Introduction to Options

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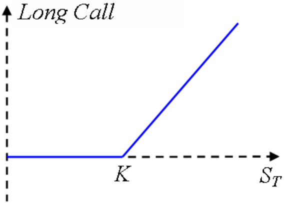
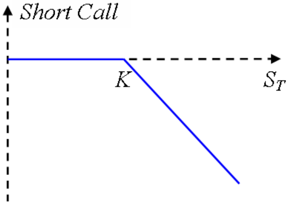
Readings: Hull Chapters 8, 9, 10

Call options

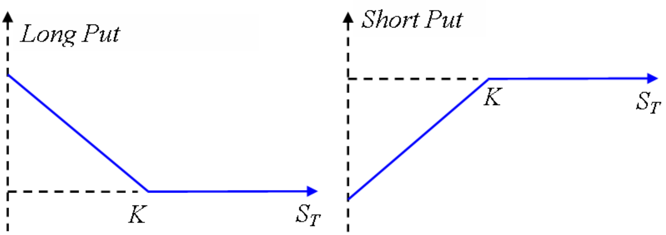
- Call option: gives the holder the **right but not the obligation** to buy an asset for price K (strike price) by a future time T (maturity)
- **Long** call: buy a call; **short** call: sell/write a call
- Long call **payoff** at T: $(S_T - K)^+$; short call payoff: $-(S_T - K)^+$





Put options

- Put option: gives the holder the **right but not the obligation** to sell an asset for price K by a future time T
- Long put **payoff** at T: $(K - S_T)^+$; short put payoff: $-(K - S_T)^+$



European and American options

- **European:** can be exercised only at maturity
  - CBOE S&P 500 index options
  - PHLX (**www.phlx.com**) British Pound options
- **American:** can be exercised any time before or at maturity
  - CBOE Microsoft stock option
  - American option = European option + right to exercise early
  - American option price $\geq$ European option price
- **Bermudan:** can be exercised at any time in a discrete set

- **Intrinsic value** of an American option at $t \leq T$

$$\text{call: } (S-K)^+, \quad \text{put: } (K-S)^+$$

- American option value $\geq$ its intrinsic value at any time $t \leq T$

$$\text{Time value} \;=\; \text{option value} \;-\; \text{intrinsic value}$$

- A call (**put**) option at time $t \leq T$ is
  - **in the money** if $S > K$ ($S < K$)
  - **out of the money** if $S < K$ ($S > K$)
  - **at the money (ATM)** if $S = K$ ($S = K$)
- Exercise a European option: if in the money at time T; exercise an American option: if in the money & time value $= 0$

Trading basics

- Underlying assets:
  - stock
  - currency
  - stock index (e.g., S&P 500, DJIA), cash settled
- Maturities: months to years (LEAPS: long-term equity anticipation securities)
- Strikes: close to current asset price initially
- Flex options: flexible strikes/maturities
- Market makers: earn ask (ready to sell) − bid (ready to buy)
- Other issues: adjustment for stock splits, commissions, margining, clearing, regulation, taxation, option like contracts

Option value function

- Option price depends on: S_0, K, T, **volatility** (σ), r, dividend (yield q or discrete dividends)
- **Current stock price**: call price higher when asset price higher, put price lower when asset price higher
- **Strike price**: call price lower when strike price higher, put price higher when strike price higher
- **Time to maturity**: American option price higher for larger maturity
- **Volatility**: option price higher for larger volatility

Arbitrage relationships for options

- Relationships must hold for option prices; arbitrage opportunities exist otherwise
- Model independent
- Call price is a decreasing function of strike price K. How do you construct an arbitrage strategy if otherwise: $K_1 < K_2$ but $c_1 < c_2$?
- Put price is an increasing function of strike price K

