

A 3D Diffusive and Advective Model of Electron Transport Applied to the Pulsar Wind Nebula HESS J1825-137

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ABSTRACT

HESS J1825-137 is one of the most powerful and luminous TeV gamma-ray pulsar wind nebulae (PWNe), making it an excellent laboratory to study particle transportation around pulsars. We present a model of the (diffusive and advective) transport and radiative losses of electrons from the pulsar PSR 1826-1334 powering HESS J1825-137 using interstellar medium gas (ISM) data, soft photon fields and a spatially varying magnetic field. We find that for the characteristic age of 21 kyr, PSR 1826-1334 is unable to meet the energy requirements to match the observed X-ray and gamma-ray emission. An older age of 40 kyr, together with an electron conversion efficiency of 0.14 and advective flow of $v = 0.002c$, can reproduce the observed multiwavelength emission towards HESS J1825-137. A turbulent ISM with magnetic field of $B = 20 \mu\text{G}$ to $60 \mu\text{G}$ to the north of HESS J1825-137 (as suggested by ISM observations) is required to prevent significant gamma-ray contamination towards the northern TeV source HESS J1826-130.

Key words: cosmic rays - ISM: evolution - gamma-rays: general - X-rays: general - ISM individual (HESS J1825-137) - pulsars: individual (PSR J1826-1334)

1 INTRODUCTION

HESS J1825-137 is a luminous pulsar wind nebula (PWN) powered by pulsar PSR 1826-1334 with spin-down power $\dot{E} = 2.8 \times 10^{36} \text{ erg s}^{-1}$ and characteristic age $\tau_c = P/2\dot{P} = 21.4 \text{ kyr}$ (Manchester et al. 2005). The distance to PSR J1826-1334 has been estimated to lie at 3.6 kpc based on dispersion measurements (Taylor & Cordes 1993; Cordes & Lazio 2002), however we will use a distance of 4 kpc in line with Van Etten & Romani (2011) and H.E.S.S. Collaboration et al. (2019). The TeV gamma-ray emission from HESS J1825-137 has a characteristic ($1/e$) radius of $0.66^\circ \pm 0.03^\circ_{\text{stat}} \pm 0.04^\circ_{\text{sys}}$, implying a radius of $\approx 46 \text{ pc}$ based on a distance of 4 kpc (H.E.S.S. Collaboration et al. 2019). Owing to its brightness in TeV gamma rays, HESS J1825-137 is an ideal laboratory to study relativistic particle transport in and around middle-aged PWNe. Several studies (e.g. Porth et al. (2016), Giacinti et al. (2020)) suggest that both diffusive and advective transport mechanisms are required to explain the extended gamma-ray morphology towards PWNe.

Situated 0.7° north of HESS J1825-137 (see Figure 1), HESS J1826-130 is a TeV gamma-ray source and possible accelerator of cosmic rays up to PeV energies (Abeysekara et al. 2020; Cao et al. 2021). Due to its close proximity to HESS J1825-137,

HESS J1826-130 was originally considered an extension of HESS J1825-137 until it was revealed to be a separate source of gamma rays (H.E.S.S. Collaboration et al. 2018a; Aharonian et al. 2005). The two nearby supernova remnants (SNRs) SNR G018.1-0.1 and SNR G018.6-0.2 (Odegard 1986; Brogan et al. 2006) were deemed to be unlikely to be associated with HESS J1826-137 due to their offset positions and small angular diameters (H.E.S.S. Collaboration et al. 2020). Instead, the Eel PWN (PWNG18.5-0.4) and PSR J1826-1256 are associated with HESS J1826-130 based on spatial coincidence (H.E.S.S. Collaboration et al. 2018a). PSR J1826-1256 has a spin-down power of $3.6 \times 10^{36} \text{ erg s}^{-1}$ and characteristic age of 14 kyr, well within the range of pulsar properties associated with TeV PWNe (Manchester et al. 2005; H.E.S.S. Collaboration et al. 2018b).

Araya et al. (2019) revealed GeV gamma-ray emission $\sim 2.5^\circ$ to the Galactic south of HESS J1825-137. The same study postulated that the GeV emission from this region originates from cosmic rays accelerated by the SNR or PWN associated with HESS J1825-137 or a star-forming region such as the Cygnus Cocoon. Comprehensive modelling of the spectral energy distribution (SED) towards the GeV region suggests that the emission may be reflective of an earlier epoch of the PWN or a combination of HESS J1825-137 and nearby compact object LS 5039 (Collins et al. 2021).

The PWN associated with HESS J1825-137 must be expanding within the progenitor SNR. A large H α rim-like structure dis-

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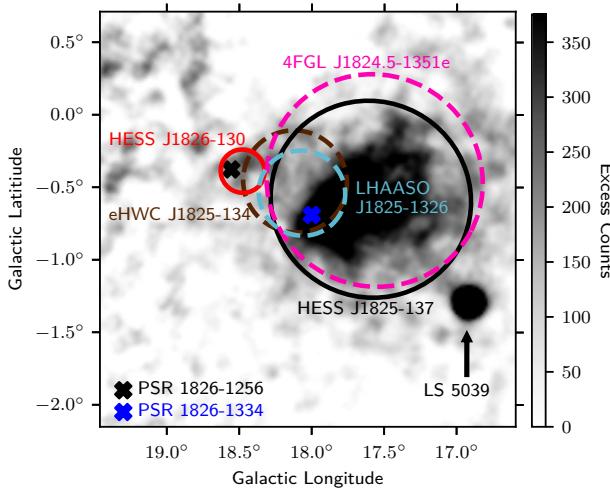


Figure 1. HESS excess counts towards HESS J1825-137 (H.E.S.S. Collaboration et al. 2019) overlaid by the regions used to extract the gamma-ray spectra towards HESS J1825-137 (black) and HESS J1826-130 (red). 4FGL J1824.5-1351e, eHWC J1825-134 and LHAASO J1825-1326 are shown by the purple, brown and cyan dashed circles respectively with the positions of PSR J1826-1334 (blue) and PSR J1826-1256 (black). The position of the nearby binary system LS 5039 is indicated by the black arrow.

covered by Stupar et al. (2008) is present towards the south of HESS J1825-137. Voisin et al. (2016) postulated a connection between this rim and another southern $H\alpha$ rim and the progenitor SNR of HESS J1825-137. Both structures lie $\approx 1.7^\circ$ away from PSR 1826-1334 (≈ 120 pc for a distance of 4 kpc), which is consistent with the predicted SNR radius of 130 pc as suggested by de Jager & Djannati-Atai (2009).

The evolution of the cosmic-ray number density distribution can be described by the Fokker-Planck equation for particle transport (e.g. Skilling (1975); Cesarsky & Volk (1978)). Henceforth, this equation will be called the transport equation. Analytical solutions of the transport equation can be found for specific cases, e.g. isotropic diffusion in a homogeneous environment (Blumenthal & Gould 1970; Atoyan et al. 1995; Aharonian & Atoyan 1996). However, it can only be solved numerically for more complex systems, e.g. anisotropic diffusion where the diffusion coefficient varies with position.

Many models of HESS J1825-137 and other PWNe assume a homogeneous nebula, where high-energy electrons are injected into a symmetric sphere of constant density and magnetic field (e.g. H.E.S.S. Collaboration et al. (2019); Principe et al. (2020)). For example, the work conducted by Van Etten & Romani (2011) treated the transport of electrons from the powering pulsar as a series of uniform, spherical ‘bubbles’. However, the observed asymmetric gamma-ray morphology observed towards HESS J1825-137 suggests a similarly asymmetric electron density and/or magnetic field and therefore requires a more complex model.

In this paper, we present a model that incorporates 3D distributions of the ISM hydrogen number density and magnetic field and solves the transport equation numerically. This model assumes PSR J1826-1334 to be a source of high-energy electrons and aims at reproducing the X-ray and gamma-ray morphology, spectrum and surface brightness radial profiles towards HESS J1825-137.

2 PARTICLE TRANSPORT AND MULTI-WAVELENGTH EMISSION

Upon the release from an accelerator, such as a SNR or PWN, cosmic rays are transported through the ISM and experience radiative losses. The evolution of the number density distribution of cosmic rays, $n \equiv n(\gamma, t, \vec{r})$, with Lorentz factor $\gamma \equiv \gamma(\vec{r})$, at position $\vec{r} \equiv (x, y, z)$ and time t after the birth of the accelerator, can be described by (e.g. Skilling (1975); Cesarsky & Volk (1978)):

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \gamma} (\dot{\gamma} n) + \nabla \cdot (\bar{D} \cdot \nabla n) - \nabla \cdot (n \vec{v}_A) - \frac{1}{3} \frac{\partial}{\partial \gamma} (\gamma (\nabla \cdot \vec{v}_A)) n \\ + \frac{\partial}{\partial \gamma} \left(\gamma^2 D_{\gamma\gamma} \frac{\partial}{\partial \gamma} \left(\frac{n}{\gamma^2} \right) \right) + S(\gamma, t, \vec{r}). \quad (1)$$

The first term in Equation 1 gives the evolution of cosmic-ray density due to radiative losses. The second term considers the spatial diffusion of cosmic rays as a second-rank tensor ($\bar{D} \equiv \bar{D}(\gamma, t, \vec{r})$), allowing preferential direction of transport. The third term describes the evolution of cosmic-ray density due to advection as a co-moving fluid with velocity $\vec{v}_A \equiv \vec{v}_A(\gamma, t, \vec{r})$. The fourth term considers losses due to adiabatic expansion. The fifth term represents the re-acceleration of cosmic rays due to stochastic processes with $D_{\gamma\gamma}$ being the acceleration rate. Finally, $S(\gamma, t, \vec{r})$ is the cosmic-ray source/injection function.

To numerically solve Equation 1, explicit finite difference techniques forward in time can be used after discretising a region of interest into a grid of voxels with dimension $\Delta x \Delta y \Delta z$ and time step Δt :

$$\frac{n(\gamma, t + \Delta t, \vec{r}) - n(\gamma + \Delta \gamma, t, \vec{r})}{\Delta t} = \left(\frac{\partial n}{\partial t} \right)'_{\text{diff}} + \left(\frac{\partial n}{\partial t} \right)'_{\text{adv}} + \left(\frac{\partial n}{\partial t} \right)'_{\text{adb}} \\ + \left(\frac{\partial n}{\partial t} \right)'_{\text{re-acc}} + S(\gamma, t, \vec{r}), \quad (2)$$

where prime represents the evolution of the number density distribution *after* radiative losses. The following discussion describes how the implemented model treats individual terms in Equation 2.

2.1 Radiation Losses

High-energy electrons interact with the ISM via inverse Compton interactions on ambient photons, via Bremsstrahlung with interstellar gas and via synchrotron interactions against magnetic fields (see Appendix A). The evolution of the electron number density with Lorentz factor γ due to radiative losses is given by:

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \gamma} (\dot{\gamma} n), \quad (3)$$

where $\dot{\gamma}$ is the cooling rate as given by Manolakou et al. (2007):

$$\dot{\gamma} = b_s \gamma^2 + b_c (3 \ln \gamma + 18.8) + 5.3 b_b + \sum_j b_{IC}^j \gamma^2 F_{KN}^j(\gamma), \quad (4)$$

for the case of ionisation or Bremsstrahlung losses in neutral hydrogen. Here, j sums over all radiation fields (CMB, infrared and optical photons), b_s , b_c , b_b and b_{IC} are the coefficients for synchrotron losses, Coulomb losses, Bremsstrahlung losses and inverse Compton losses respectively and F_{KN} is the Klein-Nishina cross section (see Equation A4). The photon fields were assumed to be constant across the 3D grid. The general solution to Equation 3 is:

$$n(\gamma, t + \Delta t) = \frac{\dot{\gamma}_0}{\dot{\gamma}} n(\gamma_0, t), \quad (5)$$

where $\gamma_0 \equiv \gamma_0(x, y, z) = \gamma + \Delta\gamma$ is the Lorentz factor at time t before electrons cool to Lorentz factor γ at time $t + \Delta t$.

2.2 Diffusion

Over distances smaller than the gyro-radius, r_g , electrons propagate through the ISM via ballistic motion. In a medium with randomised magnetic turbulence (δB), electrons scatter and the motion switches to a diffusive regime for distances larger than the gyro-radius (e.g. Prosekin et al. (2015)). For a simple case of isotropic diffusion in magnetic field $B(\vec{r})$, the diffusion tensor in Equation 1 becomes a scalar; $\overline{\overline{D}} \rightarrow D(E, \vec{r})$, where E is the energy of the cosmic ray.

Suppression of cosmic-ray diffusion is to be expected towards PWNe and SNRs where magnetic field turbulence is enhanced and the diffusion coefficient, $D(E, \vec{r})$, can be parameterised by (e.g. Gabici et al. (2007)):

$$D(E, \vec{r}) = \chi D_0 \left(\frac{E/\text{GeV}}{B(\vec{r})/3\mu\text{G}} \right)^\delta, \quad (6)$$

where $D_0 = 3 \times 10^{27} \text{ cm}^2 \text{ s}^{-1}$ is the average Galactic diffusion coefficient at 1 GeV, $\delta = 0.5$ following cosmic-ray observations (e.g. see Strong et al. (2007)) and the diffusion suppression factor, χ , takes values ≤ 1 depending on the environment (Berezinskii et al. 1990). For example, Gabici et al. (2007) found that highly suppressed diffusion ($\chi \sim 0.01$) in molecular clouds can significantly affect the shape of the observed gamma-ray spectrum. However, the diffusion suppression factor is not well constrained and a variety of χ have been found, e.g. Li & Chen (2010), Giuliani et al. (2010) and Gabici et al. (2010) found values of $\chi = 0.1, 0.01$ and 0.06 towards SNR W28, respectively. Similarly, Protheroe et al. (2008) showed that the suppression factor towards the star-forming region Sgr B2 takes values < 0.02 based on the radio synchrotron flux.

