ARRAYS

The purpose of this assignment is to give you practice writing programs with arrays.

1. **Discrete distribution.** Write a program `DiscreteDistribution.java` that takes an integer command-line argument m, followed by a sequence of positive integer command-line arguments $a_1, a_2, \ldots, a_n$, and prints m random indices (separated by whitespace), choosing each index i with probability proportional to a_i.

```
~/Desktop/arrays> java DiscreteDistribution 25 1 1 1 1 1 1
5 2 4 4 5 5 4 3 4 3 1 5 2 4 2 6 1 3 6 2 3 2 4 1 4

~/Desktop/arrays> java DiscreteDistribution 25 10 10 10 10 10 50
3 6 6 1 6 6 2 4 6 6 3 6 6 6 6 4 5 6 2 2 6 6 2 6 2

~/Desktop/arrays> java DiscreteDistribution 25 80 20
1 2 1 2 1 1 2 1 1 1 1 1 1 1 2 2 2 1 1 1 1 1 1 1 1

~/Desktop/arrays> java DiscreteDistribution 100 301 176 125 97 79 67 58 51 46
6 2 4 3 2 3 3 1 7 1 1 3 4 7 1 4 2 2 1 1 3 1 8 6 2
1 3 6 1 8 5 1 3 6 1 1 2 3 8 7 4 6 4 3 1 5 3 3 7 3
1 3 1 7 7 2 2 3 6 5 4 1 1 1 7 2 3 5 2 2 1 4 1 2 1
2 1 2 2 3 2 8 4 3 2 1 8 3 5 3 3 8 1 2 3 3 1 2 3 1
```

To generate a random index i with probability proportional to a_i:

- Define the cumulative sums $S_i = a_1 + a_2 + \ldots + a_i$ and $S_0 = 0$.
- Pick a random integer r uniformly between 0 and $S_n - 1$.
- Find the unique index i between 1 and n such that $S_{i-1} \le r < S_i$.

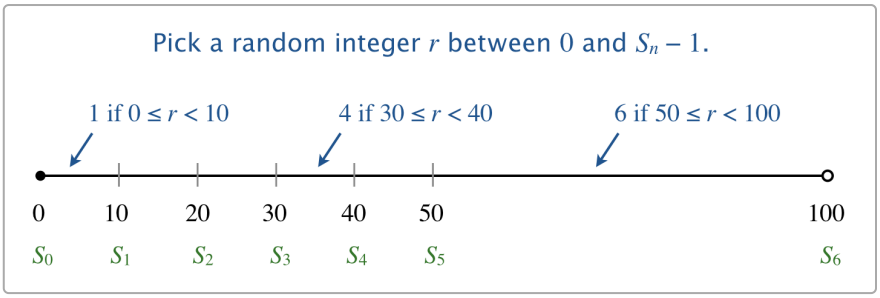
Geometrically, this subdivides the interval $[0, S_n)$ into n subintervals $[S_{i-1}, S_i)$, with the length of subinterval i proportional to a_i. For example, if the discrete distribution is defined by

$$(a_1, a_2, a_3, a_4, a_5, a_6) = (10, 10, 10, 10, 10, 50),$$

then the cumulative sums are

$$(S_1, S_2, S_3, S_4, S_5, S_6) = (10, 20, 30, 40, 50, 100)$$

and the following diagram illustrates the 6 subintervals:



In probability theory, this is known as sampling from a discrete distribution.

