THE UNIVERSITY of EDINBURGH

Multitrate Training of Neural Networks

arXiv preprint: 2106.10771

Tiffany Vlaar and Benedict Leimkuhler

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Inspiration

- Neuroscience: synapses have dynamics at different time-scales.
- Fast/slow weights in machine learning: Hinton and Plaut (1987) set each connection to have fast changing weight *[temporary memory]* and slowly changing weight *[long-term knowledge]*.

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  - Layer-wise adaptive learning rates [You et al., 2017].

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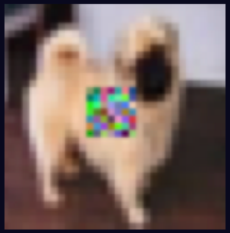
  - Use partitioned integrators for neural network training [Leimkuhler, Matthews, TV, 2019].
- Molecular dynamics: fast dynamics cheap to compute [e.g., r-RESPA Tuckerman et al. 1991, 1992] -can we obtain computational speed-up in neural network training applications?

Latent multiple time scales in deep learning

WideResNet-16 trained on patch-augmented [Li, Wei & Ma, NeurIPS 2019] CIFAR-10 data:

20% is patch-free, 16% has only the patch, and the rest has both data and patch.



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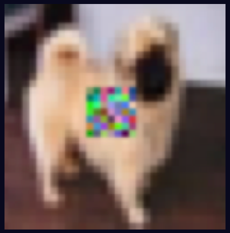

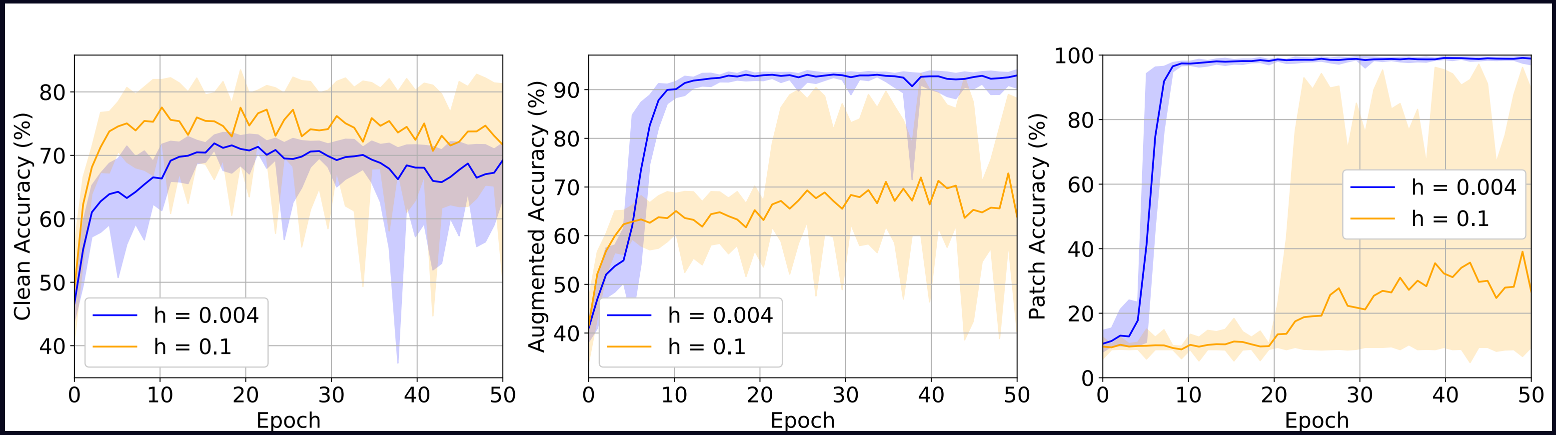

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- Small learning rate (blue) memorizes patch.

- Large learning rate (orange) gives higher accuracy on clean data.

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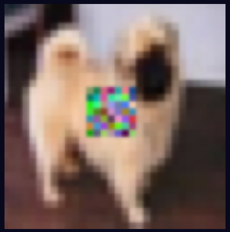

