Contents

Problem 1

```
close all
clear;clc

disp('=== Problem 1 ===')

p1 = 100;
gamma = 1.4;
theta1 = deg2rad(12);
theta2 = deg2rad(8);
mu1 = deg2rad(20);

% ----- Part (a) -----

M1 = 1/sin(mu1);

disp(['a) ' 'M1 = ' num2str(M1)])

% ----- Part (b) -----

syms m2 m3 real

eqn = theta1 == PM(m2,gamma) - PM(M1,gamma);
M2 = abs(double(vpasolve(eqn,m2)));

% p2 = gamma * M1^2 / sqrt(M1^2-1) * theta1 * p1 + p1;
p2 = p1 * ( (1+(gamma-1)/2*M1^2) ...
           /(1+(gamma-1)/2*M2^2) ) ^ (gamma/(gamma-1));

beta2 = TBM_get_beta(M2,theta2,gamma);
p3 = p2 * TBM_get_pres(M2,beta2,gamma);

M2n = M2 * sin(beta2);
M3n = sqrt( (M2n^2 + 2/(gamma-1)) / (2*gamma/(gamma-1)*M2n^2-1) );

M3 = M3n / sin(beta2-theta2);

disp(['b) ' 'M3 = ' num2str(M3)])
disp(['   ' 'p3 = ' num2str(p3)])

disp(' ')
```

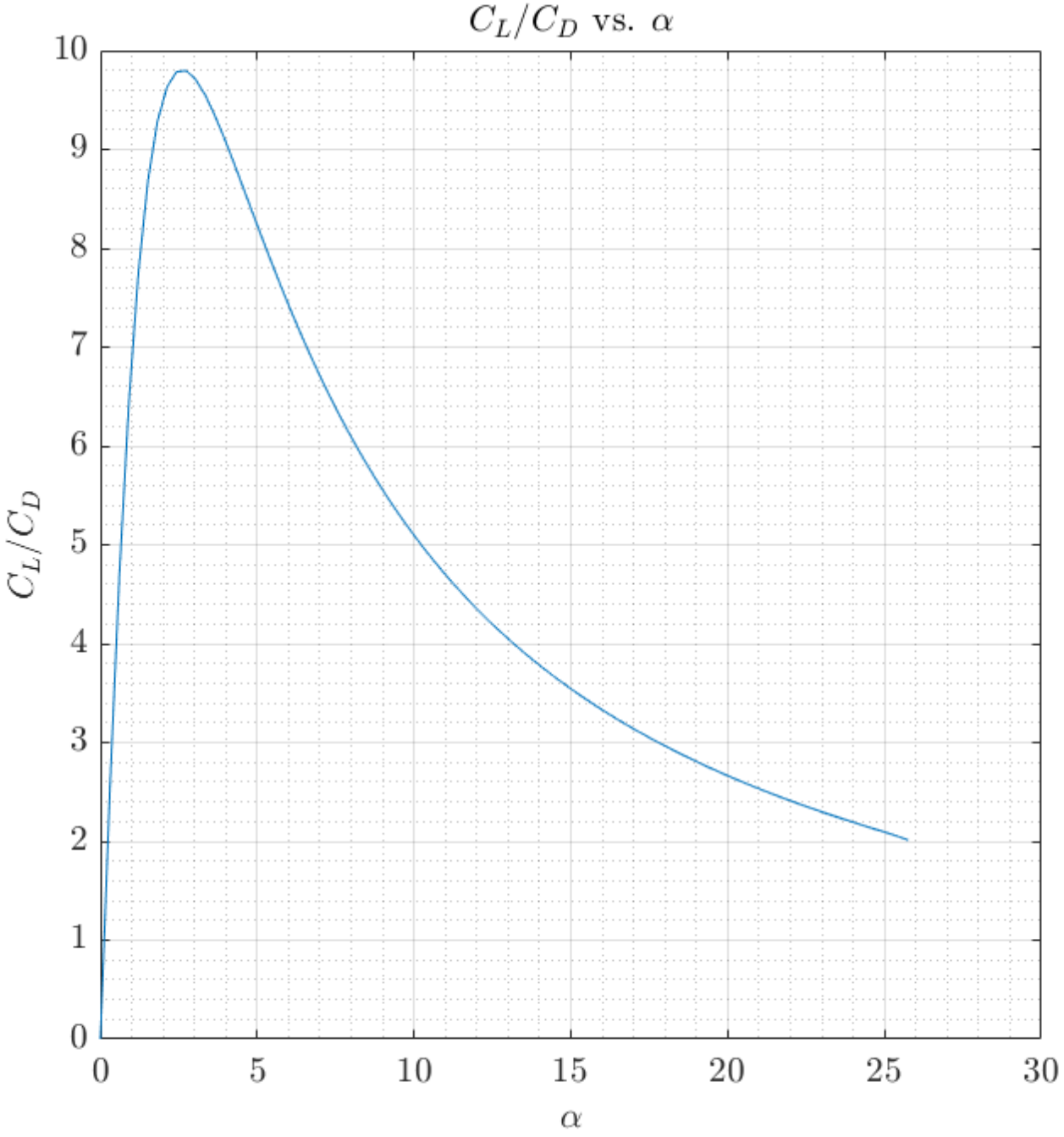
```
=== Problem 1 ===
a) M1 = 2.9238
b) M3 = 3.1136
   p3 = 72.8548
```

