CSE 015: Discrete Mathematics
Fall 2019
Homework #4
Solution

1. **Question 1:**
   (a) The domain is the set of nonnegative integers: $\{0, 1, 2, \ldots\}$. The range is the set of digits (0 through 9).
   (b) The domain is the set of positive integers: $\{1, 2, \ldots\}$. The range is the set of integers greater than 1: $\{2, 3, \ldots\}$.
   (c) The domain is the set of bit strings (of any length). The range is the set of nonnegative integers: $\{0, 1, 2, \ldots\}$. That is, the number of 1s in a bit string can be $0, 1, 2, \ldots$.
   (d) The domain is the set of all bit strings (of any length). The range is the set of nonnegative integers (we assume a bit string can have length 0, that is, it is the empty string) : $\{0, 1, 2, \ldots\}$. (Alternate solution: Also accept the range as the set of positive integers (here we assume a bit string has at least one bit): $\{1, 2, \ldots\}$).

2. **Question 2:**
   (a) The function is onto because for any $p \in \mathbb{Z}$, you can find an $m \in \mathbb{Z}$ and an $n \in \mathbb{Z}$ such that $2m - n = p$. For example, if you choose $m = 0$ and $n = -p$ then $f(m, n) = p$.
   (b) The function is not onto because there exists at least one $p \in \mathbb{Z}$ such for all $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$, $m^2 - n^2 \neq p$. One such example is $p = 2$. To see this, if $m^2 - n^2 = (m - n)(m + n) = 2$, then m and n must have same parity (both even or both odd). In either case, both $m - n$ and $m + n$ are then even, so $(m - n)(m + n)$ is divisible by 4 and hence cannot equal 2.
   (c) The function is onto because for any $p \in \mathbb{Z}$, you can find an $m \in \mathbb{Z}$ and an $n \in \mathbb{Z}$ such that $m + n + 1 = p$.
   (d) The function is onto because for any $p \in \mathbb{Z}$, you can find an $m \in \mathbb{Z}$ and an $n \in \mathbb{Z}$ such that $|m| - |n| = p$. For example, if $p < 0$ then let $m = 0$ and $n = p$ and then $f(m, n) = p$; if $p > 0$ then let $m = p$ and $n = 0$ and then $f(m, n) = p$; and if $p = 0$ then let $m = 0$ and $n = 0$ and then $f(m, n) = p$.
   (e) The function is not onto because there exists at least one $p \in \mathbb{Z}$ such for all $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$, $m^2 - 4 \neq p$. One such example is $p = -5$. Since $m^2 \geq 0$ for any $m \in \mathbb{Z}$ then $m^2 - 4 \geq -4$.

3. **Question 3:**
   (a) The function is one-to-one if each student has a unique mobile phone number. In other words, if no two students share the same mobile phone number. This is usually the case in practice since people usually don't share phones and each mobile phone has a unique phone number. Of course, if two or more students share a single phone then the function would not be one-to-one.
   (b) The function is one-to-one if each student has a unique student identification number. In other words, if no two students share the same student identification number. This is usually the case in practice since identification numbers are usually used to identify a unique student.

(c) The function is one-to-one if the students receive different grades. That is, no two student receive the same grade. This is unlikely in practice since there are only a limited number of grades (A+, A, A-, ...).

(d) This function is one-to-one if each student comes from a different hometown. That is, not two students come from the same town. This is unlikely in practice unless the class is small.

Question 4

(a) The function is a bijection because it is one-to-one and onto. It is one-to-one because if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$. It is onto because for every $y \in \mathbb{R}$ there exists an $x \in \mathbb{R}$ such that $f(x) = y$. (You can just set $x = (y - 4)/(-3)$).

(b) The function is not a bijection because it is not onto. It is not onto because there exists at least one $y \in \mathbb{R}$ such that you cannot find an $x \in \mathbb{R}$ where $f(x) = y$. One such example is $y = 8$. (The function is also not one-to-one because $f(1) = 4 = f(-1)$.)

(c) This function is not a bijection because it is not onto. It is not onto because there exists at least one $y \in \mathbb{R}$ such that you cannot find an $x \in \mathbb{R}$ where $f(x) = y$. In particular, let $y = 1$ then there is no $x \in \mathbb{R}$ such that $f(x) = 1$.

(d) This function is a bijection because it is both one-to-one and onto. It is one-to-one because if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$. It is onto because for every $y \in \mathbb{R}$ there exists an $x \in \mathbb{R}$ such that $f(x) = y$. You can just set $x = \sqrt[3]{y}$. Note this works even if y is negative. For example, $(\sqrt[3]{-1})^3 = -1$. Another way to see that $f(x) = x^3$ is one-to-one and onto is to plot the function (see below). It is one-to-one because different values of x are mapped to different values of $f(x)$. It is onto because each for each y value, there is an x value such that $f(x) = y$.