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Runge-kutta Numerical Method for Solving Nonlinear Influenza Model

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Abstract. The main object of this study is to solve a system of nonlinear ordinary differential equations (ODE) of the first order governing the epidemic model using numerical methods. The application under study is a mathematical epidemic model which is the influenza model at Australia in 1919. Runge-kutta methods of order 4 and of order 45 for solving this initial value problem(IVP) problem have been used. Finally, the results obtained have been discussed tabularly and graphically.

1. Introduction

Influenza is an infectious disease that is transmitted by a virus widespread in the population and infects people of different ages [1]. It is transmitted from person to person through the air of sneeze, cough, touch soiled objects and move them to the eye, nose or mouth, and others [2,3]. The researchers were representing them as mathematical models. Many people suffer from severe symptoms of the disease around the world. It can turn into a serious disease especially for old adults, newborns, people who have chronic diseases, pregnant women, and people who are obese [4,5]. Flu symptoms may be mild or very strong and these symptoms include cough, nausea, vomiting, fever, headache, fatigue, sore throat, runny nose, chills, muscle pain, sweating, fatigue, weakness, and others [6].

The cause of this disease is the influenza virus, which is transmitted from person to person through the respiratory system. Influenza virus may cause death for some people, especially those who suffer from other health problems [6].

There are many researchers interested in studying epidemiological models to know whether the epidemic is increasing or decreasing, such as Sabaa and Mohammed in 2019 discussed the analytic solutions of a nonlinear social epidemic model for alcohol consumption problem in Spain using Adomian decomposition and variation iteration methods [7]. Sabaa and Mohammed, in 2020 solved also the nonlinear social epidemic model that is the smoking habit problem by Adomian decomposition, variation iteration, finite difference, and Runge-Kutta approximate methods [8].

Many researchers analyzed the influenza epidemic model that is under study, such as El-Shahed and Alsaedi in 2011 studied the fractional SIRC model and influenza A [9], Shaman and Karspeck in 2012 studied Forecasting seasonal outbreaks of influenza [10]. Reynolds, et al. in 2014 studied mathematical modeling of influenza A virus dynamics within Swine Farms and the Effects of Vaccination [11]. Jódar, et al. in 2017 studied a mathematical model of Influenza: stability and treatment [12]. Zarebski et al. in 2017 analyzed a mathematical model of epidemics of seasonal influenza in Australia using the likelihood-based method [13], Kim et al. in 2017 studied the optimal control strategies of influenza epidemic model in Korea [14].

The feature of such a model is the existence of multiple parameters and multivariate. As well, it is a nonlinear system of the first-order ordinary differential equations for initial value problems that do not mostly get the exact solution. Our interest is to solve numerically such models. The choice of appropriate numerical methods is necessary. One of the numerical methods for solving this problem and the most famous method is Runge-Kutta of order 4 (RK_4) and Runge-kutta of order 45 (RK_{45}) to solve the Influenza model in Australia in 1919 depending on previous studies which is a reliable method. The solution is approximate because most mathematical models are very complicated since such models contain several variables and parameter, the exact solution of these models is not available [15]. There were concerns with the manual calculation of somewhat large data sets but those concerns disappeared with the widespread availability of computers that establish programs for these numerical methods to get the solution easy and fast [16].

This research is arranged as follows: In Section 2, the mathematical model of an influenza epidemic is presented, Section 3 constructs the numerical solutions of the influenza model by Runge-Kutta of 4th order ($RK4$) and of order 45 (RK_{45}) method, Section 4, results, and discussion are presented tabularly and graphically, Section 5 has ended the study by the conclusion.

2. Mathematical Model

Mathematical modeling is an abstract model that uses mathematical language to describe the behavior of a system. The mathematical model is also defined as a collection of equations that describes a natural phenomenon. The importance of modeling in understanding the spread of the influenza epidemic [17]. The influenza epidemic in Australia in 1919 has been studied to predict the progression of disease levels. The population contains four species of individuals, $S(t), E(t), I(t)$ and $R(t)$ called compartments, the first compartment is the healthy persons and symbolized by (S), the second compartment is the infected persons, but does not cause infection, the person is in the incubation period and symbolized by (E), the third compartment is the infected persons and the cause infection after the incubation period and symbolized by (I), the fourth compartment is the persons who recover or die and symbolized by (R). So our model is made $(SEIR)$,[1]. The governing equations for the influenza model are given as a system of the first-order nonlinear ordinary differential equations in the system (1), [1]:

$$S'(t) = -\beta \frac{IS}{N} - \mu S + rN + \delta R$$
$$E'(t) = \beta \frac{IS}{N} - (\mu + \sigma + \kappa)E$$
$$I'(t) = \sigma E - (\mu + \alpha + \gamma)I$$
$$R'(t) = \kappa E + \gamma I - \mu R - \delta R$$

where Table 1. has described the variables of the influenza model $S(t), E(t), I(t), R(t)$, Table 2 has the description of the parameters of the influenza model $\beta, \mu, r, \delta, \sigma, \kappa, \alpha, \gamma$ and the initial conditions (random variables) of the system (1): $S_0 = 0.4865$, $E_0 = 0.0000009$, $I_0 = 0.000068$, $R_0 = 0.0000000$, N is the total population such that $N = S + E + I + R$ and the specified time period $(0,70)$ is used to obtain results by Samsuzzoha, Singh, et al. [1].