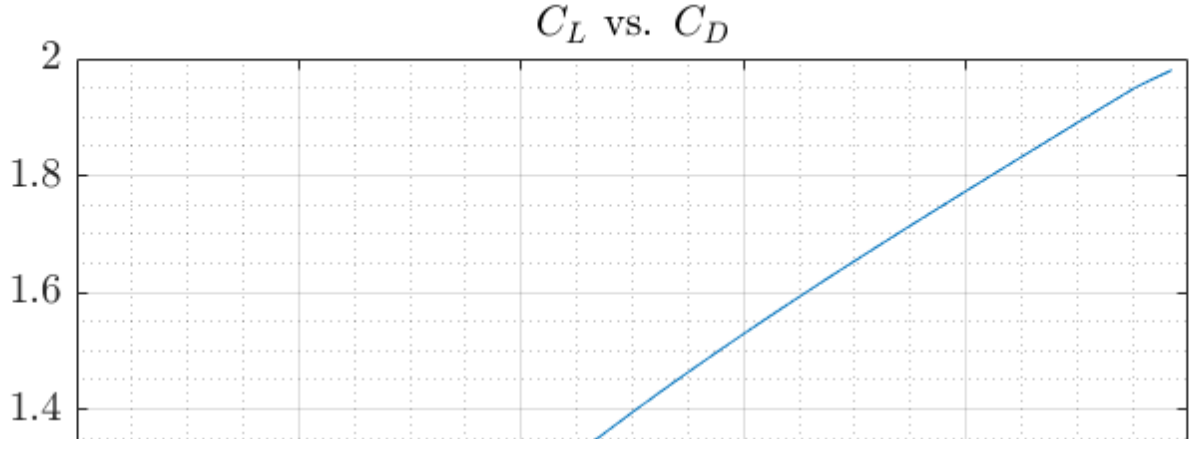
Problem 2

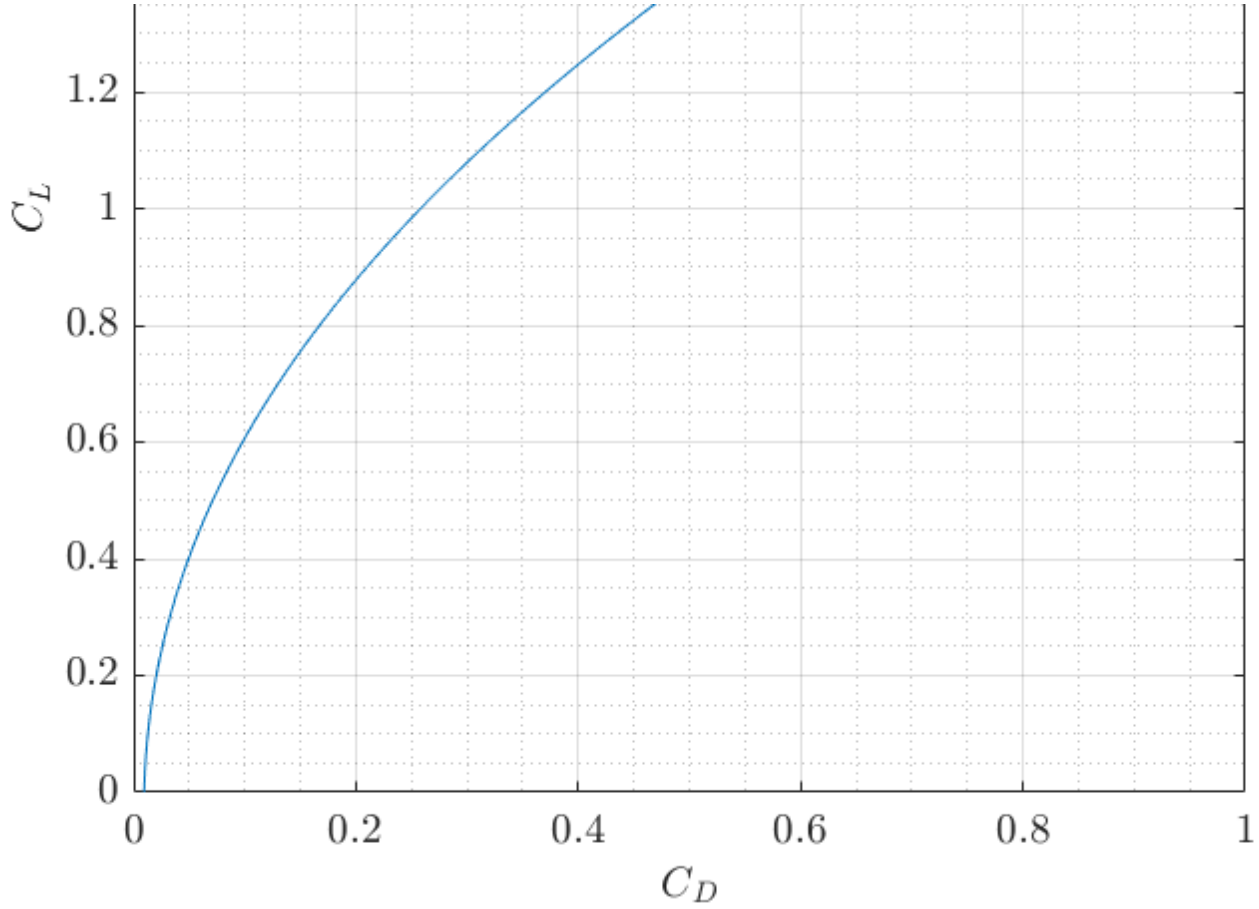
```
close all
clear;clc

disp('=== Problem 2 ===')

% ----- Part (a) -----

disp('a) See figure attached in HW submission.')

% ----- Part (b) -----

disp('b) See derivation attached in HW submission.')

% ----- Part (c) -----

disp('c) ')

alpha_vect = deg2rad(linspace(0,30,100)); % stalls after 25.5 deg
cl_vect = zeros(length(alpha_vect),1);
cd_vect = zeros(length(alpha_vect),1);

syms m1 m2 m4 real

for i = 1:length(alpha_vect)

alpha = alpha_vect(i);
theta = deg2rad(4);
gamma = 1.4;
p = 100;
Mi = 2.5;
c = 1;
t = 0.07;

if alpha <= theta % oblique shock
    t1 = theta - alpha;
    beta1 = TBM_get_beta(Mi,t1,gamma);
    p1 = p * TBM_get_pres(Mi,beta1,gamma);
    Min = Mi * sin(beta1);
    M1n = sqrt( (Min^2 + 2/(gamma-1)) / (2*gamma/(gamma-1)*Min^2-1) );
    M1 = M1n / sin(beta1-t1);
else % alpha > theta, expansion fan
    t1 = theta - alpha;
    eqn = -t1 == PM(m1,gamma) - PM(Mi,gamma);
    M1 = abs(double(vpasolve(eqn,m1)));
    p1 = p * ( (1+(gamma-1)/2*Mi^2) ...
              /(1+(gamma-1)/2*M1^2) ) ^ (gamma/(gamma-1));
end

if size(M1,1) == 0
    continue
end

% calculations for p2,p3,p4 remain the same in both cases

% p2: expansion fan
    t2 = -theta - alpha;
    eqn = -t2 == PM(m2,gamma) - PM(M1,gamma);
    M2 = abs(double(vpasolve(eqn,m2)));
    p2 = p1 * ( (1+(gamma-1)/2*M1^2) ...
