State Observer Feedback Control with Internal Model

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1 Introduction

In the midterm exam, we are asked to design a state observer feedback controller with an internal model for the galvanometer to track 400 Hz sinewave signal based on the model we identified in Lab01.

This report is organized as follows. Section 2 briefly introduces the full-order state observer, state feedback controller, and internal model principle. Section 3 elaborates on how we design a state observer feedback controller to track a step signal and the 400 Hz sinewave in the simulation environment. Furthermore, to increase the performance of tracking a 400 Hz sinewave, we add an internal model in the feedback loop. In Section 4, we implement our controller on the real galvanometer system. Finally, we draw some conclusions in the last section.

2 State Observer Feedback control with Internal Model

The block diagram of typical state observer feedback control with an internal model is shown in Fig.1. This diagram includes four parts

1. Original system
2. Full-order state observer
3. State feedback controller
4. Internal model

The original system is a closed-loop stable galvanometer scanner system. We defined it as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

The other parts will be illustrated in the following section.

2.1 Full-order state observer

The typical state observer can be expressed as

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \tag{2}$$

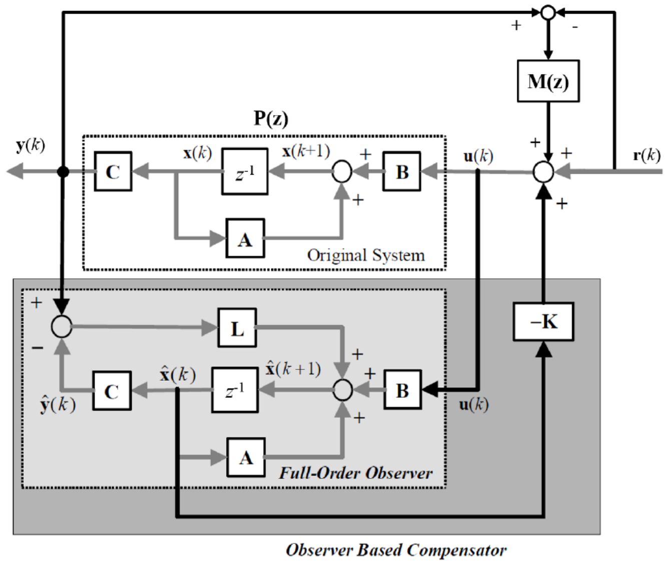


Fig. 1: Block diagram of typical state observer feedback control [1]

where $\hat{x}$ and $\hat{y}$ are the estimated states and output respectively, and L is observer gain. Here we define the estimated error as follows,

$$e = x - \hat{x} \tag{3}$$

We subtract Eqn.1 with Eqn.2, then we can obtain the following estimated error dynamic,

$$\dot{e} = (A - LC)e \tag{4}$$

Finally, we can design an observer gain L such that $(A - LC)$ is Hurwitz, which means that the $\hat{x}$ will approach x, as long as (A, C^T) is observable.

Here we provide three skills to make our observer work not only in the simulation environment but also in the experiment.

1. When L is larger, the converging rate of the error is faster, however, the larger L also means that the observer will easily affected by the disturbances and noises.
2. When implementing the state observer feedback controller, the observer gain(L) should be larger(typically 10 times) than the feedback gain(K). Detailed information about the state feedback controller will be elaborated on in the next section.
3. We should collect the step response data from the real system and ensure that the $\hat{y}$ aligns with the data with the designed observer.

2.2 State feedback controller

A state feedback controller can be represented as

$$u = -Kx \tag{5}$$

where K is the feedback gain. We can substitute Eqn.5 into Eqn.1, then obtaining the closed-loop system

$$\dot{x} = (A - BK)x \tag{6}$$

With the same manner as designing the observer gain, we can design a feedback gain K such that $(A - BK)$ is Hurwitz. It's worth mentioning that there are lots of methods to design the K or L, such as pole placement and LQR. In this report, we adopt the pole placement method.