Assuming isotropic inhomogeneous diffusion, the diffusive component of Equation 2 is given by:

$$\begin{aligned} \left(\frac{\partial n}{\partial t} \right)_{\text{diff}} &= \frac{1}{\Delta t^2} \sum_{i=x,y,z} \left[\frac{D(\gamma, i + \Delta t) - D(\gamma, i)}{2} \right] \cdot [n(\gamma, t, i + \Delta t) - n(\gamma, t, i)] \\ &\quad + \left[\frac{D(\gamma, i - \Delta t) - D(\gamma, i)}{2} \right] \cdot [n(\gamma, t, i - \Delta t) - n(\gamma, t, i)], \end{aligned} \quad (7)$$

where $D(\gamma, i)$ is the diffusion coefficient from Equation 6. The central finite difference technique used in Equation 7 only considers the transport of electrons to/from the surrounding voxels. If the time step is too large, electrons travel across more than one voxel and are lost from the system. The finite difference technique is then said to be numerically ‘unstable’. Using Von Neuman stability analysis (e.g. see Isaacson (1966)), Equation 7 is stable when:

$$\Delta t \leq \frac{\Delta i^2}{2D(i)} \Big|_{\min}. \quad (8)$$

2.3 Advection

For simplicity, the velocity due to the bulk flow of electrons ($\vec{v}_A \equiv [v_{A,x}, v_{A,y}, v_{A,z}]$) was assumed to be spatially-independent and energy-independent across the region of interest. Using explicit finite difference techniques, the advective component of Equation 2 is given by Equation 9:

$$\left(\frac{\partial n}{\partial t} \right)_{\text{adv}} = - \sum_{i=x,y,z} v_{A,i} \frac{1}{\Delta t} \begin{cases} n(\gamma, t, i + \Delta t) - n(\gamma, t, i), & v_{A,i} < 0 \\ n(\gamma, t, i) - n(\gamma, t, i - \Delta t), & v_{A,i} > 0 \end{cases}, \quad (9)$$

where $v_{A,i}$ is the component of advective velocity in the i th direction. Equation 9 uses the forward difference method to approximate the derivative in Equation 2 when $v_{A,i} < 0$ and the backward difference method when $v_{A,i} > 0$.

For Equation 9 to be numerically stable, the time step must be chosen so that an electron does not travel across more than one voxel in time Δt :

$$\Delta t \leq \frac{\Delta i}{|v_{A,i}|} \Big|_{\min}. \quad (10)$$

The time step must satisfy both Equation 8 and Equation 10 when modelling a scenario including both diffusion and advection.

2.4 Adiabatic Expansion and Re-acceleration of Electrons

The spatially-independent advective velocity assumed in our model results in zero adiabatic losses in Equation 1 ($\nabla \cdot \vec{v}_A = 0$). Moreover, studies such as Tanaka & Takahara (2010) and Porth et al. (2016) who considered spherically symmetric advection concluded that adiabatic losses are dominant over radiative losses for electrons $< 1 \text{ TeV}$ (equivalent to gamma-ray emission $< 20 \text{ GeV}$). As we are interested in the VHE gamma-ray range which is not dominated by adiabatic losses, adiabatic expansion is not considered here but is left for future work.

The termination shock (TS) of pulsars has been proposed as a site for the re-acceleration of electrons through diffusive shock acceleration (DSA). By ensuring the voxel width ($\Delta x, \Delta y, \Delta z$) is larger than the diameter of the TS (0.2 pc, (Gaensler & Slane 2006)), electrons are both injected and re-accelerated within the same voxel. Therefore, the source term in Equation 2 treats the injected electron spectra as the spectra obtained after re-acceleration due to the TS. Furthermore, magnetohydrodynamic models (e.g. Lemoine & Pelletier (2010); Sironi et al. (2015)) suggest that DSA at the TS is too suppressed for electron acceleration up to energies responsible for the TeV emission seen towards PWNe. Hence, the re-acceleration of electrons is left for future work.

2.5 Multi-wavelength Photon Production

The final electron number density distribution was obtained by solving Equation 2 in discrete time steps Δt until the desired age was reached. Based on the obtained electron number densities, the multi-wavelength photon emission was derived for each voxel and summed along the line of sight, z , to obtain the 2D photon distribution. Equations A1, A3a and A6 gives the flux from synchrotron, inverse Compton and Bremsstrahlung interactions respectively.

3 APPLICATION TO HESS J1825-137

The modelling described in section 2 was applied to the PWN HESS J1825-137 with the pulsar PSR J1826-1334 being the accelerator of high-energy electrons. PSR J1826-1334 is located at $\ell = 18^\circ$, $b = -0.69^\circ$ and has a proper motion of $\approx 440 \text{ km s}^{-1}$ (assuming a distance of 4 kpc) approximately perpendicular to the extended TeV emission (see Figure 3) (Manchester et al. 2005). Hence, the proper motion of the pulsar is unlikely to be related to the asymmetric gamma-ray emission and our model assumed that electrons are injected at the current position of the pulsar for simplicity. Two different ages of PSR J1826-1334 were considered, the characteristic age of 21.4 kyr and the older age of 40 kyr suggested by Van Etten & Romani (2011).

Each voxel in the 3D grid had a volume of $\Delta x \Delta y \Delta z$, where Δz is the voxel length in the line of sight and Δx and Δy are the voxel length along Galactic longitude and latitude respectively. For the purposes of this study, we utilised a $200 \text{ pc} \times 200 \text{ pc} \times 200 \text{ pc}$ grid consisting of voxels of size $2 \text{ pc} \times 2 \text{ pc} \times 2 \text{ pc}$ ($\approx 0.03^\circ \times 0.03^\circ \times 2 \text{ pc}$). The pulsar was located in the centre of the grid with the central z slice lying at distance 4 kpc. The time step used for the finite difference technique was $\approx 8 \text{ yr}$.

3.1 Electron Injection

High-energy electrons were injected into the 3D grid by PSR 1826-1334 and follow an exponential cutoff power-law:

$$S(E, t) = A \cdot \left(\frac{E}{1 \text{ TeV}} \right)^{-\Gamma} \exp\left(-\frac{E}{E_c}\right), \quad (11)$$

following the observed TeV gamma-ray emission (e.g. see Blumenthal & Gould (1970)) as observed by H.E.S.S. (H.E.S.S. Collaboration et al. 2019), where E_c is the cutoff energy and A is the normalisation factor such that:

$$L_{\text{inj}}(t) = \int_{E_{\min}}^{E_{\max}} S(E, t) \, dE, \quad (12)$$

with $L_{\text{inj}} \equiv \eta \dot{E}$ being the electron injection luminosity, $\eta < 1$ is the conversion efficiency of the pulsar spin-down power, $E_{\min} = 1 \text{ MeV}$ and $E_{\max} = 500 \text{ TeV}$. The spin-down power, $\dot{E}(t)$ at time t is given by (Haensel et al. 2007):

$$\dot{E}(t) = \dot{E}(t = t_{\text{age}}) \left[1 + (n - 1) \frac{\dot{P}(t - t_{\text{age}})}{P} \right]^{-\Gamma_n}, \quad (13)$$

where n is the braking index of the pulsar, $\Gamma_n \equiv (n + 1)/(n - 1)$ and $\dot{E}(t = t_{\text{age}})$, P and \dot{P} are the spin-down power, period and spin-down period of the pulsar at the current age t_{age} .

3.2 The Environment Towards HESS J1825-137

3.2.1 Magnetic Field

Following Van Etten & Romani (2011), the magnetic field due to the PWN was assumed to follow a time-independent power-law with a decreasing magnetic field strength varying with distance r from the pulsar:

$$B_{\text{PWN}}(r) = B_0 \left(\frac{r}{r_{\text{ts}}} \right)^{-\beta}, \quad (14)$$

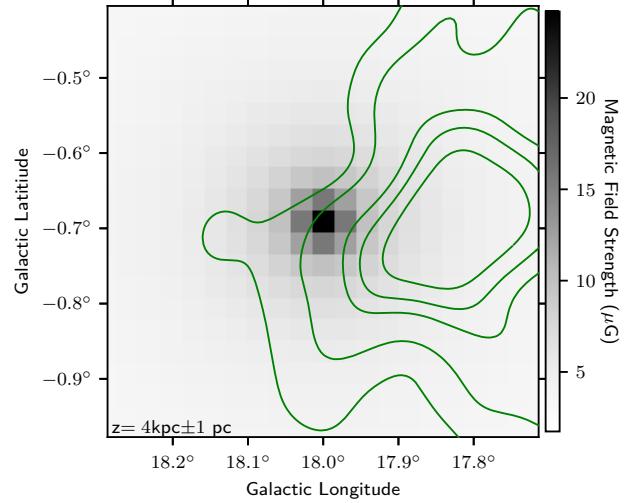


Figure 2. The radially symmetric magnetic field due to the pulsar at slice $z = 4 \text{ kpc} \pm 1 \text{ pc}$. Overlaid are the green H.E.S.S. contours at 4, 5, 6, 7, 8 and 9σ (H.E.S.S. Collaboration et al. 2019).

where $r_{\text{ts}} = 0.03 \text{ pc}$ is the radius of the termination shock, and B_0 and β are free parameters optimised to match the multi-wavelength SED of HESS J1825-137. Van Etten & Romani (2011) suggested $\beta = -0.69$ and $B_0 = 400 \mu\text{G}$ for an age of 40 kyr. Note that Van Etten & Romani (2011) considered an additional dependence on the spin-down energy of the pulsar which was not considered in this study.

3.2.2 Interstellar Medium

The Nanten $^{12}\text{CO}(1-0)$ survey (Mizuno & Fukui 2004) was used to trace the column density of molecular hydrogen towards HESS J1825-137:

$$N_{\text{H}_2} = X_{12\text{CO}} W_{12\text{CO}} \quad (15)$$

where $W_{12\text{CO}}$ is the integrated intensity of the gas. The scaling factor $X_{12\text{CO}} = 1.5 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s}$ is assumed to be constant over the Galactic plane but may vary with galactocentric radius (1.3–1.5 per kpc) (Strong et al. 2004). The length of the 3D grid (< 1 kpc) allows the assumption of a constant $X_{12\text{CO}}$ towards the region of interest. PSR 1826-1334 has a dispersion measure distance of 4 kpc, corresponding to a velocity of 50 km s^{-1} using the Galactic Rotation model (Brand & Blitz 1993). As there may be local motion of the gas unrelated to Galactic rotation, we considered a velocity range of $40 - 60 \text{ km s}^{-1}$ (3.5 kpc to 4.5 kpc) consistent with Voisin et al. (2016). Atomic hydrogen in the same velocity range contributes less than 1% to the total column density towards HESS J1825-137 and thus was not considered (Voisin et al. 2016; Collins et al. 2021). Assuming that all the gas in the $40 - 60 \text{ km s}^{-1}$ velocity range lies within the 3D grid and the density along the line of sight is constant, the number density of a voxel with column density N_{H_2} is given by:

$$n_H = \frac{N_H}{200 \text{ pc}}, \quad (16)$$

where $N_H \equiv 2.8 N_{\text{H}_2}$ considers a 20% He component.

Stellar winds from the progenitor star of PSR J1826-1334 pushes out gas in the nearby vicinity (Castor et al. 1975). The subsequent supernova explosion creates a ‘bubble’ of hot dense gas

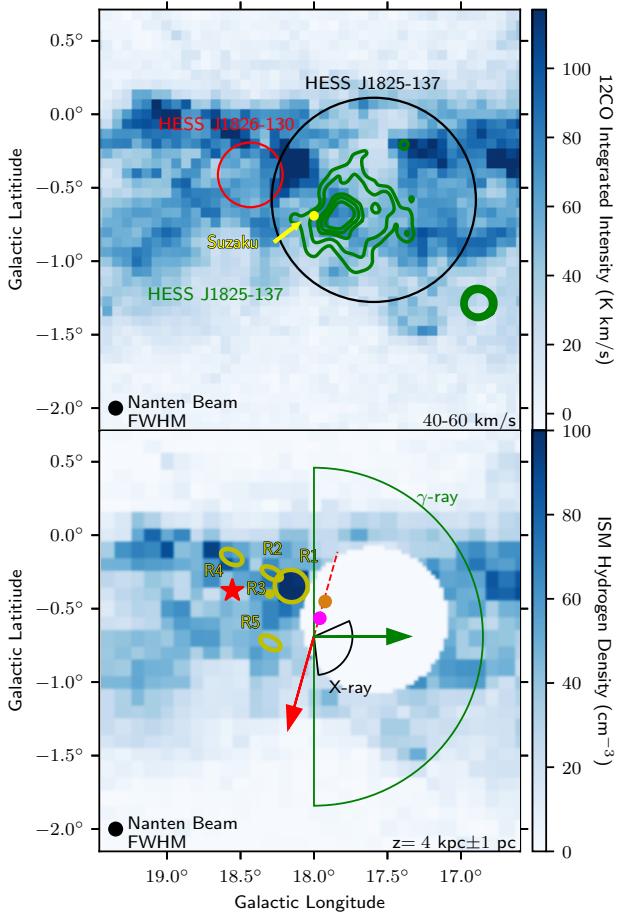


Figure 3. (top) Nanten $^{12}\text{CO}(1-0)$ integrated intensity in the velocity range $40 - 60 \text{ km s}^{-1}$ corresponding to $3.5 - 4.5 \text{ kpc}$ overlaid by green H.E.S.S. contours (at $4, 5, 6, 7, 8$ and 9σ). The yellow dot represents the Suzaku region A as defined in Uchiyama et al. (2009) and is used to extract the X-ray SED. The region used to obtain the gamma-ray spectra towards HESS J1825-137 and HESS J1826-130 are shown in black and red respectively. (bottom) Calculated ISM number density across the 3D grid at $4 \text{ kpc} \pm 1 \text{ pc}$ where the voxels within the PWN extent ($R < 0.5^\circ$) are set to a density of 0.5 cm^{-3} to represent the bubble that has been swept out by the stellar wind from the progenitor star. The proper motion of the pulsar is shown by the red arrow with the projected birthplaces indicated by the red-dashed line. The projected birthplaces for ages 21 kyr and 40 kyr are indicated by the magenta and brown dot respectively. The direction of advective particle transport suggested by H.E.S.S. Collaboration et al. (2019) is shown by the green arrow. The black and green segments represent the regions used to extract the X-ray and gamma-ray surface brightness radial profile respectively. Molecular clouds R1-R5 from Voisin et al. (2016) are shown in yellow with the position of PSR J1826-1256 indicated by the red star. The width and height of the voxel in the 2D slice is 2 pc compared to the minimum Nanten resolution of 5 pc (assuming a distance of 4 kpc).

around a low-density interior. A region of low-density gas in the $40 - 60 \text{ km s}^{-1}$ velocity range can be seen towards the centre of the TeV emission in Figure 3. To include this, any voxels lying within the extent of the PWN volume (a sphere centered on the pulsar with radius $0.5^\circ \approx 35 \text{ pc}$) was set to a density of 0.5 cm^{-3} based on the average densities expected within massive stellar wind bubbles (Weaver et al. 1977). The Nanten $^{12}\text{CO}(1-0)$ integrated intensity between $40 - 60 \text{ km s}^{-1}$ and calculated ISM number density for the central slice lying at distance 4 kpc can be seen in Figure 3. Any

difference between the bottom and top panel of Figure 3 was due to the different resolutions of the 3D grid and Nanten.

