Introduction to Optimization and Operations Research

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Some examples

Fruit Supply Chain

"Agricultural fruit supply chain (AFSC) consititues the processes from production to delivery of the fruit products from farm to market. Fruit loss in AFSC is dependent on how the products are dealt with throughout the SC processes and can happen due to problems in different stages. This may cause the production of unavoidable second and third-grade fruits, fruit damage across the chain or perished fruit. A remedy for this problem is to develop planning mathematical models in AFSC. . . .

The contribution of this research is developing a mathematical model for postharvest handling and storage of an apple SC with two different time periods of harvesting and planning and quantifying apple losses based on the time gap between their harvest and delivery."[1]

[1] https://www.springer.com/gp/book/9783319899190

Efficient Dismantling Networks for Wind Turbines

"Today, the dismantling of onshore w.b.'s is generally conducted completely on-site. The rotor blades are cracked, the tower segments are separated, and the nacelle is cut into smaller pieces. This undistributed dismantling of w.b.'s is very time-consuming, inefficient, and implies ecological and economical risks. The dismantling of a single w.b. takes around two weeks and entails costs of more than €130,000. ... An option to the undistributed dismantling is the transportation of only partly dismantled w.b.'s to specialized dismantling sites for further handling. ... However, this distributed dismantling implies higher costs for the complex transportation of large-scale components and the initialization of dismantling sites. Consequently, when planning a distributed dismantling of w.b.'s, assigned companies face the challenge of determining the optimal dismantling depth for each component as well as the optimal location of specialized dismantling sites."[2]

[2] https://www.springer.com/gp/book/9783319899190

Periodic Traveling Politician Problem (PTPP)

"The Periodic Traveling Politician Problem (PTPP) deals with determining daily routes for a party leader who holds meetings in various cities during a campaign period of τ days. On a graph with static edge costs and time-dependent vertex profits, PTPP seeks a closed or open tour for each day. The objective is the maximization of the net benefit defined as the sum of rewards collected from meetings in the visited cities minus the traveling costs normalized into a compatible unit. The reward of a meeting in a city are linearly depreciated according to the meeting date and recency of the preceding meeting in the same city. We propose a MILP formulation in which we capture many real-world aspects of the PTPP."[3]

[3]https://www.springer.com/gp/book/9783319557014

Mathematical Models

Definition

A mathematical model is the collection of variables and relationships needed to describe relevant features of a problem.

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Operations Research (OR)

Definition

Operations research (OR) is the study of how to form mathematical models of complex engineering and management problems and how to analyze them to gain insight about possible solutions.

Decisions, Constraints, and Objectives

Definition

The three fundamental concerns in forming OR models are

1. the **decisions** open to decision makers.

Decisions, Constraints, and Objectives

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2. the **constraints** limiting decision choices.

Decisions, Constraints, and Objectives

Definition

The three fundamental concerns in forming OR models are

1. the **decisions** open to decision makers.
2. the **constraints** limiting decision choices.
3. the **objectives** making some decisions preferred to others.

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Optimization Models

Definition

Optimization models (also called **mathematical programs**) represent problem choices as decision variables and seek values that maximize or minimize objective functions of the decision variables subject to constraints on variable values expressing the limits on possible decision choices.

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Optimization Models[4]

- What is it that you ideally wish to achieve?
  - e.g., maximize profit
  - e.g., minimize risk

This is the Objective.

[4] Matthew Roughan: http://www.maths.adelaide.edu.au/matthew.roughan/

Optimization Models[4]

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- What is it that you have control over?
  - e.g., who should do W?
  - e.g., how many X should I make?
  - e.g., where should I make Y?
  - e.g., when should I do Z?
  - e.g., what model of V should I choose?

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  - e.g., what model of V should I choose?

These are the Variables.

- Are there any restrictions?
  - e.g., we can't make negative numbers of items
  - e.g., we have a limited resources or time

These are the Constraints.

[4] Matthew Roughan: http://www.maths.adelaide.edu.au/matthew.roughan/

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Our first problem[5]

Examples

A manufacturing company makes three types of items: desks, chairs, and bed-frames, using metal and wood.