2.3 Internal model

In plain words, the spirit of the internal model principle is *"Any good tracking controller must stabilize the closed-loop system and must contain a model of the reference signal"*[1]. Following the materials provided by the professor, the internal model can be generalized as

$$x_m[k+1] = A_m x_m[k] + B_m[y[k] - r[k]] \tag{7}$$

$$u_m[k] = -K_m x_m[k] \tag{8}$$

For a integral action,

$$A_m = 1, B_m = 1 \tag{9}$$

For a peak filter,

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & 2cos(\omega_0) \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{10}$$

Where $\omega_0 = f/f_s \cdot 2\pi$ is the normalized frequency, f is the targeted reference frequency, and f_s is the sampling frequency. The state observer feedback law with internal model is defined in Eqn.(11)

$$u[k] = -K\hat{x}[k] + u_m[k] + r[k] \tag{11}$$

3 Simulation

Following the "DAVI" guidelines, we do the simulation as a verification before the implementation. Fig.2 shows the system block diagram made by Matlab and Simulink. Then, we verify our controller by observing the performance of the step response and tracking a 400 Hz sinewave.

3.1 Step response

We use the pole placement method to design feedback gain K and observer gain L, After the trial and error process, the location of poles of (A-LC) and (A-BK) is assigned at

$$\text{poles}_{(A-LC)} = \begin{bmatrix} 0.85\ 0.8\ 0.1\ 0.11\ 0.15 \end{bmatrix} \tag{12}$$

$$\text{poles}_{(A-BK)} = \begin{bmatrix} 0.5\ 0.55\ 0.62\ 0.63\ 0.6 \end{bmatrix} \tag{13}$$

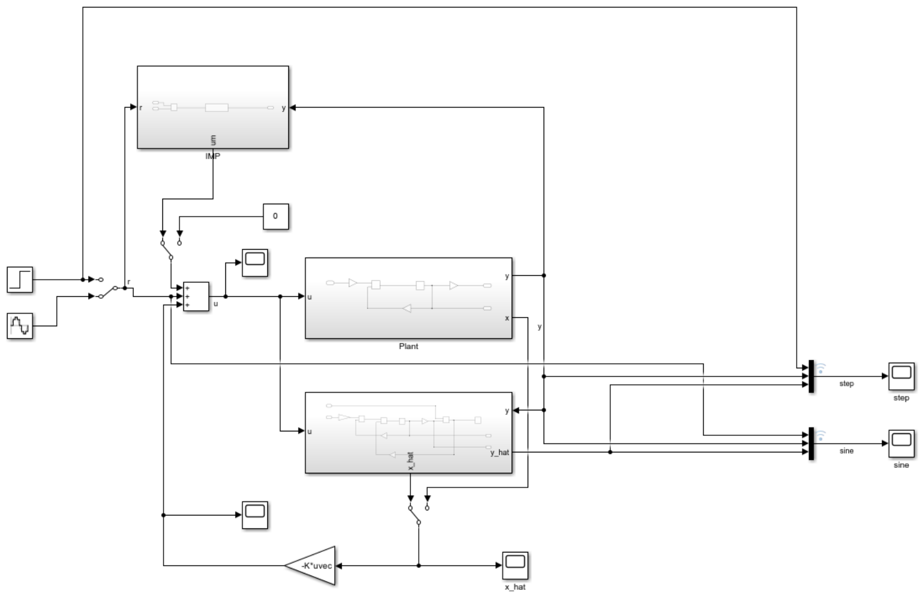


Fig. 2: System block diagram in Simulink

and K and L are,

$$K = \begin{bmatrix} 0.5450 & -1.4578 & 1.4289 & -0.6525 & 0.1360 \end{bmatrix} \tag{14}$$

$$L = \begin{bmatrix} 8.7637 & 9.3032 & 10.3930 & 13.6338 & 18.2859 \end{bmatrix} \tag{15}$$

We first check that L is roughly 10 times larger than K; as a result, it is a reasonable design. Next, we plot out the $\hat{y}$ based on the output $y_{experiment}$ collected from the real galvanometer system, and the result is shown in Fig.3. Comparing it with a very extreme case of the pole location. Let's say

$$\text{poles}_{(A-LC)_{ex}} = \begin{bmatrix} 0.1 & 0.11 & 0.12 & 0.13 & 0.14 \end{bmatrix} \tag{16}$$

