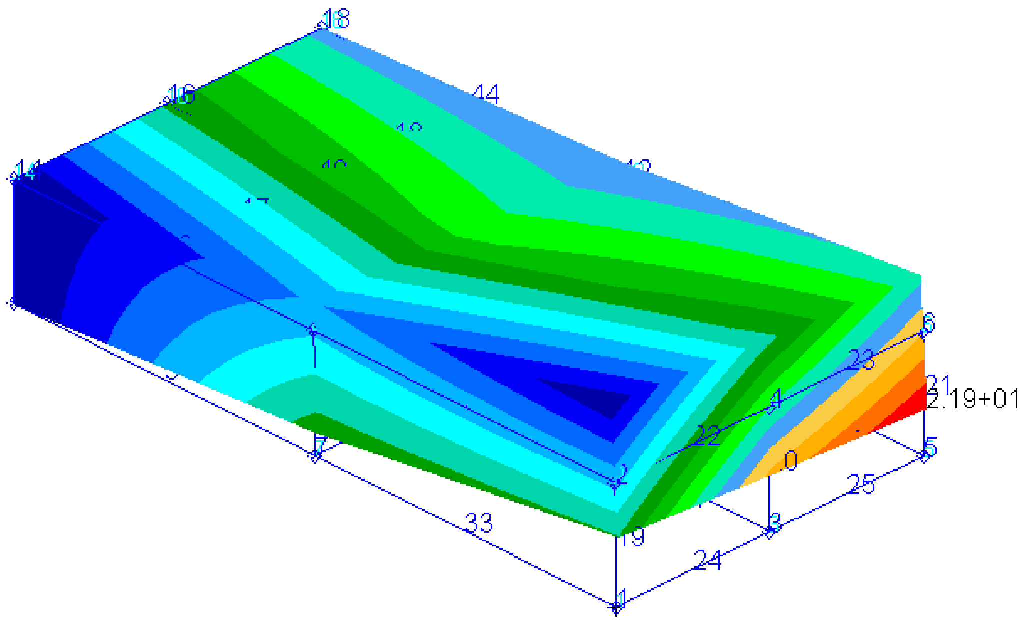
Homework 10 Stress Report



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Executive Summary

Finite-element post-processing of the two-cell, two-bay cantilever box beam confirms that the structure transfers the 1000 lb point load exactly as anticipated. Peak values collected from the stress plots (Figures 6.1.0.1 –6.2.0.5) are compiled below.

Table 0.0.1 Peak shear stresses and flows in CQUAD4 panels

Member set	Critical element(s)	τ (psi)	q (lb/in)
Shear Panels (Elm 1-18)	10	794.73	63.58

Table 0.0.2 Peak axial stresses and forces in CROD bars

Member set	Critical element(s)	σ (psi)	P (lb)
Stringers (Elm 33–44)	34 (Tension) / 36 (Compression)	1 660.30	1 660.30
Cap rods (Elm 19–25)	19	500	500
	23	108.47	108.47

Table 0.0.3 Static-equilibrium residuals for the full model

Residual	$\sum F_x$ (lb)	$\sum F_y$ (lb)	$\sum M_z$ (lb-in)
Numerical value	≈ 0	≈ 0	≈ 0

The tables demonstrate that:

1. Forward-bay panels govern the web behaviour, registering the highest shear stress of 794.73 psi and a peak flow of 63.58 lb/in.
2. Cap rod Element 19 is the most-stressed bar at 500 psi, while stringer Elements 34 and 36 top the axial charts at 1660.30 psi in tension and compression, respectively.
3. Force and moment residuals are effectively zero, confirming that the FEM satisfies global equilibrium under the applied load and boundary conditions.

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1. Introduction

This problem involves the application of the displacement method to calculate the shear flow in a two-bay cantilevered box beam. The beam is subjected to a uniform load and consists of 18 shear panels and 26 rod elements. The primary objective is to determine the shear flow in the shear panels and axial loads in the longitudinal and transverse rods of the structure, using PATRAN [1]. The structure's load distribution and geometrical configuration are provided in the example problem [2], with the beam's dimensions and material properties detailed in the problem statement.

