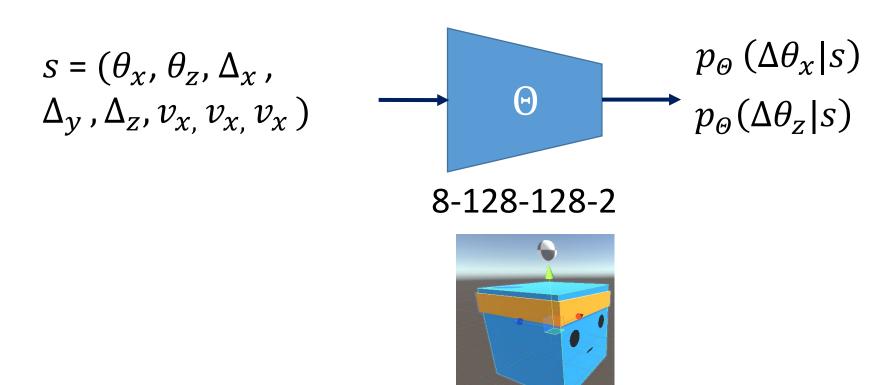
### NN to play 3D ball balancing

#### 2. NN with policy interacts with 3D Ball (MLAgent 10).ipynb

Θ: neural network weights and biases



### Calculate GAE

# 3. NN with policy interacts with 3D Ball to collect training data (MLAgent\_10).ipynb

$$\nabla \bar{R}_{\theta} \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{20} \left( \sum_{t'}^{20} \gamma^{t'-t} r_{t'}^{n} - b \right) \nabla \log p_{\theta}(a_{t}^{n} | s_{t}^{n})$$

 $\Delta$  = reward + expected accumulated reward gae =  $\Delta$  + accumulated gae Return = gae + expected accumulated reward

$$\begin{split} &\Delta_{20} = r_{20} + (\gamma * v_{21} * mask_{20} - v_{20}) \\ &gae_{20} = \Delta_{20} + \gamma * \tau * mask_{20} * gae_{initial} \\ &return_{20} = gae_{20} + v_{20} \end{split}$$

$$\Delta_{19} = r_{19} + (\gamma * v_{20} * mask_{19} - v_{19})$$

$$gae_{19\sim20} = \Delta_{19} + \gamma * \tau * mask_{19} * gae_{20}$$

$$return_{19} = gae_{19\sim20} + v_{19}$$

- - -

$$\Delta_{1} = r_{1} + (\gamma * v_{2} * mask_{1} - v_{1})$$

$$gae_{1\sim20} = \Delta_{1} + \gamma * \tau * mask_{1} * gae_{2\sim20}$$

$$return_{1} = gae_{1\sim20} + v_{1}$$

### Policy gradient to update NN

$$\max_{\Theta} \overline{R}_{\Theta} \qquad \nabla \overline{R}_{\Theta} \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\Theta}(a_t^n | s_t^n)$$

$$\nabla \bar{R}_{\Theta'} \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{I_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\Theta'}(a_t^n | s_t^n)$$

### Sampling efficiency problem

$$\nabla \bar{R}_{\Theta} \approx \boxed{\frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\Theta}(a_t^n | s_t^n)}$$

$$\Theta' \leftarrow \Theta + \eta \nabla \bar{R}_{\Theta}$$

$$S \longrightarrow P_{\Theta} (\Delta \theta_x | s)$$

$$p_{\Theta}(\Delta \theta_z | s)$$

$$\nabla \bar{R}_{\theta'} \approx \left[ \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\theta'}(a_t^n | s_t^n) \right]$$

### Important sampling

$$E_{x \sim p}[f(x)] = \int f(x)p(x)dx = \int f(x)\frac{p(x)}{q(x)}q(x)dx = E_{x \sim q}\left[f(x)\frac{p(x)}{q(x)}\right]$$

$$Var_{x \sim p}[f(x)] = E_{x \sim p}[f(x)^2] - (E_{x \sim p}[f(x)])^2$$
  $VAR[X] = E(X^2) - (E[X])^2$ 

$$Var_{x \sim q} \left[ f(x) \frac{p(x)}{q(x)} \right] = E_{x \sim q} \left[ \left( f(x) \frac{p(x)}{q(x)} \right)^2 \right] - \left( E_{x \sim q} \left[ f(x) \frac{p(x)}{q(x)} \right] \right)^2$$
$$= E_{x \sim p} \left[ f(x)^2 \frac{p(x)}{q(x)} \right] - \left( E_{x \sim p} [f(x)] \right)^2$$

### Off-policy to improve sampling efficiency

$$\nabla \bar{R}_{\theta'} = E_{\tau \sim p_{\theta'}(\tau)} \left[ R(\tau) \right] \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\theta'}(a_t^n | s_t^n)$$

