Artificial isochrons

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The rubidium–strontium isochron procedure has proven to be a powerful tool in many geochemical studies. An isochron is said to exist if a plot of ($^{87}Sr/^{86}Sr$) versus ($^{87}Rb/^{86}Sr$) produces a linear trend with a large correlation coefficient. However, the variables selected to portray the isochron have the same denominator (^{86}Sr) and Pearson long ago noted that a large correlation could be induced when such ratios are formed from uncorrelated numerators and denominators. A set of numerical experiments are described that illustrate the common denominator effects as applied specifically to rubidium–strontium systematics. For at least one previously published Rb–Sr isochron it can be shown that the common denominator effect is capable of producing a correlation coefficient that is very nearly 1.000. However, it is also shown that, for the data sets analyzed, the common denominator effect can not produce a geologically meaningful isochron. The numerical approach to assessing the common denominator effect can be applied only to those sets of isotopic analyses in which ^{87}Rb and ^{86}Sr have been determined by isotope dilution techniques. For the many data sets in which only the ratios ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$) have been determined the common denominator effect can neither be assessed nor dismissed as trivial.

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The analysis of major, minor, trace element and isotopic data often involves assessing the degree to which the measured values agree with, or depart from, an accepted geochemical model. The level of acceptance (or the investigator's confidence in conformity to the model) in many cases is controlled by the degree to which the measurements (or some combination of the measurements) follow a linear trend. In the development and application of such models there are two requirements:

(1) to establish the conditions that must be satisfied in order to demonstrate that the model is appropriate for a given data set and
(2) to define a mechanism for identifying the degree to which the model accounts for the variation observed in the data set.

Demonstration of the 'success' of a particular model often involves measuring the goodness-of-fit of the measurements to an expected or modeled relationship. The form of the variables selected to portray the variation may seriously affect the decision as to whether or not the relationship is statistically significant. For example, Chayes (1949, 1971) has shown that the selection of zero as the null value against which to test the measured correlation is unwarranted when the data are percentages of the same whole. It is well known, but oftimes ignored, that percentages formed from uncorrelated variables may be highly correlated. Ignoring this may lead one to conclude falsely that correlated percentages conform to a particular model when in fact the correlation may be due entirely to the percentage formation effect.

Conversion from percentages to ratios (such as from %MgO and %SiO_2 to the ratio MgO/SiO_2) will eliminate the percentage formation effect (Chayes 1971) but the ratio formation process itself is capable of seriously modifying the structure of the data matrix. That is, certain ratios may exhibit a high level of correlation although the parts of the ratios are uncorrelated.

Ratio formation

In 1896 Pearson published his classic examination of ratio formation effects. If X_1, X_2, X_3 and X_4 represent the measured amounts of these four variables, the correlation between V_i (X_1/X_2) and V_j (X_3/X_4) can be approximated by:

$$r_{ij} \cong \frac{r_{13}C_1C_3 - r_{14}C_1C_4 - r_{23}C_2C_3 + r_{24}C_2C_4}{(C_1^2 + C_2^2 - 2C_1C_2r_{12})^{\frac{1}{2}}(C_3^2 + C_4^2 - 2C_3C_4r_{34})^{\frac{1}{2}}} \quad (1)$$

Where C_k is the coefficient of variation of the kth variable ($C_k = s_k/\bar{X}_k$). If the correlation between all pairs of variables is zero the correlation between V_i and V_j is zero. Equation (1) is approximate rather than exact as only first order terms were used in its derivation and as expectations of the properties of ratios are involved (Pearson 1896 and Chayes 1975). Pearson (1896) was particularly interested in situations in which variable X_2 equalled variable X_4; that is, when the ratios V_i and V_j have a common denominator. If the correlations in equation (1) are all zero and if X_2 equals X_4, the approximated correlation between V_i (X_1/X_2) and V_j (X_3/X_2) is given by:

$$r_{ij} = \frac{C_2^2}{(C_1^2 + C_2^2)^{\frac{1}{2}}(C_3^2 + C_2^2)^{\frac{1}{2}}} \qquad (2)$$

and r_{ij} must have a positive value. Pearson (1896) referred to r_{ij} (equation 2) as a spurious correlation as it arises solely as a consequence of forming ratios with a common denominator from uncorrelated parts of the ratios. The magnitude of r_{ij} (equation 2) increases rapidly as C_2 exceeds C_1 and C_3; for example, if $C_2 = 3C_1$ and $C_2 = 3C_3$ then $r_{ij} = 0.90$. Clearly, zero is not an appropriate null value for testing the strength of linear association between such ratios. Pearson (1896) and Chayes (1949, 1971 and 1975) argued that the spurious correlation itself (r_{ij} in equation 2) could serve as the suitable null value.