- **Notations**

  0 : current time

  T : option maturity

  S_0 : current asset price

  K : strike price

  S_T : asset price at maturity

  r : continuous compounding risk free interest rate

  c/p : European call/put price

  C/P : American call/put price

  D : present value of income during $[0, T]$

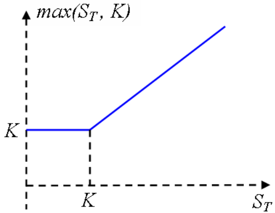
  q : continuous yield

European put-call parity

- European put-call parity (assets with no income)

$$c + Ke^{-rT} = p + S_0$$

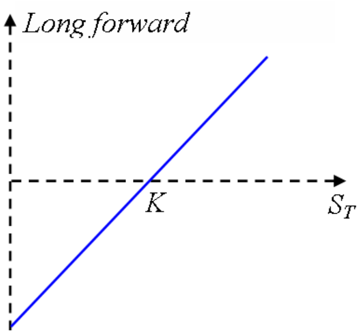
Consider two portfolios: (1). call + Ke^{-rT} deposited at rate r; (2). put + asset



Same payoff at time T $\Rightarrow$ same value at time 0

- Long forward = long call + short put:

$$S_T - K = (S_T - K)^+ - (K - S_T)^+$$



Value at time 0 of long forward contract $V_0 = c - p$

- Recall that

$$V_0 = e^{-rT}(F_0 - K)$$

where F_0 is the **forward price**

$$F_0 = \begin{cases} S_0 e^{rT} & \text{asset paying no income} \\ (S_0 - D)e^{rT} & \text{asset paying known income} \\ S_0 e^{(r-q)T} & \text{asset with continuous yield} \end{cases}$$

- **European put-call parity** (general case)

$$c + e^{-rT}K = p + \begin{cases} S_0 & \text{asset paying no income} \\ S_0 - D & \text{asset paying known income} \\ S_0 e^{-qT} & \text{asset with continuous yield} \end{cases}$$

- **Example** *consider a call and a put (European) on a non-dividend-paying stock with the same strike* $K = 30$ *and maturity* $T = 1/4$. $S_0 = 31, r = 10\%$. *The call price is* $c = 3$. *The put price is* $p = 2.25$. *Is there arbitrage opportunities?*

  Put call parity not holding:
  $c + Ke^{-rT} = 3 + 30e^{-0.1/4} = 32.26 < p + S_0 = 2.25 + 31 = 33.25$:
  buy call, sell put, (short) sell stock, deposit 30.25

  At $T = 1/4$, get $30.25e^{0.1/4} = 31.02$, **if** $S_T > 30$, exercise call and buy stock for $K = 30$, earn 1.02

  If $S_T \leq 30$, put exercised and obliged to buy stock for $K = 30$, earn 1.02

Option price bounds - calls

- **European call** option price bounds (asset with continuous yield q)
$$(S_0 e^{-qT} - Ke^{-rT})^+ \leq c \leq S_0 e^{-qT}$$
- **Upper bound**: European call option payoff $(S_T - K)^+ \leq S_T$:
$$c \leq S_0 e^{-qT}$$
Construct an arbitrage strategy otherwise

- **Low bound**: from put-call parity

$$\begin{aligned} c &= p + S_0 e^{-qT} - Ke^{-rT} \\ &\geq S_0 e^{-qT} - Ke^{-rT} \end{aligned}$$

call price is non-negative

$$c \geq 0$$

Therefore, $c \geq (S_0 e^{-qT} - Ke^{-rT})^+$

- **Example** *consider a European call on a non-dividend-paying stock.* $S_0 = 20, K = 18, r = 10\%, T = 1$. *Suppose the call price is* $c = 3$. *Construct an arbitrage strategy.*

  Lower bound $S_0 - Ke^{-rT} = 20 - 18e^{-0.1} = 3.71$: buy call, (short) sell stock, deposit $S_0 - c = 17$

  At $T = 1$: get $17e^{0.1} = 18.79$; **if** $S_T > 18$, exercise call and buy stock at $K = 18$, earn $18.79 - 18 = 0.79$