2. Geometry

Table 2.0.1 Finite-element breakdown for the cantilever box-beam model

Element set	ID range	Count	FE type	Thickness / Area
Shear panels	1–18	18	`CQUAD4`	$t = 0.08\,\text{in}$
Cap rods	19–25	7	`CROD`	$A = 1.00\,\text{in}^2$
Center rods	26–32	7	`CROD`	$A = 0.0064\,\text{in}^2$
Stringers	33–44	12	`CROD`	$A = 1.00\,\text{in}^2$

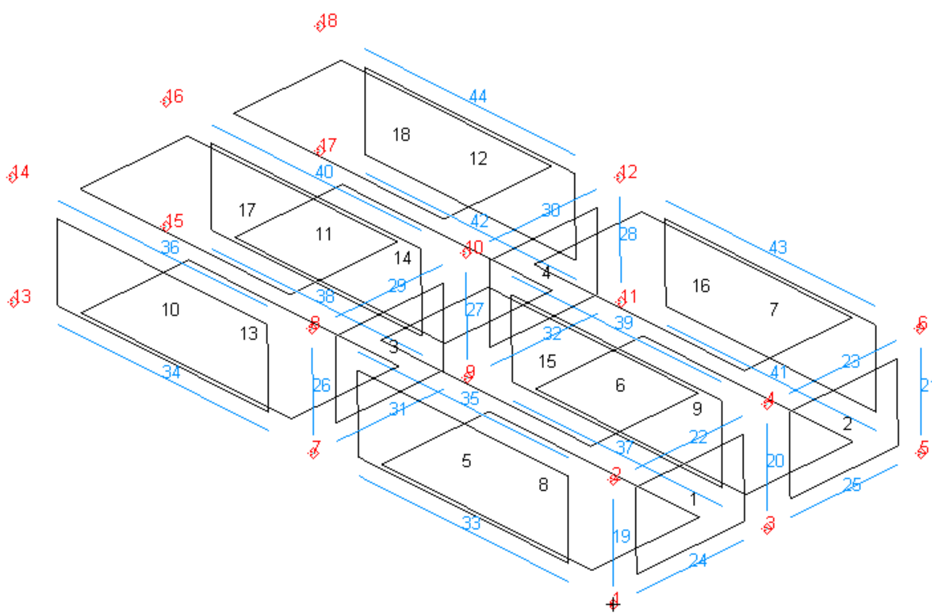
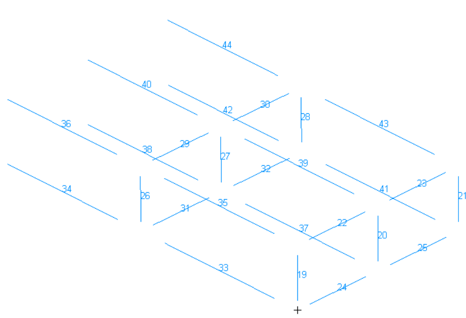
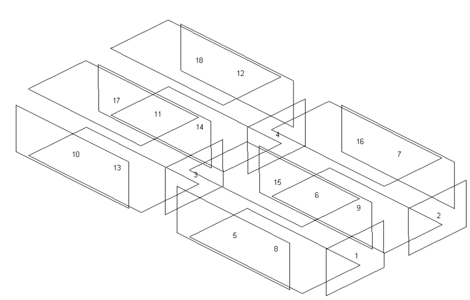


Fig. 2.0.0.1 Exploded View of Model



(a) Stringers & Rods



(b) Shear Panels

3. Loading and Boundary Conditions

3.1 Support Conditions

The two-cell, two-bay cantilever box beam is root-fixed at five adjacent nodes (Nodes 13–17), representing a rigid attachment to the wing spar carry-through. All six degrees of freedom—three translations and three rotations—are constrained at each node (Table 3.1.1).

Table 3.1.1 Boundary-condition summary

Node	u_x	u_y	u_z	θ_x	θ_y	θ_z
13–17	0	0	0	0	0	0

3.2 Applied Loads

A single point load of $P = 1000$ lb is applied at Node 1 in the positive $+z$ direction, producing upward bending in the box beam (Table 3.2.1). No additional distributed or inertial loads are considered for the present linear-static analysis.

Table 3.2.1 Nodal load definition

Node	$\mathbf{F_z}$ [lb]
1	+1000

4. Materials

The cantilever box beam is analyzed using a single, isotropic, linear-elastic material. The material is assumed to be homogeneous throughout the structure and non-linear properties such as plasticity or creep are neglected in this analysis. Its mechanical constants are summarized in Table 4.0.1 .

Table 4.0.1 Assumed mechanical properties of the structural material

Property	Value
Young's modulus, E	1.0×10^7 psi
Poisson's ratio, ν	0.25

5. Finite Element Model

The cantilever box beam has been modeled with a minimalist element palette to capture the critical load paths. As summarized in Table 5.0.1 , the skin webs are modeled with 18 `CQUAD4` shear panels (Elements 1–18), while the axial load carriers are represented by 26 `CROD` bars: cap rods and stringers (Elements 19–25 and 33–44) use a full 1.00 in^2 area, whereas the rods along the center cap (Elements 26–32) are slimmed down to 0.0064 in^2. All elements share a single isotropic, linear-elastic material ($E = 1.0 \times 10^7$ psi, $\nu = 0.25$).

