Exercises (Lecture 5)

Shuffle operads

Introduction to Operads

Note. We will solve some of these exercises during the exercise sessions. Try to solve at least one exercise you find easy and at east two exercises that you find challenging.

Exercise 1. Use the definition of shuffle trees to repeat the computation of a basis of $\mathrm{Tree}^{\amalg}_{\mathscr{X}}(4)$ in case $\mathscr{X}$ consists of a single symmetric or antisymmetric operation. What happens if the operation is not symmetric?

Exercise 2. Explain how $\mathscr{X}^f \circ_{\amalg} \mathscr{Y}^f$ fails to identify with $(\mathscr{X} \circ_{\Sigma} \mathscr{Y})^f$ in case $\mathscr{Y}$ is not reduced.

Exercise 3. Go through the definition of the shuffle compositions γ_π for shuffle tree monomials.

Exercise 4. Give an example of a shuffle operad that is not obtained from a symmetric operad through the forgetful functor.

Exercise 5. Write down a presentation of the commutative operad as a shuffle operad.

Exercise 6. Write down a presentation of the Lie operad as a shuffle operad.

Exercise 7. Write down a presentation of the associative operad as a shuffle operad.

Exercise 8. Write down a presentation of the operad of 3-ary totally commutative associative algebras as a shuffle operad.

Exercise 9. Repeat the theme of the last four exercises with any other (quadratic) operad of your choice.