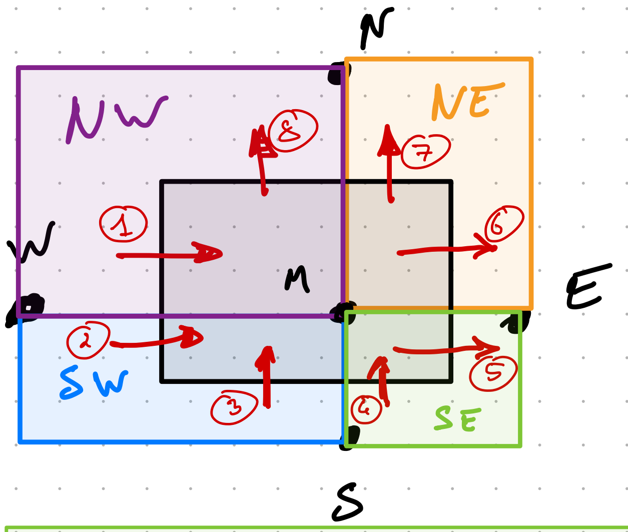


$$\overbrace{\phi_1 + \phi_2 + \phi_3 + \phi_4 - \{\phi_5 + \phi_6 + \phi_7 + \phi_8\}}^{\text{heat fluxes}} = \rho_M C_{p_M} S_M \frac{T_M^{p+1} - T_M^{p}}{\Delta t}$$

$\boxed{⚠\ \phi \text{ in } W.m^{-1}}$

remove e the thickness

check: $\rho_M C_{p_M} S_M \frac{T^{p+1} - T^{p}}{\Delta t} = \frac{\cancel{kg}}{m^{3}} \frac{J}{\cancel{kg}\,\cancel{K}} \frac{\cancel{m^{2}}\,\cancel{K}}{s} = \frac{J}{m.s} = \frac{W}{m}$!! ok



$$\phi_1 = -k_{NW}\,\frac{\Delta y_N}{2}\,\frac{T^{p+1} - T_W^{p+1}}{\Delta x_W}$$

$$+\;\phi_2 = -k_{SW}\,\frac{\boxed{\Delta y_S}}{2}\,\frac{T^{p+1} - T_W^{p+1}}{\boxed{\Delta x_W}}$$

$$+\;\phi_3 = -k_{SW}\,\frac{\boxed{\Delta x_W}}{2}\,\frac{T^{p+1} - T_S^{p+1}}{\boxed{\Delta y_S}}$$

$$+\;\phi_4 = -k_{SE}\,\frac{\Delta x_E}{2}\,\frac{T^{p+1} - T_S^{p+1}}{\Delta y_S} \qquad -\;\phi_5 = -k_{SE}\,\frac{\Delta y_S}{2}\,\frac{T_E^{p+1} - T^{p+1}}{\Delta x_E}$$

$$-\;\phi_6 = -k_{NE}\,\frac{\Delta y_N}{2}\,\frac{T_E^{p+1} - T^{p+1}}{\Delta x_E} \qquad -\;\phi_7 = -k_{NE}\,\frac{\Delta x_E}{2}\,\frac{T_N^{p+1} - T^{p+1}}{\Delta y_N}$$

$$+\;\phi_1 = -k_{NW}\,\frac{\Delta y_N}{2}\,\frac{T^{p+1} - T_W^{p+1}}{\Delta x_W} \qquad -\;\phi_8 = -k_{NW}\,\frac{\Delta x_W}{2}\,\frac{T_N^{p+1} - T^{p+1}}{\Delta y_N}$$

$$\left\{ -\frac{k_{NE}\,\Delta t}{2\,\rho_M\,C_{pM}\,S_M}\,\frac{\Delta x_E}{\Delta y_N} - \frac{k_{NW}\,\Delta x_W\,\Delta t}{2\,\rho_M\,C_p\,S_m\,\Delta y_N} \right\} T_N^{p+1}$$

$$-\frac{Fo_{NE}^{xy}}{2} \qquad -\frac{Fo_{NW}^{xy}}{2}$$

$$+$$

$$\left\{ \frac{-k_{NW}\,\Delta y_N\,\Delta t}{2\rho_M C_{pM} S_m \Delta x_W} - \frac{k_{SW}\,\Delta y_S\,\Delta t}{2\rho_M\,C_{pM}\,S_M\,\Delta x_W} \right\} T_W^{p+1}$$

