Project for Cognitive & Computational Neuroscience Summer School
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A simple firing-rate model of working memory and decision-making

Consider two excitatory neural assemblies, that compete with each other through a shared pool of inhibitory neurons. Let r_1 and r_2 be their respective population-firing rates, and the total synaptic input current I_i and the resulting firing rate r_i of the neural population i obey the following input-output relationship ($F - I$ curve):

$$r_i = F(I_i) = \frac{aI_i - b}{1 - \exp(-d(aI_i - b))} \tag{1}$$

which captures the current-frequency function of a leaky integrate-and-fire neuron. The parameter values are $a = 270$ Hz/nA, $b = 108$ Hz, $d = 0.154$ sec.

Assume that the 'synaptic drive variables' s_1 and s_2 obey

$$\frac{ds_1}{dt} = \phi(F(I_1)\gamma(1 - s_1) - s_1/\tau_s) \tag{2}$$

$$\frac{ds_2}{dt} = \phi(F(I_2)\gamma(1 - s_2) - s_2/\tau_s) \tag{3}$$

where $\gamma = 0.641$, and $\phi = 1$ except for problem (5). $I_1 = g_E s_1 - g_I s_2 + I_b 1 + g_{ext}\mu_1$, $I_2 = g_E s_2 - g_I s_1 + I_b 2 + g_{ext}\mu_2$. The synaptic time constant $\tau_s = 100$ ms. The synaptic coupling strengths $g_E = 0.2609$ nA, $g_I = 0.0497$ nA and $g_{ext} = 0.00052$ nA. Stimulus-selective inputs to populations 1 and 2 are governed by unitless parameters μ_1 and μ_2, respectively. I_b is the background input which has a mean (I_0) and a noise component described by an Ornstein-Uhlenbeck process:

$$\tau_0 \frac{dI_{b1}}{dt} = -(I_{b1} - I_0)/\tau_0 + \eta_1(t)\sqrt{\tau_0 \sigma^2} \tag{4}$$

$$\tau_0 \frac{dI_{b2}}{dt} = -(I_{b2} - I_0)/\tau_0 + \eta_2(t)\sqrt{\tau_0 \sigma^2} \tag{5}$$

where $I_0 = 0.3255$ nA, filter time constant $\tau_0 = 2$ ms, and noise amplitude $\sigma = 0.02$ nA; $\eta(t)$ is a Gaussian white-noise with zero mean and unit standard deviation.

Caution: $F(I)$ is given in Hz, but in the s-equations, it should be divided by 1000 so that it has the unit of 1/msec.

(1) 'Coin-tossing' simulations with $\mu_1 = \mu_2 = \mu_0 = 30$ ($c' = 0$). In a decision-making simulation, both μ_1 and μ_2 are presented for a time interval, say from $t_1 = 500$ ms to $t_2 = 1500$ ms ($T = t_2 - t_1 = 1$ sec), and your total simulation time should be much longer (say 3 sec). The decision choice is determined according to which of the two active attractors wins the competition.

Display time courses of $s_1(t)$ and $s_2(t)$, as well as firing rates $r_1(t) = F(I_1)$ and $r_2(t) = F(I_2)$. Also, plot s_1 against s_2, or r_1 against r_2 in the 'phase space'.

In different (n) trials (each with a different seed for the random number generator, but always with the same initial condition $s_1 = s_2 = 0.1$), what do you observe? Do you see 50-50 decision outcome if n is large, say n=100-500?

(2) Stimulus-specific stimuli are given by μ_1 and μ_2. The 'coherence level' is defined as $c' = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$. For example, if $\mu_1 = 0.84$ and $\mu_2 = 0.8$, then $c' = 0.0244$ or 2.44%. Repeat (2) with several $c' = 0.032, 0.064, 0.128, 0.256, 0.512, 0.85, 1.0$ (for example, with $\mu_1 = \mu_0(1 + c')$ and $\mu_2 = \mu_0(1 - c')$). Plot the 'psychometric function', namely the percentage of correct decisions (choice=1 is correct if $\mu_1 > \mu_2$) as a function of $\log(c')$.

(3) Reaction time task. Set a firing threshold (e.g. $\theta = 15$ Hz, but adjust it if necessary). In any trial, the decision is made whenever one of the two neural populations reaches this threshold first. Run simulations over many trials for each c' as in (2).

(a) Show sample time courses of firing rates for different coherence levels.

(b) Plot the psychometric function, namely the trial-averaged reaction time as a function of $\log(c')$.

References

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