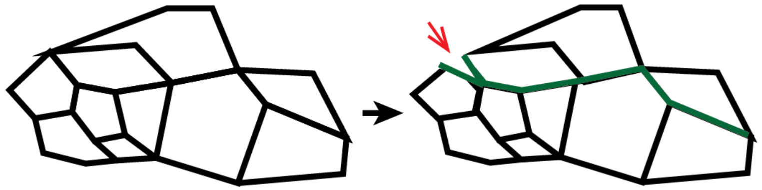
Overarching goal: investigate coupling between rheology and ice damage

How does the feedback between damage and ice rheology affect projections of glacier mass loss?

Goal of Proposed Research: How do feedbacks between ice flow and ice fracture affect rates of ice flow and projections of future sea-level rise?

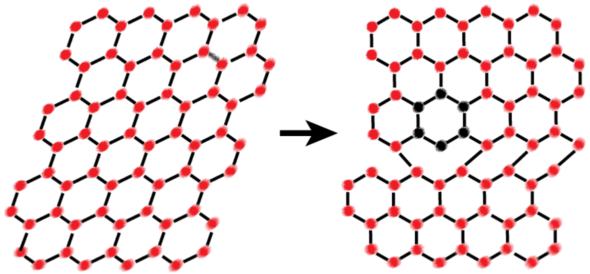
Objective 1: How does rapid ice flow affect the development of ice microfractures (microdamage)?

Ice Fracture



Ice microcracks initiate and propagate along grain boundaries, sensitive to stress, rate of flow and deformation, and ice material properties

Ice Flow



In deforming ice, microcracks and microvoids add additional anisotropy and defects that enhance ice flow rates

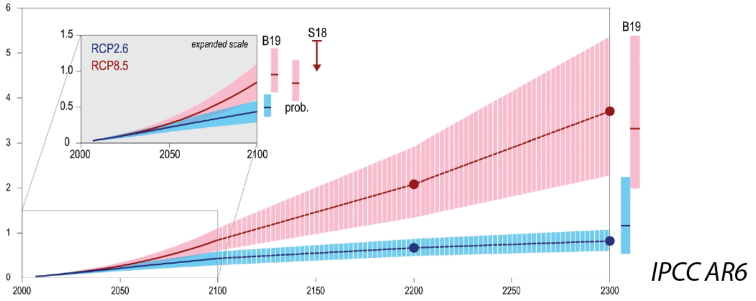
Objective 2: How does microdamage affect ice flow rates?

Objective 3: How does this affect sea-level rise projections?

Damage Model (Objective 1) → Damage Estimates (*D*) → Ice Viscosity Model η=η(D) (Objective 2)

→ Projections of ice flow in a large-scale ice sheet model

→ Updated sea-level rise projections



IPCC AR6

Part 3: Full Stokes Model

In collaboration with Ravi Duddu and Computational Physics and Mechanics Laboratory at Vanderbilt University

Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Full Stokes model equations:

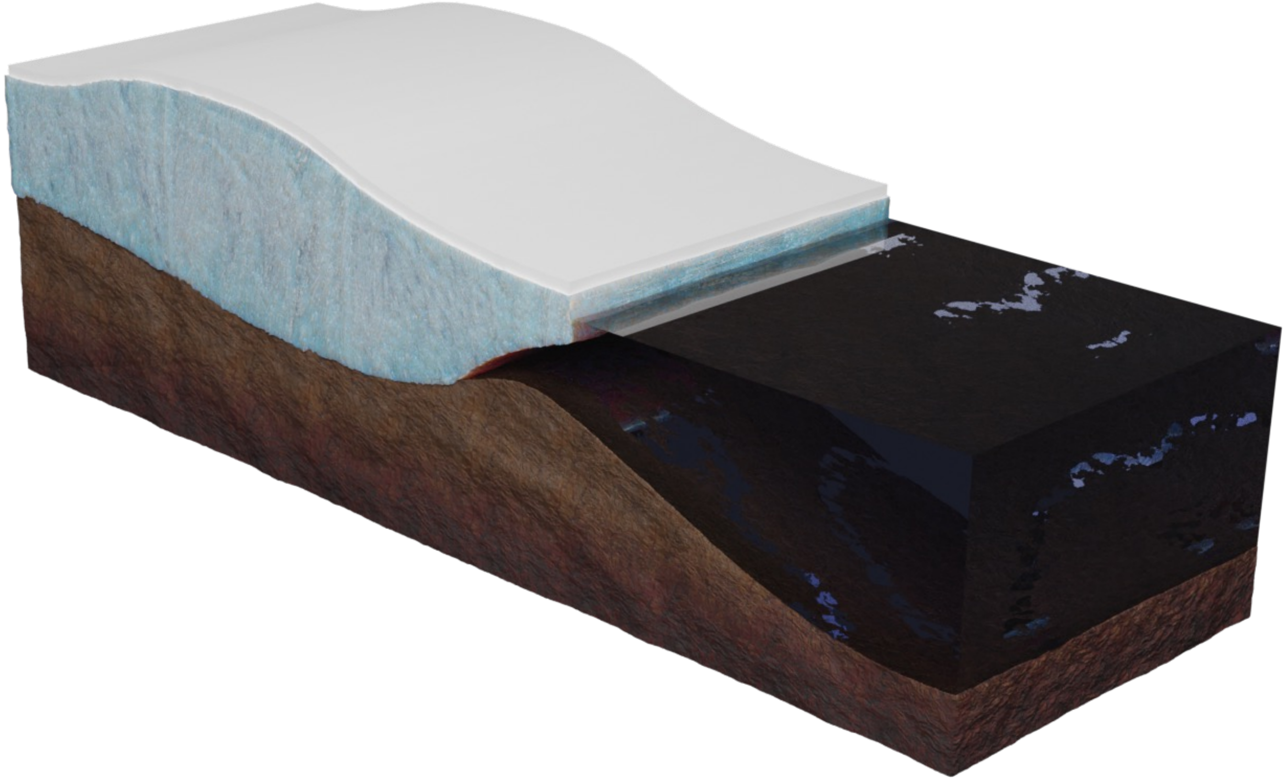
$$\nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} = \mathbf{0}$$

$$tr(\dot{\boldsymbol{\epsilon}}) = 0$$

Constitutive relation:

$$\tau = 2\,\eta\dot{\epsilon}$$

$$\eta = \frac{1}{2}A^{-\frac{1}{n}}\dot{\epsilon}_e^{(1-n)/n}$$



Full Stokes Model

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Boundary Conditions:

Stress-free surface at surface of ice sheet:

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{0}$$

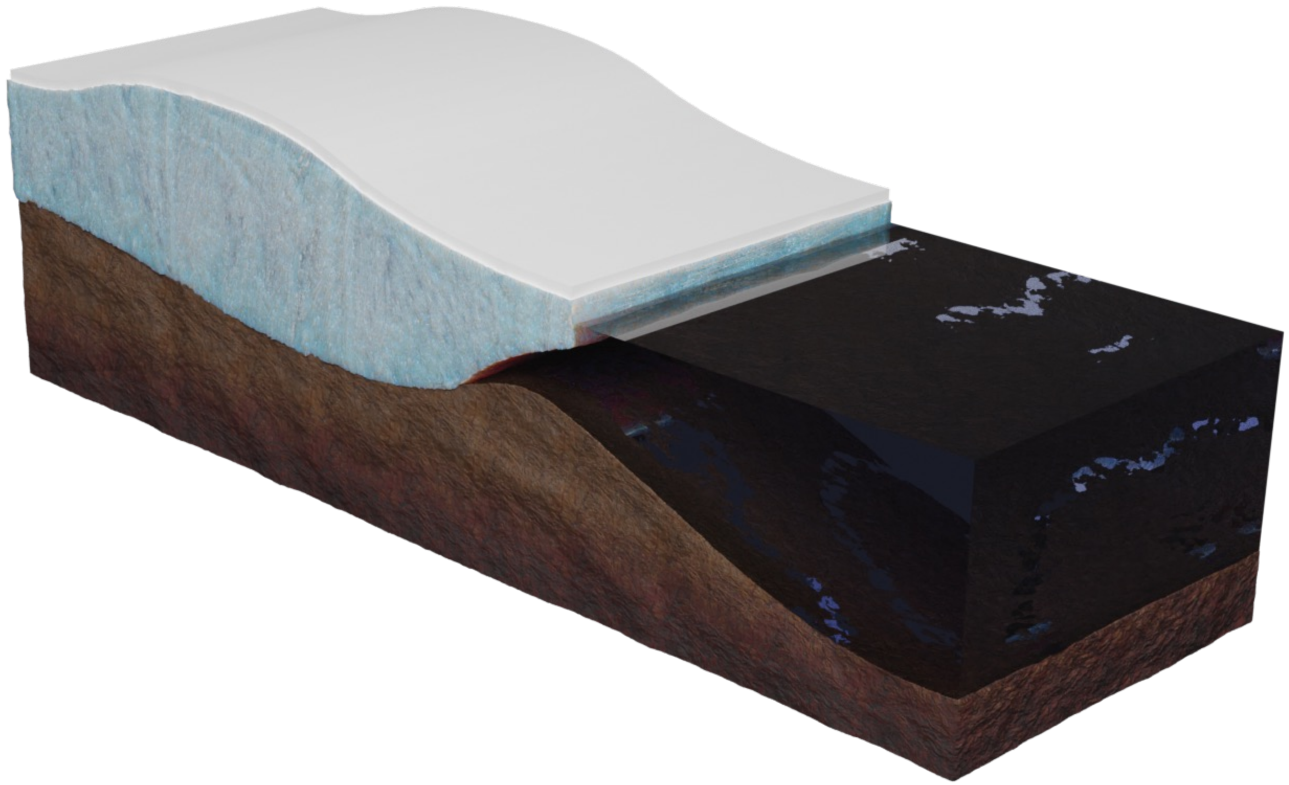
Basal boundary condition:

$$\boldsymbol{\tau_b} = C\boldsymbol{u_b^m}$$

Water pressure at ice-seawater interface:

$$\begin{aligned} \boldsymbol{\sigma} \cdot \boldsymbol{n} &= -\rho_w g \boldsymbol{z}, && z < 0 \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} &= \boldsymbol{0}, && z \geq 0 \end{aligned}$$