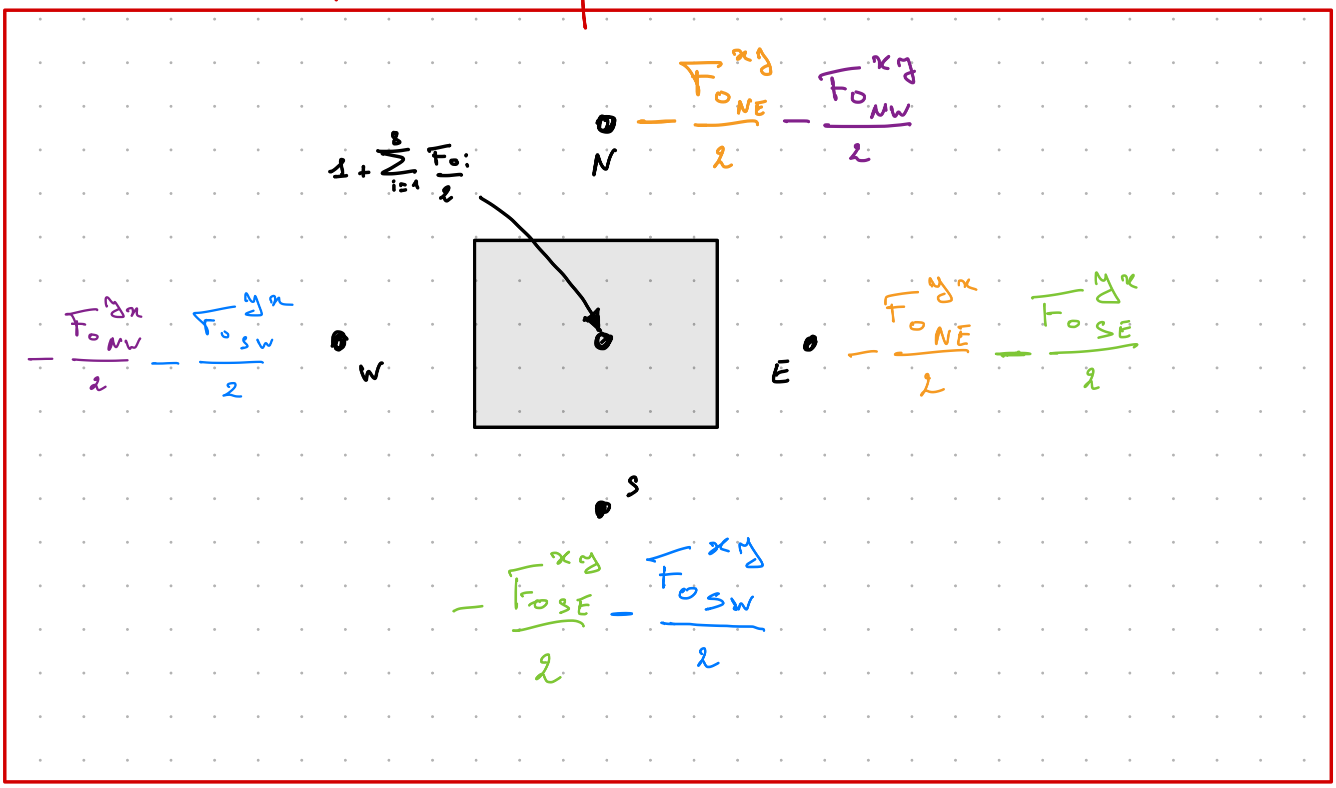
$$-\frac{Fo_{NW}^{yx}}{2} \qquad -\frac{Fo_{SW}^{yx}}{2}$$