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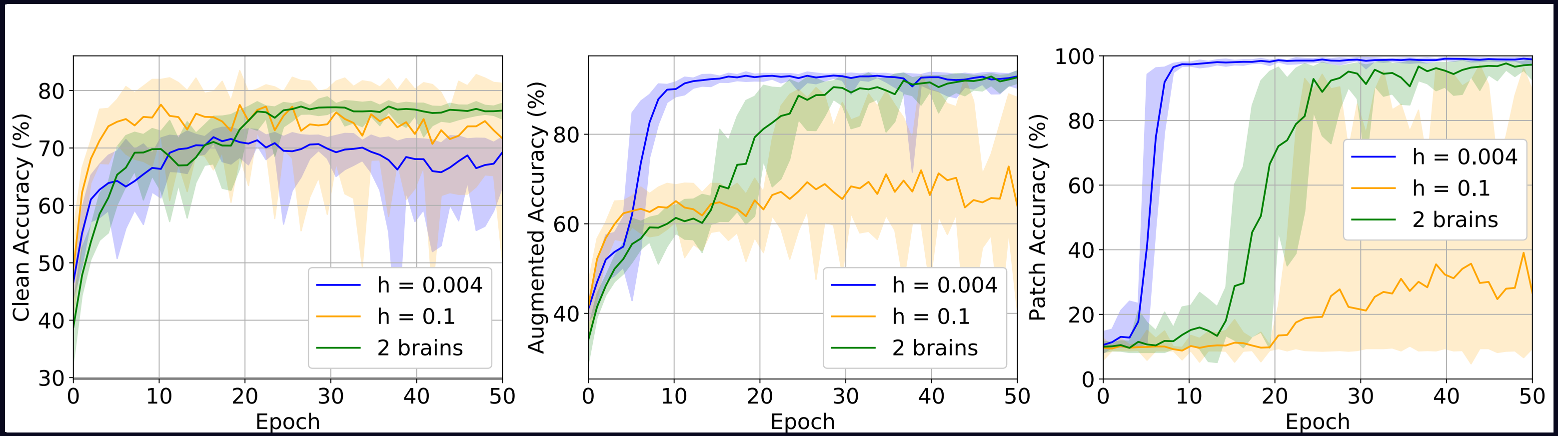

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- Our multirate approach (green) can perform well on both.

Neural network training

Basis: system of ODEs: $d\theta = G(\theta)\ dt$, with neural network parameters $\theta \in \mathbb{R}^n$

and G the negative gradient of the loss of entire dataset.

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SGLD: additionally drive with constant variance additive noise [Welling & Teh, ICML 2011].
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Underdamped Langevin Dynamics framework:

$$d\theta = p\ dt$$

$$dp = \tilde{G}(\theta)\ dt - \gamma p\ dt + \sqrt{2\gamma\tau}\ dW$$

Hyperparameter τ controls: pure optimization ($\tau = 0$) $\leftrightarrow$ pure sampling ($\tau = 1$).

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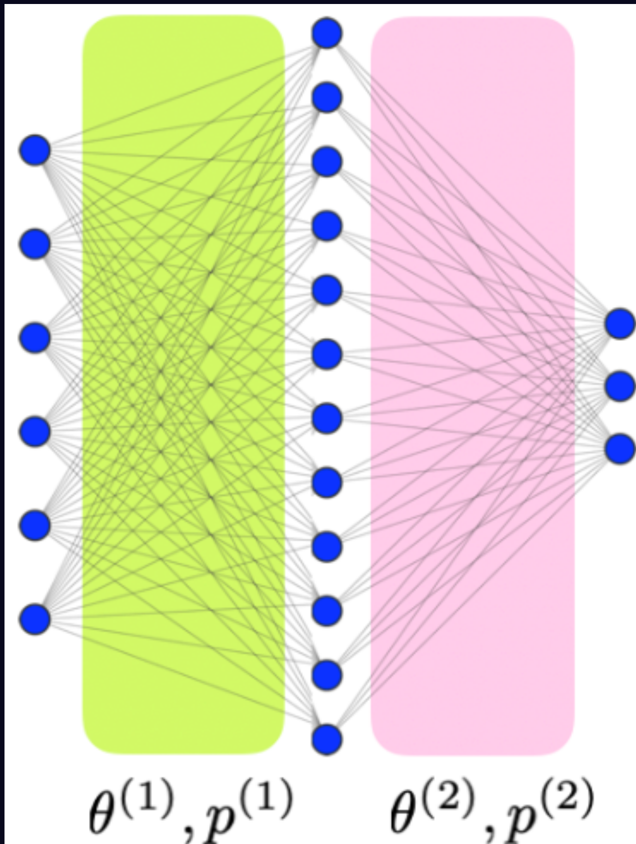

