IE 420 Financial Engineering

Liming Feng

Dept. of Industrial & Enterprise Systems Engineering
University of Illinois at Urbana-Champaign

Financial crisis of 2007-2010

- Started in 2007 with losses in subprime loans
- Bear Stearns, Lehman Brothers, Merrill Lynch
- Unemployment rate: 4.4% in Oct 2007 to 9.6% Jun 2010

- Low interest rate in late 1990s
- Real estate boom, sustained by subprime mortgages
- Packaged into mortgage-backed securities
- Trading of MBS made easier by AAA ratings by credit-rating agencies
- and by the excessive trading of default insurances

- Increases of interest rate led to simultaneous defaults, caused huge losses on investors, insurers, mortgage originators
- Risks associated with MBS and credit derivatives not fully understood
- Default correlation not correctly modeled
- Need better financial engineers: understand the risks associated with derivative securities, can model and manage such risk effectively

Financial engineering

- Interested in studying **derivative** securities: *forwards, futures, options, etc.*
- Derivative: a contract whose value is derived from the values of more basic **underlying assets/variables**
- **Financial engineering**
  - Construction of financial products from other assets/variables to meet specific requirements
  - Applications of mathematical and computational techniques to solving financial problems, in particular, the pricing and risk management of derivatives

Derivatives pricing/risk management

- Find the relationship between the underlying asset price and the derivative price
- Need to model the **dynamics of the underlying** assets; need to handle **complex structures** (satisfying specific requirements of customers)
- For the pricing and risk management of a derivative contract
  - Mathematical **modeling** (probabilistic)
  - **Computational** scheme (binomial tree, solving PDEs, monte carlo simulation, fast Fourier transform, etc.)
  - Computer **implementation** (industry standard C/C++)

Objectives of the course

- Understanding derivatives
  - e.g., what is an option
  - why options are so heavily traded
  - how to trade options
  - what are associated risks
- Pricing and risk management of derivatives
  - introduce standard models
  - associated numerical methods
  - implementation
  - limitations of the models

Potential recruiters

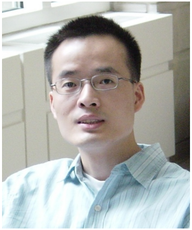
- Trading firms (trading assistants; traders; traders regularly visit IESE: potential guest speakers; campus career fairs)
- Financial institutions (IBs, hedge funds, etc.)
- Corporations (e.g. automakers, airlines, who need to hedge financial risks)
- Consulting firms
- **Regulating agencies**

Further training

- **Master of Science in Financial Engineering (MSFE)**
  1. Fin 500: Introduction to Finance
  2. Fin 501: Financial Economics
  3. IE 522: Statistical Methods in Finance
  4. IE 523: Financial Computing
  5. IE 524: Optimization in Finance
  6. IE 525: Numerical Methods in Finance
  7. Fin 512: Financial Derivatives
  8. IE 526: Stochastic Calculus in Finance
  9. Fin 517: Credit Risk and Instruments
  10. Fin 516: Term Structure Models
  11. IE 527/Fin 576: Financial Engineering Project
  12. Elective
- More info on http://msfe.illinois.edu/

Introduce yourself

- Syllabus available on compass
- Submit the following on compass (under assignments)
  - Name
  - Program/year (e.g., IE 1st year master, CS senior, etc.)
  - Attach a photo which helps me to identify you



Introduction to derivatives

Readings: Hull chapter 1

A concrete example

- A US dealer has signed a contract with a British company to buy 4000 units of a certain machine in 6 months at the price of £250/unit. The dealer will pay £1 million at the time of delivery. The current exchange rate b/w GBP and USD is 2.0 USD/GBP. How does the dealer hedge the financial risk it faces?
- Main financial risk for the dealer?
- Scenarios without hedging:
  - Exchange rate drops to $S_T = 1.8$ USD/GBP: pay \$1.8 million
  - Exchange rate rises to $S_T = 2.2$ USD/GBP: pay \$2.2 million

