Exercises (Lecture 4)

Koszul duality I

Introduction to Operads

Note. We will solve some of these exercises during the exercise sessions. Try to solve at least one exercise you find easy and at east two exercises that you find challenging.

Exercise 1. Repeat the computations done during the lecture, where we proved that Ass is Koszul self dual and that Com and Lie are Koszul dual to each other.

Exercise 2. Show that PreLie and Perm are Koszul dual to each other.

Exercise 3. Show that the Poisson operad is Koszul self-dual.

Exercise 4. The operad Nov of Novikov algebras is the quotient of the (right) pre-Lie operad by the left permutative relation

$$x_1(x_2x_3) = x_2(x_1x_3).$$

Show that Nov is Koszul dual to its "opposite" operad $\mathsf{Nov}^{\mathrm{op}}$ controlling left pre-Lie algebras satisfying the right permutative relation.

Exercise 5. Show that the Koszul dual of the operad controlling totally associative k-ary algebras is the operad controlling partially associative k-ary algebras.

Exercise 6. Show that the Koszul dual of the operad controlling commutative totally associative k-ary algebras is the operad controlling k-ary Lie algebras.

Exercise 7. Suppose that $\mathscr{P}$ is binary quadratic generated by an operation with no symmetries subject to the relation

$$x_1(x_2x_3) = \sum_{\sigma\in S_3} \lambda_\sigma \sigma(x_1(x_2x_3)).$$

Show that its Koszul dual operad is presented by the relation

$$(x_1x_2)x_3 = \sum_{\sigma\in S_3} \lambda_\sigma \sigma^{-1}((x_1x_2)x_3).$$

Exercise 8. Show that in the case of binary operads, the bilinear form we constructed during the lectures is S_3-invariant.