Name: Thien Tran
ID#: 1222245570

CSE 571: Artificial Intelligence
Assignment Week 1: Derivation of Logic Proofs Assignment
Oct-17-2021

1. $p \rightarrow (q \lor r), \neg q, \neg r \vdash \neg p$ using MT rule

1	$P \rightarrow (Q \lor R)$	
2	$\neg Q$	
3	$\neg R$	
4	$Q \lor R$	
5	Q	
6	$\bot$	¬E 2, 5
7	R	
8	$\bot$	¬E 3, 7
9	$\bot$	∨E 4, 5–6, 7–8
10	$\neg(Q \lor R)$	¬I 4–9
11	$\neg P$	MT 1, 10

2. $\vdash (p \rightarrow q) \lor (q \rightarrow r)$ without using LEM

Since $((p \rightarrow q) \lor (q \rightarrow r))$ is valid with all p, q,r {p,q,r} belonging to {0,1}, $\vdash (p \rightarrow q) \lor (q \rightarrow r)$ is valid.

p	q	r	$((p \rightarrow q) \lor (q \rightarrow r))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

3. (p → q) ∧ (q → r) ⊢ (p → r)

1	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	
2	$(P \rightarrow Q)$	∧E 1
3	$(Q \rightarrow R)$	∧E 1
4	│ P	
5	│ Q	→E 2, 4
6	│ R	→E 3, 5
7	$(P \rightarrow R)$	→I 4–6

4. (p → r) ∧ (q → ¬r) ⊢ (q → ¬p)

1	$(P \rightarrow R) \wedge (Q \rightarrow \neg R)$	
2	$P \rightarrow R$	∧E 1
3	$(Q \rightarrow \neg R)$	∧E 1
4	│ Q	
5	│ $\neg R$	→E 3, 4
6	│ │ P	
7	│ │ R	→E 2, 6
8	│ │ $\bot$	¬E 5, 7
9	│ $\neg P$	¬I 6–8
10	$Q \rightarrow \neg P$	→I 4–9

5. (p → r) ⋁ (q → r) ⊢ (p ⋀ q) → r

1	$(P \to R) \lor (Q \to R)$	
2	$\quad P \land Q$	
3	$\quad P$	∧E 2
4	$\quad Q$	∧E 2
5	$\quad\quad P \to R$	
6	$\quad\quad R$	→E 3, 5
7	$\quad\quad Q \to R$	
8	$\quad\quad R$	→E 4, 7
9	$\quad R$	∨E 1, 5–6, 7–8
10	$(P \land Q) \to R$	→I 2–9

6. ¬(p ⋁ q) is equivalent to (¬p ⋀ ¬q) (prove → in both directions)

Proof ¬(p ⋁ q) derives (¬p ⋀ ¬q)

1	$\neg(P \lor Q)$	
2	$\quad P$	
3	$\quad (P \lor Q)$	∨I 2
4	$\quad \bot$	¬E 1, 3
5	$\neg P$	¬I 2–4
6	$\quad Q$	
7	$\quad (P \lor Q)$	∨I 6
8	$\quad \bot$	¬E 1, 7
9	$\neg Q$	¬I 6–8
10	$\neg P \land \neg Q$	∧I 5, 9

Proof (¬p ∧ ¬q) derives ¬(p ∨ q)

1	$\neg P \wedge \neg Q$	
2	$\neg P$	∧E 1
3	$\neg Q$	∧E 1
4	$(P \vee Q)$	
5	P	
6	$\bot$	¬E 2, 5
7	Q	
8	$\bot$	¬E 3, 7
9	$\bot$	∨E 4, 5–6, 7–8
10	$\neg(P \vee Q)$	¬I 4–9

7. (p → ¬q) is equivalent to ¬(p ∧ q)

Proof (p → ¬q) derives ¬(p ∧ q)

1	$P \rightarrow \neg Q$	
2	$P \wedge Q$	
3	P	∧E 2
4	Q	∧E 2
5	$\neg Q$	→E 1, 3
6	$\bot$	¬E 4, 5
7	$\neg(P \wedge Q)$	¬I 2–6

Proof ¬(p ∧ q) derives (p → ¬q)

1	$\neg(P \wedge Q)$	
2	P	
3	Q	
4	$P \wedge Q$	$\wedge$I 2, 3
5	$\bot$	$\neg$E 1, 4
6	$\neg Q$	$\neg$I 3–5
7	$P \to \neg Q$	$\to$I 2–6

Part 2: Determining the Validity of Sequents, Proofs, and Truth Tables

Assuming binding priority (¬, ∧, ∨, →), are these sequents valid or not? If they are valid, how do you prove it? If they are not valid, what would the truth table be?