$$\left\{ 1 + \boxed{\frac{F_{o_{NE}}^{xy}}{2}} + \boxed{\frac{F_{o_{NW}}^{xy}}{2}} + \boxed{\frac{F_{o_{NW}}^{yx}}{2}} + \boxed{\frac{F_{o_{SW}}^{yx}}{2}} + \boxed{\frac{F_{o_{SW}}^{xy}}{2}} + \boxed{\frac{F_{o_{NE}}^{yx}}{2}} + \boxed{\frac{F_{o_{SE}}^{yx}}{2}} + \boxed{\frac{F_{o_{SE}}^{xy}}{2}} \right\} T^{p+1}$$

$$+ \left\{ -\frac{F_{o_{NE}}^{yx}}{2} - \frac{F_{o_{SE}}^{yx}}{2} \right\} T_E^{p+1}$$

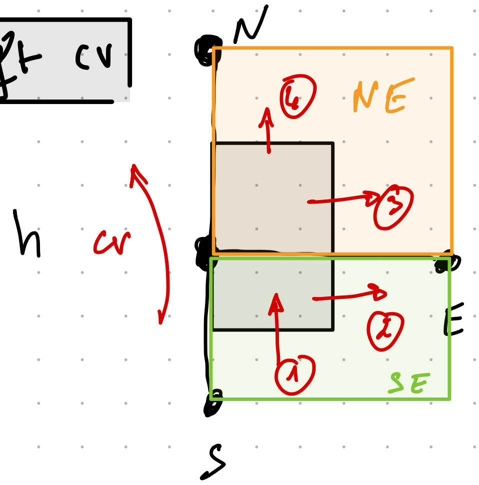
$$+ \left\{ -\frac{F_{o_{SE}}^{xy}}{2} - \frac{F_{o_{SW}}^{xy}}{2} \right\} T_S^{p+1} = T^p$$

Final recap conduction



II Vertical convection

Left CV



$r_W = 0$

$r_E = 1$

$\Delta x_W = 0$

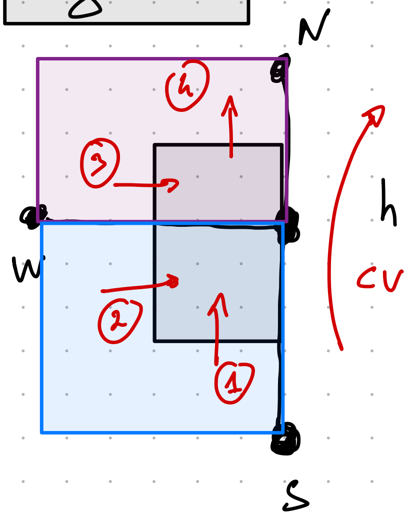
$$\phi_1 - \{\phi_2 + \phi_3 + \phi_4\} - \phi_{cv} = \rho_M c_{p_M} S_M \frac{T^{p+1} - T^p}{\Delta t}$$

$$\phi_{cv} = h \, \frac{\Delta y_S + \Delta y_N}{2} (T - T_\infty)$$

why $(-\phi_{cv})$

$\boxed{T_\infty > T \text{ heating} \Rightarrow T - T_\infty < 0 \Rightarrow -\phi_{cv} \sim \frac{dT}{dt}}$

right CV



$r_W = 1$

$r_E = 0$

$\Delta x_E = 0$

$$\phi_2 + \phi_3 + \phi_1 - \phi_4 - \phi_{cv} = \rho_M c_{p_M} S_M \frac{T^{p+1} - T^p}{\Delta t}$$

$$\left\{ - \frac{F_{oNE}^{xy}}{2} \right\} T_N^{p+1} +$$

$$\left\{ 1 + \frac{F_{oNE}^{xy}}{2} + \frac{F_{oSE}^{xy}}{2} + \frac{F_{oSE}^{yx}}{2} + \frac{F_{oNE}^{yx}}{2} + \boxed{\frac{h(\Delta y_S + \Delta y_N)\Delta t}{2 \rho_M c_{p_M} S_M}} \right\} T^{p+1} +$$