Notice: My interpretation about  $\Theta$  and  $\Theta'$  here is different from Lee, Hung-yi

$$\nabla \overline{R}_{\Theta'} = E_{\tau \sim p_{\Theta}(\tau)} \left[ \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} R(\tau) \right] \qquad E_{x \sim p}[f(x)] = E_{x \sim q} \left[ f(x) \frac{p(x)}{q(x)} \right]$$

$$\approx \frac{1}{N} \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left( \sum_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla \log p_{\Theta}(a_t^n | s_t^n)$$

$$\nabla \bar{R}_{\Theta'} = E_{(s_t, a_t) \sim \Theta} \left[ \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} A^{\Theta}(s_t, a_t) \nabla \log p_{\Theta}(a_t^n | s_t^n) \right]$$

### From gradient to objective function

$$\nabla \overline{R}_{\Theta'} = E_{(s_t, a_t) \sim \Theta} \left[ \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} A^{\Theta}(s_t, a_t) \nabla \log p_{\Theta}(a_t^n | s_t^n) \right]$$

$$\max_{\Theta'} \bar{R}_{\Theta'}$$

$$\bar{R}_{\Theta'} = E_{(s_t, a_t) \sim \Theta} \left| \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} A^{\Theta}(s_t, a_t) \right| \qquad \nabla f(x) = f(x) \nabla \log f(x)$$

### Proximal policy optimization (PPO)

$$\max_{\Theta'} \left( \overline{R}_{\Theta'} - \beta KL(\Theta', \Theta) \right) \qquad \overline{R}_{\Theta'} = E_{(s_t, a_t) \sim \Theta} \left[ \frac{p_{\Theta'}(a_t | s_t)}{p_{\Theta}(a_t | s_t)} A^{\Theta}(s_t, a_t) \right]$$

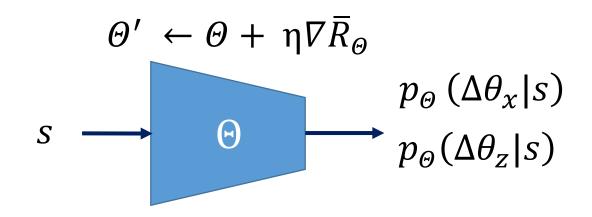
$$\max_{\Theta'} PPO2(\Theta')$$

$$PO2(\Theta') = \sum_{(S_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347.

### Proximal policy optimization (PPO)

#### 4. NN optimization with PPO (MLAgent\_10).ipynb



$$\max_{\Theta'} PPO2(\Theta')$$

$$PO2(\Theta') = \sum_{(s_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

### Combine data collected from different agents

#### Use PPO to update NN weights and biases

```
returns = torch.cat(returns).detach()
log_probs = torch.cat(log_probs).detach()
values = torch.cat(values).detach()
states = torch.cat(states)
actions = torch.cat(actions)
advantage = returns - values
```

```
print(len(returns), returns[0].shape)
print(len(log_probs), log_probs[0].shape)
print(len(values), values[0].shape)
print(len(states), states[0].shape)
print(len(actions), actions[0].shape)
print(len(advantage), advantage[0].shape)
```

```
60 torch.Size([1])
60 torch.Size([2])
60 torch.Size([1])
60 torch.Size([8])
60 torch.Size([2])
60 torch.Size([1])
```

N: no. of agents K: time horizon

```
egin{array}{c|c} ec{S}_{1,step1} & ec{a}_{1,step1} \ ec{S}_{N,step1} & ec{a}_{N,step1} \ ec{s}_{1,stepk} & ec{a}_{1,stepk} \ ec{s}_{N,stepk} & ec{a}_{N,stepk} \end{array}
```

```
\begin{bmatrix} v_{1,step1} \\ \vdots \\ v_{N,step1} \\ \vdots \\ v_{1,stepk} \\ \vdots \\ v_{N,stepk} \end{bmatrix}
```

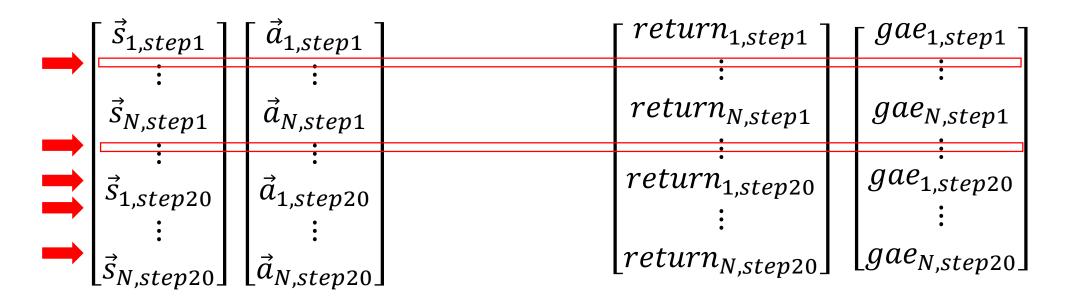
 $[return_{1,step1}]$   $\vdots$   $return_{N,step1}$   $\vdots$   $return_{1,stepk}$   $\vdots$   $return_{N,stepk}$ 