Turbulent motion in the ISM results in an amplification of the magnetic field, suppressing the diffusion of electrons as they travel through the ISM as given by Equation 6. Figure 6 of Voisin et al. (2016) shows a three-coloured image of the CS(1-0) and NH₃ integrated intensity between $40 - 60 \text{ km s}^{-1}$ and the H62 α integrated intensity between $45 - 65 \text{ km s}^{-1}$ towards the cloud defined as R1 (see Figure 3 for the position of clouds R1-R5 from Voisin et al. (2016)). This suggests that cloud R1 is highly turbulent with a minimum magnetic field strength of $21 \mu\text{G}$ based on the density of 600 cm^{-3} calculated by Voisin et al. (2016) (see Equation B1). The amplification of the magnetic field towards cloud R1 was considered in subsubsection 3.4.3. Given the likely physical proximity to HESS J1825-137, cloud R1 may act as a barrier for electrons escaping into HESS J1826-130 from PSR 1826-1334 (Voisin et al. 2016).

3.2.3 Soft Photon Fields

The photon fields around HESS J1825-137 was estimated utilising the radiation field model described by Popescu et al. (2017); the far-infrared field (FIR) with temperature $T = 40 \text{ K}$ and energy density $U = 1 \text{ eV cm}^{-3}$, near infrared field (NIR) with temperature $T = 500 \text{ K}$ and energy density 0.4 eV cm^{-3} and optical light with temperature $T = 3500 \text{ K}$ and energy density of $U = 1.9 \text{ eV cm}^{-3}$.

3.3 Multi-wavelength Observations

The modelled gamma-ray SED of HESS J1825-137 was optimised to the TeV gamma-ray energy flux presented by H.E.S.S. Collaboration et al. (2019) and the GeV spectrum from the 4FGL catalogue (Abdollahi et al. 2020). To compare the modelled surface brightness radial profile to Figure 6 from H.E.S.S. Collaboration et al. (2019), a collection area of 0.25 km^2 (Benbow 2005) and observation time of 387 hr was used. The X-ray SED and surface brightness radial profile was optimised to the results presented by Uchiyama et al. (2009) using a collection area of 0.029 m^2 .

To investigate the gamma-ray contamination of HESS J1826-130, by HESS J1825-137, we utilised the gamma-ray SED presented by H.E.S.S. Collaboration et al. (2020) and the spectrum from the 4FGL catalogue (Abdollahi et al. 2020). H.E.S.S. Collaboration et al. (2020) estimated that the gamma-ray contamination to be 40% for photon energies below 1.5 TeV and 20% above 1.5 TeV . The modelled X-ray emission towards HESS J1826-130 was constrained by the ROSAT X-ray upper limit calculated using the ROSAT X-ray background tool (Sabol & Snowden 2019). The regions used to extract the X-ray and gamma-ray SED towards HESS J1825-137 and HESS J1826-130 are shown in Figure 3.

3.4 Results

The full list of model parameters is summarised in Table 1, including any constraints based on measurements. A computationally quick single-zone model, where the electron number density is derived using a uniform spherical shell, was utilised to investigate a large range of parameters to gain insight into HESS J1825-137. The results of the single-zone modelling are summarised in Appendix C. For an age of 21 kyr, the single-zone model required electrons to follow an exponential cutoff power-law with spectral index $\Gamma = 2.1$

Table 1. Model parameters used for the application towards HESS J1825-137. Fixed parameters refer to those constrained by measurements and non-fixed refers to those that are optimised to observations discussed in subsection 3.3.

Fixed Parameters	Value	Reference
t	21 kyr & 40 kyr	a, b
d	4 kpc	a
P	101 ms	a
\dot{P}	$7.5 \times 10^{-14} \text{ s s}^{-1}$	a
\dot{E}	$2.8 \times 10^{36} \text{ erg s}^{-1}$	a
$\Delta x, \Delta y, \Delta z$	2 pc	
Δt	8 yr	
E_{\min}	1 MeV	
E_{\max}	500 TeV	
D_0	$3 \times 10^{27} \text{ cm}^2 \text{ s}^{-1}$	c
r_{ts}	0.03 pc	
$U_{\text{CMB}}, T_{\text{CMB}}$	$0.26 \text{ eV cm}^{-3}, 2.72 \text{ K}$	e
$U_{\text{NIR}}, T_{\text{NIR}}$	$1 \text{ eV cm}^{-3}, 500 \text{ K}$	e
$U_{\text{FIR}}, T_{\text{FIR}}$	$0.4 \text{ eV cm}^{-3}, 40 \text{ K}$	e
$U_{\text{Opt}}, T_{\text{Opt}}$	$1.9 \text{ eV cm}^{-3}, 3500 \text{ K}$	e
Non-fixed Parameters	Value	Reference
η	< 1	
χ	< 1	c
Γ	-	
E_c	-	
B_0	-	
β	-	
n	2-3	
$\vec{V}_A = (\ell, b, z)$	($< 0.01c, 0, 0$)	d
B_{1826}	★	

★ See subsubsection 3.4.3

a. Manchester et al. (2005)

b. Van Etten & Romani (2011)

c. Berezinskii et al. (1990)

d. H.E.S.S. Collaboration et al. (2019)

e. Popescu et al. (2017)

and cutoff $E_c = 40 \text{ TeV}$ to match the observed gamma-ray SED while an older age of 40 kyr required an index of $\Gamma = 2.1$ and cutoff of $E_c = 50 \text{ TeV}$.

In the following, we present three applications of our model towards HESS J1825-137. All models incorporated a simple assumption of isotropic diffusion and radiative losses as described in subsection 2.2. Model 1 considered both ages of PSR J1826-1334, 21 kyr and 40 kyr. Model 2 introduced an additional advective component to Model 1 with velocity $\vec{V}_A = [V_A, 0, 0]$ as suggested by H.E.S.S. Collaboration et al. (2019) to explain the asymmetric gamma-ray morphology. H.E.S.S. Collaboration et al. (2019) constrained the total flow velocity to be $< 0.01c$. Model 3 expanded on Model 2 by including turbulent ISM towards HESS J1826-130 (see subsubsection 3.2.2) to reduce the contamination by HESS J1825-137. The model parameters were chosen based on the observations discussed in subsection 3.3 with the parameter list shown in Table 1. The parameters we found to match the multi-wavelength SED and morphology are shown in Table 2.

3.4.1 Model 1 (21 & 40 kyr) - Isotropic Diffusion

Figure 4 and 5 show the modelled gamma-ray morphology in different energy bands, the multi-wavelength SED and the 1–9 keV X-ray and 0.1–91 TeV gamma-ray surface brightness radial profiles for the 21 and 40 kyr models respectively. Both models predicted that the gamma-ray morphology towards HESS J1825-137 is symmetric around the powering pulsar with some gamma-ray contribution $< 1 \text{ TeV}$ via Bremsstrahlung radiation toward the region between HESS J1825-137 and HESS J1826-130 (see Figure 4 and 5). The

40 kyr gamma-ray emission between 1–10 TeV extends further from the pulsar than the 21 kyr emission. Both models predicted a steep surface brightness radial profile for X-rays between 1–9 keV (see the bottom-middle panels of Figure 4 and 5). The 21 kyr model was able to replicate the HESS surface brightness radial profile for gamma rays between 0.133–91 TeV (see the bottom-middle right panel of Figure 4) while the 40 kyr model over-predicted the gamma-ray emission for distances $> 0.5^\circ$ from the pulsar (see the bottom-middle right panel of Figure 5).

The 21 kyr modelled gamma-ray SED predicted by the multi-zone model was able to match observations with a slight over-prediction ($\approx 94\%$) of the HESS data between 1–10 TeV. While able to predict the normalisation of X-rays produced by synchrotron emission, the model was unable to replicate the slope of the observed Suzaku SED. The multi-zone 40 kyr SED was able to predict both the X-ray and gamma-ray SED with a similar over-prediction in 1–10 TeV photons as seen in the 21 kyr model. A slight 'bump' is present in the SED for photons around 50–100 TeV for both ages.

The 21 kyr model required electrons with spectral index $\Gamma = 2.0$ and cutoff 40 TeV to be injected into the ISM with a spin-down conversion factor of 10.4 to match the multi-wavelength SED. The 40 kyr required a conversion factor of 0.14 with a spectral index and cutoff of 1.9 and 500 TeV respectively. As $\eta < 1$, Models 2 and 3 only considered an age of 40 kyr.

Figure 6 shows the modelled SED towards HESS J1826-130 due to electrons escaping from HESS J1825-137 for the 40 kyr model. The SED towards HESS J1826-130 as a result of HESS J1825-137 exceeds observations for photons below 2 TeV. In the model, too many low-energy electrons have escaped into the region towards HESS J1826-130 before losing their energy to radiative losses. It is clear that further refinement of the model is required to accurately describe the region surrounding HESS J1825-137.

3.4.2 Model 2 - Isotropic Diffusion + Advection

The gamma-ray morphology in Figure 5 shows that Model 1 (40 kyr) did not reproduce the extended TeV gamma-ray morphology towards HESS J1825-137 at lower Galactic longitudes (see the top-middle left panel of Figure 5). Thus, Model 2 introduced an additional advective component as suggested by H.E.S.S. Collaboration et al. (2019) towards lower Galactic longitudes. The modelled flux, surface brightness radial profiles and gamma-ray morphology for Model 2 (40 kyr) with an advective flow of $v = 0.002c$ are shown in Figure 7. A comparison between different advection speeds ($v = 0.001c, v = 0.002c$ and $v = 0.003c$) is shown in Figure E4. All models otherwise have the same parameters as Model 1 (40 kyr) (see Table 2).

For further comparison of the gamma-ray morphology towards HESS J1825-137, the energy flux was extracted from rectangular regions taken along Galactic longitude centred on PSR J1826-1334 and are shown in Figure 8. An advective velocity of 0.002c was chosen so that the peak in the modelled gamma-ray morphology in energy range $E < 1 \text{ TeV}$ and $1 \text{ TeV} < E < 10 \text{ TeV}$ corresponds to the HESS data (see the left-upper panel of Figure E4).

While an additional advective flow of 0.002c lowered the gamma-ray SED towards HESS J1826-130 for energies less than 2 TeV, the emission still exceeds H.E.S.S. observations.