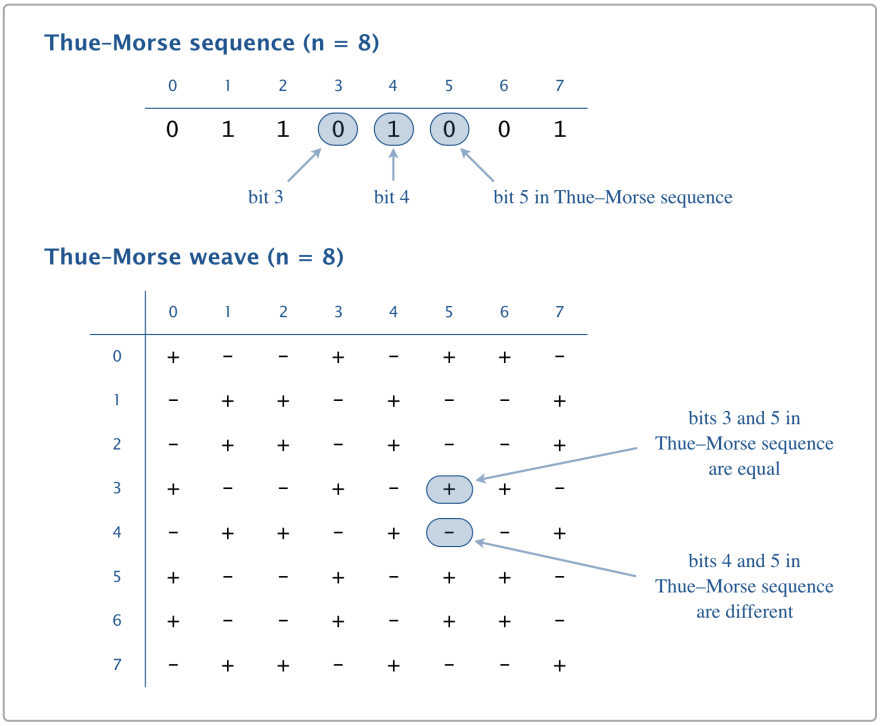
2. **Thue–Morse weave.** Write a program `ThueMorse.java` that an integer command-line argument *n* and prints an *n*-by-*n* pattern like the ones below. Include two space characters between each + and – character.

```
~/Desktop/arrays> java ThueMorse 4
+  –  –  +
–  +  +  –
–  +  +  –
+  –  –  +

~/Desktop/arrays> java ThueMorse 8
+  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +
+  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +
+  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +

~/Desktop/arrays> java ThueMorse 16
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
–  +  +  –  +  –  –  +  +  –  –  +  –  +  +  –
+  –  –  +  –  +  +  –  –  +  +  –  +  –  –  +
```

The *Thue–Morse sequence* is an infinite sequence of 0s and 1s that is constructed by starting with 0 and successively appending the *bitwise negation* (interchange 0s and 1s) of the existing sequence. Here are the first few steps of this construction:

0
0 1
0 1 1 0
0 1 1 0 1 0 0 1
0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0

To visualize the Thue–Morse sequence, create an *n*-by-*n* pattern by printing a + (plus sign) in row *i* and column *j* if bits *i* and *j* in the sequence are equal, and a – (minus sign) if they are different.



Note: you may assume that n is a positive integer (but it need not be a power of 2).

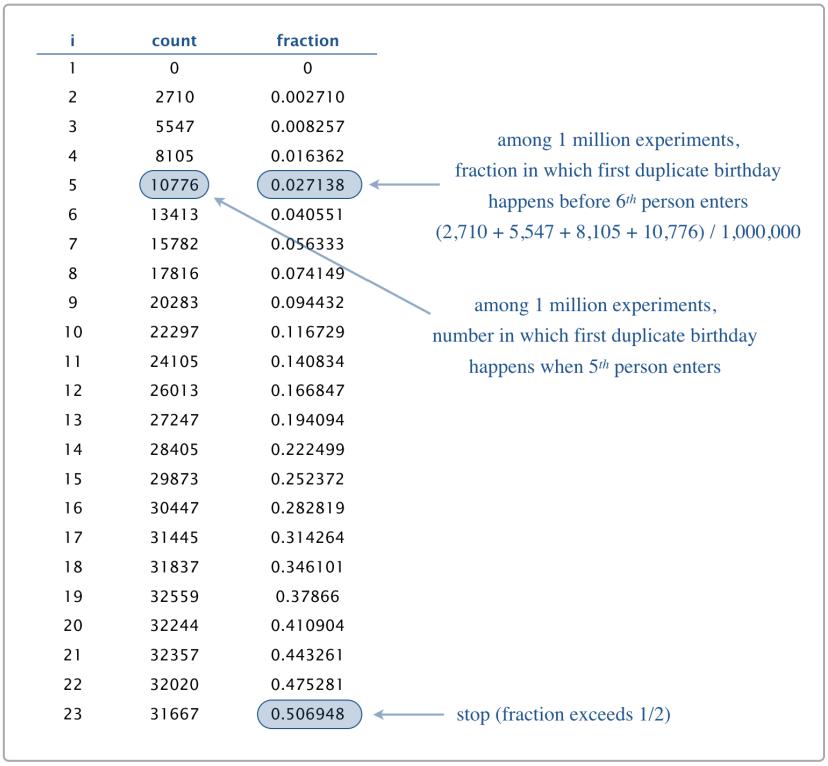
The Thue–Morse sequence has many fascinating properties and arises in graphic design and in music composition.