  If $S_T \leq 18$, abandon call and buy stock at S_T, earn $18.79 - S_T \geq 0.79$

Option price bounds - puts

- **European put** option price bounds

$$(Ke^{-rT} - S_0 e^{-qT})^+ \leq p \leq Ke^{-rT}$$

- **Upper bound**: European put option payoff is $(K - S_T)^+ \leq K$. So

$$p \leq Ke^{-rT}$$

  Construct an arbitrage strategy otherwise

- **Lower bound**: from put-call parity,

$$
\begin{aligned}
p &= c + Ke^{-rT} - S_0 e^{-qT} \\
&\geq Ke^{-rT} - S_0 e^{-qT}
\end{aligned}
$$

  Therefore, $p \geq (Ke^{-rT} - S_0 e^{-qT})^+$ since $p \geq 0$
- Otherwise, one can construct an arbitrage strategy

- **Example** *consider a European put on a non-dividend-paying stock.* $S_0 = 37, K = 40, r = 5\%, T = 0.5$. *The put price is* $p = 1$. *Construct an arbitrage strategy.*

  Lower bound: $Ke^{-rT} - S_0 = 40e^{-0.05/2} - 37 = 2.01$: buy put, buy stock, borrow $p + S_0 = 38$

  At $T = 0.5$, need to repay $38e^{0.05/2} = 38.96$, **if** $S_T < 40$, exercise put and sell stock for 40, earn 1.04

  If $S_T \geq 40$, abandon put and sell stock for S_T, earn $S_T - 38.96 \geq 1.04$

Bounds and parity for American options

- A lower bound for a European call/put is also a lower bound for an American call/put
- American option price cannot be lower than its intrinsic value

$$P \geq (K - S_0)^+, \quad C \geq (S_0 - K)^+$$

- Upper bounds: $C \leq S_0$, $P \leq K$
- **"Parity"**

$$S_0 e^{-qT} - K \leq C - P \leq S_0 - Ke^{-rT}$$

American calls on assets with no income

- **Early exercise never optimal:** suppose $S > K$
- Exercise the call to buy the share and hold the share:

  Better to exercise later to delay the cash payment K

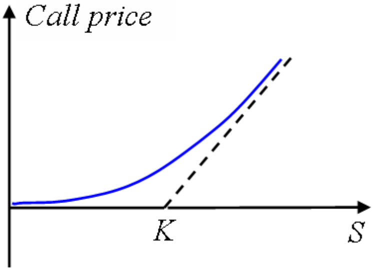
- Exercise the call to buy the share, sell the share immediately, receive $S - K$:

  Better to sell the call

  since

$$C \geq (S - Ke^{-rT})^+ > S - K$$

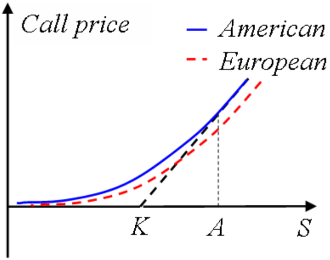
- Call on assets with no income: Early exercise is not optimal; American call price = European call price
- Positive **time value** at any time before maturity



asset with no income

American calls on assets with income

- May be optimal to early exercise an American call on an asset paying income

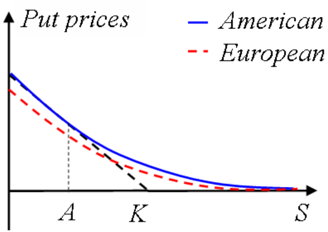


asset with income

- Find A: call should be exercised when $S > A$; **early exercise boundary**: collection of A's for varying maturities

American puts

- **Early exercise may be optimal for American puts**
- When the underlying asset price is close to zero, early exercise is optimal
  - Exercise the put and receive K immediately
  - Exercise later to receive at most K
- Find A: put should be exercised when $S < A$; find the **early exercise boundary**