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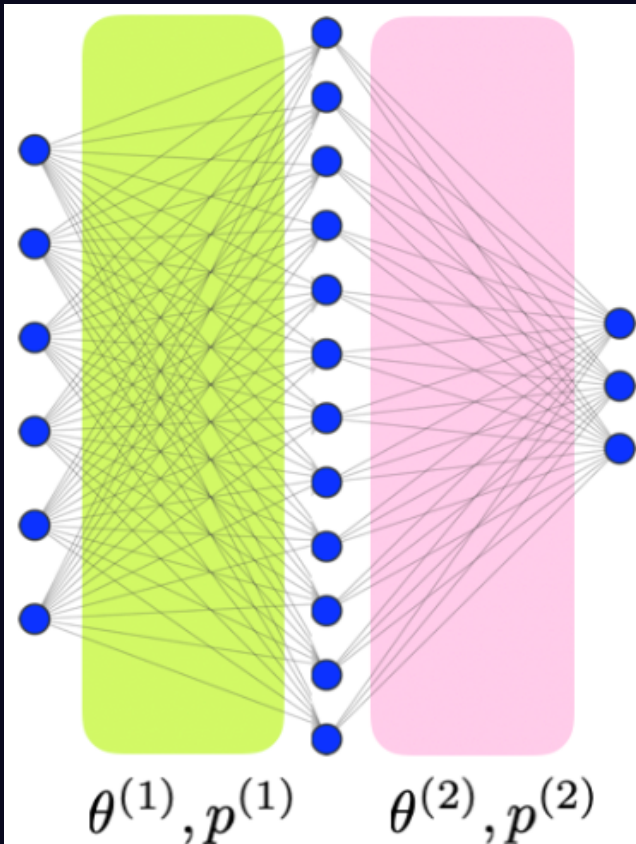

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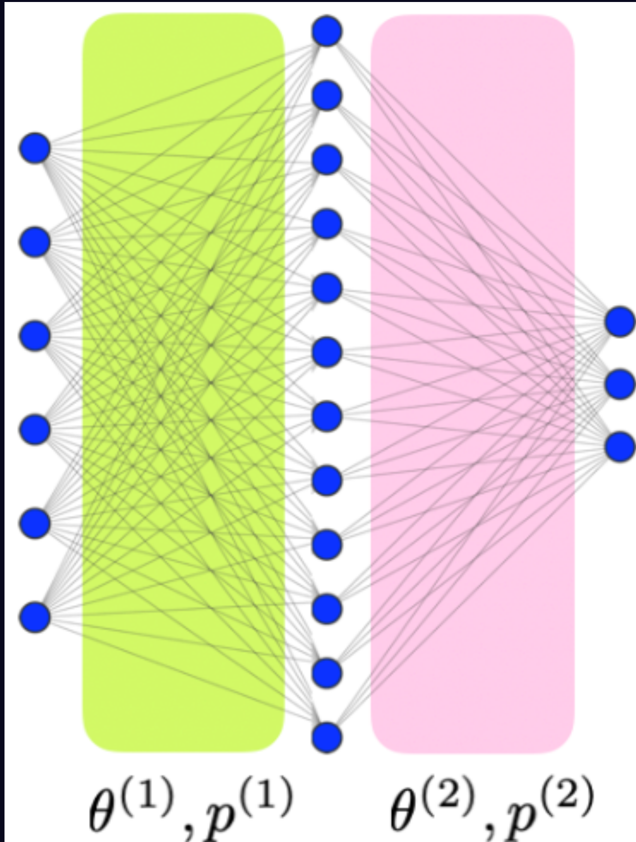
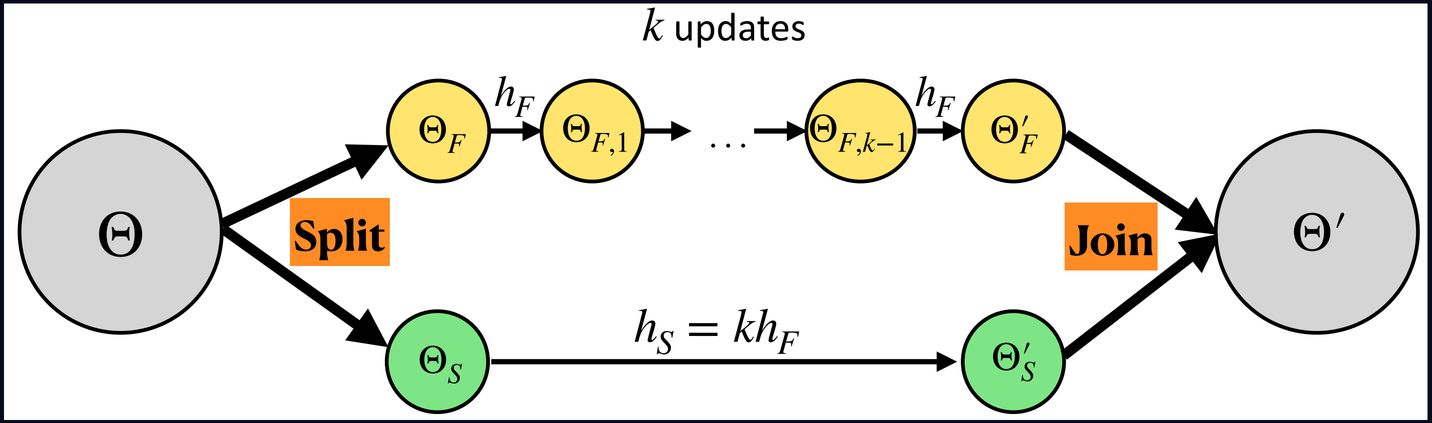


Multirate methods

Two time-scales example:

Partition model (+ accompanying momentum) parameters $\Theta = (\Theta_F, \Theta_S)$.



