Assignment 1

Preliminaries

MAU22101 — Group Theory

NAME AND SURNAME: ....

STUDENT NUMBER: .... NUMBER OF PAGES: ....

Note. Solutions to this assignment are **due** by 5:00 pm on Monday, October 19th. Remember to **fill in** all the information above All exercises are weighed equally unless otherwise stated.

Recollections. Recall that a *cycle* is a permutation of the form $i_1 \mapsto i_2 \mapsto \cdots \mapsto i_j \mapsto i_1$, which we usually write $\tau = (i_1 \cdots i_j)$. The set $I = \{i_1, \ldots, i_j\}$ is called the *support of* τ. We say two cycles are disjoint if their supports are disjoint sets. Any permutation σ can be written uniquely[1] as a product $\tau_1 \cdots \tau_j$ of disjoint cycles, which we call the *cycle decomposition of* σ.

Exercise 1. Show that for the cycle $\sigma = (123456)$ in S_6, the permutation σ^2 consists of 2 disjoint cycles, and the permutation σ^3 consists of 3 disjoint cycles.

Exercise 2. Show that if τ is any permutation in S_n and if $\sigma = (i_1 \cdots i_j)$ is a cycle, then

$$\tau\sigma\tau^{-1} = (\tau(i_1) \cdots \tau(i_j))$$

so that, in particular, $\tau\sigma\tau^{-1}$ is also a cycle. *Example:* $(123)(1324)(123)^{-1} = (2134)$.

Exercise 3. Draw a table with all the possible cycle decompositions of an element of S_4, along with a representing permutation for each decomposition. *Hint:* show that these are in bijection with the ways to write 4 as a sum of positive integers so that, for example, $(12)(34)$ corresponds to the expression $2 + 2 = 4$.

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[1] Order the cycles by looking at the minimum element in each of them, for example.