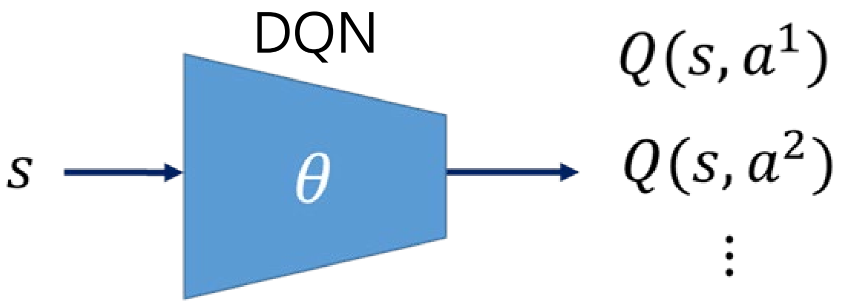
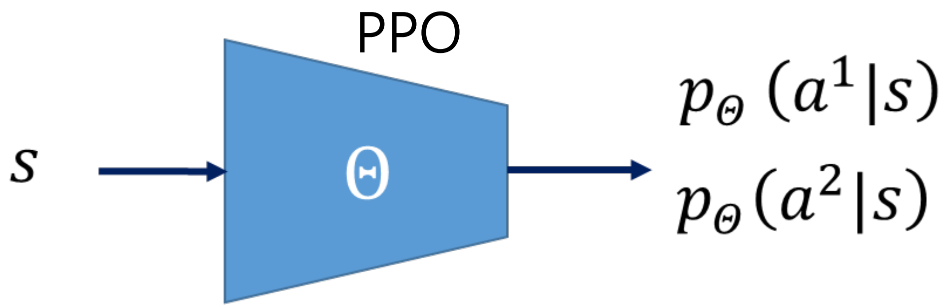
Recap: PPO, DQN and DDPG



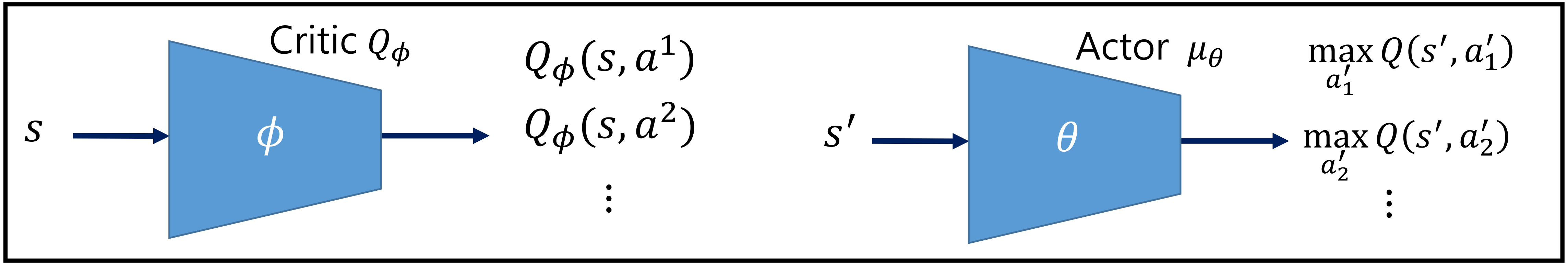
- Learn expected long-term reward $Q(s, a)$
- Use Bellman eq. to update $Q^*(s, a)$ based on $Q^*(s', a')$



- Learn policy $p_\Theta\,(a|s)$
- Use policy gradient (gradient of expected value of long-term rewards) to adjust $p_\Theta\,(a|s)$