A single desk requires

- 2 hours of labour
- 1 unit of metal
- 3 units of wood

One chair requires

- 1 hour of labour
- 1 unit of metal
- 3 units of wood

One bed frame requires

- 2 hours of labour
- 1 unit of metal
- 4 units of wood

Our first problem[6]

Examples

In a given time period, there are

- 225 hours of labour available,
- 117 units of metal, and
- 420 units of wood.

The profit on

- one desk is \$13,
- one chair is \$12, and
- one bed frame is \$17.

[6] Matthew Roughan: http://www.maths.adelaide.edu.au/matthew.roughan/

Our first problem[7]

Examples

The company wants a manufacturing schedule designed, which maximizes profits without violating any constraint on resource availability during that time period.

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Our first problem[7]

Examples

The company wants a manufacturing schedule designed, which maximizes profits without violating any constraint on resource availability during that time period.

So what are the steps in **Formulation?**

1. Read the question right through!
2. Tabulate any data
3. Identify the variables
4. Identify the objective
5. Formulate the constraints, i.e., define the feasible region
6. Write down the mathematical model

[7] Matthew Roughan: http://www.maths.adelaide.edu.au/matthew.roughan/

2. Tabulate the data

	Labour (hrs)	Metal	Wood	Profit
Desk	2	1	3	13
Chair	1	1	3	12
Bedframe	2	1	4	17
Total	225	117	420	

Our first problem

3. Define the decision variables:
   - x_1 to be the number of desks
   - x_2 to be the number of chairs
   - x_3 to be the number of bedframes

made per time period.

Our first problem

4. Identify (formulate) the objective function:

We wish to maximize the profit, z, (in dollars), so we have:

$$\underset{\boldsymbol{x}}{\text{maximize}} \quad z = 13x_1 + 12x_2 + 17x_3$$

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Our first problem

4. Identify (formulate) the objective function:

We wish to maximize the profit, z, (in dollars), so we have:

$$\underset{\boldsymbol{x}}{\text{maximize}} \quad z = 13x_1 + 12x_2 + 17x_3$$

We might sometimes write this as

$$\underset{\boldsymbol{x}}{\text{argmax}} \quad z = 13x_1 + 12x_2 + 17x_3$$

which means find the argument $\boldsymbol{x}$ that maximizes z.

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Our first problem

5. Formulate the constraints

$$\begin{aligned}
2x_1 + x_2 + 2x_3 &\leq 225 && (labour)\\
x_1 + x_2 + x_3 &\leq 117 && (metal)\\
3x_1 + 3x_2 + 4x_3 &\leq 420 && (wood)\\
x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 & && (non-negativity)
\end{aligned}$$

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Our first problem

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Question:
Should we include any other constraints on the x_i's?
What about integer constraints?

Our first problem

6. Write down the mathematical model.

(objective function)

$$\underset{\boldsymbol{x}}{\text{maximize}} \quad z = 13x_1 + 12x_2 + 17x_3$$

subject to

$$\begin{array}{rcrcrcl} 2x_1 & + & x_2 & + & 2x_3 & \leq & 225 \\ x_1 & + & x_2 & + & x_3 & \leq & 117 \\ 3x_1 & + & 3x_2 & + & 4x_3 & \leq & 420 \end{array}$$

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(non-negativity)

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

Note: This is a linear program (LP).
If the integer constraints were added, it would be an Integer Linear Program (ILP).

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Our first problem

Matrix form of the problem:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{maximize}} \quad z &= \boldsymbol{c}^T\boldsymbol{x} \\ \text{such that} \quad A\boldsymbol{x} &\leq \boldsymbol{b} \\ \text{and} \quad \boldsymbol{x} &\geq 0 \end{aligned}$$

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Feasible & Optimal Solutions

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A feasible solution is a choice of values for the decision variables that satisfies all constraints.

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A feasible solution is a choice of values for the decision variables that satisfies all constraints.

Definition

Optimal solutions are feasible solutions that achieve objective function value(s) as good as those of any other feasible solutions.

Closed-Form Solutions

- For example, the solution of the quadratic equation $ax^2 + bx + c = 0$ can be expressed as a closed-form expression:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition

Sensitivity analysis is an exploration of results from mathematical models to evaluate how they depend on the values chosen for parameters.

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Definition

Sensitivity analysis is an exploration of results from mathematical models to evaluate how they depend on the values chosen for parameters.

Principle

Closed-form solutions represent the ultimate analysis of mathematical models because they provide both immediate results and rich sensitivity analysis.