$$\left\{ - \frac{F_{oNE}^{yx}}{2} - \frac{F_{oSE}^{yx}}{2} \right\} T_E^{p+1} +$$

$$\left\{ - \frac{F_{oSE}^{xy}}{2} \right\} T_S^{p+1} =$$

$$T^p + \left(\frac{Bi_{SE} F_{oSE}^{yx}}{2} + \frac{Bi_{NE} F_{oNE}^{yx}}{2} \right) T_\infty$$

$$\frac{h \Delta y_S \Delta t}{2 \rho_M c_{p_M} S_M} + \frac{h \Delta y_N \Delta t}{2 \rho_M c_{p_M} S_M} =$$

$$\frac{h \cancel{R_{SE}} \Delta t \Delta y_S}{2 \rho_M c_{p_M} S_M \cancel{R_{SE}}} + \frac{h R_{NE} \Delta t \Delta y_N}{2 \rho_M c_{p_M} S_M R_{NE}}$$

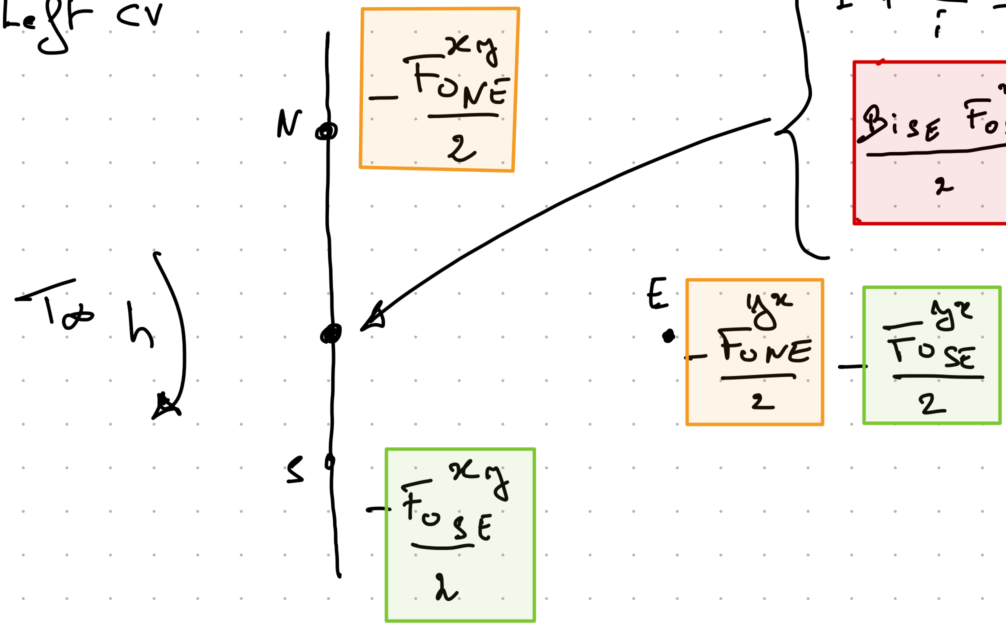
$$\frac{h \cancel{\Delta x_E}}{2 \cancel{R_{SE}}} \times \frac{\cancel{R_{SE}} \Delta t \Delta y_S}{\rho_M c_p S_M \cancel{\Delta x_E}} + \; ''$$