$$\tag{17}$$

The L_{ex} is

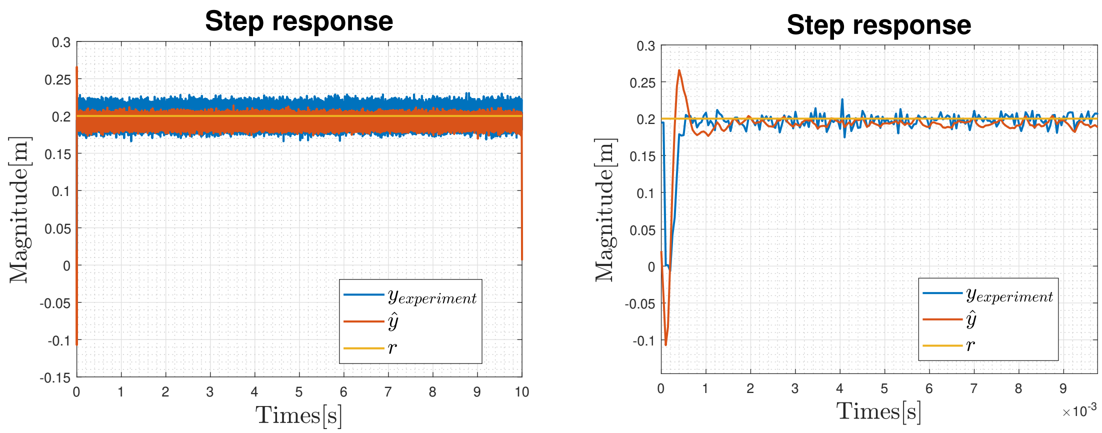
$$L_{ex} = 10^3 \cdot \begin{bmatrix} 0.892 & 1.003 & 1.129 & 1.272 & 1.438 \end{bmatrix} \tag{18}$$

$$\tag{19}$$

The result is shown in Fig.(4). It is easy to observe a drastic oscillation in the transient and steady state because the large L is easily affected by the noises of the data. The step response of the state observer feedback closed-loop system is shown in Fig.5, A steady-state error exists, and it is roughly 0.001.

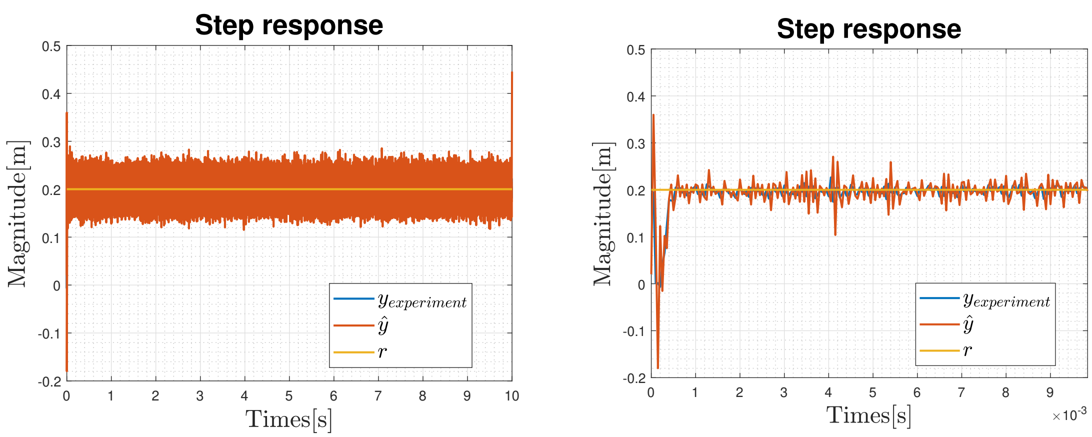
3.2 400 Hz sinewave

Next, we try to use the same feedback gain K to track a 400 Hz sinewave, and the result is shown in Fig.6. From Fig.6b, the $\hat{y}$ has a slight discrepancy with y initially, resulting from the different initial