               /(1+(gamma-1)/2*M2^2) ) ^ (gamma/(gamma-1));
```

```
% p3: oblique shock
    t3 = alpha + theta;
    beta3 = TBM_get_beta(Mi,t3,gamma);
    p3 = p * TBM_get_pres(Mi,beta3,gamma);
    Min = Mi * sin(beta3);
    M3n = sqrt( (Min^2 + 2/(gamma-1)) / (2*gamma/(gamma-1)*Min^2-1) );
    M3 = M3n / sin(beta3-t3);

% p4: expansion fan
    t4 = -theta;
    eqn = -t4 == PM(m4,gamma) - PM(M3,gamma);
    M4 = abs(double(vpasolve(eqn,m4)));
    p4 = p3 * ( (1+(gamma-1)/2*M3^2) ...
               /(1+(gamma-1)/2*M4^2) ) ^ (gamma/(gamma-1));

cl = ( -cos(theta-alpha)*p1 - cos(-theta-alpha)*p2 ...
       +cos(theta+alpha)*p3 + cos(-theta+alpha)*p4 ) / (0.5*gamma*p*Mi^2);
cd = (  sin(theta-alpha)*p1 + sin(-theta-alpha)*p2 ...
       +sin(theta+alpha)*p3 + sin(-theta+alpha)*p4 ) / (0.5*gamma*p*Mi^2);

cl_vect(i) = cl;
cd_vect(i) = cd;

end

alpha_vect(imag(cl_vect)~=0) = [];
cd_vect(imag(cl_vect)~=0) = [];
cl_vect(imag(cl_vect)~=0) = [];

alpha_vect = rad2deg(alpha_vect);

% plotting
figure(1)
plot(alpha_vect,cl_vect./cd_vect)
title('$C_L/C_D$ vs. $\alpha$')
xlabel('$\alpha$')
ylabel('$C_L/C_D$')
grid(gca,'minor')
grid on
latexify

figure(2)
plot(cd_vect,cl_vect)
title('$C_L$ vs. $C_D$')
xlabel('$C_D$')
ylabel('$C_L$')
grid(gca,'minor')
grid on
latexify

disp(' ')
```

```
=== Problem 2 ===
a) See figure attached in HW submission.
b) See derivation attached in HW submission.
c)
```