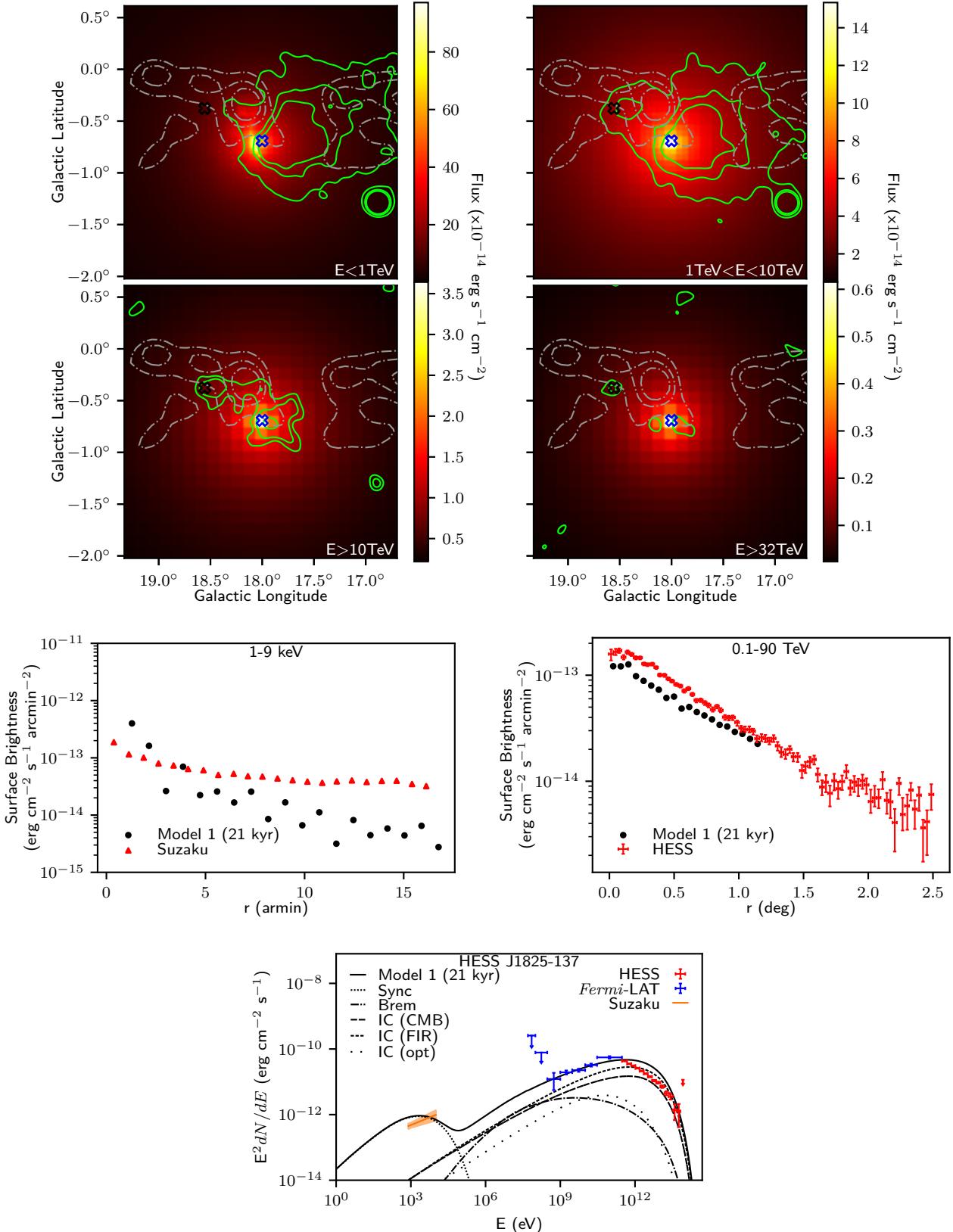


Figure 4. Model 1 (21 kyr), see Table 2 for model parameters. (top & top-middle) Modelled gamma-ray morphology towards HESS J1825-137 in different energy bands overlaid by green HESS significance contours (5, 10 and 15 σ for $E < 10 \text{ TeV}$ and 3, 5 and 10 σ for $E > 10 \text{ TeV}$) and grey 40, 50 and 60 σ Nanten 12CO integrated intensity contours. The positions of PSR J1826-1334 and PSR J1826-1256 are indicated by the empty blue and black crosses respectively. (bottom-middle) 1 – 9 keV X-ray (left) and 1 – 91 TeV gamma-ray (right) surface brightness radial profiles in comparison to Suzaku (Uchiyama et al. 2009) and HESS observations (H.E.S.S. Collaboration et al. 2019) respectively. (bottom) SED towards HESS J1825-137 with the orange Suzaku X-ray spectral fit, blue 4FGL J1824.5-135e flux observations (Abdollahi et al. 2020) and red HESS J1825-137 flux observations.

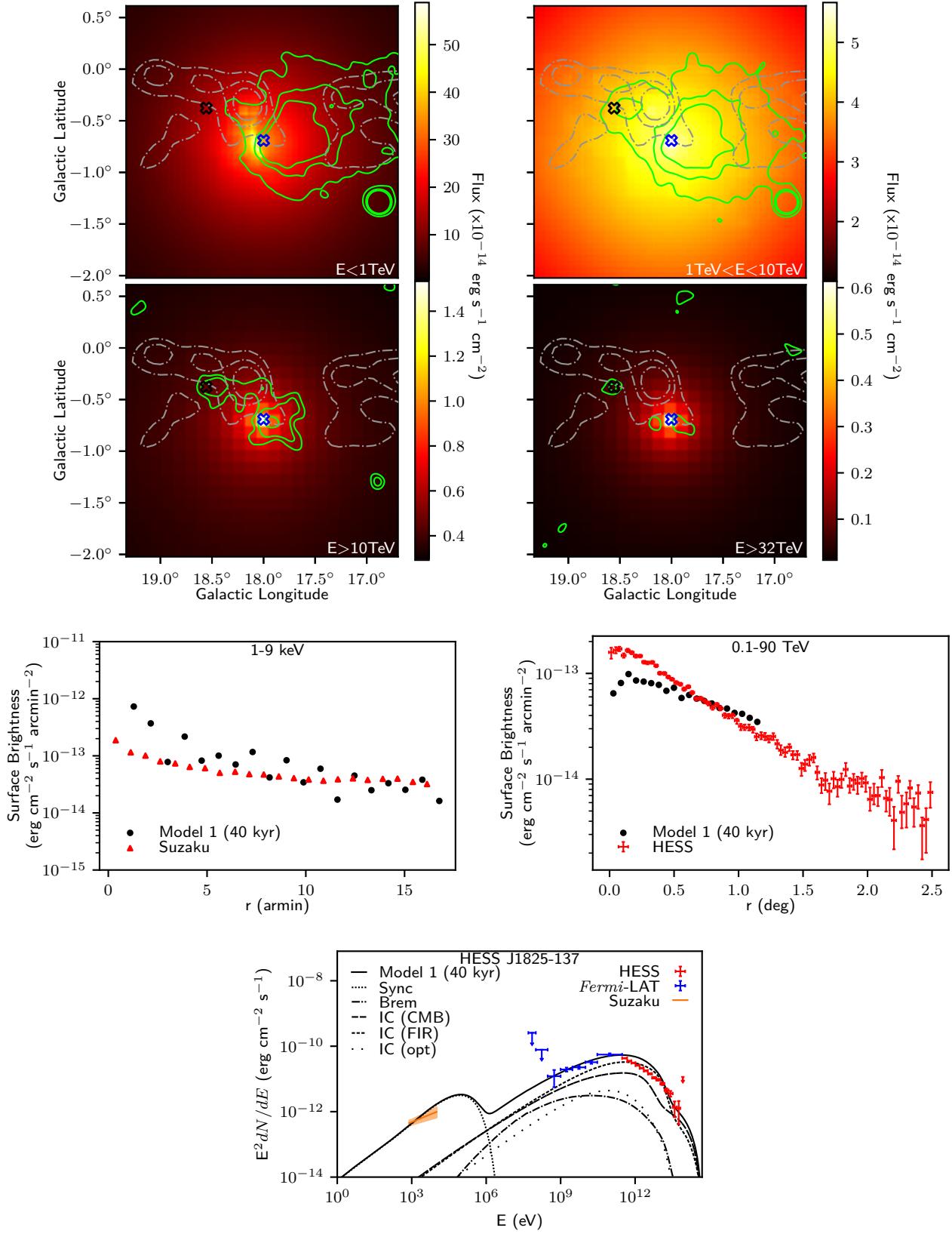
**Figure 5.** Model 1 (40 kyr), see Table 2 for model parameters. Same panel layout as in Figure 4.

Table 2. Model parameters that match the multi-wavelength SED and gamma-ray morphology towards HESS J1825-137. See section D for the 10% and 20% systematic variation of parameters.

Parameter	Model 1 (21 kyr)	Model 1 (40 kyr)	Model 2 (0.002c)	Model 3* (60 μ G)	Model 3 (60 μ G)
η	10.7	0.14	0.14	0.14	0.14
χ	0.25	0.1	0.1	0.1	0.1
Γ	2.0	1.9	1.9	1.9	1.9
E_c (TeV)	40	500	500	500	500
B_0 (μ G)	70	450	450	450	450
β	-0.9	-0.7	-0.7	-0.7	-0.7
n	2	2	2	2	2
V_A	-	-	0.002c	-	0.002c
B_{J1826} (μ G)	-	-	-	60	60*

* See subsubsection 3.5.3

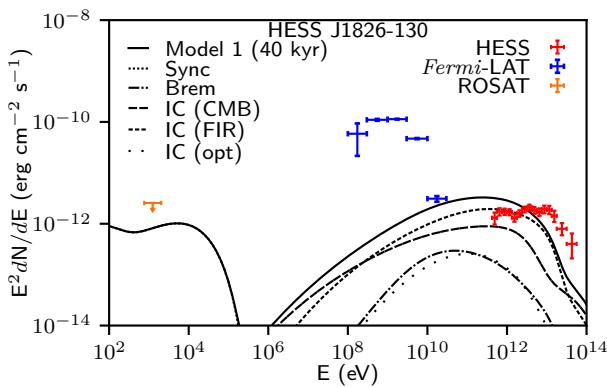


Figure 6. SED towards HESS J1826-130 from electrons accelerated by PSR 1826-1256 by Model 1 (40 kyr) against the observed flux of HESS J1826-130 (H.E.S.S. Collaboration et al. 2020) and 4FGL J1826.1-1256 (Abdollahi et al. 2020). The GeV and TeV gamma-ray flux observations towards HESS J1826-130 are represented by blue and red respectively.

3.4.3 Model 3 - Isotropic Diffusion + Advection + Magnetic Field towards HESS J1826-130

As discussed in subsubsection 3.2.2, the turbulent molecular gas between HESS J1825-137 and HESS J1826-130 can act as a barrier for electrons escaping from the PWN. As clouds R1-R5 from Voisin et al. (2016) are positioned in an approximate semi-circle around PSR J1826-1256 (see Figure 3), Model 3 expanded on Model 2 (0.002c) by including a shell of increased magnetic field strength, B_{1826} , centred on HESS J1826-130 with inner and outer radii 0.17° and 0.33° respectively. Model 3* refers to Model 1 (40 kyr) with the shell of increased magnetic field strength with no advective component ($v_A = 0$).

Figure 9 shows the SED, surface brightness radial profiles and the gamma-ray flux along Galactic longitude of HESS J1825-137 and HESS J1826-130 for Model 3 with magnetic field strengths of $B = 20, 60$ and $100 \mu\text{G}$. A comparison between Model 1 (40 kyr), Model 3* (60 μG) and Model 3 (60 μG) is shown in Figure 10 as well as the gamma-ray morphology for Model 3 (60 μG). All models otherwise have the same parameters as Model 1 (40 kyr) (see Table 2).

3.5 Discussion

3.5.1 Model 1 - Isotropic Diffusion

The 21 kyr and 40 kyr models were unable to reproduce both the X-ray and gamma-ray surface brightness radial profiles. For example, the diffusion suppression coefficient, χ , could be increased to compensate for the steep X-ray surface brightness radial profile for the 21 kyr model. Electrons would then escape the PWN at a higher rate and the gamma-ray surface brightness radial profile will flatten. This can be seen in the 40 kyr model, which assumed a lower value of χ than the 21 kyr model. The shallow 40 kyr gamma-ray surface brightness radial profile indicates that lower energy electrons have started to accumulate near the pulsar, while high-energy electrons rapidly lose their energy through radiative cooling and do not escape far from the pulsar. This is demonstrated in the upper right panel in Figure 5 where the gamma-ray flux below 10 TeV is relatively constant over the grid while the flux above 10 TeV is constrained to the pulsar. This accumulation is not as apparent in the 40 kyr X-ray surface brightness radial profile and the SED as the regions used in extracting X-ray spectra and surface brightness radial profile are smaller than the regions used for the gamma-ray analysis (see Figure 3 and top-middle right panel of Figure 4). The accumulation of lower-energy electrons is also reflected as a bump in the TeV gamma-ray SED. The bump occurs when synchrotron losses start to dominate at electron energies > 9 TeV, resulting in inverse Compton radiation > 6 TeV, and radiative energy losses are balanced by the electron injection luminosity (Manolakou et al. 2007; Hinton & Hofmann 2009). This bump is not present for a slightly younger age of 36 kyr (with the same parameters as Model 1 (40 kyr), see Figure E1), where the gamma-ray SED $\gtrsim 10$ TeV at age 36 kyr matches Model 1 (40 kyr).

The 21 kyr model required a spin-down conversion factor of 10.7. To compensate, a braking index of 3 would inject a greater quantity of electrons at earlier times (see Equation 13). However, this results in an accumulation of electrons at lower energies, consequently increasing the gamma-ray flux for photons with energies < 1 TeV (see Figure E2) and the modelled SED no longer reproduces observations. This suggests that the age of HESS J1825-137 lies between 21 kyr and 40 kyr. The 40 kyr magnetic field profile takes values of $B_0 = 450 \mu\text{G}$ and $\beta = -0.7$ (see Equation 14) in comparison to $B_0 = 400 \mu\text{G}$ and $\beta = -0.69$ used in (Van Etten & Romani 2011). Van Etten & Romani (2011) considered an evolving magnetic field $B \propto \dot{E}(t)$ where the magnetic field takes larger values at earlier times. This could explain the larger B_0 normalisation used in our modelling.