No-slip boundary conditions at margins of glacier



Full Stokes Model

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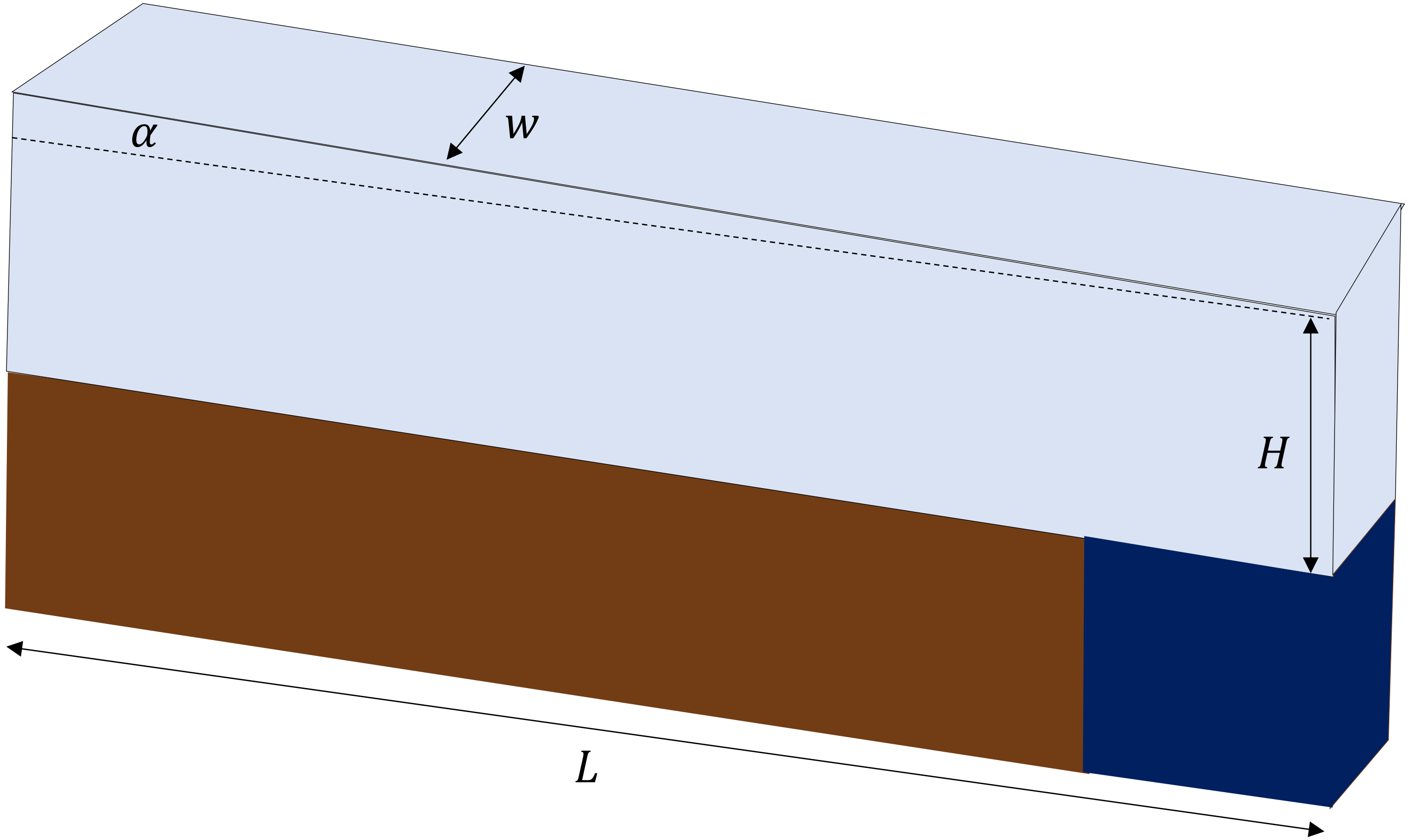
Geometry of idealized simulation:

- Flat bed
- Fixed width ($w = 20\ \mathrm{km}$)
- Initial length ($L = 100\ \mathrm{km}$)
- Initial surface slope α of 1 degree
- Initial height at calving front ($H = 400\ \mathrm{m}$)



Full Stokes Model

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Damage Evolution: follow Jimenez and others, 2017

$$\dot{D}^{loc} = \begin{cases} B\dfrac{<\chi>^{r}}{(1-D)^{k_\sigma}}, \sigma \geq \sigma_t \\ 0, \sigma < \sigma_t \end{cases}$$

$$\dot{D} - \frac{1}{2} l_c^2 \nabla^2 \dot{D} = \dot{D}^{loc}$$

$$\chi = \alpha\sigma^{(1)} + \beta\bar{\sigma}^{v} + (1-\alpha-\beta) tr[\overline{\sigma}]$$

Full Stokes Model

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Steps towards building this damage/rheology coupling; run century-scale simulations of idealized glacier to obtain damage estimates:

1. **Constant Rheology:** Start with a constant rheology parameter *B*
2. **Incorporate steady-state temperature:** Calculate steady-state temperature (see subsequent slides), incorporate into *B*
3. **Incorporate steady-state grain size:** Calculate steady-state grain size (see subsequent slides), incorporate into *B*
4. **Incorporate time-dependent temperature:** implement the full heat equation (see subsequent slides), incorporate into *B*
5. **Apply full coupling:** include damage into ice rheology (see subsequent slides)

