

# TigerQuant VWAP 02-02-2026

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## 1 Introduction

Let  $VWAP_t$  be the volume-weighted average price at time  $t$ ,  $p_t$  be the price at time  $t$ , and  $v_t$  be the volume at time  $t$ . Then,

$$VWAP_t = \frac{\sum_{i \leq t} v_i p_i}{\sum_{i \leq t} v_i},$$

and the price distance or residual  $d_t$  is given by

$$d_t = p_t - VWAP_t.$$

VWAP can be thought of as a proxy for the equilibrium price during a given period. It represents a fair value for a security. Our use case of intraday minute data implies that the VWAP is the fair price during the day. The idea is that prices will return to their mean (theory of mean reversion), and thus, VWAP is our proxy for that mean.

The general strategy is to go long when  $p_t < VWAP_t$ , short when  $p_t > VWAP_t$ , and exit when  $p_t \rightarrow VWAP_t$ . We will get back to scaling this trading logic later on.

We determine the strength of this deviation using the distribution of the residuals or deviation  $d_t$ . Later, we will use this to decide when exactly to make trades. We use the standard deviation shown below.

$$\sigma_t = \text{std}(d_t) = \text{std}(p_t - VWAP_t).$$

It is key to note that one should not use  $\sigma_t^{\text{price}} = \text{std}(p_t)$ , as this represents how volatile the price process is, not the volatility around the VWAP.

Now, for any given day, we have

$$\sigma_t^2 = \mathbb{E}[(d_t - \mathbb{E}[d_t])^2].$$

Then, intraday, we have  $\mathbb{E}[d_t] \approx 0$ , so  $\sigma_t \approx \sqrt{\mathbb{E}[r_t^2]}$ . This is the mispricing magnitude relative to the VWAP.

Then, for some tuning parameter  $k$ , we define upper and lower bands:

$$\text{Upper}_t = \text{VWAP}_t + k\sigma_t$$

$$\text{Lower}_t = \text{VWAP}_t - k\sigma_t.$$

These bands represent the strength of the deviation around our price. We'll come back to this later.

Now, we can improve a simple VWAP mean reversion strategy by adding volatility regime filtering, as well as modeling the deviations using a discrete AR(1) process. For those curious, the continuous analogue to this is an Ornstein-Uhlenbeck process. We'll just be using an AR(1) for our purposes. First, we'll explore volatility regime filtering.

Some level of volatility is needed for VWAP mean-reversion to work. We need to filter for a regime in which our strategy is likely to win. If we trade outside this regime, we may experience unwanted deviations, poor liquidity, or highly directional trends. This filter simply asks the question of whether we are in a tradable environment. We will use log returns here, as we care about relative risk, rather than the raw residual/price deviation volatility we defined above. As an aside, log returns provide greater computational efficiency.

Define log returns to be:

$$\ell_t = \log(p_t) - \log(p_{t-1})$$

Define rolling volatility for window  $v$ :

$$\hat{\sigma}_{\ell,t} = \text{std}(\ell_{t-v:t})$$

Lastly, define our acceptable range as:

$$\text{VolOK}_t = \{\sigma_{min} \leq \hat{\sigma}_{\ell,t} \leq \sigma_{max}\}.$$

Such  $\sigma_{min}$  and  $\sigma_{max}$  values may be obtained via some percentile of historical volatility levels, a backtest based on time of day, or optimized another way from historical data. Such parameter selection will be left for another discussion.

Once we determine that we are in a tradable volatility environment, we assume that our price deviations/residuals  $d_t = p_t - \text{VWAP}_t$  are in fact mean-reverting around our VWAP. But is this actually the case? Recall a previous lecture on AR(1) processes regarding stationarity and the mean-reverting properties therein. To determine if the regime we are in actually supports mean reversion, we will use an AR(1) process:

$$d_{t+1} = \phi d_t + \varepsilon_{t+1}$$

Note that  $\varepsilon_t$  is just noise. For mean reversion, we need  $|\phi| < 1$ . Recall that  $\phi$  is a parameter representing the strength of the linear relationship between  $d_t$  and  $d_{t-1}$ , i.e., how much influence the past has on the future. Thus, for  $\phi$  close to 0, we have quick reversion; for  $\phi$  close to 1, we have slow reversion; and for  $\phi \geq 1$ , we have no reversion. In this last case, we would in fact have more of a momentum or trend regime. From this, we can calculate the half-life of a price deviation as:

$$h_{1/2} = \frac{\ln(1/2)}{\ln(\phi)}$$

A short half-life is great for VWAP. A long half-life means your capital can get tied up. Sizing trades proportionally to the half-life can be a further implementation of this.