- Strategy I: a simple approach
  - Buy £1 million today at the price of $2.0 USD/GBP
  - Drawback: the dealer may not have access to $2.0 million today
- Strategy II: a forward approach
  - Enter a **forward contract** with a bank to buy £1 million in 6 months at the rate of 2.01 USD/GBP
  - **Forward price** 2.01 is so determined that the cost of the contract today is 0

When the forward contract matures in 6 months

- The dealer delivers $2.01 million
- The bank delivers £1 million
- The dealer completely eliminates the exchange rate risk it faces

- Scenarios when the forward contract is used
  - $S_T = 2.2$ USD/GBP: instead of paying \$ 2.2 million, only needs to pay \$2.01 million
  - $S_T = 1.8$ USD/GBP: instead of paying \$1.8 million, has to pay \$2.01 million
- The forward provides **(one-sided) protection** from unfavorable rate movements

- Strategy III: an option approach
  - Purchase **an option** contract from a bank to buy £1 million in 6 months at the **strike price** 2.0 USD/GBP

  When the option contract matures in 6 months
  - $S_T = 2.2$ USD/GBP: exercise the option and buy £1 million at the rate 2.0 from the bank
  - $S_T = 1.8$ USD/GBP: abandon the option and buy £1 million on the market
- The option holder has the **RIGHT BUT NOT THE OBLIGATION** to buy USD at the rate 2.0
- The option provides protection from unfavorable rate movements, allows profit from favorable rate movements
- Insurance: the option is not free

Derivatives

- A **derivative** is a contract whose value depends on the values of some underlying assets or variables
  - Both the forward and option contracts are derivatives
  - The underlying asset is the GBP
- Possible underlying assets or variables
  - Stocks: Microsoft, IBM
  - Stock indices: DJIA, S&P 500
  - Bonds: treasury bills, notes, bonds
  - Foreign currencies: GBP, Euro
  - Commodities: copper, corn
  - Weather conditions: temperature, snowfall

- Derivatives typically do not incur the transferring of the underlying at initiation (for the forward and option contracts, USD and GBP are exchanged in 6 months)
- **Spot market**: an asset is transferred immediately. **Spot price**: the price of the asset for immediate delivery
- **Financial derivative**: with financial instruments, interest rates, exchange rates as the underlying
- World centers for derivatives trading: Chicago, NY, London

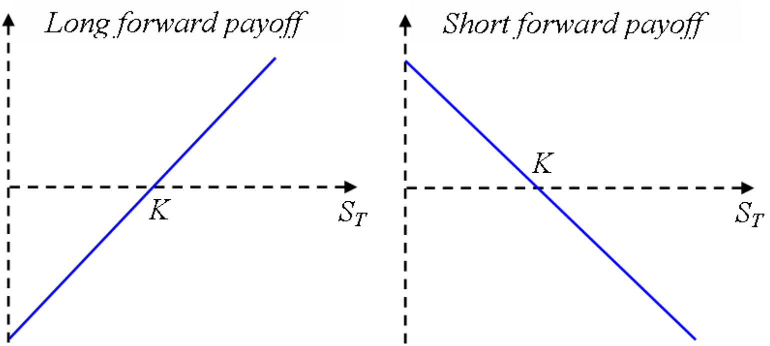
Forward Contracts

- A **forward contract** is an agreement to buy or sell an asset at a certain future time T (maturity or expiration) for a certain price K (delivery price)
- **Forward price**: the delivery price that makes the forward contract zero cost
- **Long position:** agree to buy; **Short position:** agree to sell
- **Example (long forward)**: *The dealer agrees to buy GBP in 6 months at the rate of* 2.01 *USD/GBP. The dealer has a long forward position.* $T = 6$ *months*, $K = 2.01$ *USD/GBP*