C_L/C_D vs. α
C_L/C_D
α
0
1
2
3
4
5
6
7
8
9
10
0
5
10
15
20
25
30


C_L vs. C_D
1.4
1.6
1.8
2



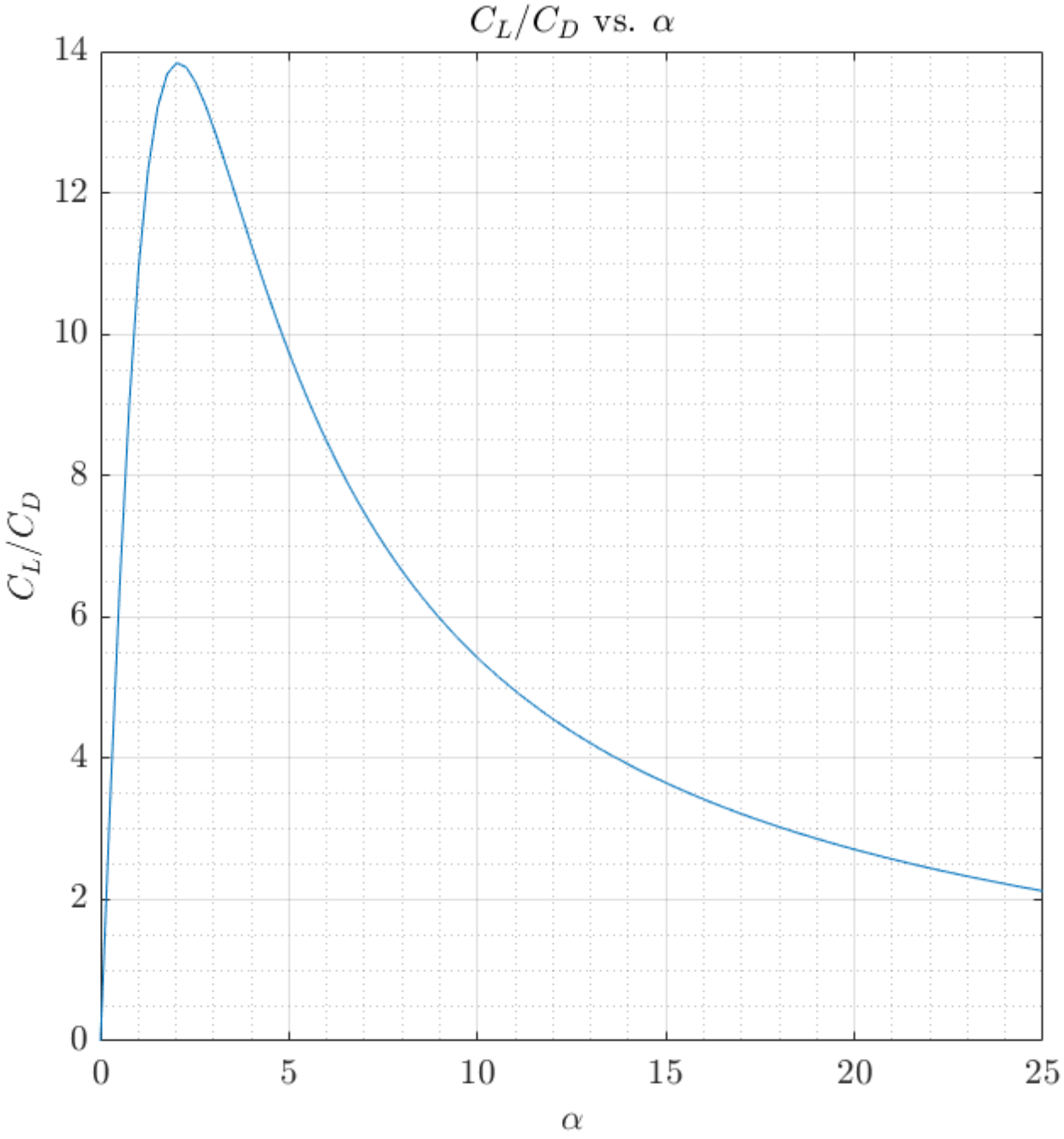
Problem 3

```
close all
clear;clc

disp('=== Problem 3 ===')

syms x real

t = 0.03;
c = 1;
Mi = 2.5;
p = 100;
gamma = 1.4;

h = t - t.*(x./c).^2;
theta = diff(h,x);
theta0 = atan(double(subs(theta,x,-1))); % theta at the leading edge

pres_points = 25;
syms m1 real
mU = sym('mU', [pres_points,1]) ;
mL = sym('mL', [pres_points,1]) ;

alpha_vect = deg2rad(linspace(0,25,100));
cl_vect = zeros(length(alpha_vect),1);
cd_vect = zeros(length(alpha_vect),1);

% alpha_vect = deg2rad([0.1]);
```

```
for i = 1:length(alpha_vect)

    alpha = alpha_vect(i);

% top surface LE oblique shock / expansion fan
if alpha <= theta0 % oblique shock
    t1 = theta0 - alpha;
    beta1 = TBM_get_beta(Mi,t1,gamma);
    p1 = p * TBM_get_pres(Mi,beta1,gamma);
    Min = Mi * sin(beta1);
    M1n = sqrt( (Min^2 + 2/(gamma-1)) / (2*gamma/(gamma-1)*Min^2-1) );
    M1 = M1n / sin(beta1-t1);
else % alpha > theta0, expansion fan
    t1 = theta0 - alpha;
    eqn = -t1 == PM(m1,gamma) - PM(Mi,gamma);
    M1 = abs(double(vpasolve(eqn,m1)));
    p1 = p * ( (1+(gamma-1)/2*Mi^2) ...
              /(1+(gamma-1)/2*M1^2) ) ^ (gamma/(gamma-1));
end

% bottom surface LE oblique shock
    t3 = theta0 + alpha;
    beta3 = TBM_get_beta(Mi,t3,gamma);