(a) $y_{experiment}$ vs $\hat{y}$ (b) $y_{experiment}$ vs $\hat{y}$: zoomin

Fig. 3: Open-loop step response with L : $y_{experiment}$ vs $\hat{y}$

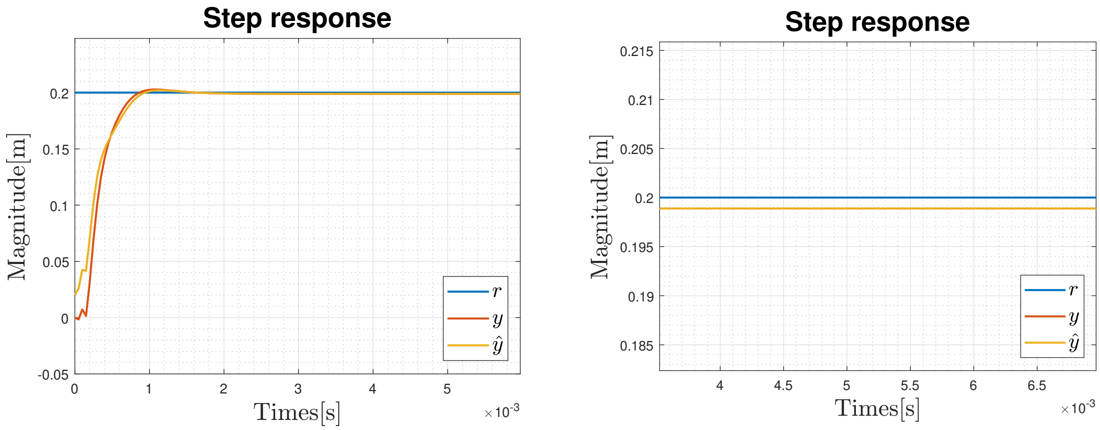


(a) $y_{experiment}$ vs $\hat{y}$ (b) $y_{experiment}$ vs $\hat{y}$: zoomin

Fig. 4: Open-loop step response with L_{ex} : $y_{experiment}$ vs $\hat{y}$

values between the real system and the state observer. However, the error converges to zero after passing a little time. On the other hand, there is a huge phase delay and a steady state error in our tracking results, as a result, we try to eliminate these phenomena by adding an internal model into our controller.

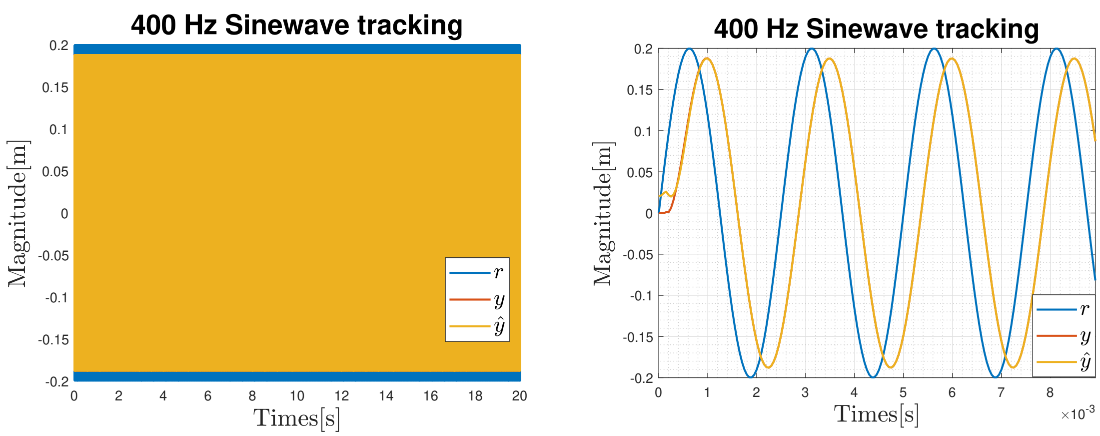
Because our control objective is to track a 400 Hz sinewave, we add a peak filter (10) at 400Hz where $\omega_0 = 400/f_s \cdot 2\pi$, and $f_s = 2 \cdot 10^5$. To obtain K and K_m, we augment the plant and internal



