# Introduction to Quantum Computing

## Module 2 — Part III

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# Part I

# Error Correction

# Errors in Classical Computation and Communication

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  - It can well be, instead, that long-distance *communication channels* are subject to errors.
- One can deal with errors occurring along communication channels in a variety of ways, as studied in **information theory**
  - For example, one can encode every logical bit to be sent as three instances of *the same* bit:
$$0 \longrightarrow 000 \qquad\qquad 1 \longrightarrow 111$$
  - This way, if errors affect either 0 or 1 of the bits in the triple, errors can be not only recognized, but also corrected.
  - This is the so-called **repetition code**.

# Errors in Quantum Computating

- When we switch from *classical* to *quantum* communication/computation, a number of additional problems show up:
  - Already at the level of quantum **computation**, error correction is absolutely crucial, because qubits are implemented by individual atomic-scale physical systems.
  - Quantum error correction is problematic, given the very nature of quantum computing, because the typical way of *detecting* an error in any piece of data consists in *observing it*, but doing this would **disturb** the underlying quantum state.
  - The kind of errors one can observe are more general: not only the state of a qubit can be flipped, but there could be, e.g. *phase errors*.
- Despite all these additional problem, quantum error correction is indeed possible, and there is a whole research field whose objective is precisely that of designing **quantum error correcting codes** (QECCs for short).

# Part II

# QECC in a Simplified Setting

# Quantum Repetition Code?

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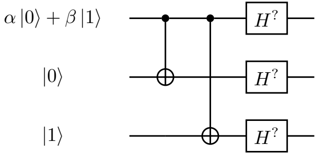

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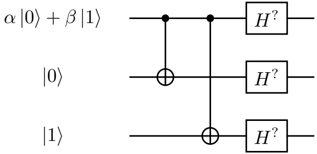


- How can we **detect** and **correct** errors, then?

# A Simple QECC