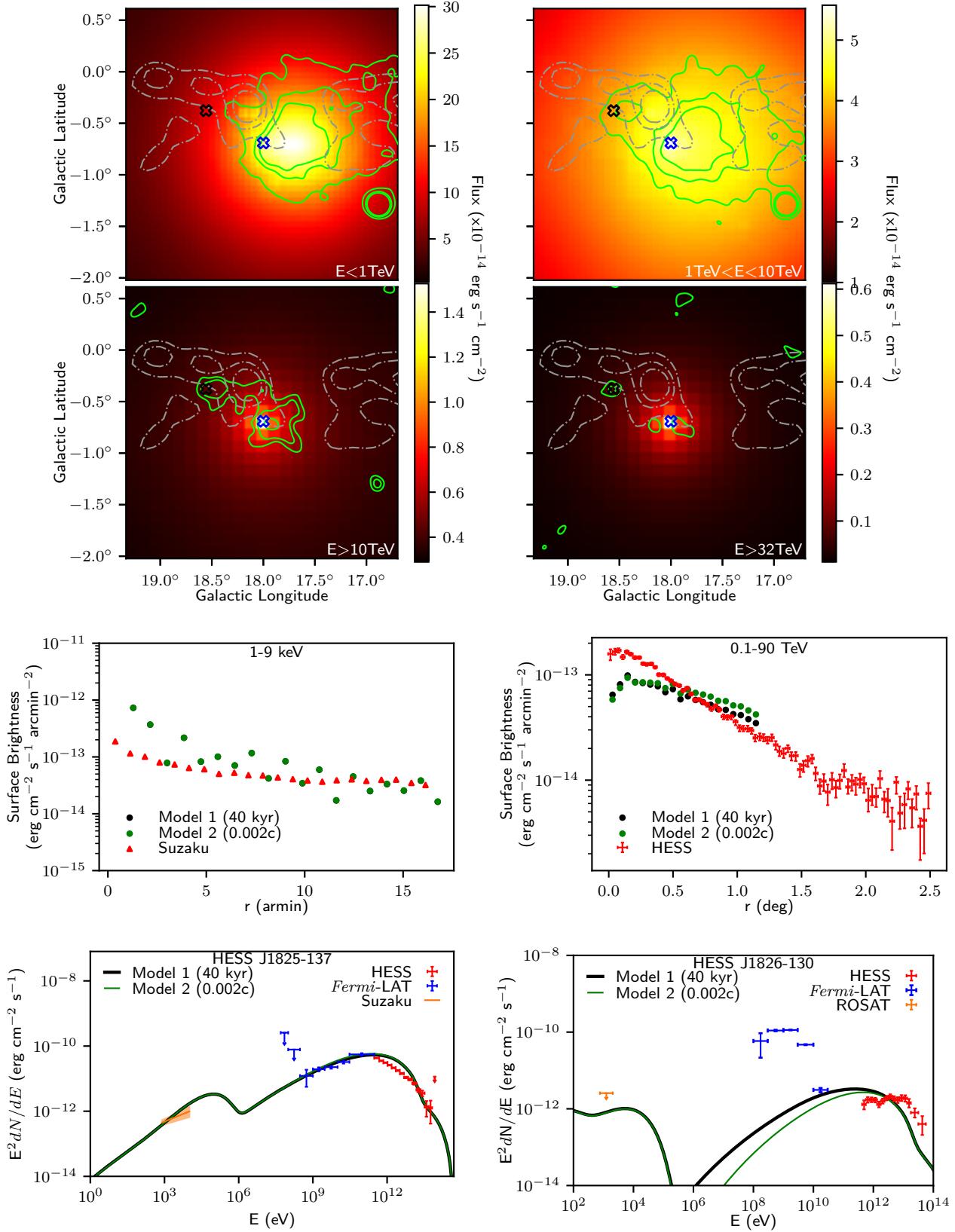


Figure 7. Model 2 (0.002c) vs Model 1 (40 kyr), see Table 2 for model parameters. (top and top-middle) panels show the gamma-ray morphology for Model 2 (0.002c). (bottom-middle) 1 – 9 keV X-ray (left) and 1 – 91 TeV gamma-ray (right) surface brightness radial profiles. (bottom-left) SED towards HESS J1825-137. (bottom-right) SED towards HESS J1826-130.

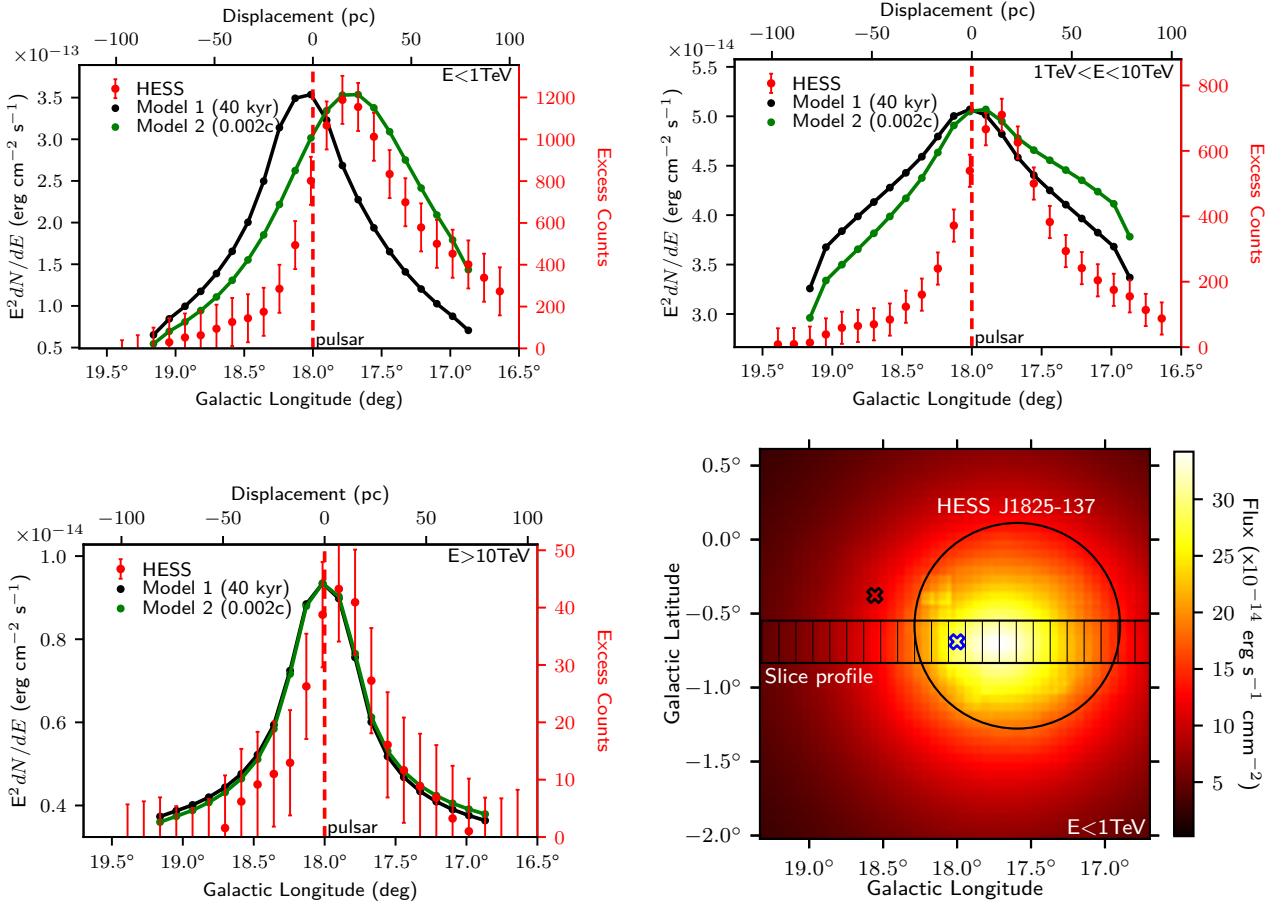


Figure 8. The energy flux along Galactic longitude for Model 1 (40 kyr, black) and Model 2 (0.002c, green) for energy bands $E < 1$ TeV (top-left), $1 < E < 10$ TeV (top-right) and $E > 10$ TeV (bottom-left) vs HESS excess counts (H.E.S.S. Collaboration et al. 2019). (bottom-right) Model 2 (40 kyr) gamma-ray morphology for energies < 1 TeV. The rectangular regions used to take the slice profile are indicated in black while the region used to extract the gamma-ray SED towards HESS J1825-137 is shown by the black circle. The positions of PSR J1826-1334 and PSR J1826-1256 are depicted by the blue and black empty crosses respectively. See Table 2 for model parameters.

3.5.2 Model 2 - Isotropic Diffusion + Advection

An advective component of $0.002c$ towards lower Galactic longitudes was included into Model 1 (40 kyr). The SED and X-ray surface brightness radial profile with an advective transportation component remains unchanged to Model 1 (40 kyr). Electrons rapidly escape the small ($r = 0.05^\circ$) X-ray region, hence the subsequent X-ray SED and surface brightness radial profile depends more on the injected electron spectrum rather than the method of transport. On the other hand, for both Model 1 (40 kyr) and Model 2 (0.002c), the majority of electrons remain within the large (0.7°) HESS region leaving the gamma-ray SED unchanged. However, the electrons in Model 2 have migrated further from the pulsar while remaining within the HESS region. Subsequently, the gamma-ray profile for Model 2 (0.002c) is flatter than Model 1 (40 kyr). Figure E3 shows the distance that electrons are transported before losing their energy to radiation. It can be seen that advection is the dominant particle transport method for electron energies less than 10 TeV, resulting in IC emission below 1 TeV. Diffusion is dominant for electrons above 10 TeV. However, these high-energy electrons do not travel far from their birthplace before losing their energy to radiative cooling.

At all energies, the gamma-ray energy flux along Galactic longitude for Model 1 (40 kyr) appears symmetric around the pulsar

position and shows no preferential direction of transportation. However, the HESS uncorrelated excess data indicates that electrons are preferentially transported to lower Galactic longitudes. With the addition of an advective flow of $0.002c$, the peak in the 40 kyr gamma-ray slices for photons less than 1 TeV is now offset from the pulsar and follows the shape of the uncorrelated excess. For the $1 \text{ TeV} < E < 10$ TeV energy band, both Model 1 and Model 2 show a flatter slice profile compared to HESS observations. Electrons resulting from this emission appear to be contained near their birthplace before escaping into the nebula. Our model assumed that the diffusion lies within the Kraichnan regime with the index being fixed at $\delta = 0.5$. The top-right panel of Figure 8 shows that the modelled gamma-ray slice morphology is broader than that observed by HESS, suggesting that electrons are constrained within the PWN. This suggests that the diffusion index inside the PWN may be somewhat less than the $\delta = 0.5$ value we assumed.

By assuming that diffusion was isotropic in Equation 2, any preferential direction for particle transport was a result of magnetic field irregularities and/or advective flow. The highly asymmetric morphology towards HESS J1825-137 could be explained if diffusion is anisotropic with preferential diffusion towards lower Galactic longitudes. However, an anisotropic diffusion model can be approximated by an isotropic diffusion + advection model (i.e. Model 2).

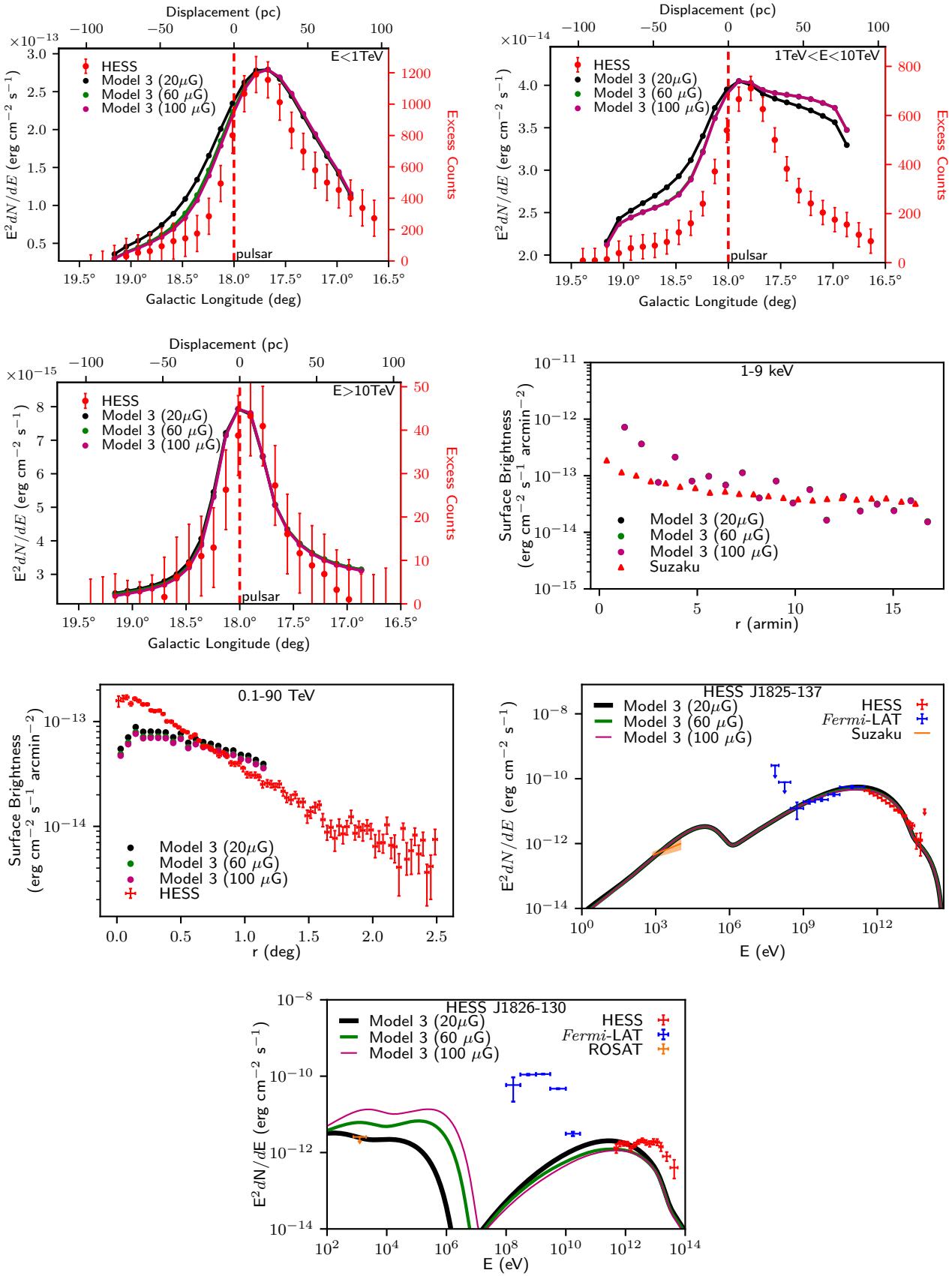


Figure 9. The energy flux along Galactic longitude profiles (top & top-middle left), surface brightness radial profiles (top-middle right & bottom-middle left) and SED towards HESS J1825-137 (bottom-middle right) and HESS J1826-130 (bottom) for Model 3 (20 μG , black), (60 μG , green) and (100 μG , purple) around HESS J1826-130. See Table 2 for model parameters.