Below, we show a few assumptions and derivations for the AR(1) process and the half-life result.

We assume that  $\mathbb{E}[\varepsilon_t] = 0$ , and take conditional expectations on  $d_t$ :

$$\mathbb{E}[d_{t+1}|d_t] = \mathbb{E}[\phi d_t + \varepsilon_{t+1}|d_t] = \mathbb{E}[\phi d_t|d_t] + \mathbb{E}[\varepsilon_{t+1}|d_t] = \phi d_t.$$

Think of this as the expected price deviation one minute into the future is just a decrease of the current minute by factor  $\phi$ .

Now, how do we find the half-life? We first need  $\mathbb{E}[d_{t+h}|d_t] = \phi^h d_t$ . Clearly, we have

$$d_{t+2} = \phi d_{t+1} + \varepsilon_{t+2}$$

Substitute  $d_{t+1} = \phi d_t + \varepsilon_{t+1}$ :

$$d_{t+2} = \phi(\phi d_t + \varepsilon_{t+1}) + \varepsilon_{t+2} = \phi^2 d_t + \phi \varepsilon_{t+1} + \varepsilon_{t+2}.$$

Take conditional expectations given  $d_t$ :

$$\mathbb{E}[d_{t+2}|d_t] = \phi^2 d_t + \phi \mathbb{E}[\varepsilon_{t+1}|d_t] + \mathbb{E}[\varepsilon_{t+2}|d_t].$$

Assuming,  $\mathbb{E}[\varepsilon_{t+1}|d_t] = 0$  and  $\mathbb{E}[\varepsilon_{t+2}|d_t] = 0$ , then

$$\mathbb{E}[d_{t+2}|d_t] = \phi^2 d_t.$$

We can generalize this and get:

$$D_{t+h} = \phi^h D_t + \sum_{j=1}^h \phi^{h-j} \varepsilon_{t+j},$$

and

$$\mathbb{E}\left[\sum_{j=1}^h \phi^{h-j} \varepsilon_{t+j} \mid D_t\right] = \sum_{j=1}^h \phi^{h-j} \mathbb{E}[\varepsilon_{t+j} \mid D_t] = 0.$$

Now, we can say:

$$\mathbb{E}[d_{t+h}|d_t] = \phi^h d_t.$$

Now, to get our half-life, we want the  $h$  such that the expected deviation halves, i.e.,

$$\mathbb{E}[d_{t+h}|d_t] = \frac{1}{2}d_t \Rightarrow \phi^h d_t = \frac{1}{2}d_t.$$

Trivially, and assuming  $d_t \neq 0$ , we get:

$$\phi^h = \frac{1}{2} \Rightarrow h_{1/2} = \frac{\ln(1/2)}{\ln(\phi)}.$$

Now, enough math, and back to the strategy. This AR(1) model allows us to get a glimpse at whether or not we are in a mean-reverting environment in the first place. Then we can trade.

Now that our assumptions are met, and we are in the right environment, let's discuss trading logic. We already know when to long and short, and we have volatility-scaled bands:

$$\text{Upper}_t = \text{VWAP}_t + k\sigma_t$$

$$\text{Lower}_t = \text{VWAP}_t - k\sigma_t.$$

These are strictly for measuring an imbalance and not for telling us to trade. If we enter the market when  $d_t < -k\sigma_t$ , we might be entering right when the flow is still moving away from the VWAP. So we'll add another trading condition and call it entry confirmation. This will answer: has the force that pushed us away from the VWAP weakened?

Define:

$$\Delta d_t = d_t - d_{t-1}.$$

Then, we enter long when:

$$d_t < -k\sigma_t \text{ AND } \Delta d_t > 0.$$

Similarly, we enter short when:

$$d_t > k\sigma_t \text{ AND } \Delta d_t < 0.$$

The rules are that we need a large enough band, that the band must be shrinking, and that we are in the right market environment. Now, you may be asking yourself: when do we exit?

We exit long when  $d_t \geq 0$  and exit short when  $d_t \leq 0$ . In other words, we exit when we cross back over the VWAP.