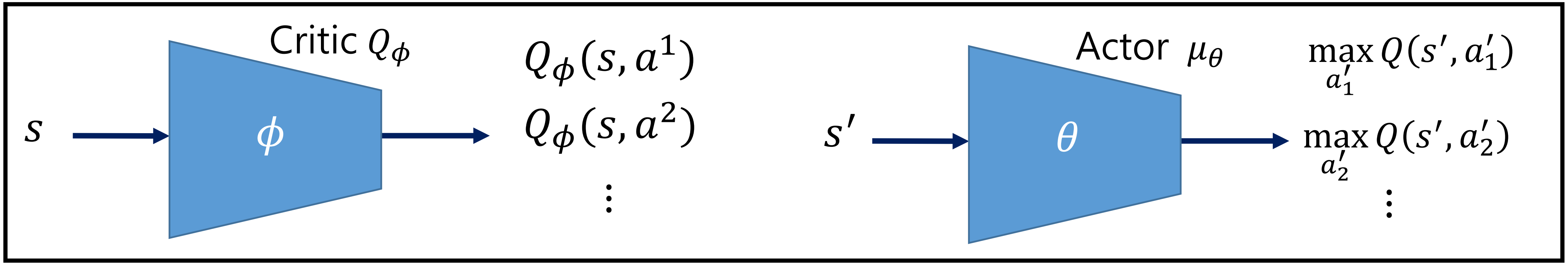
DDPG

- Learn $Q(s, a)$ and a' that max $Q(s', a')$
- Use Bellman eq. to update $Q^*(s, a)$ and use $Q(s, a)$ to update policy network



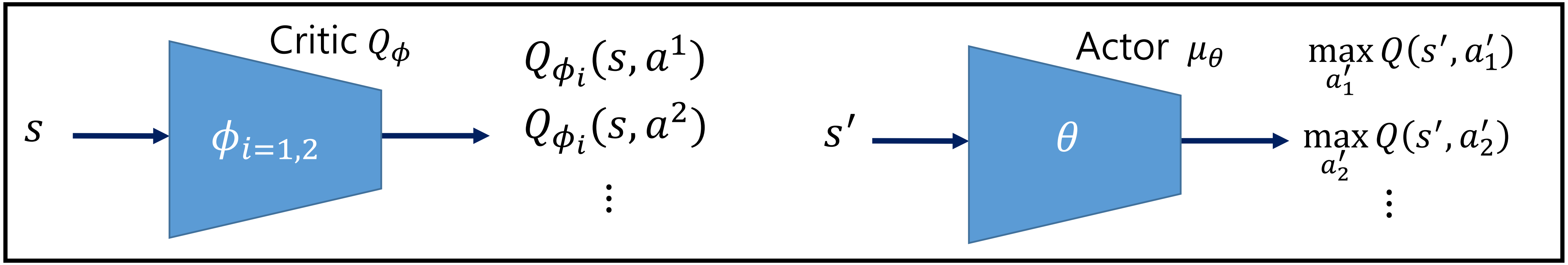
Recap: DDPG → TD3

DDPG



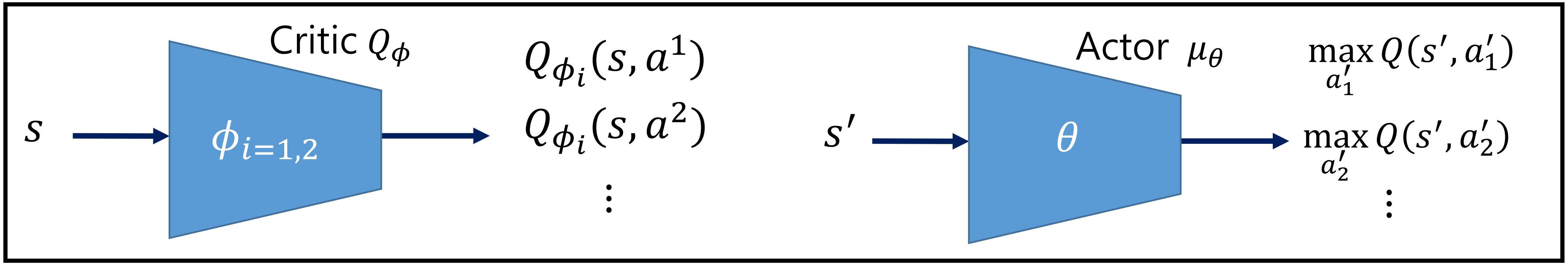
TD3

- Learn two Q functions $\phi_{i=1,2}$ and a' that max$\phi_{1,}(s', a')$
- Use Bellman eq. to update $Q^*(s, a)$ use $Q(s, a)$ to update policy network