(a) Closed-loop step response

(b) Closed-loop step response: steady-state

Fig. 5: Closed-loop step response with K and L



(a) Sinewave tracking

(b) Sinewave tracking: zoom-in

Fig. 6: 400 Hz sinewave tracking

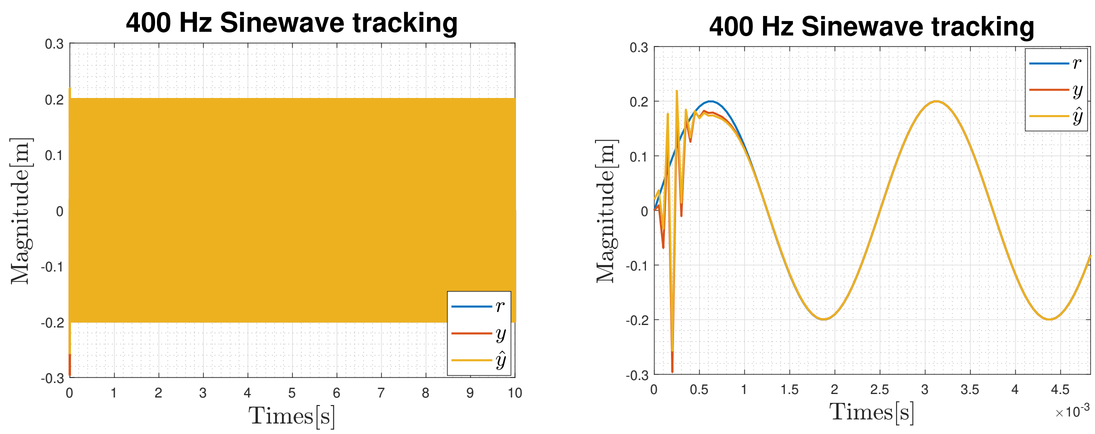
model,

$$x_{aug}[k] = \begin{bmatrix} x[k+1] \\ x_m[k+1] \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_m C & A_m \end{bmatrix} \begin{bmatrix} x[k] \\ x_m[k] \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u[k] + \begin{bmatrix} 0 \\ -B_m \end{bmatrix} r[k] \tag{20}$$

$$y[k] = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ x_m[k] \end{bmatrix} \tag{21}$$

$$K_{aug} = \begin{bmatrix} K & K_m \end{bmatrix} \tag{22}$$

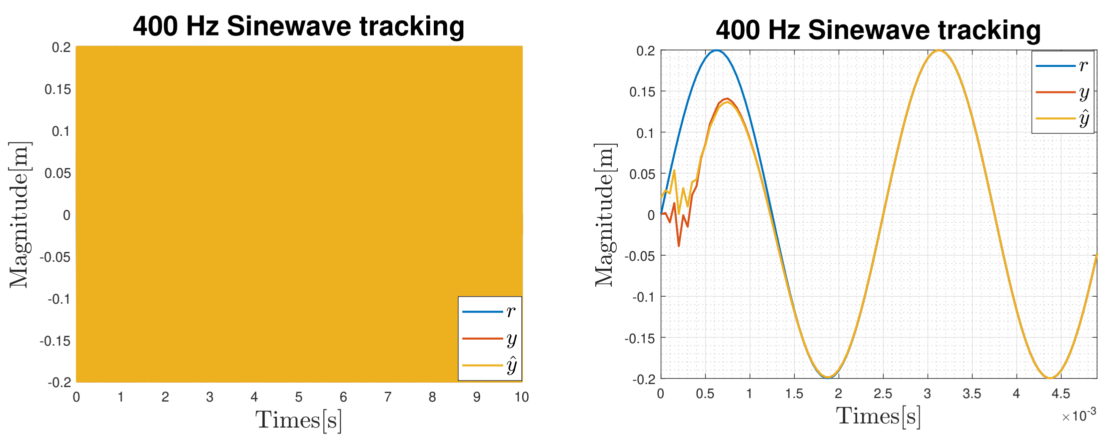
By applying the pole placement method again, we consider two cases of pole location. One is in the



(a) Sinewave tracking with internal model and aggressive poles

(b) Sinewave tracking with internal model and aggressive poles: zoom-in

Fig. 7: 400 Hz sinewave tracking with internal model : aggressive poles



(a) Sinewave tracking with internal model and non-aggressive poles

(b) Sinewave tracking with internal model and non-aggressive poles: zoom-in

Fig. 8: 400 Hz sinewave tracking with internal model: non-aggressive poles

aggressive location and the other is in the non-aggressive location. Both cases work in the simulation environment, however, one of them doesn't work in the experiment. We can observe this in the next section. The two pole locations are listed below

$$\text{poles}_{agg} = \left[0.85\ 0.8\ 0.1\ 0.11\ 0.15\ -0.45\ -0.4\right] \tag{23}$$

$$\text{poles}_{non} = \left[0.85\ 0.8\ 0.7\ 0.65\ 0.6\ -0.6\ -0.55\right] \tag{24}$$