Fast components Θ_F are updated every step with step size h_F
Slow components Θ_S are updated every k steps with step size $h_S = kh_F$

Multirate methods for neural network training

We consider multirate methods both *within* and *across* networks:

Within: Partition network parameters into fast and slow parts.

Across: Train multiple copies of a network on multiple time scales simultaneously.

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Across: Train multiple copies of a network on multiple time scales simultaneously.

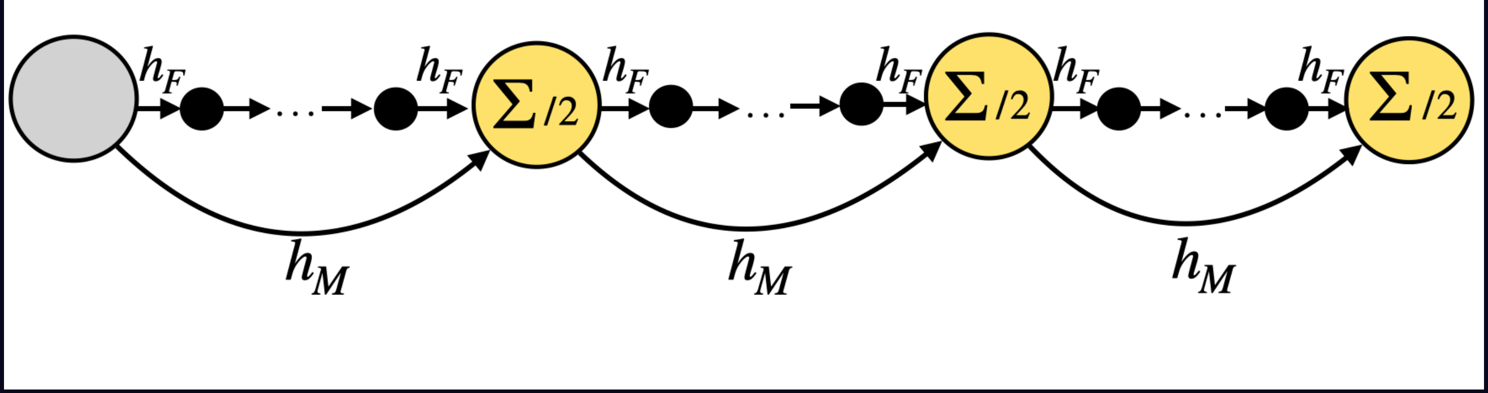
Act as an add-on to existing optimization schemes: they can be combined with any desired base algorithm.

Across networks: "multi-brain"

Train independent copies (brains) of the same network using different learning rates.
Merge parameters whenever two or more copies reach the same time threshold.

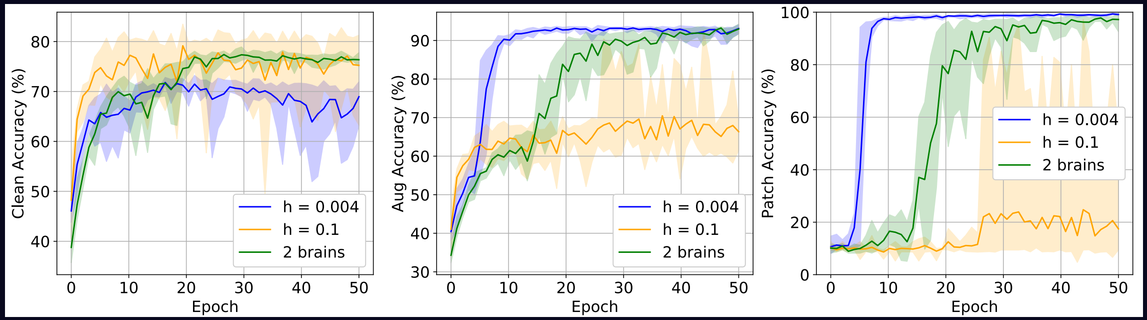
Can be used with optimizer of your choice.