Figure 5.0.0.1 complements Table 5.0.1 with an exploded wireframe of the finite-element model, showing Element ID numbers, constraints, and the applied point load.

Table 5.0.1 Concise definition of the FE model

Aspect	Details
Element types	18 × `CQUAD4` shear panels, $t = 0.08$ in
	26 × `CROD` bars:
	⇒ Cap Rods (Elm 19-25) & Stringers (Elm 33-44): $A = 1.00$ in^2
	⇒ Center rods (Elm 26-32): $A = 0.0064$ in^2
Material	Isotropic, linear elastic;
	$E = 1.0 \times 10^7$ psi, $\nu = 0.25$
Boundary conditions	Nodes 13–17 fixed in all 6 DOF (cantilever root)
Loading	+1000 lb at Node 1 in global $+z$ direction (LC-01)

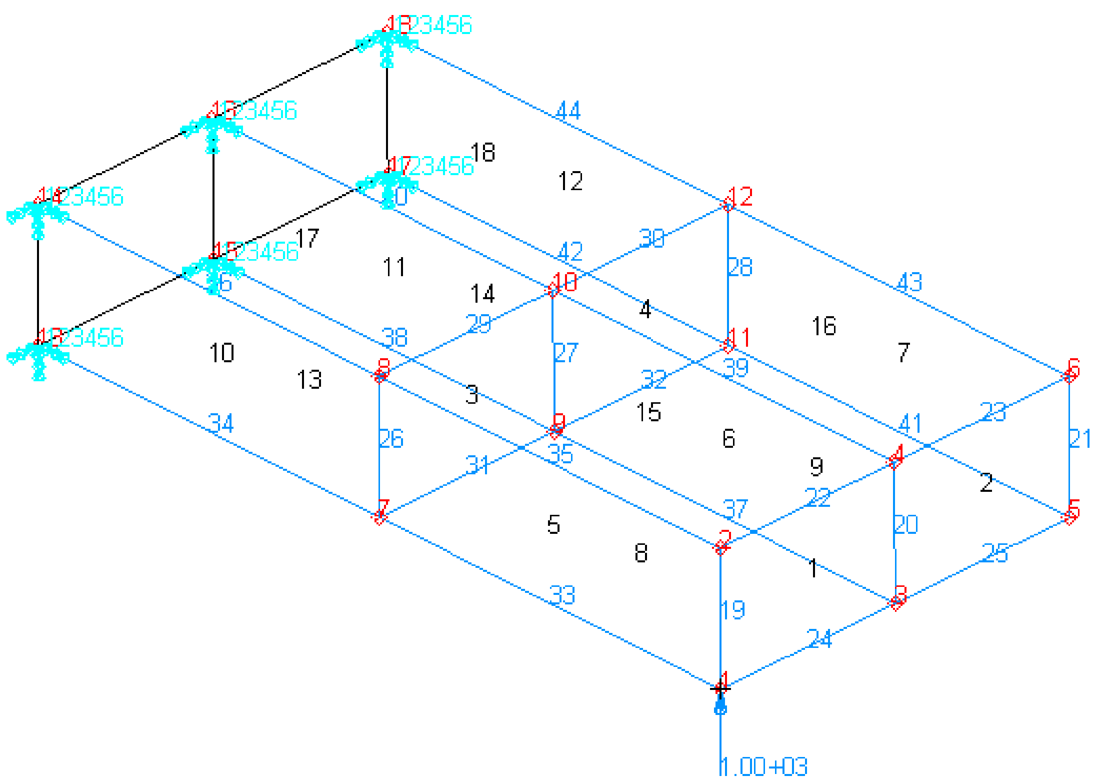


Fig. 5.0.0.1 Finite Element Model

6. Results

6.1 Shear Flow in Panels

For thin-walled shear panels, the shear flow q is related to the in-plane shear stress and the panel thickness as:

$$q = \tau \cdot t$$

where:

- τ is the average shear stress in the panel (from F06 results),
- t is the thickness of the panel.

The stresses in the shear panels are shown in Figure 6.1.0.1 , with average shear stresses in the forward bay highlighted in yellow, and the average shear stresses in the rear bay highlighted in blue.