We can derive K_{aug}

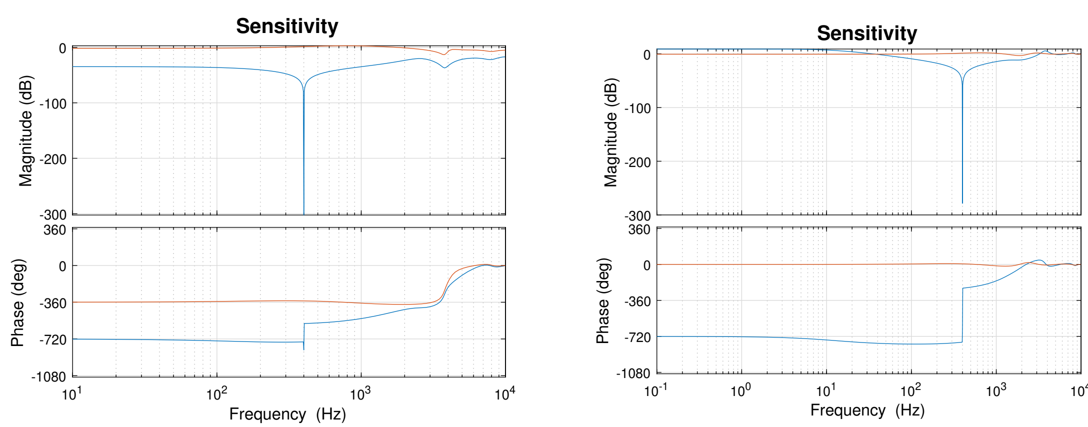
$$K_{aug_{agg}} = \begin{bmatrix}3.3950 & -2.1665 & 1.3868 & 1.1696 & -2.5367 - 15.1447 & 17.1860\end{bmatrix} \tag{25}$$
$$K_{aug_{non}} = \begin{bmatrix}2.0892 & -3.3342 & 2.0622 & -0.4409 & -0.1826 - 1.4641 & 1.4513\end{bmatrix} \tag{26}$$

Then we can also derive K and K_m.

$$K_{agg} = \begin{bmatrix}3.3950 & -2.1665 & 1.3868 & 1.1696 & -2.5367\end{bmatrix} \tag{27}$$
$$K_{m_{agg}} = \begin{bmatrix}-15.1447 & 17.1860\end{bmatrix} \tag{28}$$
$$K_{non} = \begin{bmatrix}2.0892 & -3.3342 & 2.0622 & -0.4409 & -0.1826\end{bmatrix} \tag{29}$$
$$K_{m_{non}} = \begin{bmatrix}-1.4641 & 1.4513\end{bmatrix} \tag{30}$$
$$\tag{31}$$

Fig.7 and Fig.8 show the tracking result using state observer feedback with the internal model. We can observe that the phase delay is eliminated in both cases, however, with the aggressive poles, the system has a more drastic transient response than the one with the non-aggressive poles, which also causes problems in the experiment.

Fig.9 shows the bode plot of the sensitivity function after applying the internal model. The drop in the magnitude of the bodeplot at 400Hz means that we have designed our internal model correctly.



(a) Sensitivity function: aggressive

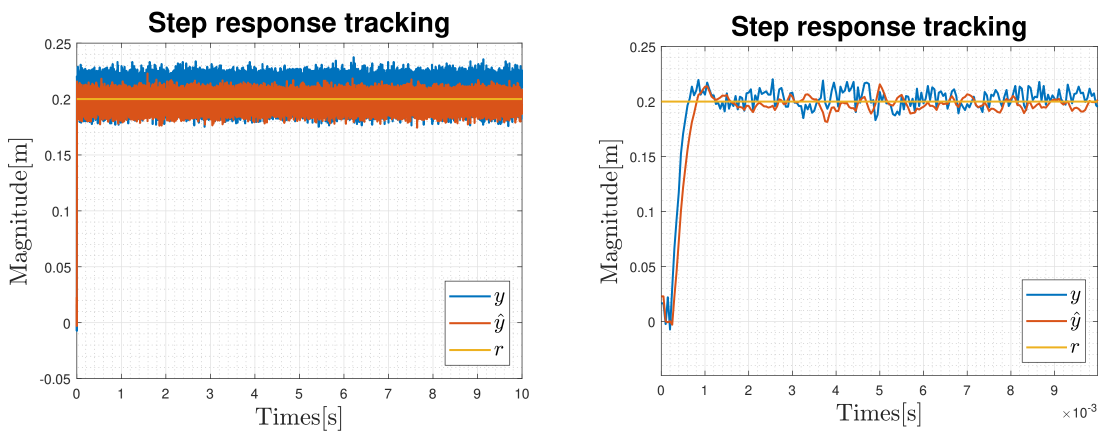
(b) Sensitivity function: non-aggressive

Fig. 9: Bode plot of sensitivity function after applying internal model

4 Experiment

4.1 Step response

We use the same L and $K(15)$ as in simulation, and the step response experiment result is shown in Fig.10. We can see that the $\hat{y}$ catch up the y immediately meaning that the feedback controller works well.