Payoff for each GBP

- S_T: the underlying asset price at time T
- Payoff for the long forward position (dealer)
  - $S_T = 2.2$ USD/GBP: payoff $= S_T - K = 2.2 - 2.01 = 0.19$
  - $S_T = 1.8$ USD/GBP: payoff $= S_T - K = 1.8 - 2.01 = -0.21$
  - **long forward payoff** at maturity: $S_T - K$
- Payoff for the short forward position (bank)
  - $S_T = 2.2$ USD/GBP: payoff $= K - S_T = 2.01 - 2.2 = -0.19$
  - $S_T = 1.8$ USD/GBP: payoff $= K - S_T = 2.01 - 1.8 = 0.21$
  - **short forward payoff** at maturity: $K - S_T$

- Payoffs of long and short forward positions



Futures Contracts

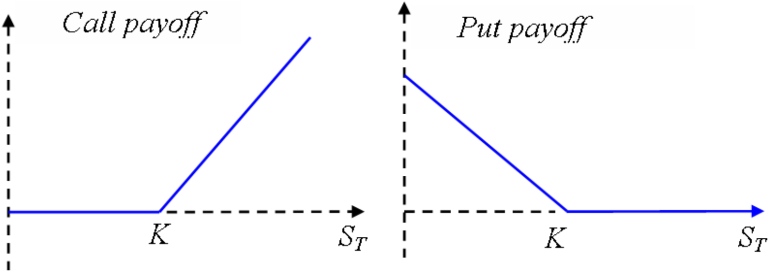
- Forward contracts are traded in the **over-the-counter (OTC) market**
    - Trades are done between two counterparties (financial institutions, clients, etc.)
    - **Credit risk**: default may occur
- Futures contracts are normally **traded on exchanges**
    - Highly standardized: the exchange defines the terms of a contract (less flexible)
    - Credit risk mostly eliminated
- **The Chicago Board of Trade** (CBOT, since 1848), World's first futures exchange (Part of CME since July 07)
- **The Chicago Mercantile Exchange** (CME, since 1919), one of the world's largest derivatives exchanges

Options

- A **call option** gives its holder **the right but not obligation** to buy an asset for a certain price K (**strike price**) by a certain future time T (**expiration time or maturity**)
- **Example (call options)**: *The dealer enters a call option with a bank to buy GBP in 6 months at the rate of 2.0 USD/GBP. Maturity $T = 6$ months, strike price $K = 2.0$ USD/GBP*

- Call payoff for the option holder (dealer)
  - $S_T = 2.2$ USD/GBP: payoff $= S_T - K = 2.2 - 2.0 = 0.2$
  - $S_T = 1.8$ USD/GBP: exercising the option leads to negative payoff $S_T - K = 1.8 - 2.0 = -0.2$. The option holder should not exercise the option: payoff $= 0$
  - **Call payoff for the option holder**: $\max(S_T - K, 0) = (S_T - K)^+$
- Call payoff for the **option writer** (bank): $-(S_T - K)^+$
- Option holder has all the rights, option writer has all the obligations

- Call and put option payoffs at option maturity



- A **put option** gives its holder the right but not obligation to sell an asset by a certain time T for a certain price K
- **Example (put options)**: *Suppose an investor purchases a put option on the IBM stock with maturity 3 months and strike price* **$100**. *What will be the payoff of the put?*
- Maturity $T = 3$ months, strike price $K = \$100$, $S_T =$ IBM stock price in 3 months

- Put payoff for the investor
  - $S_T = 80$: payoff $= K - S_T = 100 - 80 = 20$
  - $S_T = 120$: exercising the option leads to negative payoff $K - S_T = 100 - 120 = -20$, the option should not be exercised: payoff $= 0$
  - **Put payoff for the option holder**: $\max(K - S_T, 0) = (K - S_T)^+$
- Put payoff for the option writer: $-(K - S_T)^+$
- **European options** can be exercised only at maturity
- **American options** can be exercised any time at or before maturity
- *The Chicago Board Options Exchange* (CBOE, since 1973, World's first options exchange and one of the largest)

Types of derivative traders

- **Hedgers** use derivatives to reduce risk
- **Speculators** use derivatives to bet on the future direction of a market variable
- **Arbitrageurs** are looking for risk free profit
- **Market makers**: maintain a two way market, execute trades for others and earn bid/ask spread
  Bid price: price at which the market maker is willing to buy
  Ask price: price at which the market maker is willing to sell