Analyses of isotopes are often cast into, or gathered as, ratios with a common denominator; for example, ($^3He/^{21}Ne$) and ($^{22}Ne/^{21}Ne$) or ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$). Interpretation of bivariate plots of such ratios depends upon the degree to which the ratios define a straight line and the presence of a linear trend with a large correlation coefficient is often taken as evidence of conformity to some geochemical model.

The preceding brief discussion of ratio formation with a common denominator suggests that not only must an investigator be intimately familiar with the constraints of each geochemical model but must also be able to take into account the potential contribution of ratio formation. As long as the parts of the ratio have been measured separately the effects of ratio formation can be evaluated. If, however, the parts of the ratios are not directly measured, the investigator is able to assess neither the significance of a linear trend on a scatter diagram nor the significance of some computed statistical measure (such as the correlation coefficient or the slope of a best-fit straight line).

Strontium-rubidium systematics

The use of strontium–rubidium isotopic variations for dating a geologic event and for estimating the ratio of ^{87}Sr to ^{86}Sr at $t = 0$ is widespread. Faure (1979) provides an excellent introduction to both theory and practice; From Faure (1979), the total number of atoms of ^{87}Sr (produced by the decay of ^{87}Rb through emission of negative beta particles) in a mineral of age t is given by:

$$^{87}Sr = {^{87}Sr_o} + {^{87}Rb}(e^{\lambda t} - 1) \qquad (3a)$$

where $^{87}Sr_o$ is the number of atoms of ^{87}Sr present at time $t = 0$, ^{87}Rb is the number of atoms of ^{87}Rb at the present time and λ is the decay constant of ^{87}Rb. Equation (3a) is usually modified by dividing each term by the number of atoms of ^{86}Sr (assumed to be a constant for the sample):

$$(^{87}Sr/^{86}Sr) = (^{87}Sr/^{86}Sr)_o + (^{87}Rb/^{86}Sr)\ (e^{\lambda t} - 1) \qquad (3b)$$

Faure & Powell (1972) note that ratios of isotopes are more easily measured than are the numbers of atoms of each isotope.

The basis for the strontium–rubidium isochron method is as follows; if the strontium present in a parent magma of a comagmatic suite were isotopically homogeneous, the rocks produced by processes of magmatic differentiation would have had the same initial ($^{87}Sr/^{86}Sr$) ratio. When t is a constant, equation (3b) reduces to the equation of a straight line with slope of $(e^{\lambda t} - 1)$ and intercept of $(^{87}Sr/^{86}Sr)_o$. Therefore, all comagmatic rocks that:

(1) have the same age
(2) have the same initial ratio of ^{87}Sr to ^{86}Sr
(3) have remained closed to Rb and Sr since crystallization

will plot as a straight line in ($^{87}Sr/^{86}Sr$)–($^{87}Rb/^{86}Sr$) space; the straight line is the isochron. From the literature it would appear that if a set of analyses plot as a straight line that the investigator often assumes that all of the conditions listed above are satisfied.