(a) Step response (b) Step response: zoomin

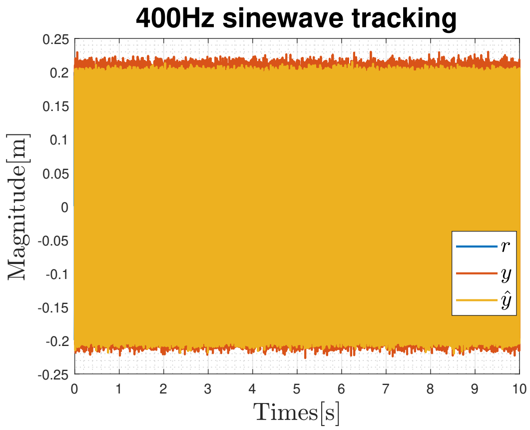
Fig. 10: Step response with state observer feedback controller

4.2 400 Hz sinewave

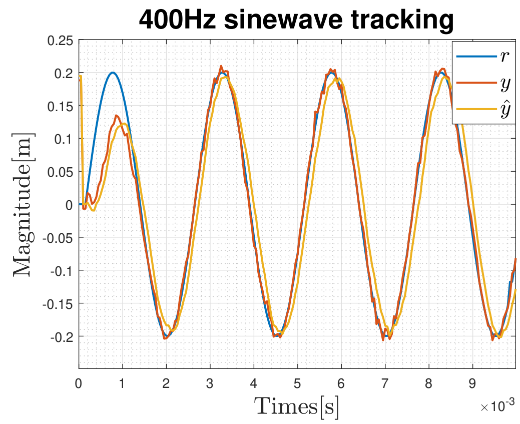
In the experiment of tracking a 400 Hz sinewave, we also consider two cases; the aggressive poles location and non-aggressive poles location. The tracking results are shown in Fig.11. From the non-aggressive case, Fig.11a and Fig.11b show that the tracking result performs well, the y almost aligns with the reference after running $0.001s$. On the other hand, for the aggressive case, Fig.11c shows that the system is unstable. We can have a brief conclusion that although both cases work in the simulation, they do not also represent that they will work in the experiment. This has a lot to do with how large the noise and disturbance in the experiment. The best way is to reduce the oscillation of the estimated value in the transient state as much as possible when we are doing the simulation.

5 Conclusion

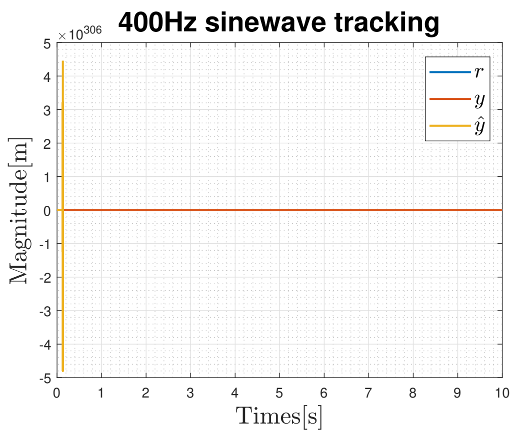
This report introduced state feedback control, state observer, and internal model principle in the first place. Afterward, focusing on our targeting system, we designed our digital state observer feedback controller with an internal model based on the identified system from Lab01 to track a 400 Hz sinewave signal. Furthermore, we verify our controller in the simulation and implement it in the



(a) Sinewave tracking



(b) Sinewave tracking: zoomin



(c) Sinewave tracking: aggressive poles

Fig. 11: Sinewave tracking with state observer feedback controller

real galvanometer system. Finally, some practical issues are occurred in the experiments and solved by tuning the parameters with reasonable inference. As a closing remark, we claim
"We did this exam by ourselves without working with or getting help from any other group."

References

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