3. **Birthday problem.** Suppose that people enter a room one at a time. How people must enter until two share a birthday? Counterintuitively, after 23 people enter the room, there is approximately a 50–50 chance that two share a birthday. This phenomenon is known as the *birthday problem* or *birthday paradox*.

   Write a program `Birthday.java` that takes two integer command-line arguments n and *trials* and performs the following experiment, *trials* times:

   - Choose a birthday for the next person, uniformly at random between 0 and $n - 1$.
   - Have that person enter the room.
   - If that person shares a birthday with someone else in the room, stop; otherwise repeat.

   In each experiment, count the number of people that enter the room. Print a table that summarizes the results (the count i, the number of times that exactly i people enter the room, and the fraction of times that i or fewer people enter the room) for each possible value of i from 1 until the fraction reaches (or exceeds) 50%.



i	count	fraction
1	0	0
2	2710	0.002710
3	5547	0.008257
4	8105	0.016362
5	10776	0.027138
6	13413	0.040551
7	15782	0.056333
8	17816	0.074149
9	20283	0.094432
10	22297	0.116729
11	24105	0.140834
12	26013	0.166847
13	27247	0.194094
14	28405	0.222499
15	29873	0.252372
16	30447	0.282819
17	31445	0.314264
18	31837	0.346101
19	32559	0.37866
20	32244	0.410904
21	32357	0.443261
22	32020	0.475281
23	31667	0.506948


```
~/Desktop/arrays> java Birthday 365 1000000
1       0       0.0
2       2710    0.00271
3       5547    0.008257
4       8105    0.016362
5       10776   0.027138
6       13413   0.040551
7       15782   0.056333
8       17816   0.074149
9       20283   0.094432
10      22297   0.116729
11      24105   0.140834
12      26013   0.166847
13      27247   0.194094
14      28405   0.222499
15      29873   0.252372
16      30447   0.282819
17      31445   0.314264
18      31837   0.346101
19      32559   0.37866
20      32244   0.410904
21      32357   0.443261
22      32020   0.475281
23      31667   0.506948

~/Desktop/arrays> java Birthday 31 1000000
1       0       0.0
2       32270   0.03227
3       62580   0.09485
4       87582   0.182432
5       105596  0.288028
6       114427  0.402455
7       115494  0.517949
```

The birthday problem arises in many computer science applications, including hashing, cryptographic attacks, and testing random number generators.

4. **Minesweeper.** *Minesweeper* is a 1960s era video game played on an *m*-by-*n* grid of cells. The goal is to deduce which cells contain hidden mines using clues about the number of mines in neighboring cells. Write a program `Minesweeper.java` that takes three integer command-line arguments *m*, *n*, and *k* and prints an *m*-by-*n* grid of cells with *k* mines, using asterisks for mines and integers for the neighboring mine counts (with two space characters between each cell). To do so,

    - Generate an *m*-by-*n* grid of cells, with exactly *k* of the *mn* cells containing mines, uniformly at random.
    - For each cell not containing a mine, count the number of neighboring mines (above, below, left, right, or diagonal).

```
~/Desktop/arrays> java Minesweeper 9 9 10
0  1  *  1  0  0  0  1  *
1  3  2  2  0  0  0  1  1
*  2  *  1  0  0  1  1  1
1  2  2  2  1  0  1  *  1
0  1  2  *  1  0  1  1  1
1  2  *  3  3  1  1  0  0
1  *  3  *  2  *  1  0  0
1  1  2  1  2  1  1  0  0
0  0  0  0  0  0  0  0  0
```

Submission. Submit a `.zip` file containing `DiscreteDistribution.java`, `ThueMorse.java`, `Birthday.java`, and `Minesweeper.java`. You may not call library functions except those in the `java.lang` (such as `Integer.parseInt()` and `Math.sqrt()`). Use only Java features that have already been introduced in the course (e.g., loops and arrays, but not functions).

This assignment was developed by Kevin Wayne.