$$\boxed{\frac{Bi_{SE} F_{oSE}^{yx}}{2} + \frac{Bi_{NE} F_{oNE}^{yx}}{2}}$$

with $Bi_{SE} = \dfrac{h \, \Delta x_E}{R_{SE}}$

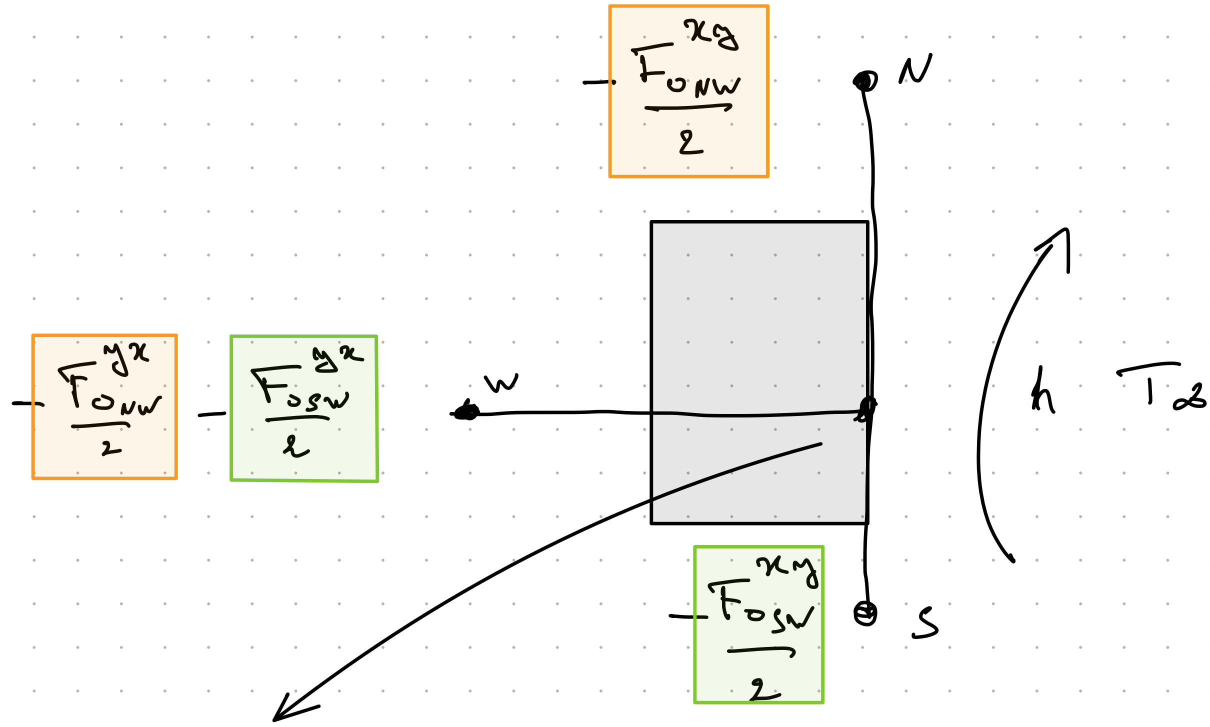
$Bi_{NE} = \dfrac{h \, \Delta x_E}{R_{NE}}$

Left cv



$$1 + \sum_i \frac{F_{oi}}{2} + \boxed{\frac{Bi_{SE}\, F_{oSE}^{yx}}{2} + \frac{Bi_{NE}\, F_{oNE}^{yx}}{2}}$$

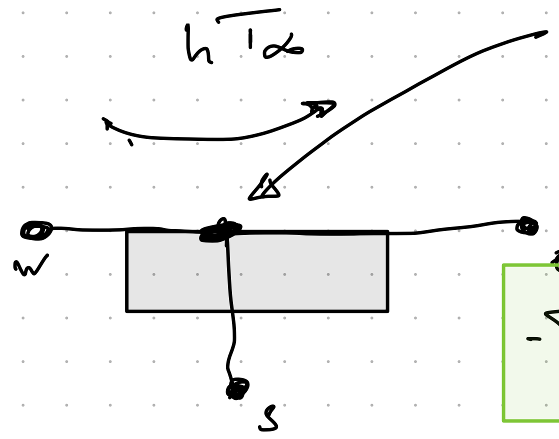
Right cv



$$1 + \sum_i \frac{F_{oi}}{2} + \boxed{Bi_{SW}\frac{F_{oSW}^{yx}}{2} + Bi_{NW}\frac{F_{oNW}^{yx}}{2}}$$

$$Bi_{SW} = \frac{h\,\Delta x_v}{k_{SW}} \qquad Bi_{NW} = \frac{h\,\Delta x_w}{k_{NW}}$$