Faure & Powell (1972:15) note that the goodness-of-fit of the ratios to the isochron can be used to confirm that the suite is indeed comagmatic and that a combination of geological information and statistical tests is required for the conformation. However, from equation (3b) it is clear that one is dealing with ratios with a common denominator and this should raise the suspicion that some or all of the observed variation on the scatter diagram may be due to the effects of having formed ratios with a common denominator. Is it possible to produce an isochron that would violate the conditions listed above? From the preceding discussion concerning ratio formation in general, it would appear to be evident that such a possibility cannot be dismissed unless the common denominator effect can be eliminated. In an extremely interesting literature exchange Chayes (in Chayes & Brooks et al. 1977) argued that in the absence of information concerning the properties of the numerators and denominators of the ratios it was unwise to seek statistical justification for the interpretation of Sr-Rb isochrons. Brooks et al. (1976) worked with the system ($^{87}Sr/^{86}Sr$) – (Rb/Sr) – a more complicated Sr–Rb system as total Sr includes contributions from both ^{87}Sr and ^{86}Sr. Although Brooks et al. (in Chayes & Brooks et al. 1977) describe a set of numerical experiments, it is not at all clear that they knew the statistical properties of the parts of the ratios they were concerned with, and therefore their claim that they had demonstrated statistical significance is unwarranted.

The ability to relate statistics of a ratio to the statistics of its numerator and denominator will be needed for the following discussion. If the numerator and denominator of a ratio are correlated, and if second order terms are included in the approximation procedure (Pearson 1896 and Butler 1979), the mean of the ratio V_i (X_1/X_2) can be approximated as:

$$\bar{V}_i \simeq (\bar{X}_1/\bar{X}_2)\ (1 + C_2 - C_1C_2r_{12}) \quad (4)$$

If numerator and denominator were perfectly positively correlated, the mean of the ratio will be minimized; a value of r_{12} of -1.0 will maximize the mean of the ratio. The variance of the ratio V_i can be approximated by:

$$s_i^2 = (\bar{X}_1^2/\bar{X}_2^2)\ (C_1^2 + C_2^2 - 2C_1C_2r_{12})^{\frac{1}{2}} \quad (5)$$

(Pearson 1896). From equation (5) it is clear that if the correlation between numerator and denominator is positive and if the coefficients of variation are approximately equal, the variance of the ratio will approach 0.0 as r_{12} approaches 1.0. If numerator and denominator are perfectly positively correlated the variance of their ratio will be minimized; perfect negative correlation will maximize the variance of the ratio.

In order to assess the effects of using ^{86}Sr as a common denominator one must know the coefficients of variation of ^{87}Sr, ^{86}Sr, and ^{87}Rb, as well as the correlations between these variables. Most of the data published in the literature gives only the values of the ratios ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$); for such sets the common denominator effect can be neither dismissed as trivial nor properly evaluated. More recently, isotope dilution procedures have been developed that allow the direct measurement of ^{86}Sr and ^{87}Rb. For example, Halpern & Fuenzalida (1978) give measured values for ($^{87}Sr/^{86}Sr$), ($^{87}Rb/^{86}Sr$), ^{87}Rb (μm/g) and ^{86}Sr (μm/g) for several suites of igneous rocks from southern Chile. The ratio of ^{87}Rb (μm/g) to ^{86}Sr (μm/g) is the reported ($^{87}Rb/^{86}Sr$) ratio. Given the ratio of ^{87}Sr to ^{87}Sr and the measured amount of ^{86}Sr one can compute the amount of ^{87}Sr. Statistics for the measured amounts of ^{86}Sr, ^{87}Rb, ($^{87}Sr/^{86}Sr$), ($^{87}Rb/^{86}Sr$) and the computed ^{87}Sr values from the Simpson River adamellite (samples 2A, 2B, 2C, 2D and 3; Halpern & Fuenzalida 1978:62) are given in Table 1. The correlation between the parts of the ratios are of interest; for ^{87}Sr and ^{86}Sr the correlation is nearly 1.000 whereas the correlations between ^{86}Sr and ^{87}Rb and between ^{87}Sr and ^{87}Rb are nearly 0.000. This pattern has been observed in other Sr–Rb isotopic data sets examined to date by the author.

A modified version of RTCRSM2 (Chayes 1975) has been developed by the author to enable numerical experimentation with a variety of transformation processes. One can employ a Monte Carlo solution approach by drawing a large number of samples or a Monte Carlo simulation in which the number of samples drawn is kept small and the experiment is repeated many times so that the results will have applicability in the world of small sample sizes. The population from which the samples are drawn can be based on a variety of distributions (normal, log normal, exponential or empirical). Means and variances of ^{86}Sr, ^{87}Sr, and ^{87}Rb from Table 1 serve as the input to RTCRSM2 for the following Monte Carlo studies.