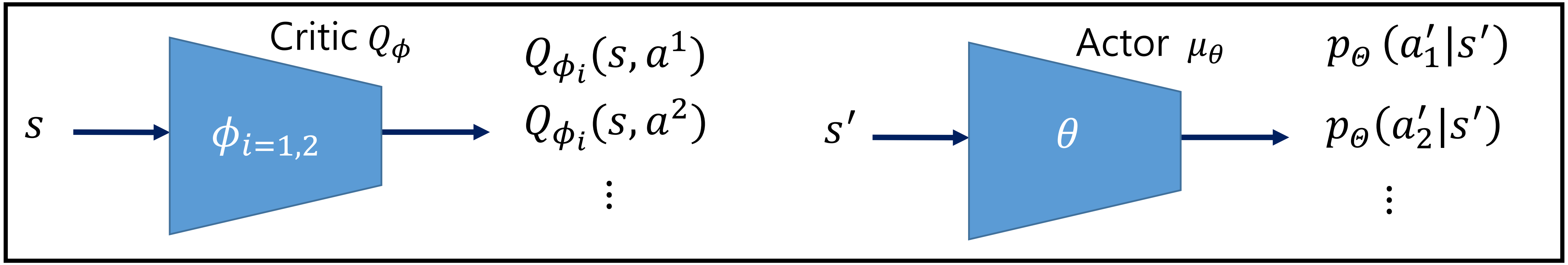
TD3 → SAC (Soft Actor Critic)

TD3



SAC

- Learn two Q functions $\phi_{i=1,2}$ and a stochastic policy
- Use Bellman eq. to update $Q^*(s, a)$ and use $Q(s, a)$ + entropy regularization to update policy network



Policy, Q_1, Q_2 networks

```
self.Normal    = NormalPolicyNet(input_dim=input_dim, action_dim=action_dim)
self.Normal_optimizer = optim.Adam(self.Normal.parameters(), lr=1e-3)

self.Q1        = QNet(input_dim=input_dim, action_dim=action_dim)
self.Q1_targ   = QNet(input_dim=input_dim, action_dim=action_dim)
self.Q1_targ.load_state_dict(self.Q1.state_dict())
self.Q1_optimizer = optim.Adam(self.Q1.parameters(), lr=1e-3)

self.Q2        = QNet(input_dim=input_dim, action_dim=action_dim)
self.Q2_targ   = QNet(input_dim=input_dim, action_dim=action_dim)
self.Q2_targ.load_state_dict(self.Q2.state_dict())
self.Q2_optimizer = optim.Adam(self.Q2.parameters(), lr=1e-3)
```

μ_θ

Q_{ϕ_1}

$Q_{\phi_{1,target}}$

Q_{ϕ_2}

$Q_{\phi_{2,target}}$

code: https://github.com/zhihanyang2022/pytorch-sac

Introduction

Soft Actor Critic (SAC) is an algorithm that optimizes a stochastic policy in an off-policy way, forming a bridge between stochastic policy optimization and DDPG-style approaches.

Reference: https://spinningup.openai.com/en/latest/algorithms/sac.html

Stochastic policy in SAC

Unlike in TD3, there is no explicit target policy smoothing. TD3 trains a deterministic policy, and so it accomplishes smoothing by adding random noise to the next-state actions. SAC trains a stochastic policy, and so the noise from that stochasticity is sufficient to get a similar effect.

Stochastic policy in SAC $\tilde{a}' \backsim \pi_\theta(\cdot\,|s')$

Target policy smoothing in TD3 $a'(s') = clip\left(\mu_{\theta_{target}}(s') + clip(\epsilon, -c, c), a_{Low}, a_{High}\right)$

Target policy network in DDPG $\mu_{\theta_{targ}}(s') = arg\max_{a'} Q(s', a')$

Entropy regularization

A central feature of SAC is entropy regularization. The policy is trained to maximize a trade-off between expected return and entropy, a measure of randomness in the policy. This has a close connection to the exploration-exploitation trade-off: increasing entropy results in more exploration, which can accelerate learning later on. It can also prevent the policy from prematurely converging to a bad local optimum.

Entropy-regularized RL

Entropy is a quantity which, roughly speaking, says how random a random variable is. If a coin is weighted so that it almost always comes up heads, it has low entropy; if it's evenly weighted and has a half chance of either outcome, it has high entropy.

$$H(P) = \underset{x \backsim P}{\mathrm{E}}[-log P(x)]$$

$$\pi^* = arg\max_{\pi} \underset{x \backsim \pi}{\mathrm{E}} \left[\sum_{t=0}^{\infty} \gamma^t \left(R(s_t, a_t, s_{t+1}) + \alpha H(\pi(\cdot\,|s_t)) \right) |s_0 = s \right]$$

