108-1 PDSA Midterm

Nov. 4, 2019

學號：_____

姓名：_____

1. a. (5%) Please define what a **stable sort** is.
   b. (5%) Explain how **insertion sort** achieves the stable property.
   c. (5%) Explain how **mergesort** achieves the stable property.

2. (20%) **Dequeue**. A double-ended queue or deque (pronounced "deck") is a generalization of a stack and a queue that supports adding and removing items from either the front or the back of the data structure. Please implement the following two functions of the Deque class with the specified API:

```
public class Deque implements Iterable {
   …
   public Item removeFirst() // remove and return the item from the front
   public Item removeLast() // remove and return the item from the end
   …
}
```

3. (15%) (a) Please describe the procedures of the 'Knuth Shuffle' algorithm that produces a uniformly random permutation of the input array. (b) Prove that the 'Knuth Shuffle' algorithm is unbiased, so that every permutation is equally likely.

4. (10%) Please describe the procedures of the 'Quick-select' algorithm that takes linear time on average. Justify your answer.

5. (5%) What is a binary heap?

6. (10%) Provide the id[] array that results from the following sequence of 9 union operations on a set of 10 items using the weighted quick-union algorithm from lecture.

   `8-4 8-6 3-5 1-7 0-8 8-2 7-3 6-7 9-2`

   Recall: when joining two trees of equal size, our weighted quick union convention is to make the root of the second tree point to the root of the first tree. Also, our weighted quick union algorithm uses union by size (number of nodes), not union by height.

7. (15%) **Write a function** that reads in a sequence of characters (stored in a char array), and determines whether its parentheses, braces, and curly braces are "balanced."

   `boolean balanced(char[] a) // e.g. return true for [()]{}{[()()]()}, false for [(])`

8. (10%) Solve the recurrence

$$C_N = C_{N/2} + N^2, \text{ for } N \geq 2 \text{ with } C_1 = 0,$$

   where N is a power of 2.