 $\begin{bmatrix} gae_{1,step1} \\ \vdots \\ gae_{N,step1} \\ \vdots \\ gae_{1,stepk} \\ \vdots \\ gae_{N,stepk} \end{bmatrix}$ 

### Sampling a batch of data to train NN

```
batch_size = states.size(0)
for _ in range(batch_size // mini_batch_size):
    rand_ids = np.random.randint(0, batch_size, mini_batch_size)
    break
print(rand_ids)
print(states[rand_ids, :].shape)
print(actions[rand_ids, :].shape)
print(log_probs[rand_ids, :].shape)
print(returns[rand_ids, :].shape)
print(advantage[rand_ids, :].shape)
```

```
[39 52 11 8 45]
torch.Size([5, 8])
torch.Size([5, 2])
torch.Size([5, 2])
torch.Size([5, 1])
torch.Size([5, 1])
```



$$PO2(\Theta') = \sum_{(s_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

select one batch and perform PPO optimization

```
for batch_state, batch_action, batch_old_log_probs, batch_re
    break

print(batch_state.shape, batch_action.shape)

torch.Size([5, 8]) torch.Size([5, 2])

dist = net(batch_state.to(device))
print(dist)

Normal(loc: torch.Size([5, 2]), scale: torch.Size([5, 2]))
```

$$PO2(\Theta') = \sum_{(s_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

```
batch_action = dist.sample()
batch new log probs = dist.log prob(batch action)
print(batch_new_log_probs.shape)
torch.Size([5, 2])
ratio = (batch_new_log_probs - batch_old_log_probs.to(device)).exp()
print(ratio)
tensor([[1.2427, 0.6327],
        [0.4962, 1.2191],
        [0.8360, 1.0056],
        [1.4339, 0.3089],
        [1.3568, 0.3191]], device='cuda:0', grad_fn=<ExpBackward>)
```

$$PO2(\Theta') = \sum_{(s_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

```
surr1 = ratio * batch_advantage.to(device)
print(surr1)
tensor([[0.0606, 0.0309],
        [0.0242, 0.0595],
        [0.0408, 0.0491],
        [0.0699, 0.0151],
        [0.0662, 0.0156]], device='cuda:0', grad fn=<MulBackward0>)
clip param=0.2
surr2 = torch.clamp(ratio, 1.0 - clip param, 1.0 + clip param) * batc
print(surr2)
tensor([[0.0585, 0.0390],
        [0.0390, 0.0585],
        [0.0408, 0.0491],
        [0.0585, 0.0390],
        [0.0585, 0.0390]], device='cuda:0', grad_fn=<MulBackward0>)
actor_loss = - torch.min(surr1, surr2).mean()
print(actor loss)
tensor(-0.0410, device='cuda:0', grad fn=<NegBackward>)
```

$$PO2(\Theta') = \sum_{(s_t, a_t)} min\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}A^{\Theta}(s_t, a_t), clip\left(\frac{p_{\Theta'}(a_t|s_t)}{p_{\Theta}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon\right)A^{\Theta}(s_t, a_t)\right)$$

```
net = Net().to(device)
optimizer = optim.Adam(net.parameters(), lr=0.001)
actor_loss = - torch.min(surr1, surr2).mean()
print(actor loss)
tensor(-0.0410, device='cuda:0', grad_fn=<NegBackward>)
optimizer.zero_grad()
actor_loss.backward()
optimizer.step()
```

### 3DBall.yaml

```
network_settings:
behaviors:
                                                    normalize: true
 3DBall:
                                                    hidden units: 128
  trainer_type: ppo
                                                    num_layers: 2
  hyperparameters:
                                                    vis_encode_type: simple
   batch size: 64
                                                   reward_signals:
   buffer size: 12000
                                                    extrinsic:
   learning_rate: 0.0003
                                                      gamma: 0.99 Y
   beta: 0.001
                                                      strength: 1.0
   epsilon: 0.2 \varepsilon
                                                   keep checkpoints: 5
    lambd: 0.99 	au
                                                   max_steps: 50000
   num epoch: 3
    learning_rate_schedule: linear
                                                   time horizon: 1000
                                                   summary freq: 5000
                                                   threaded: true
```

## Proximal policy optimization (PPO)

5. PPO (MLAgent\_10) .ipynb