Entropy-regularized RL

$$V^{\pi}(s) = \underset{\tau \backsim \pi}{\mathrm{E}}\left[\sum_{t=0}^{\infty} \gamma^{t}\left(R(s_t, a_t, s_{t+1}) + \alpha H(\pi(\cdot\,|s_t))\right) |s_0 = s\right]$$

$$Q^{\pi}(s,a) = \underset{\tau \backsim \pi}{\mathrm{E}}\left[\sum_{t=0}^{\infty} \gamma^{t}\left(R(s_t, a_t, s_{t+1}) + \alpha H(\pi(\cdot\,|s_t))\right) |s_0 = s\right]$$

$$V^{\pi}(s) = \underset{\tau \backsim \pi}{\mathrm{E}}[Q^{\pi}(s,a)] + \alpha H(\pi(\cdot\,|s))$$

$$\mathrm{Q}^{\pi}(s,a) = \underset{\substack{s' \backsim P \\ a' \backsim \pi}}{\mathrm{E}}[R(s,a,s') + \gamma(\mathrm{Q}^{\pi}(s',a') + \alpha H(\pi(\cdot\,|s')))]$$

$$= \underset{s' \backsim P}{\mathrm{E}}[R(s,a,s') + \gamma V^{\pi}(s')]$$

Learning the Q functions

Unlike in TD3, the next-state actions used in the target come from the current policy instead of a target policy.

Q learning in SAC

$$\tilde{a}' \backsim \pi_\theta(\cdot\,|s')$$

$$y(r,s',d) = r + \gamma(1-d)\left(\min_{j=1,2} Q_{\phi_{targ,j}}(s',\tilde{a}') - \alpha log\pi_\theta\,(a'|s')\right)$$

$$L(\phi_{i=1,2},\mathcal{D}) = \underset{(s,a,r,s',a')\backsim\mathcal{D}}{\mathrm{E}}\left[\left(\phi_{i=1,2}(s,a) - y(r,s',d)\right)^2\right])$$

Q learning in TD3

$$y(r,s',d) = r + \gamma(1-d)\min_{j=1,2} Q_{\phi_{targ,j}}(s',a'(s'))$$

$$L(\phi_{i=1,2},\mathcal{D}) = \underset{(s,a,r,s',a')\backsim\mathcal{D}}{\mathrm{E}}\left[\left(Q_{\phi_{i=1,2}}(s,a) - y(r,s',d)\right)^2\right])$$

Learning the Q functions

```
na, log_pi_na_given_ns = self.sample_action_and_compute_log_pi(b.ns, use_reparametrization_trick=False)
```

$$\tilde{a}' \backsim \pi_\theta(\cdot\,|s')$$

```
targets = b.r + self.gamma * (1 - b.d) * \
          (self.min_i_12(self.Q1_targ(b.ns, na), self.Q2_targ(b.ns, na)) - self.alpha * log_pi_na_given_ns)
```

$$y(r,s',d) = r + \gamma(1-d)\left(\min_{j=1,2} Q_{\phi_{targ,j}}(s',\tilde{a}'\,) - \alpha log\pi_\theta\,(a'|s')\right)$$

```
Q1_predictions = self.Q1(b.s, b.a)
Q1_loss = torch.mean((Q1_predictions - targets) ** 2)
```

$$L(\phi_1,\mathcal{D}) = \underset{(s,a,r,s',a')\backsim\mathcal{D}}{\mathrm{E}}\left[\left(Q_{\phi_1}(s,a) - y(r,s',d)\right)^2\right])$$

```
Q2_predictions = self.Q2(b.s, b.a)
Q2_loss = torch.mean((Q2_predictions - targets) ** 2)
```

$$L(\phi_2,\mathcal{D}) = \underset{(s,a,r,s',a')\backsim\mathcal{D}}{\mathrm{E}}\left[\left(Q_{\phi_2}(s,a) - y(r,s',d)\right)^2\right])$$

code: https://github.com/zhihanyang2022/pytorch-sac

Reparameterization trick to sample next action

The way we optimize the policy makes use of the reparameterization trick, in which a sample from $\pi_\theta(\cdot\,|s')$ is drawn by computing a deterministic function of state, policy parameters, and independent noise. Following the authors of the SAC paper, we use a squashed Gaussian policy, which means that samples are obtained according to

$$\tilde{a}_\theta = tanh(\mu_\theta(s) + \sigma_\theta(s) \odot \xi) \qquad \xi \sim \mathcal{N}(0, \mathcal{I})$$

```
def sample_action_and_compute_log_pi(self, state: torch.tensor, use_reparametrization_trick: bool):
        mu_given_s = self.Normal(state)
        u = mu_given_s.rsample() if use_reparametrization_trick else mu_given_s.sample()
        a = torch.tanh(u)
```