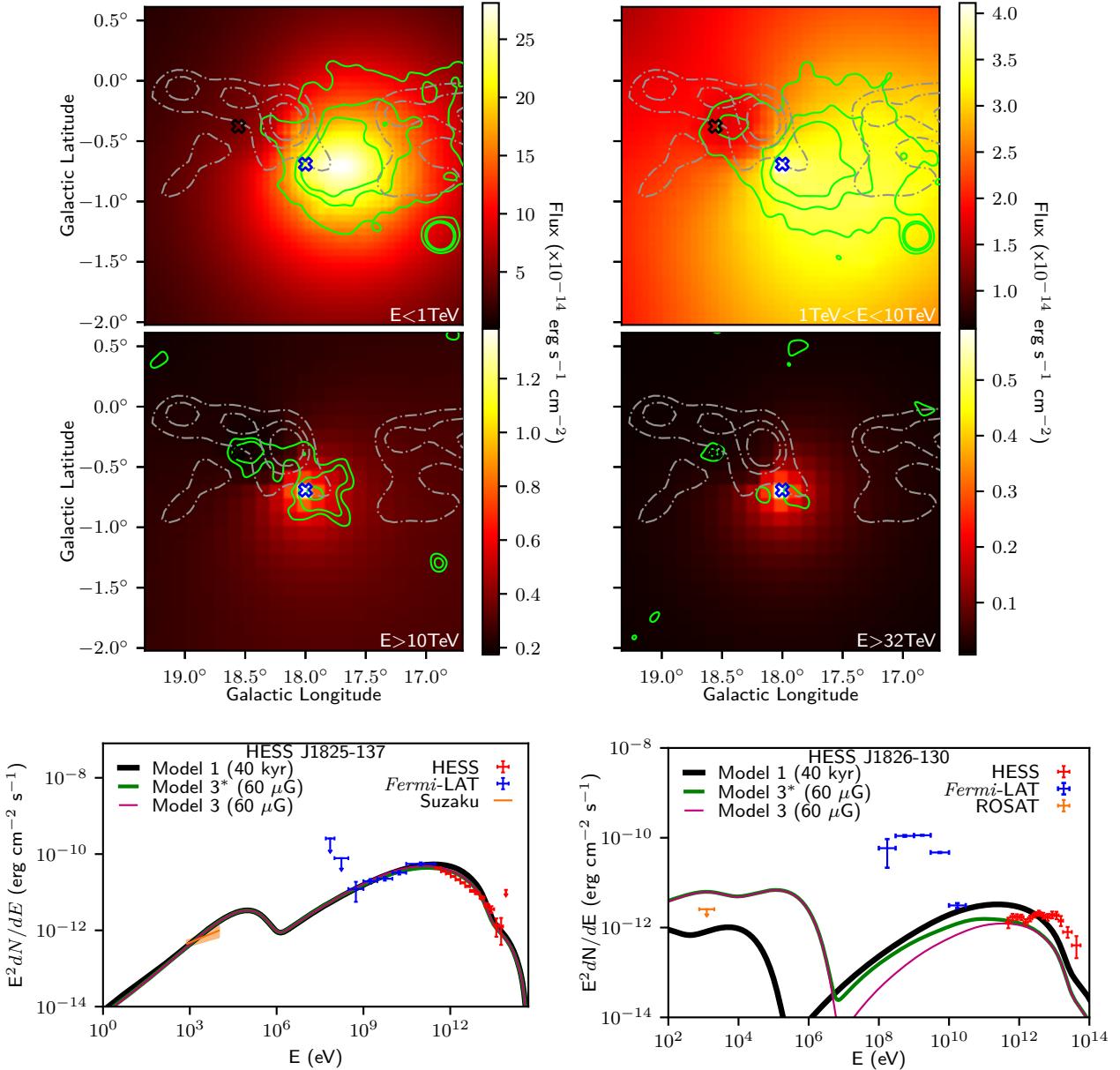


Figure 10. Comparison of the 40 kyr isotropic diffusion - Model 1 (40 kyr, black), Model 3* (Model 1 + 60 μG , green) and Model 3 (60 μG , purple). The morphology plots towards HESS J1825-137 for Model 3 (60 μG) are shown in the top and top-middle panels. See Table 2 for model parameters.

3.5.3 Model 3 - Isotropic Diffusion + Advection + Magnetic Field towards HESS J1826-130

Here, a spherical shell of increased magnetic field strength around HESS J1826-130 was considered to replicate the turbulent gas towards cloud R1 from Voisin et al. (2016).

The bottom-right panel of Figure 8 shows that cloud R1 lies within the area used to determine the gamma-ray SED of HESS J1825-137. The ratio of synchrotron to inverse Compton flux is proportional to the magnetic field (Aharonian et al. 1997). Hence, as the magnetic field around HESS J1826-130 increases, electrons lose more energy through synchrotron losses and the inverse Compton flux decreases at equivalent gamma-ray energies. This can be seen in the SED for HESS J1825-137 in the bottom-middle right panel of Figure 9. This has the effect of improving the match

to HESS observations between 1 – 10 TeV compared to Model 1 (40 kyr) as shown in the bottom-middle-left panel of Figure 10.

The gamma-ray energy flux along Galactic longitudes are shown in the top and top-middle panels of Figure 9. As the magnetic field around HESS J1826-130 increases, gamma-ray emission less than 1 < TeV and > 10 TeV remains unchanged at lower longitudes, with a decrease at higher longitudes. Between 1 TeV < E < 10 TeV, the gamma-ray slice profile drops off at a shallower rate compared to the HESS data at lower Galactic longitudes. However the gamma-ray emission at higher longitudes, representing the area towards HESS J1826-130, increases proportionally with the magnetic field.

Additionally, the bottom-left panel of Figure 9 indicates that increasing the magnetic field strength around HESS J1826-130 lowers the contamination of HESS J1826-130 by the PWN associated with HESS J1825-137 for energies < 2 TeV. Regions of high mag-

netic field strength experience a slower rate of diffusion (see Equation 6) and high synchrotron losses. Hence regions of high magnetic field tend to ‘block’ cosmic rays from passing through. The model implies that a minimum magnetic field strength of $60\,\mu\text{G}$ is required to successfully lower the contamination of HESS J1826-130 according to H.E.S.S. observations. The bottom-right panel of Figure 9 shows the multi-wavelength SED towards HESS J1826-130. An upper-limit to the X-ray emission towards HESS J1826-130 can be obtained using HEARSAC’s X-Ray background tool utilising ROSAT data (Sabol & Snowden 2019). The estimated synchrotron flux towards HESS J1826-130 combined with the ROSAT X-ray upper limit (obtained from the same region used to extract the SED of HESS J1826-130) implies a maximum magnetic field strength of $\approx 20\,\mu\text{G}$ around HESS J1826-130. This constraint violation suggests that the model is not fully encapsulating the transport of particles between PSR 1826-1334 and HESS 1826-130.

3.6 Future Work

Presently, our model only considers isotropic diffusion and does not account for diffusion parallel and perpendicular to the magnetic field (Drury 1983; Lazarian et al. 2023). The magnetic fields of PWNe are believed to be toroidal in nature (Schmidt et al. 1979; Kothes et al. 2006), hence diffusion is expected to be suppressed perpendicular to the magnetic axis of the pulsar. Additionally, the recent detection of TeV halos (Abeysekara et al. 2017) implies that the region surrounding the PWN experiences a higher diffusion suppression compared to the average Galactic diffusion coefficient (Schroer et al. 2023). While current models of particle transport suggest that advection dominates particle transport within the PWN and diffusion dominates at the edges, this could be described by two different regions of diffusion suppression. Model 2 and 3 in this study considered spatially-independent advection towards lower Galactic longitudes to explain the asymmetric TeV gamma-ray morphology towards HESS J1825-137. As a result, losses due to adiabatic expansion were neglected. Future modelling of PWNe, in particular HESS J1825-137, could investigate the effects of inhomogeneous anisotropic diffusion and an azimuthal and surface brightness dependent advective velocity on the gamma-ray morphology and SED. This can then be applied to model the formation of the TeV halo around the PWN (Principe et al. 2020).

Our model assumed a time-independent magnetic field with decreasing strength from the distance to the pulsar (see subsubsection 3.2.1). However, the average magnetic field of PWNe are expected to decrease over time from the conservation of magnetic energy density (e.g. Tanaka & Takahara (2010)) and the normalisation, B_0 , obtained from the modelling (see Table 2) can be considered as the time-averaged normalisation. Any future predictions of the formation of the TeV halo around HESS J1825-137 must consider time-dependency on the magnetic field.

The implementation of a time-dependent source position will not affect the gamma-ray morphology $\gtrsim 13\,\text{TeV}$ around PSR J1826-1334 due to high synchrotron losses. However, lower energy photons will appear to originate at a position offset to the current position of the pulsar as seen in Principe et al. (2020). This does not explain the extended TeV gamma-ray morphology towards lower Galactic longitudes as discussed in section 3, but could affect the modelled formation of the TeV halo. Future work could investigate the effects of an evolving source position on the gamma-ray surface brightness radial profile and morphology towards PWNe and other sources. For example, the application of our model to

SNRs would require cosmic rays to be injected by an expanding shell to model diffusive shock acceleration by the expanding SNR.

The model presented in this study is not limited to HESS J1825-137 and can be used to model the transport of cosmic-rays (electrons and protons) from other PWNe and cosmic ray sources.

4 SUMMARY

By modelling the transport of electrons across a 3D Cartesian grid of varying ISM density and magnetic field, we are able to reproduce the main characteristics of the multi-wavelength spectrum and morphology towards HESS J1825-137. Three different models were considered. Model 1 assumed a simple case of isotropic diffusion and radiative losses for the characteristic age of 21 kyr and an older age of 40 kyr as suggested by Van Etten & Romani (2011). Model 2 included an additional advective component to Model 1 (40 kyr) and Model 3 introduced turbulent ISM towards HESS J1826-130 to Model 2.

The best fit 21 kyr and 40 kyr Model 1 consisted of a pulsar injecting electrons into the surrounding medium with a spin-down conversion factor of 10.7 and 0.14% respectively, indicating that the true age of the system is older than the characteristic age of PSR J1826-1334. While able to reproduce the multi-wavelength SED, neither model was able to sufficiently reproduce the gamma-ray flux along Galactic longitude described in H.E.S.S. Collaboration et al. (2019) for photons with energies $1\,\text{TeV} < E < 10\,\text{TeV}$. However, the morphological profile could be matched for gamma-rays with energies $< 1\,\text{TeV}$ (with an offset of 0.3° towards higher Galactic longitudes compared to the HESS data) and energies $> 10\,\text{TeV}$.

Applying an advective bulk flow (with velocity $v = 0.002c$) of electrons towards lower Galactic longitudes did not alter the photon SED predicted by Model 1. By extracting the energy flux along Galactic longitude, we were able to compare the energy-dependent morphology towards HESS J1825-137. Model 2 (0.002c) was able to reproduce the energy flux for photons $< 1\,\text{TeV}$ and $> 10\,\text{TeV}$, however photons with energies $1 < E < 10\,\text{TeV}$ experience a shallower drop-off compared to the uncorrelated HESS excess slices as revealed by H.E.S.S. Collaboration et al. (2019). This suggests that the parent electrons are constrained within the PWN before escaping into the interstellar medium to form a TeV halo.

As the gamma-ray emission associated with PSR 1826-1334 cannot exceed the observed emission towards HESS J1826-130, HESS J1826-130 can be used to constrain the model. Model 1 and Model 2 were found to over-predict the SED of HESS J1826-130 for photons $< 1.5\,\text{TeV}$. By placing a shell of increased magnetic field strength of at least $60\,\mu\text{G}$ around HESS J1826-130, representing the turbulent gas between the two HESS sources (Voisin et al. 2016), the contamination was successfully lowered below the levels closer to those estimated by HESS. By combining the modelled synchrotron flux with the ROSAT X-ray upper limit towards HESS J1826-130, we were able to constrain the magnetic field shell to have maximum strength of $20\,\mu\text{G}$. This constraint violation suggests that further modelling of the turbulent gas is needed to fully disentangle the particle transport towards HESS J1825-137.

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DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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APPENDIX A: NON-THERMAL EMISSION

This section will provide an overview of leptonic interactions and the subsequent photon emission via synchrotron, Bremsstrahlung and inverse Compton Processes.