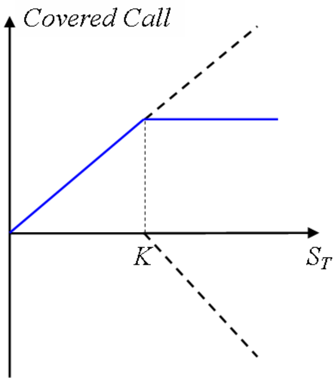


Option trading strategies

- Trading strategies involving an option and the underlying, multiple calls or puts (**spreads**), combinations of calls and puts (**combinations**)
- Different strategies reflecting different market views (bull spreads, bear spreads, butterfly spreads, etc.)
- Understand the risks

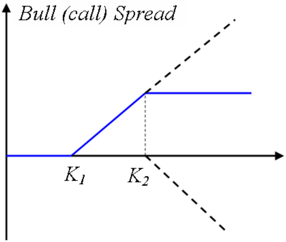
Covered calls

- **Covered call:** sell call, buy asset (with no income), cost $S - c > 0$, payoff $S_T - (S_T - K)^+$



Bull spreads

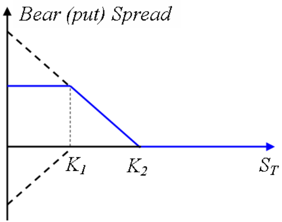
- **Bull spreads:** buy call with strike K_1, sell call with strike $K_2 > K_1$, cost $c_1 - c_2 > 0$



- Give up upside potential profit by selling a call with strike K_2
- Appropriate if expect a moderate increase in the asset price
- Bull spreads **using puts** (long put with strike K_1, short put with strike K_2)

Bear spreads

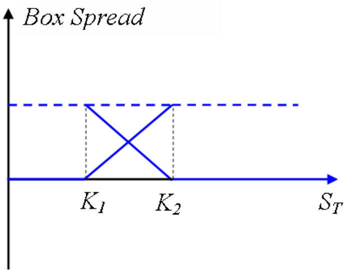
- **Bear spreads:** buy put with strike K_2, sell put with strike K_1 cost $p_2 - p_1 > 0$



- Give up some potential profit by selling a put with strike K_1
- Appropriate if expect a moderate decrease in the asset price
- Bear spreads **using calls**: long call with strike K_2, short call with strike K_1

Box spreads

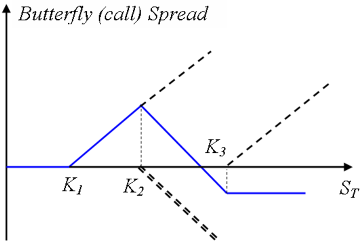
- **Box spreads:** bull (call) spread + bear (put) spread



- Cost $c_1 - c_2 + p_2 - p_1 > 0$, *payoff:* $K_2 - K_1$
- No arbitrage $\Rightarrow c_1 - c_2 + p_2 - p_1 = (K_2 - K_1)e^{-r(T-t)}$ (European options used!)

Butterfly spreads

- **Butterfly spreads:** buy calls with strikes K_1 and K_3, sell two calls with strike $K_2 \in (K_1, K_3)$, cost $c_1 + c_3 - 2c_2$



- Appropriate if expect no significant change in the asset price
- Butterfly spreads using puts

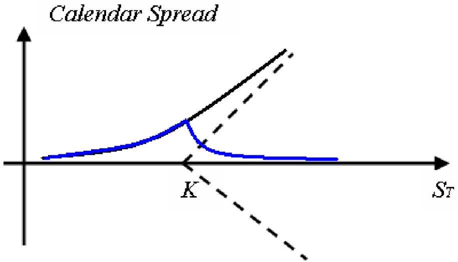
- **Example** *three European calls on a stock are available with strikes* $K_1 = 55, K_2 = 60, K_3 = 65$. *Call prices are* $c_1 = 10, c_2 = 7, c_3 = 5$. *Analyze the P&L of a butterfly spread constructed using the calls.*