Monte Carlo solutions for 2000 and 1000 entries yield null correlations of 0.843 and 0.840 respectively for the Halpern & Fuenzalida (1978) set of measurements of ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$). Selecting 0.843 as the null value against which to test the observed correlation of 0.995 requires accepting the null hypothesis; that is, there is no significant difference between the null value and the observed correlation at the 95% confidence level.

Table 1. Statistics for ^{86}Sr, ^{87}Rb, ^{87}Sr (computed), ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$) for a set of five adamellite analyses (Halpern et al. 1978:62).

	^{86}Sr	^{87}Rb	^{87}Sr	($^{87}Sr/^{86}Sr$)	($^{87}Rb/^{86}Sr$)
Mean	0.056	0.693	0.043	0.7227	12.02
Standard dev.	0.013	0.045	0.009_3	0.0034	2.48
Coefficient var.	0.220	0.065	0.215	0.005	0.206
Skewness	0.289	0.598	0.286	−0.212	−0.268

Correlation coefficient matrix

	^{86}Sr	^{87}Rb	^{87}Sr	($^{87}Sr/^{86}Sr$)	($^{87}Rb/^{86}Sr$)
^{86}Sr	1.000	0.071	1.000	−0.928	−0.957
^{87}Rb		1.000	0.079	0.249	0.213
^{87}Sr			1.000	−0.945	−0.956
($^{87}Sr/^{86}Sr$)				1.000	0.995
($^{87}Rb/^{86}Sr$)					1.000

A comparison of the means and standard deviations of the measured data set with means and standard deviations from the Monte Carlo solution with 2000 entries is given in Table 2. There is very good agreement for the parts of the ratios but for the ratios themselves the agreement is less than satisfactory; especially for the ratio of ^{87}Sr and ^{86}Sr. Both the means and standard deviations of the simulated ratios exceed those of the measured data set; note, for example, that the standard deviation of the simulated ($^{87}Sr/^{86}Sr$) ratio is approximately 100 times larger than its measured counterpart.

As only five samples comprise the measured data set and as the null value is close to 1.0 a series of Monte Carlo simulations were undertaken to assess sampling variations in means and standard deviations of the ratios and parts of the ratios and in the correlation coefficient between the ratios with a common denominator. A summary for an experiment in which 200 sets of five samples each were drawn from the populations summarized in Table 1 is given in Table 3 and a histogram depicting the distribution of the null correlation is given in Fig. 1. Again, means and standard deviations of the three parts of the ratios agree quite closely with those in the measured data set but the properties of the ratios exceed those of the population. The average correlation for the 200 sets is 0.69 and values range

Table 2. Comparison of means and standard deviations of the measured Sr–Rb data set with means and standard deviations of the Monte Carlo solution (N=2000).

Means		
Variable	Measured	Simulated
^{87}Sr	0.0403	0.0432
^{87}Rb	0.6936	0.6921
^{86}Sr	0.0560	0.0561
$^{87}Sr/^{86}Sr$	0.7227	0.8296
$^{87}Rb/^{86}Sr$	12.02	13.27
Standard deviations		
Variable	Measured	Simulated
^{87}Sr	0.0093	0.0093
^{87}Rb	0.0453	0.0443
^{86}Sr	0.0135	0.0134
$^{87}Sr/^{86}Sr$	0.0035	0.3543
$^{87}Rb/^{86}Sr$	2.483	4.838

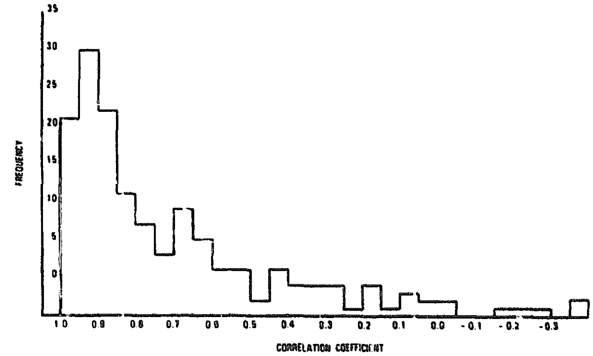

Fig. 1. Histogram portraying the distribution of 200 simulated correlations between ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$) for the Halpern & Fuenzalida (1978) data set.