Example:



Two brains with step sizes $h_M = kh_F$, $k \in \mathbb{Z}_{>0}$

Across networks:



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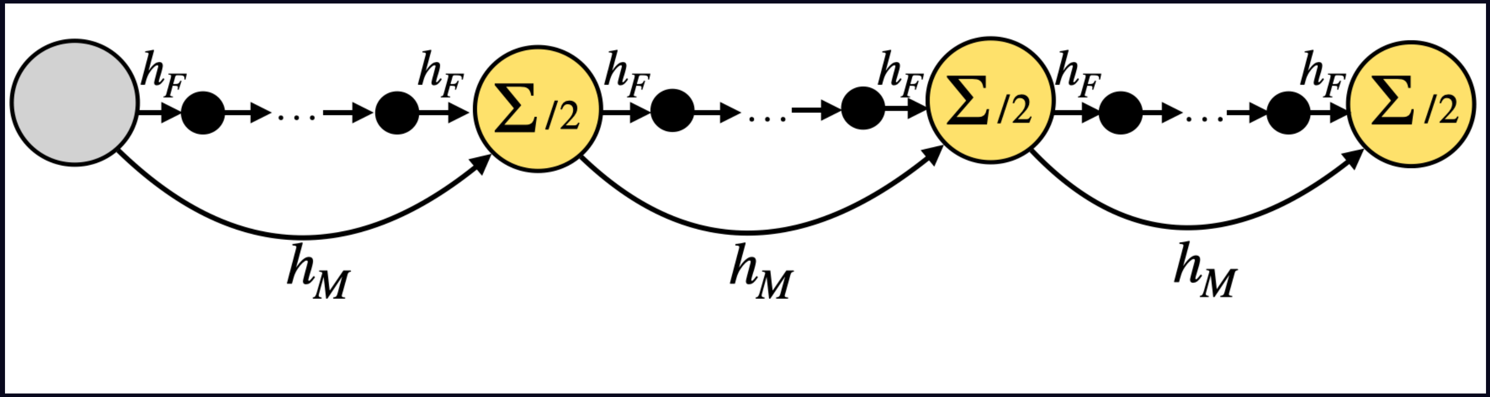

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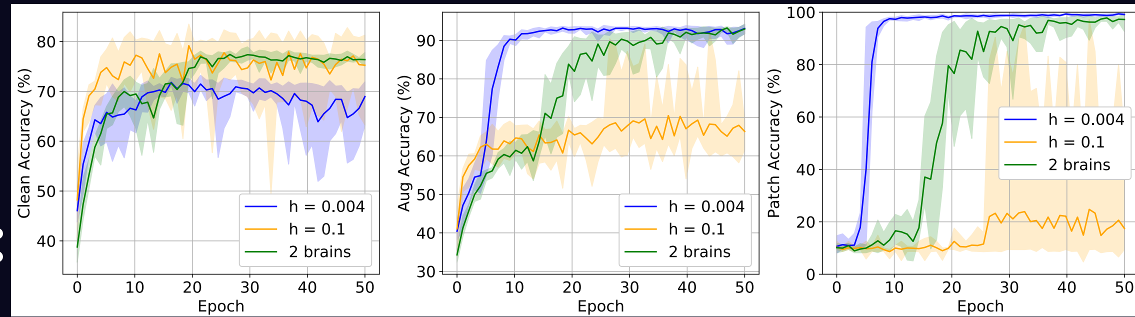

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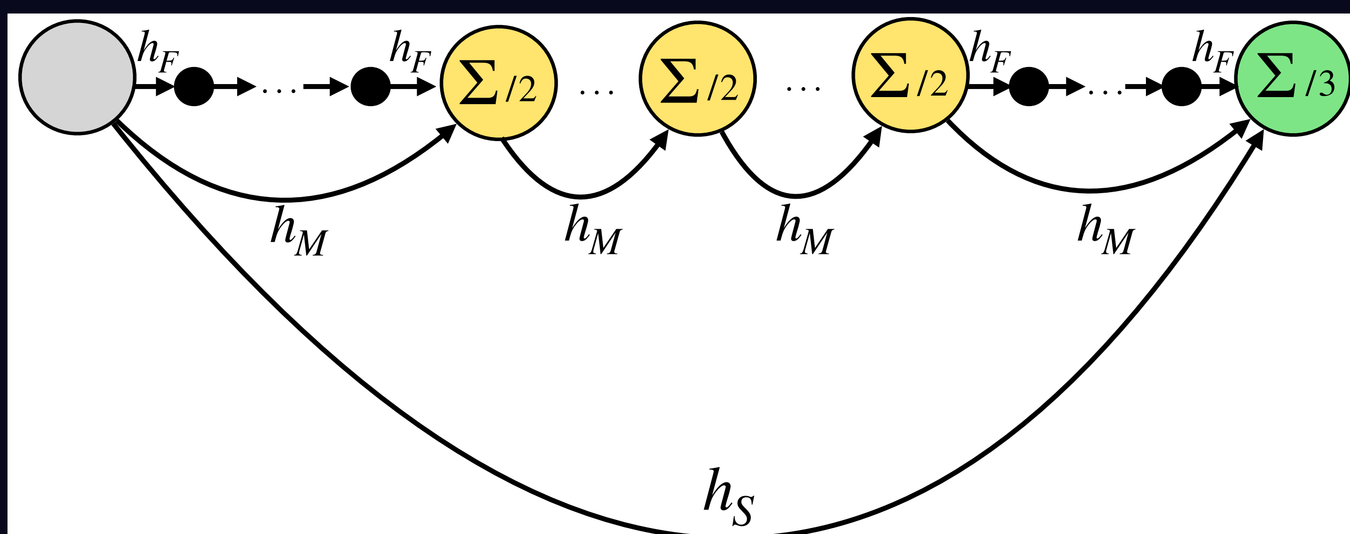
Example: Training with *three* brains with stepsizes $h_S = k_2 h_M = k_1 h_F$