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The **validity** of a model is the degree to which inferences drawn from the model hold for the real system.

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The **validity** of a model is the degree to which inferences drawn from the model hold for the real system.

Principle

OR analyst almost always confront a **trade-off** between validity of models and their tractability to analysis.

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Two Crude Petroleum Example

- Two Crude Petroleum distills crude from: Saudi Arabia & Venezuela.

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- Yields

	gasoline	jet fuel	lubricants
Saudi	0.3 barrel	0.4 barrel	0.2 barrel
Venezuela	0.4 barrel	0.2 barrel	0.3 barrel

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- Availability and cost

	availability	cost
Saudi	9000 barrels	\$100/barrel
Venezuela	6000 barrels	\$75/barrel

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- How can these requirements be fulfilled most efficiently?

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Decision Variables

Definition

Variables in optimization models represent the decisions to be taken.

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Decision Variables

Definition

Variables in optimization models represent the decisions to be taken.

- Cost, availability, yield, and requirement are all **input parameters** - can be assumed as fixed quantities.
- $x_1 \triangleq$ barrels of Saudi crude refined per day (in thousands).
- $x_2 \triangleq$ barrels of Venezuelan crude refined per day (in thousands).

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Variable-Type Constraints

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Variable-type constraints specify the domain of definition for decision variables: the set of values for which the variables have meaning.

e.g., variables may be limited to non-negative values, or to non-negative integer values, or totally unrestricted?

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e.g., variables may be limited to non-negative values, or to non-negative integer values, or totally unrestricted?

- x_1, x_2 in Two Crude example are subject to the most common variable-type constraint form: non-negativity.

$$x_1, x_2 \geq 0$$

- Formal methods for solving mathematical programs enforce only constraints explicitly stated in the model formulation.

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Main Constraints

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Main constraints of optimization models specify the restrictions and interactions, other than variable type, that limit decision decision variable values.

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- Two Crude production constraints:

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\begin{array}{lcccll}
0.3x_1 & + & 0.4x_2 & \geq & 2.0 & \text{(gasoline)} \\
0.4x_1 & + & 0.2x_2 & \geq & 1.5 & \text{(jet fuel)} \\
0.2x_1 & + & 0.3x_2 & \geq & 0.5 & \text{(lubricants)}
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- Two Crude availability constraints:

$$\begin{array}{lcll} x_1 & \leq & 9 & \text{(Saudi)} \\ x_2 & \leq & 6 & \text{(Venezuelan)} \end{array}$$

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Objective Functions

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Objective functions in optimization models quantify the decision consequences to be maximized or minimized.

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- Objective or criterion functions tell us how to rate decisions.

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- Two Crude: what makes on choice of decision variable values preferable to another?

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Definition

Objective functions in optimization models quantify the decision consequences to be maximized or minimized.

- Objective or criterion functions tell us how to rate decisions.
- Two Crude: what makes on choice of decision variable values preferable to another?
- Cost. The best solution seeks to minimize total cost:

$$\underset{x_1, x_2}{\text{minimize}} \quad 100x_1 + 75x_2 \quad \text{(thousands of dollars per day)}$$

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Standard Model

Principle

The standard statement of an optimization model has the form

$$
\begin{aligned}
\text{minimize or maximize} \quad & \text{(objective function)} \\
\text{s.t.} \quad & \text{(main constraints)} \\
& \text{(variable-type constraints)}
\end{aligned}
$$

where “s.t.” stands for “subject to.”

Standard Model

Two Crude model:

$$
\begin{array}{llll}
\text{minimize} & 100x_1 + 75x_2 & & \text{(total cost)} \\
\text{s.t.} & 0.3x_1 + 0.4x_2 & \geq 2 & \text{(gasoline requirement)} \\
& 0.4x_1 + 0.2x_2 & \geq 1.5 & \text{(jet fuel requirement)} \\
& 0.2x_1 + 0.3x_2 & \geq 0.5 & \text{(lubricant requirement)} \\
& x_1 & \leq 9 & \text{(Saudi availability)} \\
& x_2 & \leq 6 & \text{(Venezuelan availability)} \\
& x_1, x_2 \geq 0 & & \text{(non-negativity)}
\end{array}
$$

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Feasible Sets

Definition

The **feasible set** (or **region**) of an optimization model is the collection of choices for decision variables satisfying all model constraints.