$\mu_\theta(s) + \sigma_\theta(s)$

$\tilde{a}_\theta = tanh\left(\mu_\theta(s) + \sigma_\theta(s)\right)$

Learning the policy

The policy should, in each state, act to maximize the expected future return plus expected future entropy. That is, it should maximize $V^{\pi}(s)$.

$$\max_{\theta} V^{\pi_\theta}(s) = \mathop{\mathrm{E}}_{a \backsim \pi}[Q^{\pi}(s,a)] + \alpha H\big(\pi(\cdot\,|s)\big)$$
$$= \mathop{\mathrm{E}}_{a \backsim \pi}[Q^{\pi}(s,a) - \alpha \log \pi(a|s)]$$

$$H(P) = \mathop{\mathrm{E}}_{x \backsim P}[-\log P(x)]$$

Policy learning in TD3 $\quad \max_{\theta} \mathop{\mathrm{E}}_{s \backsim \mathcal{D}}\left[Q_{\phi_1}(s, \mu_\theta(s))\right]$

Policy learning in DDPG $\quad \max_{\theta} \mathop{\mathrm{E}}_{s \backsim \mathcal{D}}\left[Q_{\phi_1}(s, \mu_\theta(s))\right]$

Sample action and calculate log probability

$$\tilde{a}_\theta = tanh(\mu_\theta(s) + \sigma_\theta(s) \odot \xi) \qquad \xi \sim \mathcal{N}(0, \mathcal{I})$$

$$\max_\theta V^{\pi_\theta}(s) = \mathop{\mathrm{E}}_{a \backsim \pi}[Q^\pi(s, \tilde{a}_\theta) - \alpha log\pi(a|s)]$$

```
def sample_action_and_compute_log_pi(self, state: torch.tensor, use_reparametrization_trick: bool):

        mu_given_s = self.Normal(state)

        u = mu_given_s.rsample() if use_reparametrization_trick else mu_given_s.sample()

        a = torch.tanh(u)

        log_pi_a_given_s = mu_given_s.log_prob(u) - (2 * (np.log(2) - u - F.softplus(-2 * u))).sum(dim=1)

        return a, log_pi_a_given_s
```

$\mu_\theta(s) + \sigma_\theta(s)$

$\tilde{a}_\theta = tanh(\mu_\theta(s) + \sigma_\theta(s))$

$log\pi(a|s)$

Learning the policy

```
a, log_pi_a_given_s = self.sample_action_and_compute_log_pi(b.s, use_reparametrization_trick=True)
```

$$\tilde{a}_\theta = tanh(\mu_\theta(s) + \sigma_\theta(s) \odot \xi)$$

```
policy_loss = - torch.mean(self.min_i_12(self.Q1(b.s, a), self.Q2(b.s, a)) - self.alpha * log_pi_a_given_s)
```

$$\max V^\pi(s) = \underset{a \backsim \pi}{\mathrm{E}}[Q^\pi(s, \tilde{a}_\theta) - \alpha log\pi(\tilde{a}_\theta|s)]$$

Learning the Policy

SAC policy has two key differences from the policies we use in the other policy optimization algorithms:

1. **The squashing function**. The $tanh$ in the SAC policy ensures that actions are bounded to a finite range. This is absent in the PPO policies. It also changes the distribution: before the $tanh$ the SAC policy is a factored Gaussian like the other algorithms' policies, but after the $tanh$ it is not.
2. **The way standard deviations are parameterized**. In PPO, we represent the log std devs with state-independent parameter vectors. In SAC, we represent the log std devs as outputs from the neural network, meaning that they depend on state in a complex way. SAC with state-independent log std devs, in our experience, did not work

Update the target nets

$$\phi_{target} \leftarrow \rho \phi_{target} + (1 - \rho)\phi$$

ρ is a hyperparameter between 0 and 1 (usually close to 1). (This hyperparameter is called polyak in our code).

```
with torch.no_grad():
    self.polyak_update(old_net=self.Q1_targ, new_net=self.Q1)
    self.polyak_update(old_net=self.Q2_targ, new_net=self.Q2)
```

Exploration vs. Exploitation

SAC trains a stochastic policy with entropy regularization, and explores in an on-policy way. The entropy regularization coefficient α explicitly controls the explore-exploit trade off, with higher α corresponding to more exploration, and lower α corresponding to more exploitation. The right coefficient (the one which leads to the stablest/highest-reward learning) may vary from environment to environment, and could require careful tuning.