Top cv



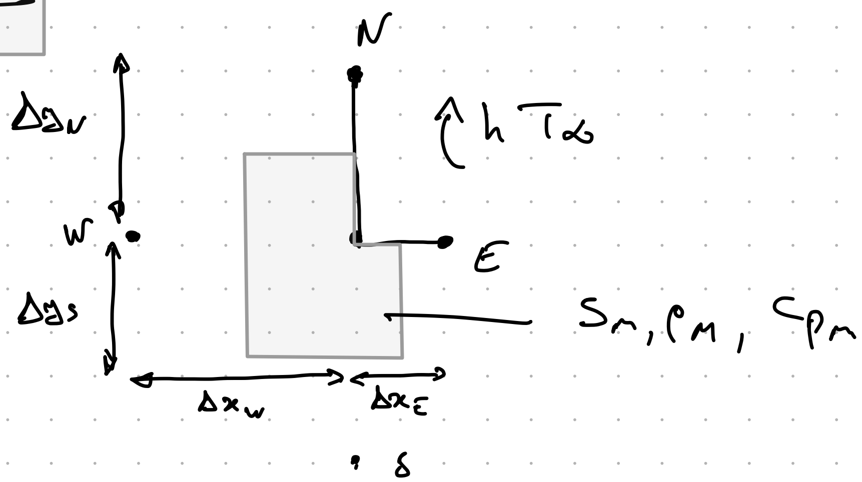
$$1 + \sum_i \frac{Fo_i}{2} + Bi_{WS}\frac{Fo_{SW}^{xy}}{2} + Bi_{ES}\frac{Fo_{SE}^{xy}}{2}$$

$$-\frac{Fo_{SW}^{yx}}{2}$$

$$-\frac{Fo_{SE}^{yx}}{2}$$

$$-\frac{Fo_{SW}^{xy}}{2} \qquad -\frac{Fo_{SE}^{xy}}{2}$$

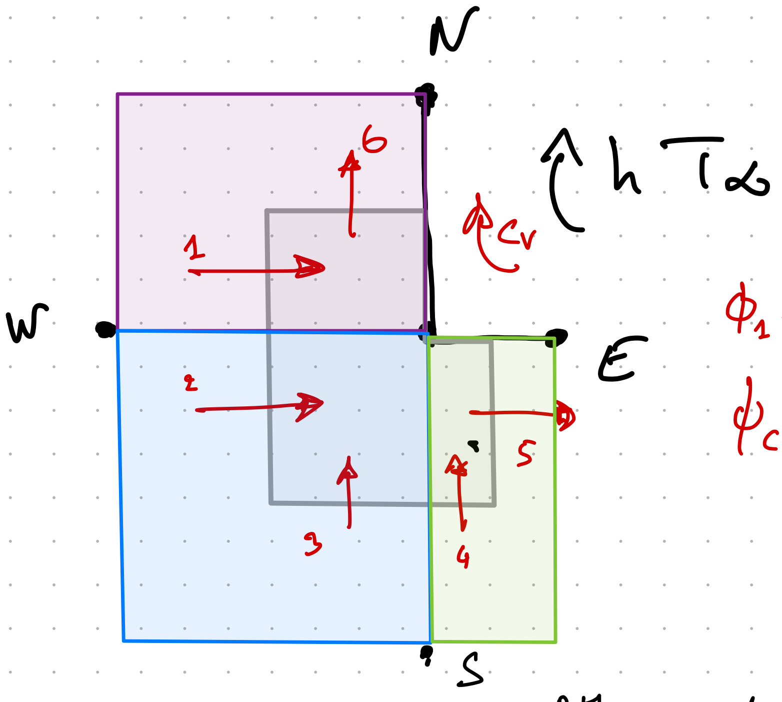
III Corner



$$S_M = S_M - \frac{\Delta x_E \Delta y_N}{4}$$

$$\rho_M = \rho_{NW} r_N r_W + \overset{0}{\cancel{\rho_{NE} r_N r_E}} + \rho_{SW} r_S r_W + \rho_{SE} r_S r_E$$