Hedging using derivatives

- **Derivatives can be used to reduce financial risks**

  "GM is exposed to market risk from changes in **foreign currency exchange rates, interest rates, and certain commodity prices**. *In the normal course of business, GM enters into a variety of* **foreign exchange, interest rate, and commodity forward contracts, swaps, and options**, *with the objective of minimizing exposure arising from these risks. A risk management control system is utilized to monitor foreign exchange, interest rate, commodity, and related hedge positions."* – From *GM 2005 annual report*

- **Example (hedging using forwards):** A US company, which is exporting goods to UK, will receive £1 million 6 months later. The company decides to enter a forward contract to sell £1 million at an exchange rate of 2.01 USD/GBP in 6 months.

	$S_T = 1.8$ \$/£	$S_T = 2.2$ \$/£
no hedging	\$1.8 million	\$2.2 million
hedging	\$2.01 million	\$2.01 million
Cost of hedging	No cost	

  - The forward guarantees a receipt of \$2.01 million in 6 month
  - Provides a protection if the USD/GBP rate **decreases**
  - Hedging does not necessarily improve the overall financial outcome, but reduces risks by **making the outcome more certain** (so that the hedger can focus on its core business)

- **Example (hedging using options):** *An investor holding 1000 IBM shares worries that IBM stock price may drop in a month. Current IBM stock price is $100. The investor decides to buy 1000 put options with one month maturity and strike price $95. The put option price is $1.*

	$S_T = 80$	$S_T = 120$
no hedging	$80,000 - 100,000 = -20,000$	$120,000 - 100,000 = 20,000$
hedging	$95,000 - 100,000 - 1000 = -6000$	$120,000 - 100,000 - 1000 = 19,000$
cost of hedging	1000	

  - The hedging provides a protection when IBM stock price drops
  - Still allows profit when IBM stock price increases
  - Need to pay for such protection (insurance)

Speculation using derivatives

- **Example (leverage):** *An investor believes that IBM stock price will increase in a month. Current IBM stock price is $100. The investor decides to buy 1000 call options with one month maturity and strike price $105. The call option price is $1.5.*

	Initial investment $1500	
	$S_T = 90$	$S_T = 110$
Buy 15 shares Return	$15(90-100) = -150$ -10%	$15(110-100) = 150$ 10%
Buy 1000 calls Return	-1500 -100%	$1000(110-105) - 1500 = 3500$ $3500/1500 = 233\%$

- **Leverage:** the use of options magnifies gain/loss
- Writing calls: unlimited potential loss, could be disastrous

Arbitrage

- **Arbitrage:** a risk free trading strategy which requires no initial cost and yields non-negative payoff with probability one and strictly positive payoff with positive probability
- **Example (arbitrage):** *The current price of a stock is \$1 per share. A call option to buy the stock in 1 year at \$0.8 per share is traded at \$2 per call. \$1 deposited for 1 year will earn \$0.05*

- Arbitrageurs **buy low sell high**: sell a call, buy a stock, deposit \$1; 1 year later,
  - if $S_T \geq 0.8$, sell one share to the call option holder and gain \$1.05+0.8
  - if $S_T < 0.8$, option not exercised, gain \$1.05+S_T
- Market response: share price increases, call price decreases
- Arbitrage opportunities are very **short-lived in liquid markets**
- In pricing derivatives, we assume no arbitrage

Derivative market size

- Market size (based on recent data by *the Bank for International Settlements*, **http://www.bis.org**) by the end of Dec 2009:
- **Total notional amount** of outstanding derivatives in the global OTC market is USD 614.7 trillion
- The notional amount of a derivative: the value of the underlying asset

  Example: *Consider a 1-year call option on the IBM stock. The IBM stock price is $100. The notional amount of the option is then $100*
- **Gross market value** of outstanding derivatives in the global OTC market is $21.6 trillions