Synchrotron radiation occurs when an electron interacts with background magnetic fields. The resulting photon emission from a single electron with Lorentz factor γ with pitch angle α to the magnetic field B is given by:

$$\frac{dN}{dE} = \frac{\sqrt{3}e^3B}{mc^2} \frac{\nu}{\nu_c} \int_{\frac{\nu}{\nu_c}}^{\infty} K_{\frac{5}{3}}(x) dx , \quad (A1)$$

where e and m are the charge and mass of an electron respectively, $K_{\frac{5}{3}}$ is the modified Bessel Function, ν is the frequency of the gamma ray and ν_c is the critical frequency of the emission:

$$\nu_c = \gamma^2 \frac{3eB \sin \alpha}{4\pi mc} . \quad (A2)$$

The inverse Compton gamma-ray emission from an electron with energy E_e scattering off a target photon with energy in range $(\epsilon + d\epsilon)$ and number density $n(\epsilon)$ can be found using:

$$\frac{dN}{dE_\gamma} = \frac{3\sigma_T mc^3}{4\gamma} \int_{E_\gamma/4\gamma^2}^{E_\gamma} \frac{n(\epsilon) d\epsilon}{\epsilon} f(q, \Gamma) \quad (\text{A3a})$$

$$f(q, \Gamma) = 2q \ln q + (1+2q)(1-q) + \frac{1}{2} \frac{(\Gamma q)^2}{1+\Gamma q} (1-q) \quad (\text{A3b})$$

$$q = \frac{E_\gamma}{\Gamma(E_e - E_\gamma)}, \quad \Gamma = \frac{4\epsilon\gamma}{m_e c^2} \quad (\text{A3c})$$

where $\sigma_T = (3/8\pi)r_0^2$ is the Thompson cross section, r_0 is the classical electron radius and F_{KN} takes account the full Klein-Nishina cross section for inverse Compton scattering (Manolakou et al. 2007):

$$F_{\text{KN}} = \frac{1}{u_0} \int_0^\infty \ell(\gamma, \epsilon) u_\epsilon d\epsilon, \quad \ell(\gamma, \epsilon) = (1+4\gamma\epsilon)^{-\frac{3}{2}}. \quad (\text{A4})$$

For a Planck distribution of photon energies, F_{KN} can be approximated by:

$$F_{\text{KN}} = (1+4\gamma\epsilon_{\text{eff}})^{-3/2}, \quad \epsilon_{\text{eff}} = \frac{2.8kT}{m_e c^2}. \quad (\text{A5})$$

Finally, the photon emission from Bremsstrahlung interactions is given by:

$$\frac{dN}{dE_\gamma} = nc \int d\sigma(E_e, E_\gamma, Z) dE_e, \quad (\text{A6})$$

where Z is the atomic number of the target material and $d\sigma$ is the Bremsstrahlung differential cross section as defined in Blumenthal & Gould (1970).

The coefficients for leptonic losses in Equation 4 are:

- $b_s \equiv 1.292 \times 10^{-15} (B/10^3 \mu\text{G})^2 \text{ s}^{-1}$ is the synchrotron loss coefficient
- $b_c \equiv 1.491 \times 10^{-14} (n_H/1\text{cm}^{-3}) \text{ s}^{-1}$ is the Coulomb loss coefficient
- $b_b \equiv 1.37 \times 10^{-16} (n_H/1\text{cm}^{-3}) \text{ s}^{-1}$ is the Bremsstrahlung loss coefficient
- $b_{\text{IC}} \equiv 5.204 \times 10^{-20} (u_0/\text{eV cm}^{-3}) \text{ s}^{-1}$ is the IC loss coefficient with the energy density of photons given by u_0
- n_H is the density of the ambient hydrogen gas

The diffusion length for electrons (Atoyan et al. 1995):

$$R_{\text{diff}} = \sqrt{\frac{4D(\gamma)}{b_s \gamma(1-\delta)}} \left[1 - (1 - \gamma b_s t)^{1-\delta} \right]. \quad (\text{A7})$$

APPENDIX B: MAGNETIC FIELD DUE TO TURBULENT ISM GAS

The magnetic field due to the ISM gas with number density n is given through Crutcher's relation (Crutcher et al. 2010):

$$B_{\text{gas}}(n) = \begin{cases} B_{0,\text{gas}}, & n < 300 \text{ cm}^{-3} \\ B_{0,\text{gas}}(n/300 \text{ cm}^{-3})^\alpha, & n > 300 \text{ cm}^{-3} \end{cases}, \quad (\text{B1})$$

where $B_{0,\text{gas}} = 10 \mu\text{G}$ and $\alpha = 0.65$.

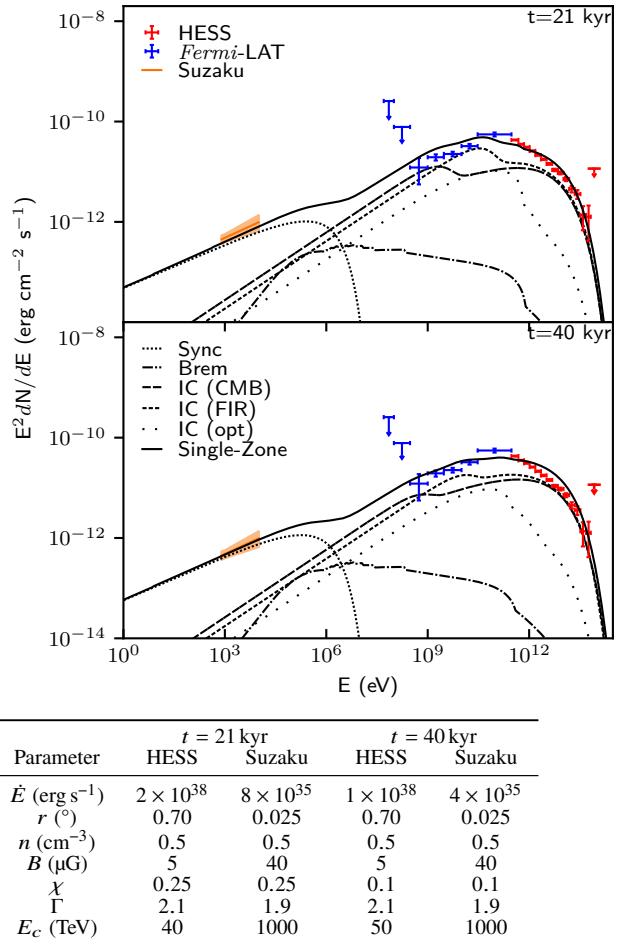


Figure C1. SED for leptonic interactions towards HESS J1825-137 using single-zone modelling for ages 21 kyr (top) and 40 kyr (bottom). The X-ray and gamma-ray spectra are fitted separately due to the different coverage areas of HESS and Suzaku. The orange line shows the Suzaku observations of X-rays between 1–9 keV towards the pulsar associated with HESS J1825-137 (Uchiyama et al. 2009). Blue data points represent the spectrum from the *Fermi*-LAT 4FGL source catalogue towards HESS J1825-137 while the red data shows the H.E.S.S. energy flux towards HESS J1825-137 (H.E.S.S. Collaboration et al. 2019). The corresponding model parameters are shown in the table.

APPENDIX C: SINGLE-ZONE MODELLING

Here we considered a ‘single-zone’ model, where electrons are injected into a spherical region of constant number density and magnetic field (Sano et al. 2017; Collins et al. 2021). The final electron number density is calculated by solving Eq. C7 from Collins et al. (2021) over the age of the system where electrons escape the region at a rate dependent on diffusion. The multi-wavelength SED from this region is then calculated. While unable to encapsulate the complexity towards HESS J1825-137, a general insight of the system was gained before more detailed modelling of the morphology and time evolution.

C1 Method

The X-ray and gamma-ray emission was modelled separately using two spheres with radii $r_{\text{X-ray}}$ and r_{gamma} (see Figure 3) following the extraction regions used by Uchiyama et al. (2009) and H.E.S.S.

Collaboration et al. (2019). Electrons were injected into the spherical region at a constant rate \dot{E} and followed a power-law spectrum with an exponential cutoff: $\frac{dN}{dE} \propto E^{-\Gamma} \cdot \exp(E/E_c)$, where Γ is the spectral index and E_c is the cutoff energy. Two different ages were modelled, 21 kyr based on the characteristic age of the pulsar and 40 kyr based on modelling conducted by (Van Etten & Romani 2011).

The HESS region adopted a uniform magnetic field of 5 μG as suggested by Principe et al. (2020) from comparing the estimated synchrotron emission to the Suzaku X-ray emission. Subsequently, it was assumed that the smaller X-ray region has a higher magnetic field strength than the HESS region due to the proximity of the pulsar and was left as a free parameter. Both the gamma-ray and X-ray region assumed a constant ISM density of 0.5 cm^{-3} .

The fits to the SED towards HESS J1825-137 as well as the modelled parameters can be seen in Figure C1.

C2 Discussion

Figure C1 shows the modelled SED with corresponding parameters to the gamma and X-ray spectra towards HESS J1825-137. The majority of gamma rays in this model are predicted to come from inverse Compton interactions from the infrared and CMB photon fields. An electron injection luminosity of $2 \times 10^{38} \text{ erg s}^{-1}$ and $1 \times 10^{38} \text{ erg s}^{-1}$ is needed to match the gamma-ray spectra at ages 21 and 40 kyr respectively. This is a factor ten times greater than the spin-down power of PSR 1826-1334 ($\dot{E} = 2.8 \times 10^{36} \text{ erg s}^{-1}$). The single-zone model assumes a time-independent injection luminosity, whereas the spin-down power of the pulsar decreases over time. The spin-down power of PSR 1826-1334 could have been as high as $10^{39} \text{ erg s}^{-1}$ at a pulsar age of 1 kyr (see subsection 3.1). Therefore the modeled injection luminosities represents the average electron injection luminosity over the age of the pulsar.

The X-ray emission towards PSR 1826-1334 can be predicted with an injection luminosity of $8 \times 10^{35} \text{ erg s}^{-1}$ and $4 \times 10^{35} \text{ erg s}^{-1}$ for the 21 and 40 kyr model respectively. The single-zone model can reasonably predict both the X-ray and gamma-ray SED, yet the X-ray and gamma-ray photon models require different injection spectra for both ages of the system. The single-zone model assumes constant density and magnetic field strength across the region of interest. However, the magnetic field structures towards PWNe have been suggested to be toroidal in nature but the viewing angle results in magnetic fields appearing radially dependent or tangled (Kothes et al. 2006). If the dense clouds towards HESS J1825-130, as seen in Figure 3, lie at the same distance as the pulsar, diffusion will be suppressed towards this region with electrons losing their energy to bremsstrahlung losses. As previously mentioned, the spin-down power of the pulsar decreases over time which has an effect on the injection luminosity of electrons in the ISM. While the single-zone model is able to predict the X-ray and gamma-ray SED towards HESS J1825-137, it is unable to encapsulate the complexity of the PWN.

APPENDIX D: SYSTEMATIC VARIATION OF MULTIZONE PARAMETERS

Figure D1 and Figure D2 shows the 10% and 20% systematic variation of the free parameters β , B_0 , χ , Γ , E_c and η for the 21 kyr and 40 kyr models. These figures show that the spectral index of injection electrons, Γ , has the largest systematic variation, where the X-ray SED and surface brightness radial profiles show more

sensitivity than the gamma-ray emission. This is a result of the smaller region used to extract the X-ray and SED (see Figure 3 and Figure 4).

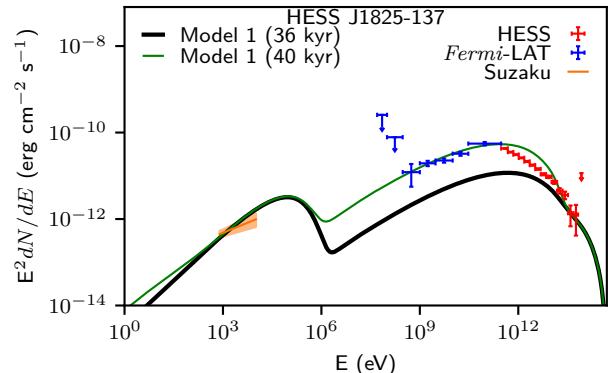
The modelled X-ray surface brightness radial profiles for the 21 and 40 kyr models are steeper than observations, indicating that the model over-predicts the synchrotron emission closer to PSR 1826-1334. This may be corrected by decreasing the rate at which the magnetic field drops off with distance from the pulsar (β), allowing electrons to escape the PWN at a faster rate. The outer edges of the PWN experiences greater synchrotron losses at the cost of TeV gamma-ray emission from IC interactions, flattening out the gamma-ray surface brightness radial profile. This is demonstrated in the 10 and 20% variation of β shown in the top row of Figure D1 and Figure D2. Alternatively, decreasing the overall magnetic field strength, B_0 , decreases synchrotron losses towards HESS J1825-137 at the cost of increasing the gamma-ray to X-ray flux ratio. With flux being dependent on the observational area, any changes to the gamma-ray and X-ray ratio will be more prominent in the X-ray SED as shown in Figure D1 and Figure D2.

To better fit the X-ray surface brightness radial profile, the diffusion suppression coefficient, χ , towards HESS J1825-137 could be increased to allow electrons to escape further from the pulsar before losing their energy to synchrotron radiation. High-energy electrons rapidly lose their energy to radiative losses and remain close to the pulsar, resulting in a shallower gamma-ray surface brightness radial profile as shown in Figure D1 and Figure D2. As the region used to extract the X-ray data is small ($< 2pc$) compared to the HESS region ($\approx 49 \text{ pc}$), electrons quickly escape the X-ray region while remaining in the HESS region. Thus the X-ray SED far more sensitive to the value of χ than the gamma-ray SED.

Both the surface brightness radial profiles and SED are very sensitive to the injected electron spectral index, Γ , as seen in Figure D1 and Figure D2. If β , B_0 or χ was altered to fit the observed Suzaku X-ray surface brightness radial profile, the predicted SED from the model will no longer fit to the data. In turn, the spectral index can be modified to refit the modelled SED. Consequently, the X-ray surface brightness radial profile will no longer match the Suzaku observations.

The cutoff energies for the 21 kyr and 40 kyr models are 45 TeV and 500 TeV respectively. As the cutoff energy for an exponential cutoff power-law increases, the energy spectra starts to follow a power-law. Hence, the systematic variation of E_c is less apparent for 40 kyr than 21 kyr as seen in the fourth row of Figure D1 and Figure D2 respectively.