S T R E S S E S   I N   S H E A R   P A N E L S   (C S H E A R)

ELEMENT ID.	MAX SHEAR	AVG SHEAR	SAFETY MARGIN	ELEMENT ID.	MAX SHEAR	AVG SHEAR	SAFETY MARGIN
1	4.998890E+02	-4.998890E+02		2	1.150987E+02	-1.150987E+02	
3	4.461595E+01	4.461595E+01		4	5.645013E+01	5.645013E+01	
5	7.501110E+02	-7.501110E+02		6	3.847903E+02	-3.847903E+02	
7	1.150987E+02	-1.150987E+02		8	3.191080E+02	3.191080E+02	
9	2.958797E+02	2.958797E+02		10	7.947270E+02	-7.947270E+02	
11	3.966245E+02	-3.966245E+02		12	5.864857E+01	-5.864857E+01	
13	2.731469E+02	-2.731469E+02		14	2.407747E+02	-2.407747E+02	
15	3.191080E+02	-3.191080E+02		16	2.958797E+02	-2.958797E+02	
17	2.731469E+02	-2.731469E+02		18	2.407747E+02	-2.407747E+02	

Fig. 6.1.0.1 Stresses in Shear Panels

Calculating the shear flows in each bay results in the following figures. Figure 6.1.0.2 showing the shear flows in each bay, and Figure 6.1.0.3 showing the shear flow in every panel.

