CS 6301.503 Spring 2019 Scott C. Waggener (scw180000)

Homework 8 (Implementation) Problem 1

Readings

Solution: Complete ■

CS 6301.503 Spring 2019 Scott C. Waggener (scw180000)
Homework 8 (Implementation) Problem 2

Write out all of the terms of Strassen based matrix matrix multiplication for $C = AB$ with BLAS dimensions $M = N = K = 4$ by applying the Strassen decomposition twice (an initial decomposition then a recursive decomposition)

Solution: Recall that the Strassen matrix decomposition algorithm defines the following new matrices.

$$\begin{aligned}
\boldsymbol{M}_1 &= \left(\boldsymbol{A}_{0,0} + \boldsymbol{A}_{1,1}\right)\left(\boldsymbol{B}_{0,0} + \boldsymbol{B}_{1,1}\right) \\
\boldsymbol{M}_2 &= \left(\boldsymbol{A}_{1,0} + \boldsymbol{A}_{1,1}\right)\boldsymbol{B}_{0,0} \\
\boldsymbol{M}_3 &= \boldsymbol{A}_{0,0}\left(\boldsymbol{B}_{0,1} - \boldsymbol{B}_{1,1}\right) \\
\boldsymbol{M}_4 &= \boldsymbol{A}_{1,1}\left(\boldsymbol{B}_{1,0} - \boldsymbol{B}_{0,0}\right) \\
\boldsymbol{M}_5 &= \left(\boldsymbol{A}_{0,0} + \boldsymbol{A}_{0,1}\right)\boldsymbol{B}_{1,1} \\
\boldsymbol{M}_6 &= \left(\boldsymbol{A}_{1,0} - \boldsymbol{A}_{0,0}\right)\left(\boldsymbol{B}_{0,0} + \boldsymbol{B}_{0,1}\right) \\
\boldsymbol{M}_7 &= \left(\boldsymbol{A}_{0,1} - \boldsymbol{A}_{1,1}\right)\left(\boldsymbol{B}_{1,0} + \boldsymbol{B}_{1,1}\right)
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
\boldsymbol{C}_{0,0} &= \boldsymbol{M}_1 + \boldsymbol{M}_4 - \boldsymbol{M}_5 + \boldsymbol{M}_7 \\
\boldsymbol{C}_{0,1} &= \boldsymbol{M}_3 + \boldsymbol{M}_5 \\
\boldsymbol{C}_{1,0} &= \boldsymbol{M}_2 + \boldsymbol{M}_4 \\
\boldsymbol{C}_{1,1} &= \boldsymbol{M}_1 - \boldsymbol{M}_2 + \boldsymbol{M}_3 + \boldsymbol{M}_6
\end{aligned}$$

We begin by applying a partition over $\boldsymbol{C}$ into four block matrices

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{0,0} & \boldsymbol{C}_{0,1} \\ \boldsymbol{C}_{1,0} & \boldsymbol{C}_{1,1} \end{bmatrix}$$

where the block matrices are given by

$$\boldsymbol{C}_{0,0} = \begin{bmatrix} A_{0,0} & A_{0,1} \\ A_{1,0} & A_{1,1} \end{bmatrix} \cdot \begin{bmatrix} B_{0,0} & B_{0,1} \\ B_{1,0} & B_{1,1} \end{bmatrix} \qquad \boldsymbol{C}_{1,0} = \begin{bmatrix} A_{2,0} & A_{2,1} \\ A_{3,0} & A_{3,1} \end{bmatrix} \cdot \begin{bmatrix} B_{2,0} & B_{2,1} \\ B_{3,0} & B_{3,1} \end{bmatrix}$$

$$\boldsymbol{C}_{0,1} = \begin{bmatrix} A_{0,2} & A_{0,3} \\ A_{1,2} & A_{1,3} \end{bmatrix} \cdot \begin{bmatrix} B_{0,2} & B_{0,3} \\ B_{1,2} & B_{1,3} \end{bmatrix} \qquad \boldsymbol{C}_{1,1} = \begin{bmatrix} A_{2,2} & A_{2,3} \\ A_{3,2} & A_{3,3} \end{bmatrix} \cdot \begin{bmatrix} B_{2,2} & B_{2,3} \\ B_{3,2} & B_{3,3} \end{bmatrix}$$

Next we recursively compute the matrix multiplications in this decomposition. Note the use of non-bold symbols reflecting that we are now working with scalar elements at this level of recursion.