APPENDIX E: ADDITIONAL FIGURES



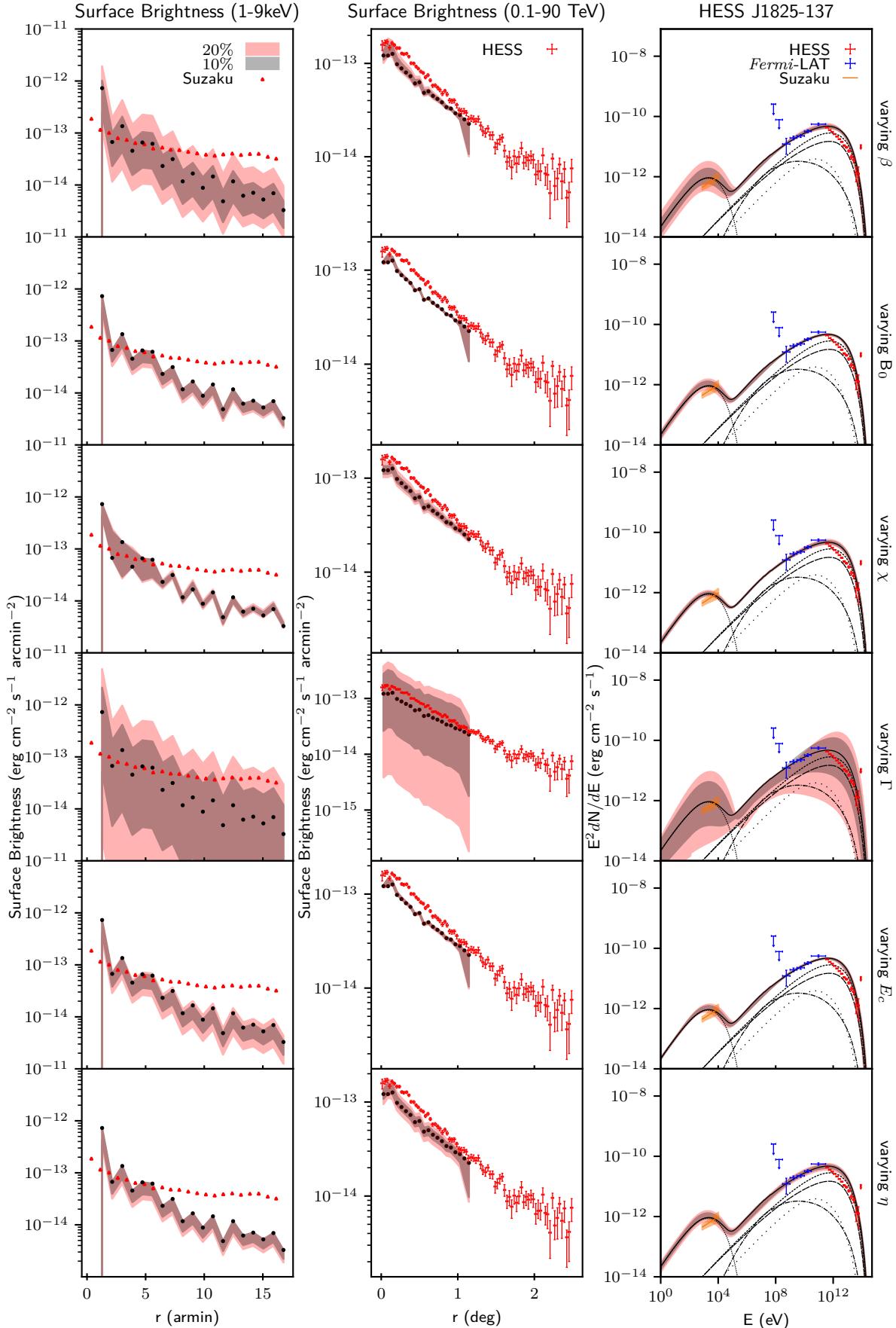


Figure D1. Model 1 (21 kyr) as in Figure 4 but with 10% (grey shaded band) and 20% (pink shaded band) variation in parameters.

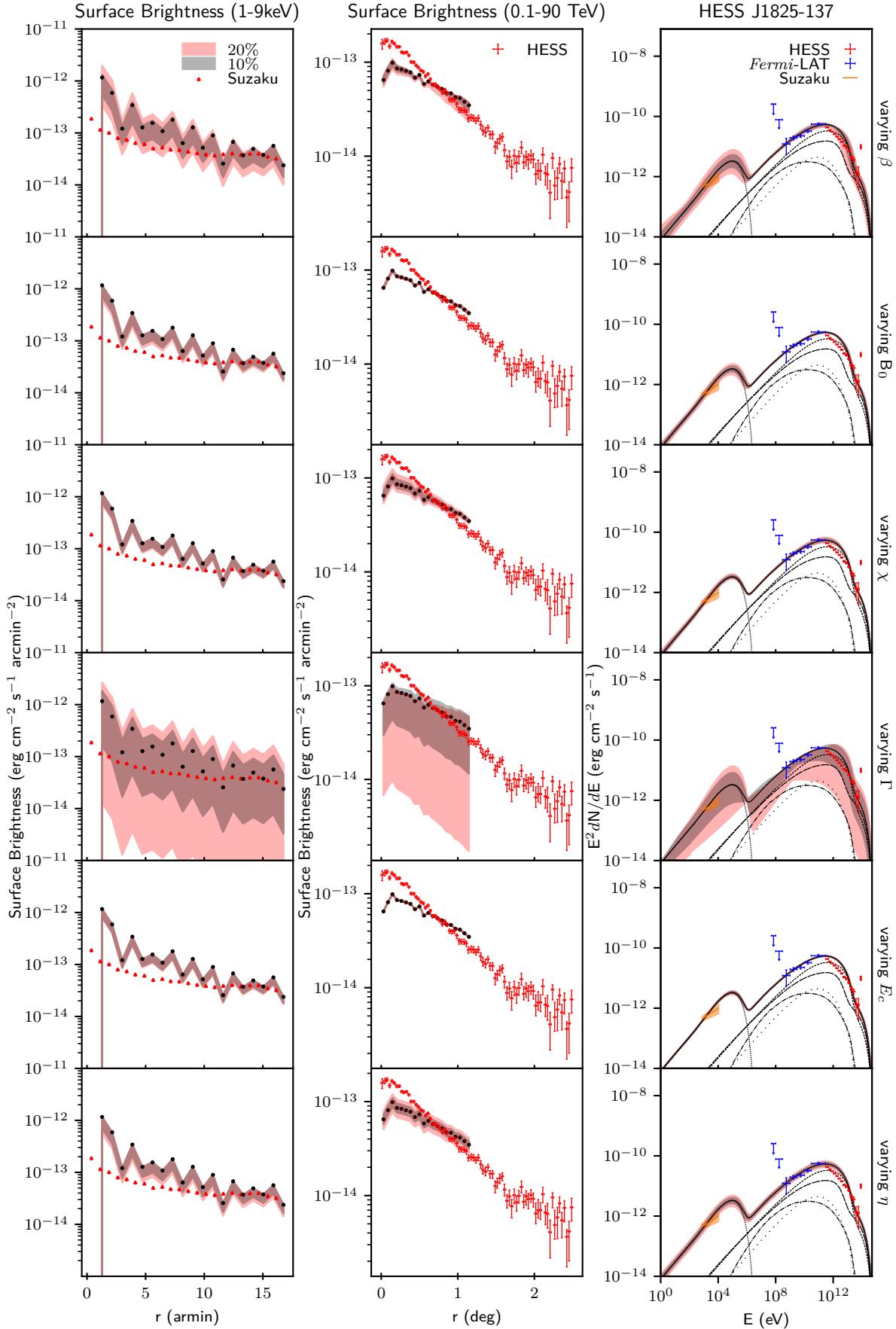


Figure D2. Model 2 (40 kyr) as in Figure 5 but with 10% (grey shaded band) and 20% (pink shaded band) variation in parameters.

Figure E1: SED towards HESS J1825-137 for Model 1 (36 kyr, green) shown vs Model 1 (40 kyr). A ‘bump’ is present in the SED above 10 TeV for Model 1 (40 kyr) where radiative losses are balanced by the electron injection luminosity. The 36 kyr model has the same parameters as the 40 kyr model (see Table 2).

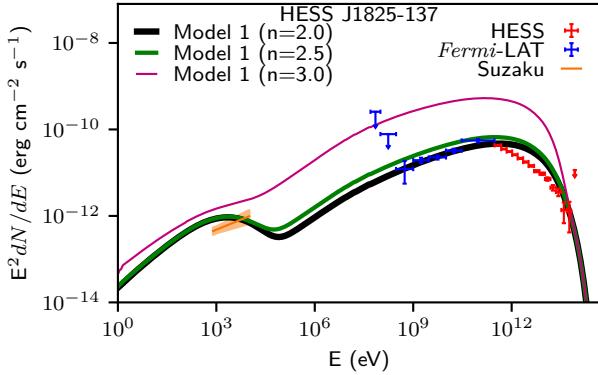


Figure E2: SED towards HESS J1825-137 for Model 1 (21 kyr, $n = 2$, black, see Table 2) vs Model 1 (21 kyr, $n = 2.5$, green) and Model 1 (21 kyr, $n = 3$, purple).

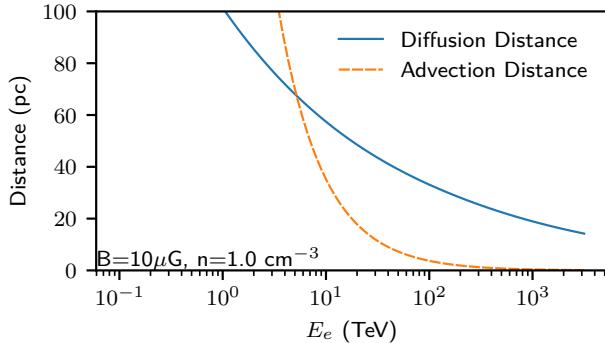


Figure E3: The distance that electrons are transported before losing their energy through radiative cooling (synchrotron, inverse Compton and Bremsstrahlung) assuming purely diffusive (solid line) or advective transport (dashed line).

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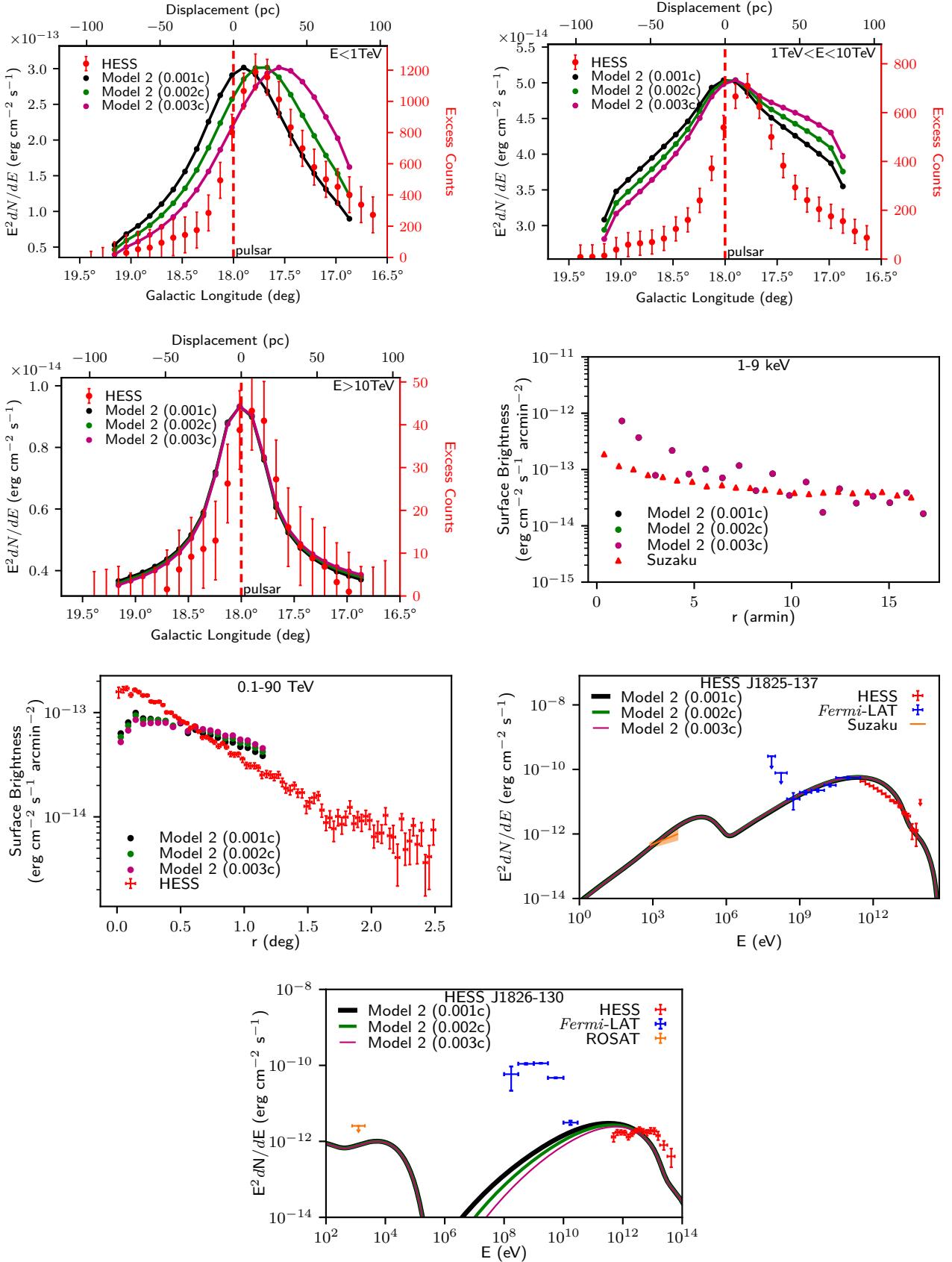


Figure E4. The slice profiles (top & top-middle left), surface brightness radial profiles (top-middle right & bottom-middle left) and SED towards HESS J1825-137 (bottom-middle right) and HESS J1826-130 (bottom) for Model 2 0.001c (black), 0.002c (green) and 0.003c (purple). See Table 2 for model parameters.