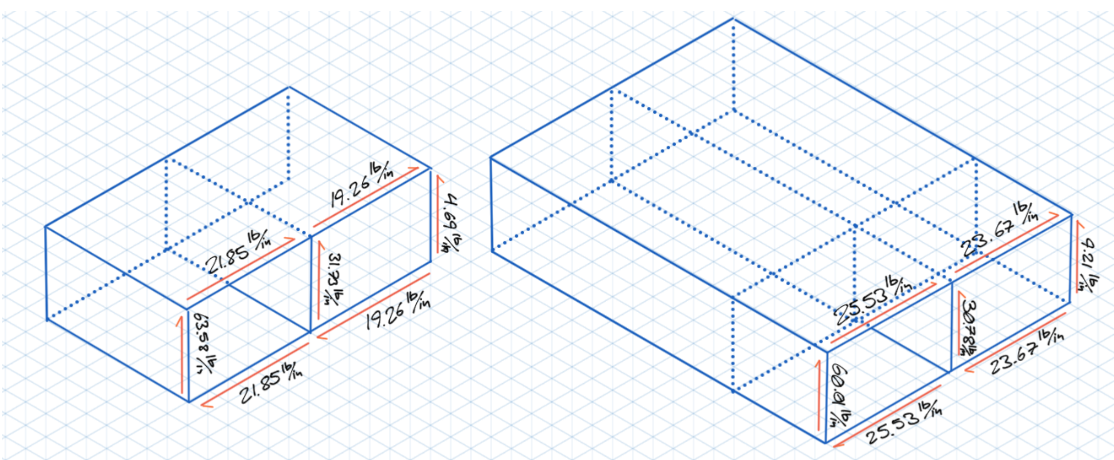


Fig. 6.1.0.2 Shear Flow in each Bay

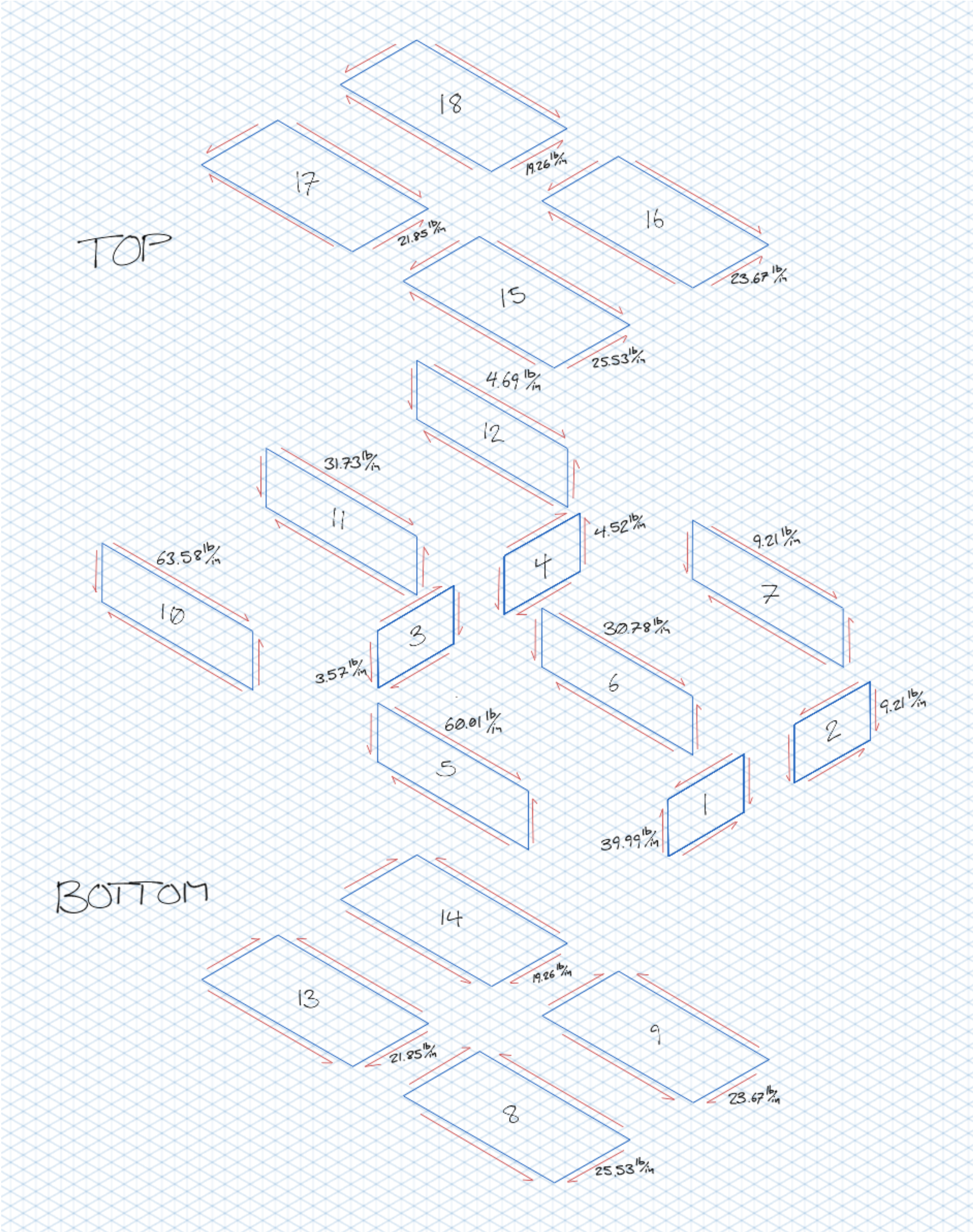


Fig. 6.1.0.3 Shear Flow in Panels

6.2 Axial Stress in Rods and Stringers

Axial stress σ in rod or stringer elements is related to axial force F through:

$$\sigma = \frac{F}{A}$$

The axial stresses in each rod member are shown in Figure 6.2.0.1 , where Elements 19-32 are rods, and Elements 33-44 are stringers.

S T R E S S E S   I N   R O D   E L E M E N T S   (C R O D)

ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN	ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN
19	-5.000000E+02		0.0		20	-7.275958E-12		0.0	
21	-7.275958E-12		0.0		22	-1.084686E+02		0.0	
23	-1.084686E+02		0.0		24	1.084686E+02		0.0	
25	1.084686E+02		0.0		26	3.637979E-12		0.0	
27	0.0		0.0		28	9.094947E-12		0.0	
29	-1.261052E+02		0.0		30	-1.261052E+02		0.0	
31	1.261052E+02		0.0		32	1.261052E+02		0.0	
33	5.172036E+02		0.0		34	1.660303E+03		0.0	
35	-5.172036E+02		0.0		36	-1.660303E+03		0.0	
37	4.896223E+02		0.0		38	1.494041E+03		0.0	
39	-4.896223E+02		0.0		40	-1.494041E+03		0.0	
41	4.931741E+02		0.0		42	1.345656E+03		0.0	
43	-4.931741E+02		0.0		44	-1.345656E+03		0.0	

Fig. 6.2.0.1 Axial Stress in Rods

The Stringers with the largest loads applied are highlighted in yellow, with Element 34 being in tension and Element 36 being in compression. The Rod with the largest load applied is Element 19, highlighted in blue. Using the stresses highlighted in Figure 6.2.0.1 , Figure 6.2.0.2 and Figure 6.2.0.3 were created, showing all stresses in the Elements 19-44.