    p3 = p * TBM_get_pres(Mi,beta3,gamma);
    Min = Mi * sin(beta3);
    M3n = sqrt( (Min^2 + 2/(gamma-1)) / (2*gamma/(gamma-1)*Min^2-1) );
    M3 = M3n / sin(beta3-t3);

% sample a whole bunch of points for pressure distribution
x = linspace(-1,1,pres_points)';
theta_vect = double(subs(theta,x));

thetas = theta_vect-theta_vect(1);
eqnU = -thetas == PM(mU,gamma) - PM(M1,gamma);
eqnL = -thetas == PM(mL,gamma) - PM(M3,gamma);

MU = struct2cell(vpasolve(eqnU,mU));
ML = struct2cell(vpasolve(eqnL,mL));

m_uu = zeros(length(MU),1);
m_ll = zeros(length(ML),1);

for k = 1:length(MU)
    m_uu(k) = abs(double(MU{k}));
    m_ll(k) = abs(double(ML{k}));
end

pu = p1 .* ( (1+(gamma-1)./2.*  M1.^2) ...
           ./(1+(gamma-1)./2.*m_uu.^2) ) .^ (gamma./(gamma-1));
pl = p3 .* ( (1+(gamma-1)./2.*  M3.^2) ...
           ./(1+(gamma-1)./2.*m_ll.^2) ) .^ (gamma./(gamma-1));

cl = sum( pl.*cos(theta_vect+alpha)...
         -pu.*cos(theta_vect-alpha) )...
    /length(pu)/(0.5*gamma*p*Mi^2);
cd = sum( pl.*sin(theta_vect+alpha)...
         +pu.*sin(theta_vect-alpha) )...
    /length(pu)/(0.5*gamma*p*Mi^2);

cl_vect(i) = cl;
cd_vect(i) = cd;
```

```
end

figure(3)
plot(rad2deg(alpha_vect),cl_vect./cd_vect)
title('$C_L/C_D$ vs. $\alpha$')
xlabel('$\alpha$')
ylabel('$C_L/C_D$')
grid(gca,'minor')
grid on
latexify
```

```
=== Problem 3 ===
```