1. $A \to B, C \to D \vdash A \vee C \to B \wedge D$

1 is not valid. Proof by a counterexample:

Consider an instance {A, B, C, D} == {false, false, true, true}

$(A \to B) \wedge (C \to D)$ is true, whereas $A \vee C \to B \wedge D$ is false

We conclude that this is not valid.

A	B	C	D	$((A \to B) \wedge (C \to D))$	$((A \vee C) \to (B \wedge D))$
F	F	F	F	T	T
F	F	F	T	T	T

F	F	T	F	F	F
F	F	T	T	T	F
F	T	F	F	T	T
F	T	F	T	T	T
F	T	T	F	F	F
F	T	T	T	T	T
T	F	F	F	F	F
T	F	F	T	F	F
T	F	T	F	F	F
T	F	T	T	F	F
T	T	F	F	T	F
T	T	F	T	T	T
T	T	T	F	F	F
T	T	T	T	T	T

2. $A \land \neg A \vdash \neg(B \to C) \land (B \to C)$

1	$A \land \neg A$	
2	A	$\land$E 1
3	$\neg A$	$\land$E 1
4	$\bot$	$\neg$E 2, 3
5	$\neg(B \to C) \land (B \to C)$	$\bot$E 4

3. $(A \land B) \to C,\ C \to D,\ B \land \neg D \vdash \neg A$

1	$(A \land B) \to C$	
2	$C \to D$	
3	$B \land \neg D$	
4	A	
5	B	$\land$E 3
6	$\neg D$	$\land$E 3
7	$A \land B$	$\land$I 4, 5
8	C	$\to$E 1, 7
9	D	$\to$E 2, 8
10	$\bot$	$\neg$E 6, 9
11	$\neg A$	$\neg$I 4–10

Part 3: Drawing Parse Trees

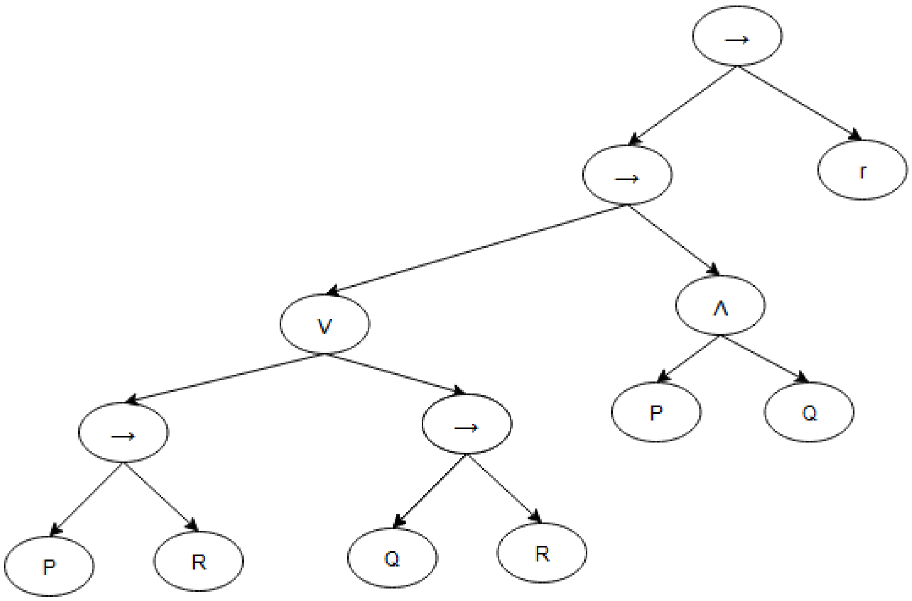
Assuming binding priority ($\neg$, $\land$, $\lor$, $\to$), what are **all** the possible parse trees for each formula? Draw the trees for each formula.