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Linear Functions

Definition

A function is **linear** if it is a constant-weighted sum of decision variables. Otherwise, it is **nonlinear**.

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A function is **linear** if it is a constant-weighted sum of decision variables. Otherwise, it is **nonlinear**.

- The objective function of Two Crude example:

$$f(x_1, x_2) \triangleq 100x_1 + 75x_2$$

is linear because it simply applies weights 100 and 75 in summing decision variables x_1 and x_2.

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  is linear because it simply applies weights 100 and 75 in summing decision variables x_1 and x_2.
- The following objective function:

$$f(w_1, w_2, w_3) \triangleq (w_1)^2 + 8w_2 + (w_3)^2$$

  is nonlinear because it includes second powers of some decision variables.

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Definition (Linear Programs)

An optimization model in functional form is a **linear program (LP)** if its objective function f **and** all constraint functions $g_1, \ldots, g_m$ are linear in the decision variables. Also, decision variables should be able to take on whole-number or fractional values.

Linear and Nonlinear Programs

Definition (Linear Programs)

An optimization model in functional form is a **linear program (LP)** if its objective function f **and** all constraint functions $g_1, \ldots, g_m$ are linear in the decision variables. Also, decision variables should be able to take on whole-number or fractional values.

Definition (Nonlinear Programs)

An optimization model in functional form is a **nonlinear program (NLP)** if its objective function f **or** any of the constraint functions $g_1, \ldots, g_m$ is nonlinear in the decision variables. Also, decision variables should be able to take on whole-number or fractional values.

E-mart Example

- Emart sells products in m major merchandise groups, e.g., children's wear, candy, music, toys, and electric.
- Advertising is organized into n campaign formats promoting specific merchandise groups (e.g., catalog, press, or television).
- For example, one campaign advertises children's wear in catalog, another promotes the same product line in newspapers while a third sells toys with television.
- The profit margin (fraction) for each merchandise group is known.
- E-mart wishes to maximize the profit gained from allocating its advertising budget across the campaign alternatives.

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E-mart Example

- Two main dimensions of the problems:

$$g \triangleq \text{merchandise group number } (g = 1, \ldots, m)$$
$$c \triangleq \text{campaign type number } (c = 1, \ldots, n)$$

- Input parameters:

$$b \triangleq \text{available advertising budget}$$
$$p_g \triangleq \text{profit, as a fraction of sales, from merchandise group } g$$
$$s_{g,c} \triangleq \text{parameter relating advertising cost spent in campaign } c$$
$$\text{to sales growth in merchandise group } g$$

- Decision variables:

$$x_c \triangleq \text{amount spent on campaign type } c$$

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E-mart Example

- How sales in each group g are affected by advertising cost on each campaign c.
- If the relationship is linear:

$$\text{sales increased in } g \text{ due to } c = s_{g,c}x_c$$

- When there is an option, linear constraint and objective functions are preferred to nonlinear ones in optimization models because each nonlinearity of an optimization model usually reduces its tractability as compared to linear forms.

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Linear and Nonlinear Programs

Equal returns to scale

Linear functions implicitly assume that each unit increase in a decision variable has the same effect as the preceding increase.

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- However, advertising data exhibit **decreasing returns to scale**, i.e., each dollar of advertising in a particular campaign yields less than the preceding dollar did.

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- Linear functions produce what economists call **equal returns to scale**.
- However, advertising data exhibit **decreasing returns to scale**, i.e., each dollar of advertising in a particular campaign yields less than the preceding dollar did.
- Thus, non-linear form:

$$\text{sales increased in } g \text{ due to } c = s_{g,c} \log(x_c + 1)$$

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- Logarithms grow at a declining rate as x_c becomes large.

E-mart Example

The complete E-mart model

$$
\begin{array}{rll}
\text{maximize} & \displaystyle\sum_{g=1}^{m} p_g \sum_{c=1}^{n} s_{g,c} \log(x_c + 1) & \text{(total profit)} \\
\text{s.t.} & \displaystyle\sum_{c=1}^{n} x_c \leq b & \text{(budget limit)} \\
& x_c \geq 0, \quad c = 1, \ldots, n & \text{(non-negative expenditures)}
\end{array}
$$