The *Iterated Elimination of Strictly Dominated Strategies (IESDS)* process is to eliminate strictly dominated strategies to get a smaller game, then repeat this procedure. Prove that for a finite game, this process is order independent.
Remark: The reason that this property of the IESDS process is of particular interest is that we have proven by example that this does not hold for the Iterated Elimination of Weakly Dominated Strategies (IEWDS) process.
It does not hold for the IEWDS process because for a strategy a_i to be weakly dominated by another strategy b_i, $\exists x_{-i}$ s.t. $u_i(a_i, x_{-i}) < u_i(b_i, x_{-i})$, and $u_i(a_i, c_{-i}) = u_i(b_i, c_{-i}).\forall c_{-i} \setminus \{x_{-i}\}$. However, if x_{-i} is weakly dominated by some y_{-i} and x_{-i} is removed before a_i, then now $u_i(a_i, c_{-i}) = u_i(b_i, c_{-i}), \forall c_{-i}$. $\Rightarrow a_i$ is no longer weakly dominated by b_i and will not be removed, resulting in a different game.
Thus, in order to prove order independence of IESDS, I prove that if a strategy is strictly dominated at any point in the game, it will remain strictly dominated, no matter what other strategies are removed.

Proof. Let there be 2 players, Alice and Bob. The set of all Alice's strategies is $A := \{a_i, \ldots, a_n\}$ and the set of all Bob's strategies is $B := \{b_1, \ldots, b_m\}$.
If $a_{dominated}$ is strictly dominated by a_l, then $\forall j \in J \subseteq \{1, \ldots, m\}$, $u_a(a_{dominated}, b_j) < u_a(a_l, b_j)$. Let S be the set of all strategies that can be removed while $a_{dominated}$ remains strictly dominated. We want to prove that every strategy in the game is in S.
Remove an arbitrary strategy g.
If $g \in A \setminus \{a_l\}$, then $\forall j \in J$, $u_a(a_{dominated}, b_j) < u_a(a_l, b_j)$ remains unaffected, since none of the relevant payoffs are changed $\Rightarrow A \setminus \{a_l\} \in S$.
If $g = \{a_l\}$, this means that a_l is dominated by an arbitrary strategy $a_r \Rightarrow \forall k \in K \subseteq J$, $u_a(a_l, b_k) < u_a(a_r, b_k) \Rightarrow \forall k \in K, u_a(a_{dominated}, b_k) < u_a(a_r, b_k)$. Thus, if a_l is removed because it is strictly dominated, then the strategy that strictly dominates it, a_r, also strictly dominates $a_{dominated}$. Thus, $a_l \in S$.
Finally, if $g \in B$, then let $g = b_i$, where $i \in J$. Since $\forall j \in J, u_a(a_{dominated}, b_j) < u_a(a_l, b_j)$, then $\forall j \in J \setminus \{i\}$, the above inequality holds as well $\Rightarrow B \in S$.
$\Rightarrow A \cup B \in S$. Any strategy that is revealed as strictly dominated in the game will remain strictly dominated, and will eventually be removed. If a strategy is not revealed to be strictly dominated after removing all other strictly dominated strategies, then it is part of the unique final game.
Without loss of generality, the above logic applies to any $b_{dominated}$ as well, taking $j \in J \subseteq \{1, \ldots, n\}$. □