1. $(p \to r) \lor (q \to r) \to (p \land q) \to r$

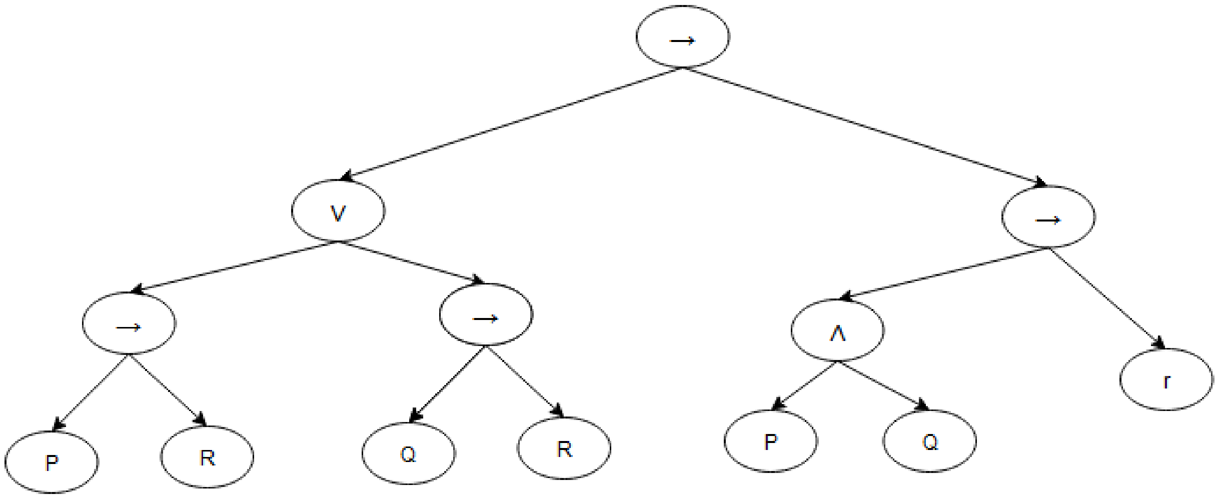
Parse tree 1: $(\ (p \to r) \lor (q \to r) \to (p \land q)\ ) \to (r)$

Parse tree 2: $(\ (p \to r) \lor (q \to r)\ ) \to (\ (p \land q) \to r\ )$

Parse tree 1: $((p \rightarrow r) \vee (q \rightarrow r) \rightarrow (p \wedge q)) \rightarrow (r)$



Parse tree 2: $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$

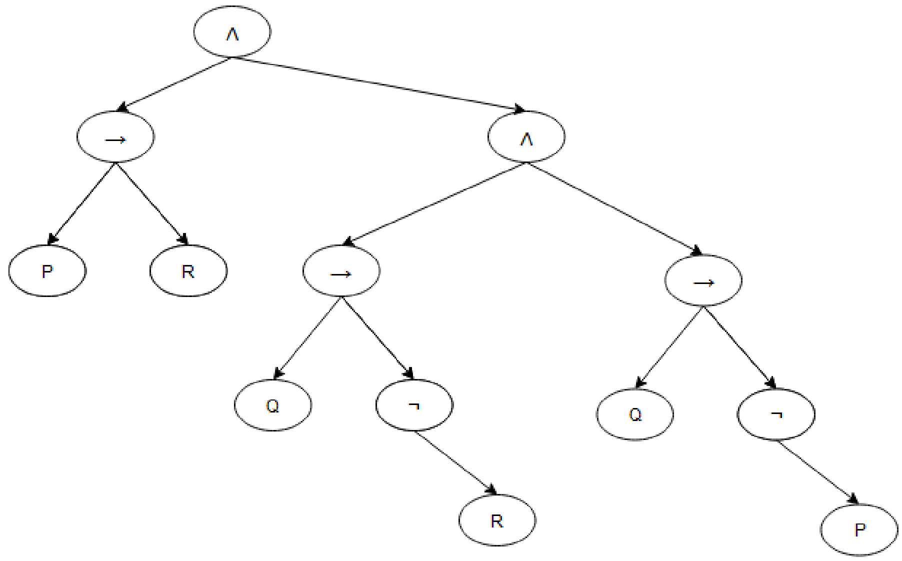


2. $(p \rightarrow r) \land (q \rightarrow \neg r) \land (q \rightarrow \neg p)$

Parse tree 1: $((p \rightarrow r) \land (q \rightarrow \neg r)) \land (q \rightarrow \neg p)$

Parse tree 2: $(p \rightarrow r) \land ((q \rightarrow \neg r) \land (q \rightarrow \neg p))$

Parse tree 1: ((p → r) ∧ (q → ¬r)) ∧ (q → ¬p)



Parse tree 2: (p → r) ∧ ((q → ¬r) ∧ (q → ¬p))