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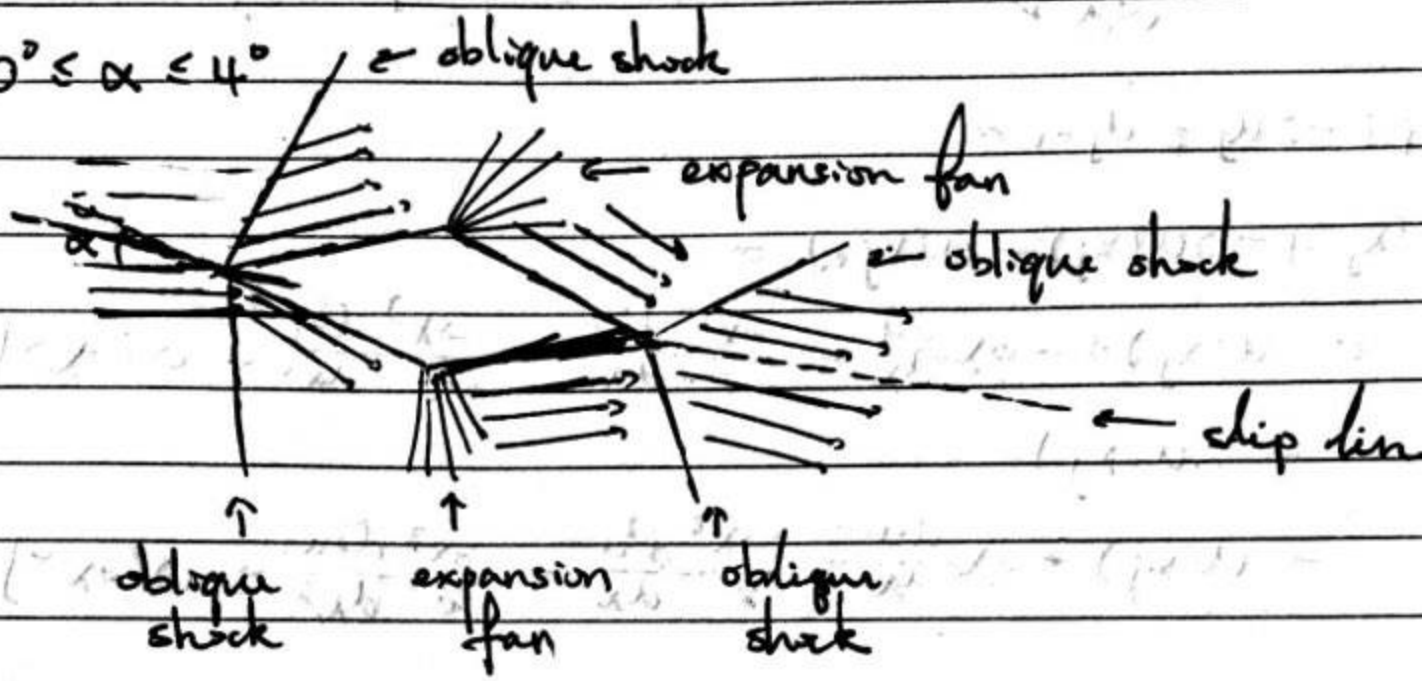