Within networks: partition-based multirate approach

Separate neural network parameters into different parts.
You have a choice!

Examples: layer-wise, weight vs. biases, or random subgroups.

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If base algorithm SGD (as used in PyTorch code):

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\begin{aligned}
&p_S := \mu p_S + \nabla_{\theta_S} \mathcal{L}(\theta_S, \theta_F) \\
&\theta_S := \theta_S - h p_S \\
&\textbf{for } i = 1, 2, \ldots, k \textbf{ do} \\
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with momentum p and loss $\mathscr{L}$

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Basics of transfer learning:

- Start with pre-trained model on large datasets, e.g., ImageNet.
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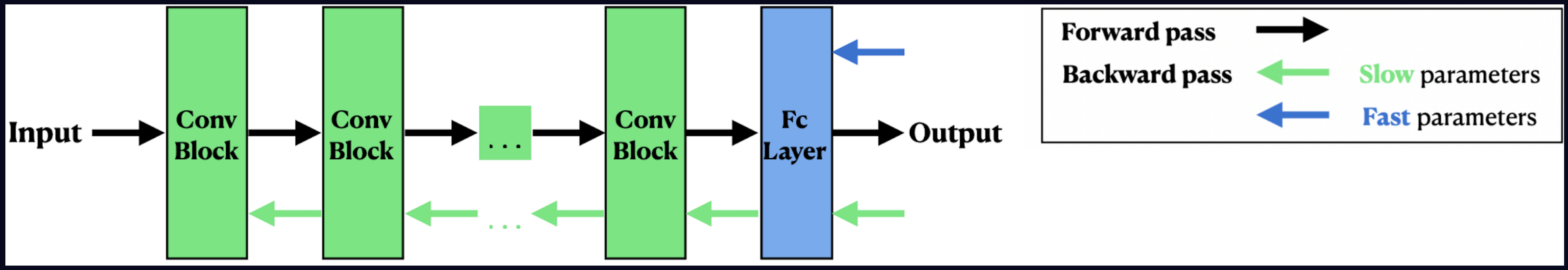
Example for a ResNet architecture:



For a ResNet-34 architecture fast parameters are only 0.024% of total.

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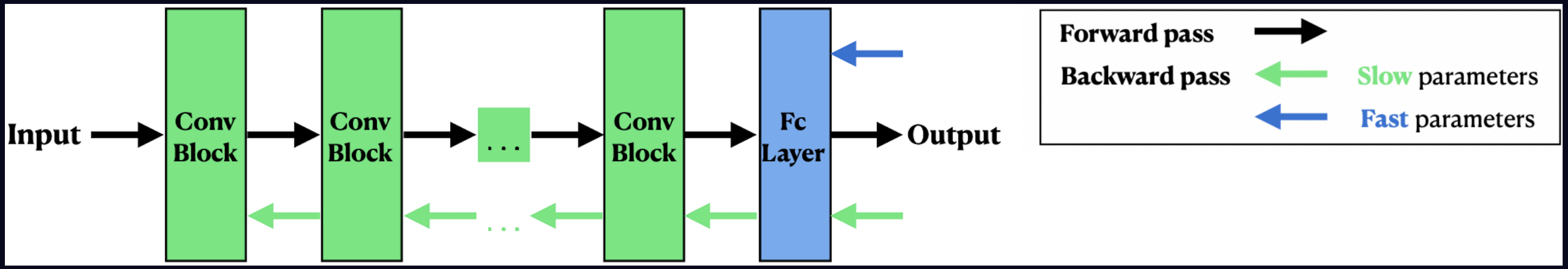
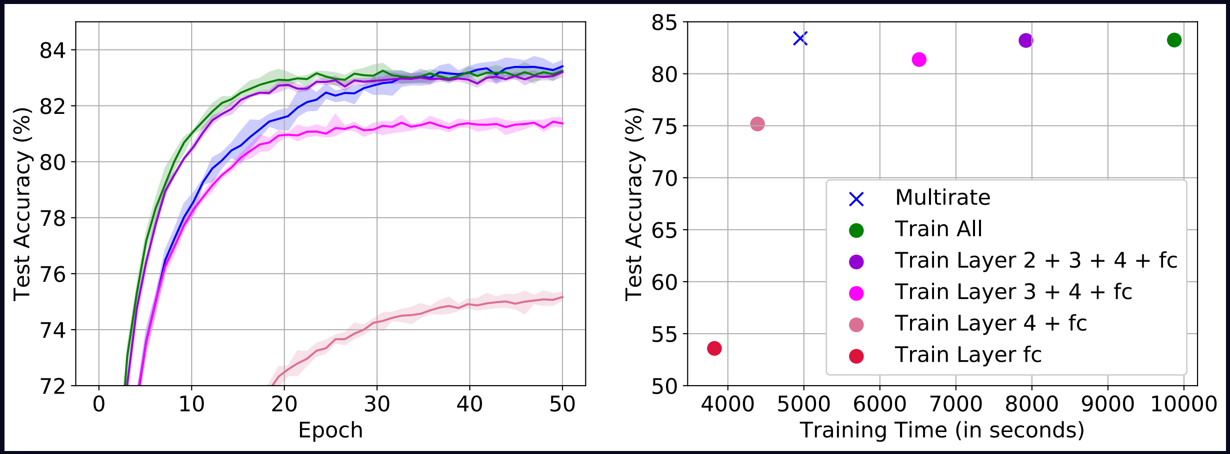