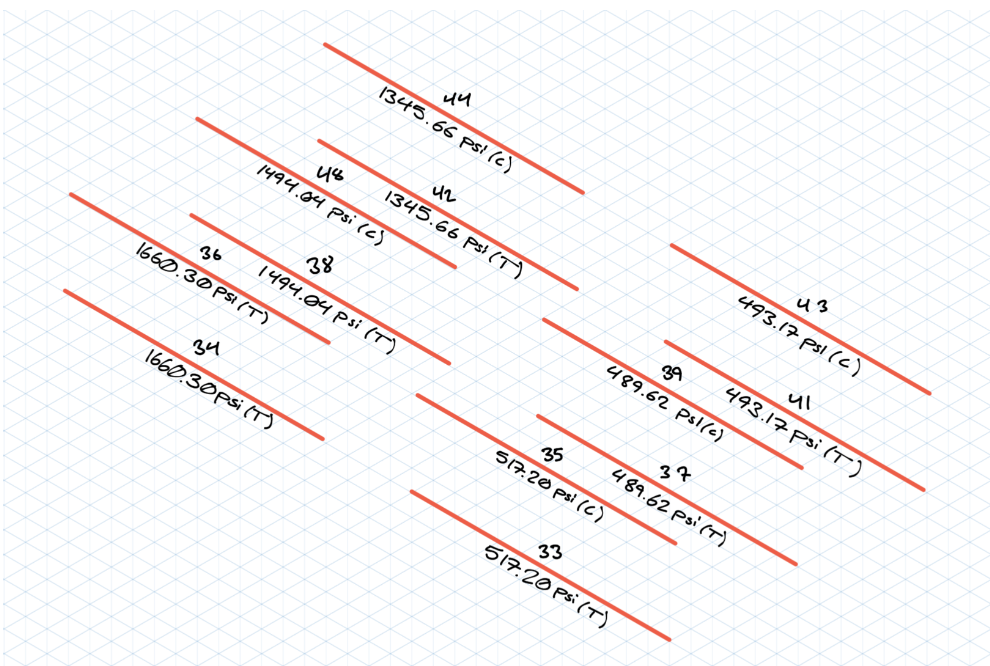


Fig. 6.2.0.2 Stresses in Stringers

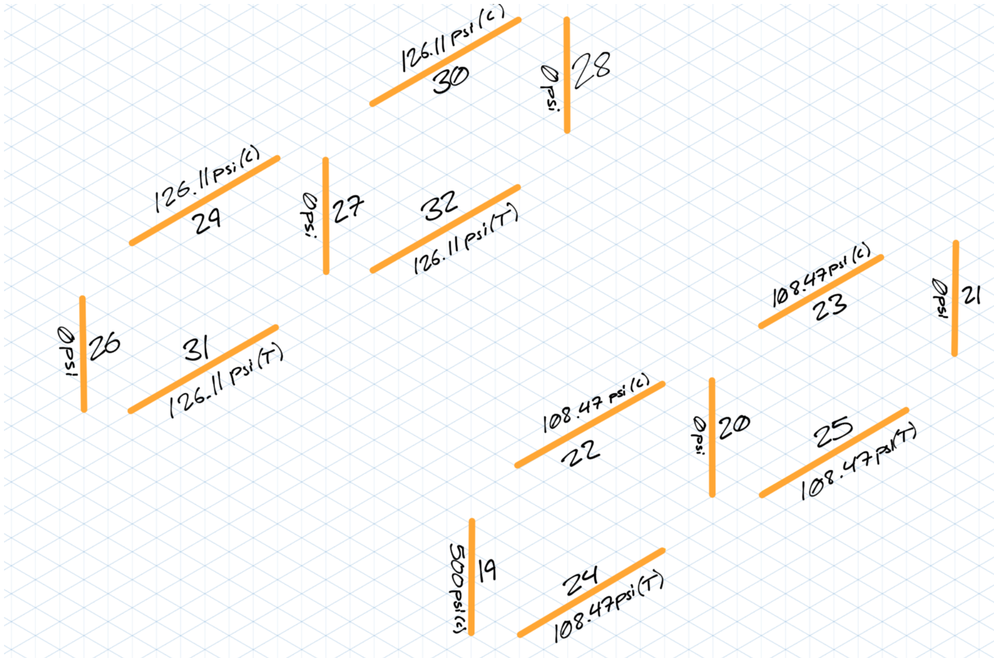


Fig. 6.2.0.3 Stresses in Rods

Solving for internal force:

$$F = \sigma \cdot A$$

where:

- σ is the axial stress reported in rod elements,
- A is the cross-sectional area of the rod or stringer,
- F is the internal axial force.

Calculating force using the equation above produces the Figures 6.2.0.4 and 6.2.0.5 on the target elements.