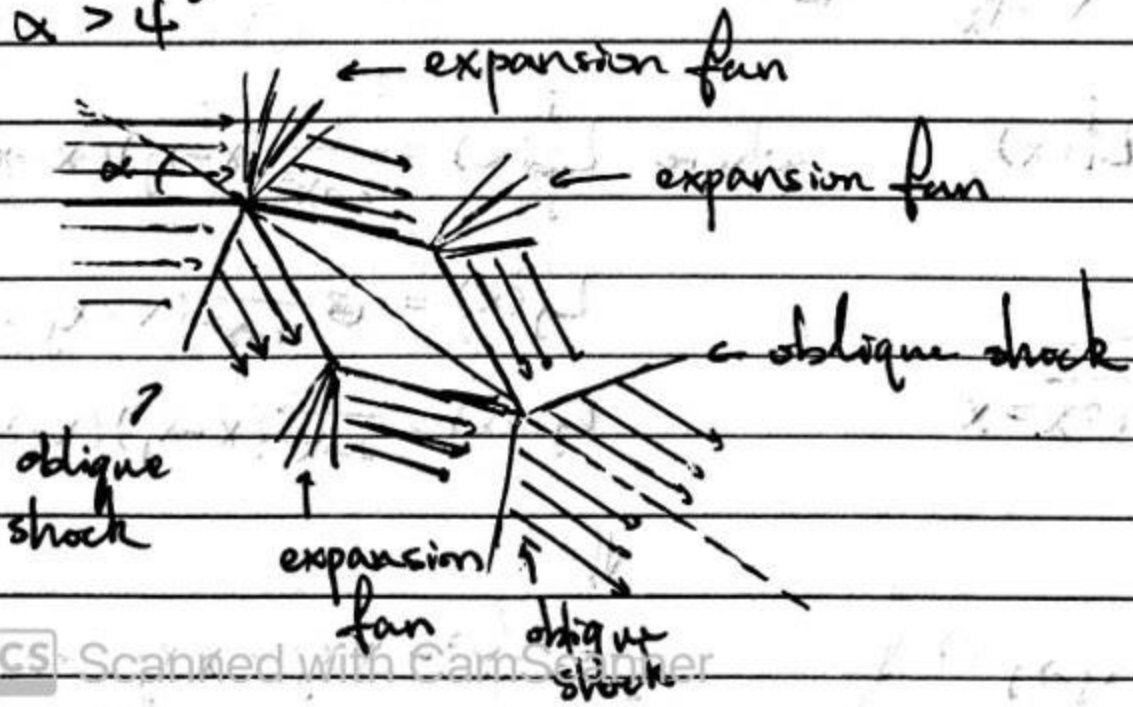
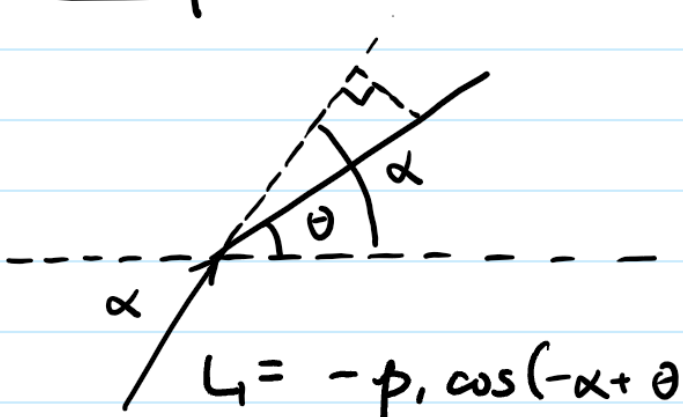
$0° \leq \alpha \leq 4°$