$$c_{pM} = \frac{\rho_{NW} r_N r_W c_{pNW} + \overset{0}{\cancel{\rho_{NE} r_N r_E c_{pNE}}} + \rho_{SW} r_S r_W c_{pSW} + \rho_{SE} r_S r_E c_{pSE}}{\rho_M}$$



$$\phi_1+\phi_2+\phi_3+\phi_4-\left\{\phi_5+\phi_6\right\}-\phi_{cv}=\rho_M C_{pM} S_M \frac{T^{p+1}-T^{p}}{\Delta t}$$

$$\phi_1=-k_{NW}\frac{\Delta y_N}{2}\frac{T^{p+1}-T_W^{p+1}}{\Delta x_W} \quad ; \quad \phi_2=-k_{SW}\frac{\Delta y_S}{2}\frac{T^{p+1}-T_W^{p+1}}{\Delta x_W}$$

$$\phi_3=-k_{SW}\frac{\Delta x_W}{2}\frac{T^{p+1}-T_S^{p+1}}{\Delta y_S} \quad ; \quad \phi_4=-k_{SE}\frac{\Delta x_E}{2}\frac{T^{p+1}-T_S^{p+1}}{\Delta y_S}$$

$$\phi_5=-k_{SE}\frac{\Delta y_S}{2}\frac{T_E^{p+1}-T^{p+1}}{\Delta x_E} \quad ; \quad \phi_6=-k_{NW}\frac{\Delta x_W}{2}\frac{T_N^{p+1}-T^{p+1}}{\Delta y_N}$$

$$\phi_{cv}=h\,\frac{(\Delta x_E+\Delta y_N)}{2}\left(T^{p+1}-T_\infty\right)$$

rq:

$$\phi_{cv}\times\frac{\Delta t}{\rho_M C_{pM} S_M}=\left\{\frac{h\,\Delta t\,\Delta x_E}{2\rho_M C_{pM} S_M}+\frac{h\,\Delta t\,\Delta y_N}{2\rho_M C_{pM} S_M}\right\}\left(T^{p+1}-T_\infty\right)$$

$$=\left\{\frac{h\,\cancel{\Delta y_S}}{\cancel{k_{SE}}}\times\frac{\cancel{k_{SE}}\,\Delta t\,\Delta x_E}{2\rho_M C_{pM} S_M\,\cancel{\Delta y_S}}+\frac{h\,\cancel{\Delta x_W}}{\cancel{k_{NW}}}\times\frac{\cancel{k_{NW}}\,\Delta t\,\Delta y_N}{2\rho_M C_{pM} S_M\,\cancel{\Delta x_W}}\right\}\left(T^{p+1}-T_\infty\right)$$

$$=\left\{\boxed{Bi_{SE}\,\frac{F_{o\,SE}^{xy}}{2}+Bi_{NW}\times\frac{F_{o\,NW}^{yx}}{2}}\right\}\left(T^{p+1}-T_\infty\right)$$

Corner CV

$$-\boxed{\frac{Fo^{xy}_{NW}}{2}} \quad T_N^{p+1}$$

$$\left\{ -\boxed{\frac{Fo^{yx}_{NW}}{2}} - \boxed{\frac{Fo^{yx}_{SW}}{2}} \right\} \quad T_W^{p+1}$$

$$\left\{ 1 + \sum_i \frac{Fo_i}{2} + \boxed{Bi_{SE} \frac{Fo^{xy}_{SE}}{2} + Bi_{NW} \times \frac{Fo^{yx}_{NW}}{2}} \right\} T^{p+1}$$

$$\left\{ -\boxed{\frac{Fo^{xy}_{SW}}{2}} - \boxed{\frac{Fo^{xy}_{SE}}{2}} \right\} \quad T_S^{p+1}$$

$$-\boxed{\frac{Fo^{yx}_{SE}}{2}} \quad T_E^{p+1}$$

$$= T^p + \left\{ \boxed{Bi_{SE} \frac{Fo^{xy}_{SE}}{2} + Bi_{NW} \times \frac{Fo^{yx}_{NW}}{2}} \right\} T_\infty$$