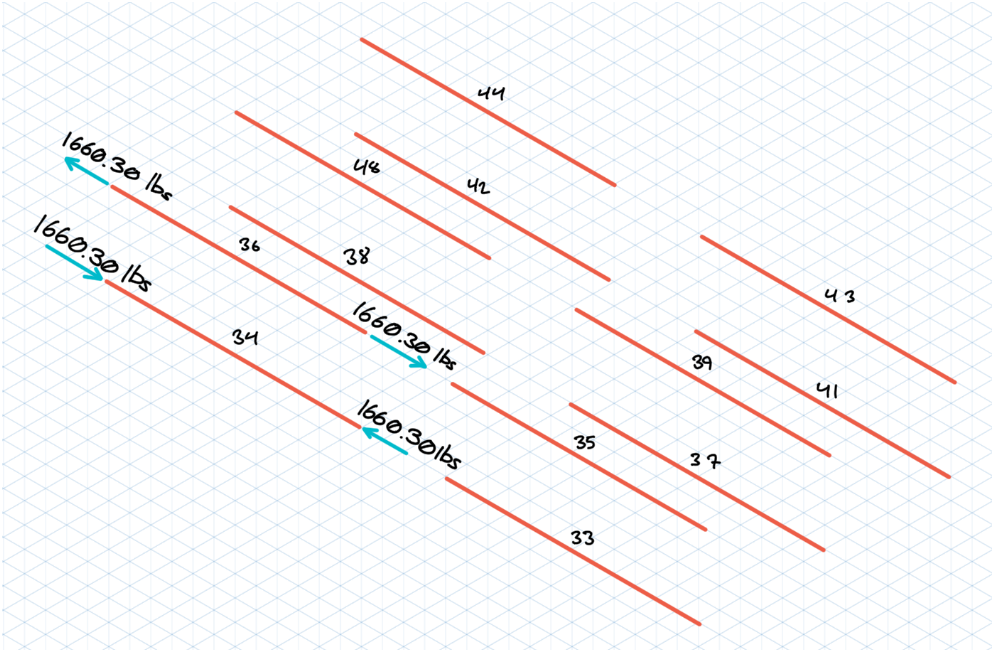


Fig. 6.2.0.4 Forces on Target Stringers

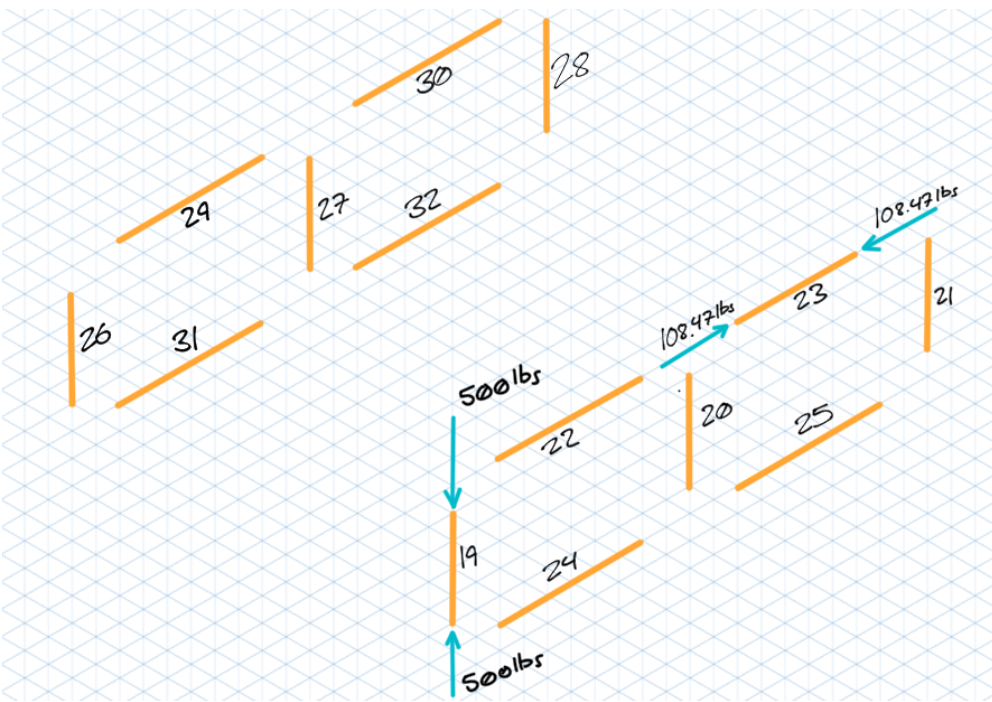


Fig. 6.2.0.5 Forces on Target Rods

6.3 Equilibrium Verification

The reaction forces reported in the F06 file for the six root nodes (Nodes 13–18) are reproduced in Table 6.3.1 . Summing these forces and comparing them with the external +1 000 lb point load applied at Node 1 confirms that the model satisfies global equilibrium.