ResNet-50, CIFAR-100 data



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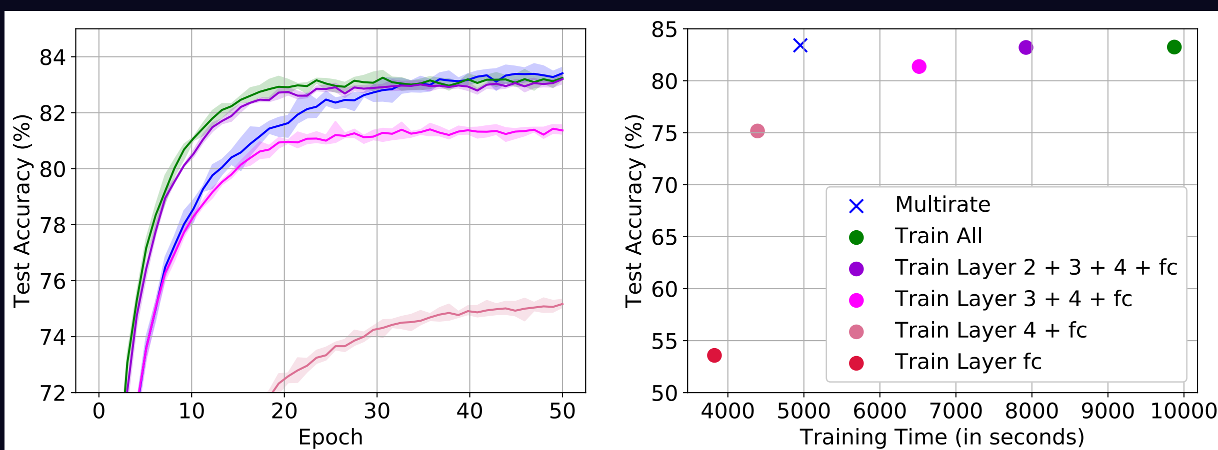
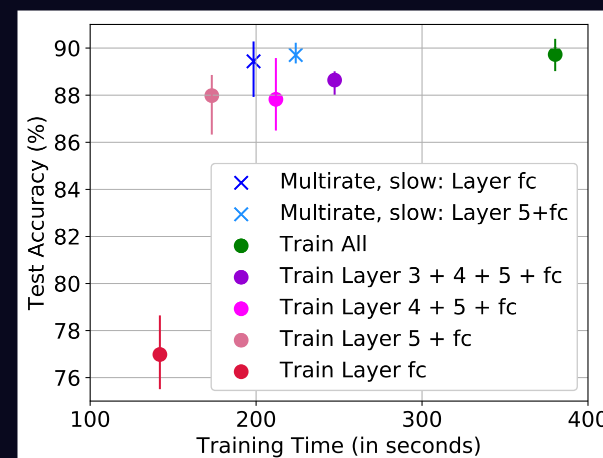