For $\boldsymbol{C}_{0,0}$ we have

$$\begin{aligned}
M_1 &= \left(A_{0,0} + A_{1,1}\right)\left(B_{0,0} + B_{1,1}\right) \\
M_2 &= \left(A_{1,0} + A_{1,1}\right)B_{0,0} \\
M_3 &= A_{0,0}\left(B_{0,1} - B_{1,1}\right) \\
M_4 &= A_{1,1}\left(B_{1,0} - B_{0,0}\right) \\
M_5 &= \left(A_{0,0} + A_{0,1}\right)B_{1,1} \\
M_6 &= \left(A_{1,0} - A_{0,0}\right)\left(B_{0,0} + B_{0,1}\right) \\
M_7 &= \left(A_{0,1} - A_{1,1}\right)\left(B_{1,0} + B_{1,1}\right)
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
C_{0,0} &= M_1 + M_4 - M_5 + M_7 \\
C_{0,1} &= M_3 + M_5 \\
C_{1,0} &= M_2 + M_4 \\
C_{1,1} &= M_1 - M_2 + M_3 + M_6
\end{aligned}$$

For $\boldsymbol{C}_{0,1}$ we have

$$\begin{aligned}
M_1 &= \left(A_{0,2}+A_{1,3}\right)\left(B_{0,2}+B_{1,3}\right)\\
M_2 &= \left(A_{1,2}+A_{1,3}\right)B_{0,2}\\
M_3 &= A_{0,2}\left(B_{0,3}-B_{1,3}\right)\\
M_4 &= A_{1,3}\left(B_{1,2}-B_{0,2}\right)\\
M_5 &= \left(A_{0,2}+A_{0,3}\right)B_{1,3}\\
M_6 &= \left(A_{1,2}-A_{0,2}\right)\left(B_{0,2}+B_{0,3}\right)\\
M_7 &= \left(A_{0,3}-A_{1,3}\right)\left(B_{1,2}+B_{1,3}\right)
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
C_{0,2} &= M_1+M_4-M_5+M_7\\
C_{0,3} &= M_3+M_5\\
C_{1,2} &= M_2+M_4\\
C_{1,3} &= M_1-M_2+M_3+M_6
\end{aligned}$$

For $\boldsymbol{C}_{1,0}$ we have

$$\begin{aligned}
M_1 &= \left(A_{2,0}+A_{3,1}\right)\left(B_{2,0}+B_{3,1}\right)\\
M_2 &= \left(A_{3,0}+A_{3,1}\right)B_{2,0}\\
M_3 &= A_{2,0}\left(B_{2,1}-B_{3,1}\right)\\
M_4 &= A_{3,1}\left(B_{3,0}-B_{2,0}\right)\\
M_5 &= \left(A_{2,0}+A_{2,1}\right)B_{3,1}\\
M_6 &= \left(A_{3,0}-A_{2,0}\right)\left(B_{2,0}+B_{2,1}\right)\\
M_7 &= \left(A_{2,1}-A_{3,1}\right)\left(B_{3,0}+B_{3,1}\right)
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
C_{2,0} &= M_1+M_4-M_5+M_7\\
C_{2,1} &= M_3+M_5\\
C_{3,0} &= M_2+M_4\\
C_{3,1} &= M_1-M_2+M_3+M_6
\end{aligned}$$

For $\boldsymbol{C}_{1,1}$ we have

$$\begin{aligned}
M_1 &= \left(A_{2,2}+A_{3,3}\right)\left(B_{2,2}+B_{3,3}\right)\\
M_2 &= \left(A_{3,2}+A_{3,3}\right)B_{2,2}\\
M_3 &= A_{2,2}\left(B_{2,3}-B_{3,3}\right)\\
M_4 &= A_{3,3}\left(B_{3,2}-B_{2,2}\right)\\
M_5 &= \left(A_{2,2}+A_{2,3}\right)B_{3,3}\\
M_6 &= \left(A_{3,2}-A_{2,2}\right)\left(B_{2,2}+B_{2,3}\right)\\
M_7 &= \left(A_{2,3}-A_{3,3}\right)\left(B_{3,2}+B_{3,3}\right)
\end{aligned}
\qquad \text{where} \qquad
\begin{aligned}
C_{2,2} &= M_1+M_4-M_5+M_7\\
C_{2,3} &= M_3+M_5\\
C_{3,2} &= M_2+M_4\\
C_{3,3} &= M_1-M_2+M_3+M_6
\end{aligned}$$

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