Table 3. Distribution of the means, standard deviations and correlation coefficient between ratios with common denominator for the 200 sets (five samples per set) drawn from the Rb-Sr data.

Distribution of the sample means					
	^{87}Sr	^{87}Rb	^{86}Sr	$(^{87}Sr/^{86}Sr)$	$(^{87}Rb/^{86}Sr)$
Mean	0.043	0.691	0.056	0.831	13.35
Standard dev.	0.004	0.021	0.006	0.142	2.218
Coefficient var.	0.091	0.030	0.112	0.183	0.166
Skewness	−0.332	−0.059	0.132	1.516	3.021
Distribution of the sample standard deviations					
	^{87}Sr	^{87}Rb	^{86}Sr	$(^{87}Sr/^{86}Sr)$	$(^{87}Rb/^{86}Sr)$
Mean	0.009	0.043	0.013	0.283	3.745
Standard dev.	0.003	0.014	0.005	0.189	3.642
Coefficient var.	0.370	0.339	0.358	0.669	0.973
Skewness	0.509	0.497	0.393	4.802	8.169
Distribution of the correlation between $(^{87}Sr/^{86}Sr)$ – $(^{87}Rb/^{86}Sr)$					
	r $(^{87}Sr/^{86}Sr)$ – $(^{87}Rb/^{86}Sr)$				
Mean	0.690				
Standard deviation	0.327				
Coefficient variation	0.474				
Maximum	0.999				
Minimum	−0.784				
Skewness	−1.869				

from 1.000 to −0.397. Thus, randomly drawn, uncorrelated values for as few as five samples from a population with parameters equal to those of the Halpern & Fuenzalida (1978) data set may yield relatively large positive correlations. The effect of increasing the sample size from five to ten to twenty samples per set is to increase the average null correlation and to decrease the number of negative null correlations. As noted previously, there is a very strong correlation between ^{87}Sr and ^{86}Sr (Table 1); nevertheless, ratios formed from uncorrelated measures frequently exhibit correlations as strong as that observed (or stronger) (Fig. 1).

A second aspect of the role of the common denominator effect on Rb–Sr isochrons requires determining whether or not randomly drawn absolute measures can produce an isochron that is realistic and one that could be misinterpreted as being geologically meaningful. Fitting a straight line to a set of paired values can be accomplished in several ways. For chemical information it is usually impossible to designate one variable as independent of the other; therefore, the line of organic correlation (Kruskal 1953) is a suitable descriptor, as it is based upon the assumption of co-dependency. The general form of the line of organic correlation is:

$$\hat{Y}_i = \gamma_j + \beta_j Y_j \qquad (6a)$$

where β_j (the slope of the line) is given by:

$$|\beta_j| = {}'s_i/s_j \qquad (6b)$$

The sign of the slope is that of the correlation between variables i and j. The intercept of the line (γ_j) is given by:

$$\gamma_j = \bar{Y}_i - \beta_j \bar{Y}_j \qquad (6c)$$

For ratios with a common denominator it is apparent that the slope and intercept of the best-fit straight line are implicit in the properties of the ratios (equations 4 and 5).

For the Halpern & Fuenzalida data set (1978) the isochron can be expressed as:

$$(^{87}Sr/^{86}Sr) = 0.7057 + 0.00141\ (^{87}Rb/^{86}Sr) \qquad (7a)$$
Age = 101 million years

Isochrons for the Monte Carlo solution and the average of the Monte Carlo simulations (Table 3) are:

$$(^{87}Sr/^{86}Sr) = -0.1422 + 0.07323\ (^{87}Rb/^{86}Sr)$$
(Table 2) (7b)
Age = 5085 million years

and

$$(^{87}Sr/^{86}Sr) = -0.1791 + 0.07562\ (^{87}Rb/^{86}Sr)$$
(Table 3) (7c)
Age = 5244 million years

It is evident that the simulated isochrons (equations 7b and 7c) are in very poor agreement with the observed isochron (equation 7a) and, in fact, the simulated isochrons do not make 'geological sense'.