DistilBERT, SST-2 data



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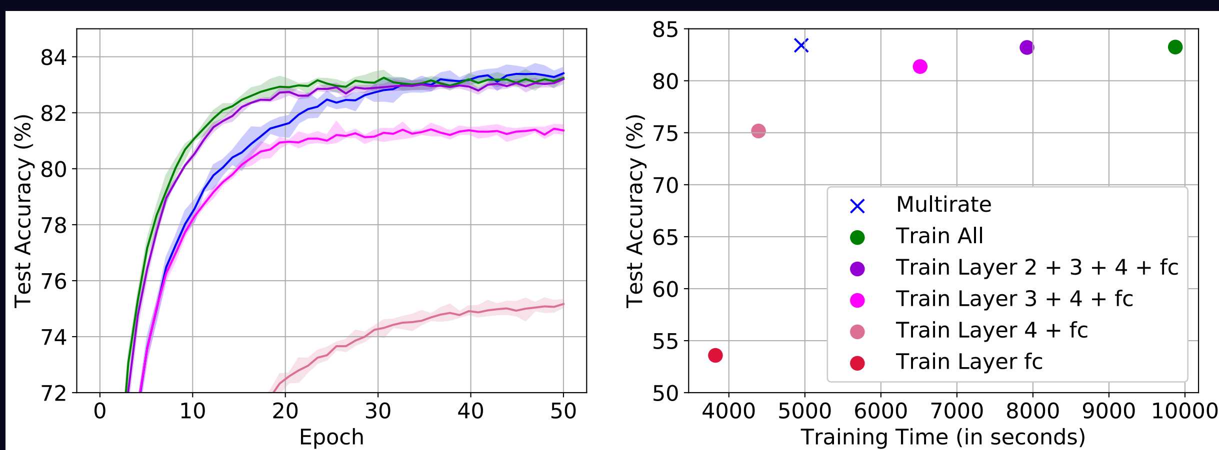

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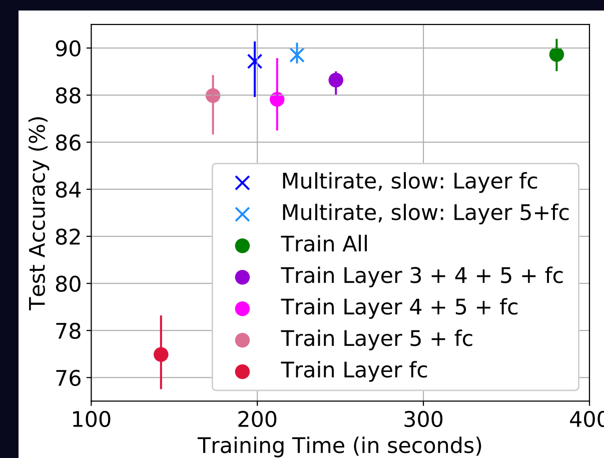

Can train in half the time, without losing much (if any) performance!

Full ablation studies in paper.

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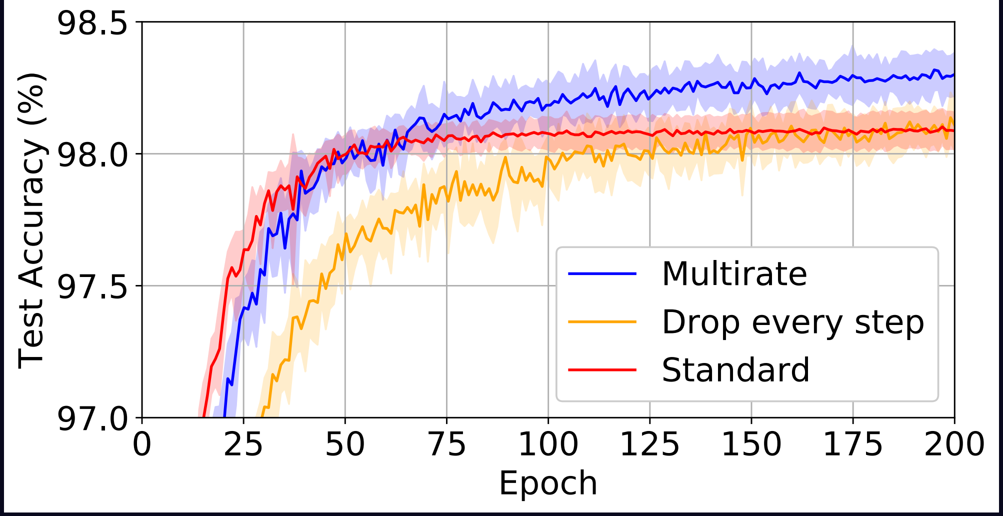

Base algorithm: SGD
Model: SHLP
Data: MNIST

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The proposed techniques:

- Act as add-on to existing optimizers.
- Can learn different features by training the net on different time scales simultaneously.
- Can train deep nets for transfer learning settings in half the time, without losing accuracy.

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Also… looking for a postdoc.

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