  Long calls with strikes 55 and 65, short two calls with strike 60, initial cost 1

  At maturity, **if** $S_T \leq 55$, all calls expire worthless, profit of -1
  If $55 < S_T \leq 60$, profit of $S_T - 55 - 1 = S_T - 56$
  If $60 < S_T \leq 65$, profit of $S_T - 55 - 2(S_T - 60) - 1 = 64 - S_T$
  If $65 < S_T$, profit of $S_T - 55 - 2(S_T - 60) + S_T - 65 - 1 = -1$

Calendar spreads

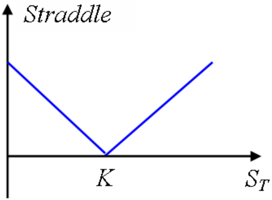
- **Calendar spreads:** sell call (maturity T), buy call (maturity $T_1 > T$), cost $c_1 - c$, payoff at time T



- Similar as butterfly spreads, using 2 calls only
- Calendar spreads using puts
- **Diagonal spreads**: different strikes, different maturities

Straddles

- **Straddle:** buy call & put with strike K, cost $c + p > 0$



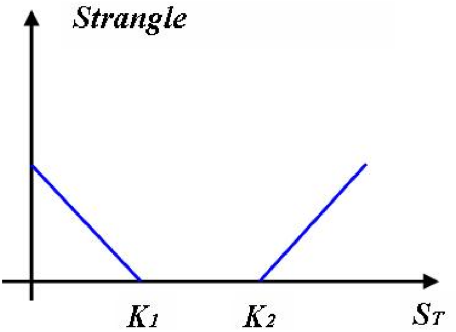
- Appropriate if expect a large move in the asset price (don't know the direction)
- Short straddles: expect no large moves, **Highly risky!**

Barings Bank

- Oldest investment bank in UK
- One of the two companies who facilitated the **Louisiana Purchase** in 1803 (15m =3m down payment + bonds)
- **Nick Leeson** of Barings Bank: **short straddles** on Nikkei 225, betting the Japanese stock market would not move significantly
- Kobe earthquake on Jan 17 1995
- **Long futures** to manipulate the market; caused a loss of $1.4b; Barings bankrupted

Strangles

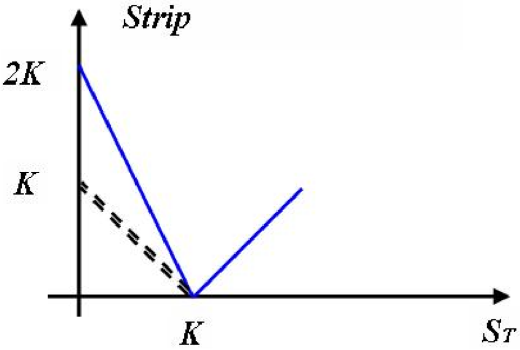
- **Strangle:** buy put with strike K_1 and call with strike $K_2 > K_1$, cost $c + p > 0$



- Less expensive than straddle

Strips

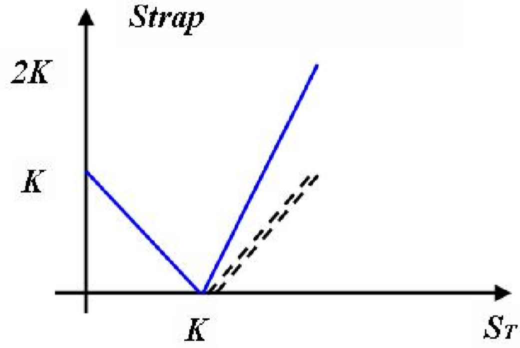
- **Strip:** buy two puts & one call with strike K, cost $c + 2p > 0$



- Appropriate when expecting a large move, more likely a drop

Straps

- **Strap:** buy two calls & one put with strike K, cost $2c + p > 0$



- Appropriate when expecting a large move, more likely a rise