Failure of the simulation procedures to produce reasonable isochrons can be attributed to the high correlation between ^{87}Sr and ^{86}Sr (Table 1). Means and standard deviations of the simulated ratio of ^{87}Sr to ^{86}Sr depart markedly from those for the measured data set (Tables 2 and 3), whereas there is much closer agreement for the ratio of ^{87}Rb to ^{86}Sr due to the fact that the correlation between this pair of variables is close to zero (Table 1). The slope of the isochron (expressed as the line of organic correlation) is the ratio of the standard deviation of (^{87}Sr/^{86}Sr) to that of (^{87}Rb/^{86}Sr); thus, the greatly enlarged slopes and 'ages' of the simulated isochrons. The intercepts of the isochrons are greatly reduced (as compared with the observed isochron – equation 7a) due to the enlarged slopes of the simulated isochrons (equations 6b and 6c). A negative estimate of the initial ratio of ^{87}Sr to ^{86}Sr does not make sense. In fact, of the 200 simulated isochrons (Table 3), none produce intercepts in the range 0.60 to 0.80.

Several Monte Carlo simulations were undertaken in which the correlation between ^{86}Sr and ^{87}Sr was constrained to large positive values. In general, such simulations were not successful. When the correlation between ^{86}Sr and ^{87}Sr is preselected as 0.50 the means and standard deviations of the ratios (^{87}Sr/^{86}Sr) and (^{87}Rb/^{86}Sr) are enlarged as compared with those actually observed (Table 1) and as compared with the Monte Carlo simulations in which all correlations were held at values of 0.0 (Table 3). Preliminary experimentation suggests that as the correlation between ^{87}Sr and ^{86}Sr approaches 1.0 the null correlation between (^{87}Sr/^{86}Sr) and (^{87}Rb/^{86}Sr) approaches 0.0 and the individual correlations appear to approach a uniform distribution between 1.0 and –1.0. Additional experiments are presently being designed.

In summary, although randomly drawn uncorrelated variables yield correlations as large as those observed in the Halpern & Fuenzalida (1978) data set, the ratios of the simulated absolute measures do not produce isochrons that either agree with the observed isochron or make geological sense.

Other Rb–Sr systems

Several other data arrays have been examined during the course of this study. Isotopic analyses (using isotope dilution for the analysis of ^{87}Rb and ^{86}Sr) for five samples of charnockitic rocks from Mirnyy Station, east Antarctica illustrate a slightly different behavior of Rb–Sr systems (McQueen et al. 1972:436). Correlation coefficients between the ratios and parts of the ratios are given in Table 4. As in the Halpern & Fuenzalida (1978) data set, the correlation between ^{87}Sr and ^{86}Sr is almost 1.000 but the correlations between the other pairs of variables differ significantly from 0.000. A Monte Carlo solution with 2000 samples yielded a null value of 0.453 for the correlation between (^{87}Sr/^{86}Sr) and (^{87}Rb/^{86}Sr). Using this null value, the observed correlation of 0.999_6 rejects the null hypothesis at the 95% confidence level. A Monte Carlo simulation with 200 sets of five samples each produced an average null correlation of 0.391 and only 27 of the 200 samples have correlations that would require rejection of the null hypothesis when tested against the null of 0.453.

One can conclude that randomly drawn, uncorrelated variables with properties equal to those of the McQueen et al. (1972) data set will not yield null values as large as the observed correlation.

Table 4. Correlation coefficients for the McQueen et al. (1972) data set.

	^{87}Sr	^{87}Rb	^{86}Sr	(^{87}Sr/^{86}Sr)	(^{87}Rb/^{86}Sr)
^{87}Sr	1.000	–0.497	0.998	–0.882	–0.874
^{87}Rb		1.000	–0.544	0.842	0.850
^{86}Sr			1.000	–0.906	–0.899
^{87}Sr/^{86}Sr				1.000	0.999
^{87}Rb/^{86}Sr					1.000

Summary and conclusions

As developed by Pearson (1896) and adopted to geological problem solving by Chayes (1949, 1971 and 1975) the concept of ratio correlation should be an integral part of the arsenal of techniques that geologists bring to bear on problems that involve the analysis of chemical data. The most difficult problems are those that arise when ratios are measured directly and the parts of the ratios are unavailable for examination and measurement. Without knowledge of the statistics of the parts of the ratios one cannot assess the possibility that the formulation of ratios with common denominators is capable of producing correlations as large as those found in the measured data sets.