Eventually…apply to real glacier or catchment in Antarctica

Full Stokes Model: Ice Temperature

In collaboration with Ravi Duddu and Computational Physics and Mechanics Laboratory at Vanderbilt University

Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Steady State Ice Temperature (see: Meyer and Minchew 2018 for full derivation of this 1D model):

Define critical strain-rate for temperate ice formation:

$$\dot{\epsilon}_{lat}^{*} = \left[\frac{\frac{1}{2}Pe^{2}}{Pe\ -1+\exp\{-Pe\}}+\frac{1}{2}\Lambda\right]^{n/(n+1)} \left[\frac{K\Delta T}{A^{-\frac{1}{n}}H^{2}}\right]^{n/(n+1)}$$

Define the (nondimensionalized) thickness of temperate ice zone:

$$\frac{\xi}{H} = \begin{cases} 1-\frac{Pe}{Br-\Lambda}-\frac{1}{Pe}\left[1+W\left(-\exp\left\{-\frac{Pe^{2}}{Br-\Lambda}-1\right\}\right)\right], \dot{\epsilon}_{lat} > \dot{\epsilon}_{lat}^{*} \\ 0,\ \dot{\epsilon}_{lat} \leq \dot{\epsilon}_{lat}^{*} \end{cases}$$

Define ice temperature:

$$T = \begin{cases} T_s+\Delta T\frac{Br-\Lambda}{Pe}\left[1-\frac{z}{H}+\frac{1}{Pe}\exp\left\{Pe\left(\frac{\xi}{H}-1\right)\right\}-\frac{1}{Pe}\exp\left\{Pe\left(\frac{\xi-z}{H}\right)\right\}\right], \xi \leq z \leq H \\ T_m, 0 \leq z < \xi \end{cases}$$

Full Stokes Model: Ice Temperature

In collaboration with Ravi Duddu and Computational Physics and Mechanics Laboratory at Vanderbilt University

Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Full Heat Equation:

$$\frac{\partial \boldsymbol{T}}{\partial t} + \nabla \cdot (-K\nabla \boldsymbol{T} + \boldsymbol{u}\boldsymbol{T}) = \boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}}$$

Full Stokes Model: Grain Size

In collaboration with Ravi Duddu and Computational Physics and Mechanics Laboratory at Vanderbilt University

Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Steady State Grain Size (see: Ranganathan and others, 2021c for full derivation of this model):

$$d = \left[\frac{4kp^{-1}c\gamma\mu^2 + \tau_s^4 D^p \left(\frac{p}{2}\right) M}{8(1-\Theta)\tau_s\dot{\epsilon}_s\mu^2}\right]^{1/(1+p)}$$

Full Stokes Model: Define *B* in terms of temperature, grain size

In collaboration with Ravi Duddu and Computational Physics and Mechanics Laboratory at Vanderbilt University

Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Include temperature into *B*: rate of damage evolution is dependent upon ice temperature as

$$B(T) = B(T_m) \exp\left[-\frac{Q}{R}\left(\frac{1}{T} - \frac{1}{T_m}\right)\right]$$

Include grain-size into *B*: rate of damage evolution is dependent upon grain size as

TBD...

Full Stokes Model: Full Coupling

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Set up idealized, 3D glacier simulation in FEniCS: to evaluate the effect of damage/rheology coupling on the evolution of ice sheets

Constitutive relation dependent upon ice damage:

$$\tau = 2\,\eta\dot{\epsilon}$$

$$\eta = \frac{1}{2}\boldsymbol{A}(\boldsymbol{D})^{-\frac{1}{n}}\dot{\epsilon}_e^{(1-n)/n}$$

Ice damage evolution dependent upon ice rheology:

$$\dot{D}^{loc} = \begin{cases} \boldsymbol{B}(\boldsymbol{\eta})\dfrac{<\chi>^r}{(1-D)^{k_\sigma}}, \sigma \geq \sigma_t \\ 0, \sigma < \sigma_t \end{cases}$$