$\alpha > 4°$

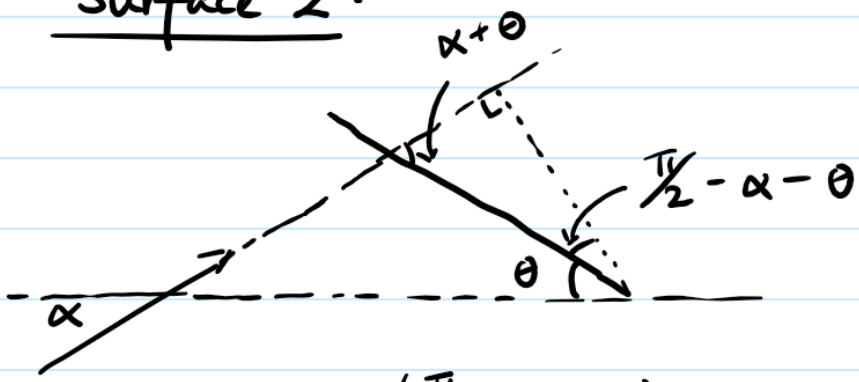


Surface 1:



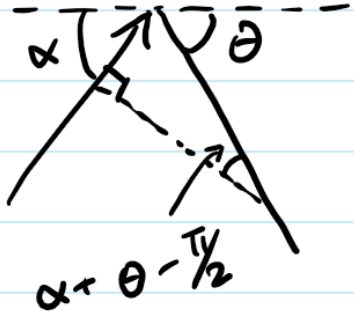
$$L_1 = -p_1 \cos(-\alpha+\theta)$$
$$D_1 = p_1 \sin(-\alpha+\theta)$$

Surface 2:



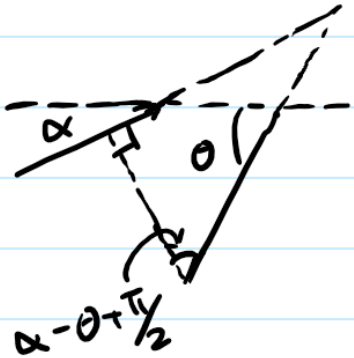
$$L_2 = -p_2 \sin(\pi/2-\alpha-\theta) = -p_2\cos(-\alpha-\theta)$$
$$D_2 = |p_2 \sin(\alpha+\theta)| = p_2 \sin(-\alpha-\theta)$$

Surface 3:



$$L_3 = p_3 \sin(\alpha+\theta-\pi/2) = p_3\cos(\alpha+\theta)$$
$$D_3 = p_3 \cos(\alpha+\theta-\pi/2) = p_3 \sin(\alpha+\theta)$$

Surface 4:



$$L_4 = p_4 \sin(\alpha-\theta+\pi/2) = p_4\cos(\alpha-\theta$$
$$D_4 = p_4 \cos(\alpha-\theta+\pi/2) = p_4 \sin(\alpha-\theta)$$

Summarizing:

$$L = (L_1+L_2+L_3+L_4)c$$
$$D = (D_1+D_2+D_3+D_4)c$$

$$C_l = \frac{2L}{\rho V_\infty^2 c} = \frac{2L}{\gamma p M_\infty^2 c}$$

$$C_d = \frac{2D}{\rho V_\infty^2 c} = \frac{2D}{\gamma p M_\infty^2 c}$$

* 'c' cancels.