For the Rb–Sr systems one can compute a value for ^{87}Sr if ^{86}Sr and the ($^{87}Sr/^{86}Sr$) ratio have been measured. The two data arrays examined in this note lead to two different conclusions:

(1) for the Halpern & Fuenzalida (1978) data set the correlation between ($^{87}Rb/^{86}Sr$) and ($^{87}Sr/^{86}Sr$) is not significant when tested against the simulated null value.

(2) for the McQueen et al. (1972) data set the correlation between ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$) is significant when tested against the simulated null value.

For both data sets, however, the simulated values of the ratios do not define meaningful isochrons. Therefore, the preliminary conclusion is that both systems have an information content that exceeds that expected if the absolute measures were uncorrelated. This should answer some of the questions raised by Chayes (in Chayes & Brooks et al. 1977).

The ratio ($^{87}Sr/^{86}Sr$) is virtually a constant in the Sr–Rb systems examined to date; the range of values being a few parts in 700. This plus the fact that the variance of ^{87}Rb is almost 90% of the total variance of the system (Table 1) suggests that attention should be focused on ^{87}Rb as the component of prime interest. From the known genetic relationship one would expect a nearly perfect (if not perfect) correlation between ^{87}Rb and ^{86}Sr if the system had remained closed since time $t = 0$. Chayes (1980, personal communication) has suggested that local fluctuations in total Sr may tend to obscure the relationship between ^{87}Sr and ^{87}Rb. If true, one cannot help but be concerned about the large correlation between ($^{87}Sr/^{86}Sr$) and ($^{87}Rb/^{86}Sr$). If the correlation between the numerators of the ratios is zero (or weakly negative) then the common denominator effect cannot be ignored.

Faure (1979) noted that fictitious isochrons could be produced by mixing processes and that the 'isochrons' would not give a meaningful age in such a situation. To such fictitious isochrons the author would add the possibility of producing artificial isochrons in which the linearity is due to the formation of ratios with a common denominator. In the cases considered in this note the possibility of artificial isochrons can be discounted; however, no blanket statement can be made concerning isochrons in general. Each case must be tested by itself. There is no universal common denominator null value. The author would encourage investigators to conduct their own Monte Carlo type studies with their own isotopic data. Such studies focus attention on fluctuations that might be expected due to sampling and may identify truely artificial isochrons.

During the review process a correspondence was begun with M. H. Dodson of the University of Leeds. Dr. Dodson's paper on 'On "Spurious" Correlations in Rb–Sr Isochron Diagrams' appears as a companion paper in this issue. The prime area of disagreement appears to be associated with Dodson's (1982) null hypothesis for isotope geochemistry: '. . . variations in the measured isotopic composition of an element are unrelated to its concentration – or to any other petrochemical property of the materials sampled'. Equations presented by Dodson (1982) hold for the case of zero correlation between a ratio and its denominator but not for the case of correlation between a ratio and its numerator. Also, the data sets examined by the author (Butler) do not conform to Dodson's (1982) model; namely, the requirement that the correlation between ^{87}Sr and ^{86}Sr be equal to the ratio of their coefficients of variation.

Readers of both papers should keep in mind that the purpose of this note (Butler) was to consider whether the formation of ratios from uncorrelated parent variables (having the statistical properties of measured or computed isotopes of strontium and rubidium) could produce strong positive correlations when ^{86}Sr is used as the common denominator and it is the author's opinion that this has successfully been demonstrated The question of the appropriateness of the Rb–Sr isochron method has not been raised; rather, what was of concern was whether all of the external influences that could affect the strength of the isochron have been considered. No investigator would be 'fooled' by an isochron in which the component samples were collected from different geographic areas of different known geologic ages. Similarly, an investigator should eliminate or recognize the contribution that the common denominator effect may play in assessing an isochron.

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