Table 6.3.1 Constraint forces extracted from the `F06` file

Node	F_x (lb)	F_y (lb)	F_z (lb)
13	163.8882	2 286.199	−317.8908
14	−163.8882	−2 286.199	−317.8908
15	308.3530	2 008.837	−158.6498
16	−308.3530	−2 008.837	−158.6498
17	144.4648	1 704.964	−23.4594
18	−144.4648	−1 704.964	−23.4594
Totals	**0.0**	**0.0**	**−1 000.0**

Applying the static–equilibrium relations

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z + P_z = 0$$

$$0 + 0 = 0, \qquad 0 + 0 = 0, \qquad (-1\,000) + (+1\,000) = 0$$

confirms that the internal reactions exactly counterbalance the external +1 000 lb load in the global $+z$ direction.

7. Summary

This problem applies computational methods to calculate the shear flow in a two-bay cantilevered box beam, subjected to a point load. The beam consists of 18 shear panels and 26 rod elements. By using this method, the shear stresses and internal forces in the structure can be determined, helping to assess the beam's structural integrity under the applied load. The problem involves detailed calculations based on the beam's geometry, material properties, and load distribution to evaluate how the stresses are distributed across the shear panels and rods.

Appendix

Exerpts from F06 File

STRESSES IN SHEAR PANELS (CSHEAR)

ELEMENT ID.	MAX SHEAR	AVG SHEAR	SAFETY MARGIN	ELEMENT ID.	MAX SHEAR	AVG SHEAR	SAFETY MARGIN
1	4.998890E+02	-4.998890E+02		2	1.150987E+02	-1.150987E+02	
3	4.461595E+01	4.461595E+01		4	5.645013E+01	5.645013E+01	
5	7.501110E+02	-7.501110E+02		6	3.847903E+02	-3.847903E+02	
7	1.150987E+02	-1.150987E+02		8	3.191080E+02	3.191080E+02	
9	2.958797E+02	2.958797E+02		10	7.947270E+02	-7.947270E+02	
11	3.966245E+02	-3.966245E+02		12	5.864857E+01	-5.864857E+01	
13	2.731469E+02	-2.731469E+02		14	2.407747E+02	-2.407747E+02	
15	3.191080E+02	-3.191080E+02		16	2.958797E+02	-2.958797E+02	
17	2.731469E+02	-2.731469E+02		18	2.407747E+02	-2.407747E+02	

Panel Stress F06

STRESSES IN ROD ELEMENTS (CROD)

ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN	ELEMENT ID.	AXIAL STRESS	SAFETY MARGIN	TORSIONAL STRESS	SAFETY MARGIN
19	-5.000000E+02		0.0		20	-7.275958E-12		0.0	
21	-7.275958E-12		0.0		22	-1.084686E+02		0.0	
23	-1.084686E+02		0.0		24	1.084686E+02		0.0	
25	1.084686E+02		0.0		26	3.637979E-12		0.0	
27	0.0		0.0		28	9.094947E-12		0.0	
29	-1.261052E+02		0.0		30	-1.261052E+02		0.0	
31	1.261052E+02		0.0		32	1.261052E+02		0.0	
33	5.172036E+02		0.0		34	1.660303E+03		0.0	
35	-5.172036E+02		0.0		36	-1.660303E+03		0.0	
37	4.896223E+02		0.0		38	1.494041E+03		0.0	
39	-4.896223E+02		0.0		40	-1.494041E+03		0.0	
41	4.931741E+02		0.0		42	1.345656E+03		0.0	
43	-4.931741E+02		0.0		44	-1.345656E+03		0.0	

Rod Stress F06

FORCES OF SINGLE-POINT CONSTRAINT

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	G	1.638882E+02	2.286199E+03	-3.178908E+02	0.0	0.0	0.0
14	G	-1.638882E+02	-2.286199E+03	-3.178908E+02	0.0	0.0	0.0
15	G	3.083530E+02	2.008837E+03	-1.586498E+02	0.0	0.0	0.0
16	G	-3.083530E+02	-2.008837E+03	-1.586498E+02	0.0	0.0	0.0
17	G	1.444648E+02	1.704964E+03	-2.345943E+01	0.0	0.0	0.0
18	G	-1.444648E+02	-1.704964E+03	-2.345943E+01	0.0	0.0	0.0

Reaction Forces

References

[1] "MSC Patran," , 2024.

[2] Curtis, H. D., *Fundemaentals of Aircraft Structures Analysis*, 1999, Chap. 11.