Table 1: Describing the variables of the influenza model

Variables	Description
$S(t)$	Proportion of susceptible population
$E(t)$	Proportion of exposed population
$I(t)$	Proportion of infective population
$R(t)$	Proportion of recovered population

Table 2: Description of the parameters of the influenza model and value of parameters [1]

Parameters	Description	Value of Parameters
β	Contact rate	0.5020000
μ	Natural mortality rate	0.0003671
r	Birth rate	0.0006762
δ	Duration of immunity loss	0.0027400
σ	Mean duration of latency	0.6990000
κ	Recovery rate of latent	0.0001500
α	Flu induced mortality rate	0.0300000
γ	Mean recovery time for clinically ill	0.3600000

3. Numerical Method for Solving Influenza Model

A numerical method is a procedure that produces approximate solutions at specific points using the operations of addition, subtraction, multiplication, division, and functional appraisals. It can be applied to most first_order initial value problems [18]. They are important tools for investigating the systems [19]. Runge_Kutta is a set of iterative, implicit, and explicit methods [20]. Runge-Kutta techniques were introduced around 1900 by Carl Runge and Wilhelm Kutta [21]. In this section, two types of numerical methods can be discussed: (RK_4 and RK_{45} to solve the influenza epidemic problem in Australia in 1919, [1].

3.1. Runge-Kutta of Order 4 (RK_4) Method

RK_4 method is a reliable and accurate iteration numerical method that is used to solve the linear and nonlinear system or equation of the first ODE for initial value problems, it gives accurate results. It is a built-in method from the Runge-Kutta family. The nonlinear system of the influenza model under study can be solved by the RK method at present work. RK method has different forms according to the order of the method, in the present work, RK of the fourth order in this section can be used to solve the nonlinear system of the system (1). This work needs four steps to find the final form, [22].

The general formula of y_{i+1} for the RK_4 method [22] is:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1}$$

In order to solve the system of system (1) using RK_4 to obtain numerical results, first write the form of RK_4 as below, in equations (2-5) the general formula of $S_{i+1}, E_{i+1}, I_{i+1}$ and R_{i+1} for the RK_4 method is:

$$S_{i+1} = S_i + \frac{1}{6}(k_{S1} + 2k_{S2} + 2k_{S3} + k_{S4}) \tag{2}$$

$$E_{i+1} = E_i + \frac{1}{6}(k_{E1} + 2k_{E2} + 2k_{E3} + k_{E4}) \tag{3}$$

$$I_{i+1} = I_i + \frac{1}{6}(k_{I1} + 2k_{I2} + 2k_{I3} + k_{I4}) \tag{4}$$

$$R_{i+1} = R_i + \frac{1}{6}(k_{R1} + 2k_{R2} + 2k_{R3} + k_{R4}) \tag{5}$$

where $i = 0,1, \dots, n-1$.

The general formula of k_1, k_2, k_3, k_4 for the RK_4 method [22] is:

$$k_1 = hf_1(t_i, y_i) \tag{6}$$

$$k_2 = hf_2(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}) \tag{7}$$

$$k_3 = hf_3(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \tag{8}$$

$$k_4 = hf_4(t_i + h, y_i + k_3) \tag{9}$$

where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size.

Now, to find k_{S1}, k_{E1}, k_{I1} and k_{R1}, we following the next steps in equations (10-13).

$$k_{S1} = hf_1(t_i, S_i, E_i, I_i, R_i) = h\left(\frac{-\beta}{N_{i1}} I_i S_i - \mu S_i + r N_{i1} + \delta R_i\right) \tag{10}$$

$$k_{E1} = hf_2(t_i, S_i, E_i, I_i, R_i) = h\left(\frac{\beta}{N_{i1}} I_i S_i - (\mu + \sigma + \kappa)E_i\right) \tag{11}$$

$$k_{I1} = hf_3(t_i, S_i, E_i, I_i, R_i) = h(\sigma E_i - (\mu + \alpha + \gamma)I_i) \tag{12}$$

$$k_{R1} = hf_4(t_i, S_i, E_i, I_i, R_i) = h(\kappa E_i + \gamma I_i - \mu R_i - \delta R_i) \tag{13}$$

where $i = 0,1, \dots, n-1$.

In the same way, k_{S2}, k_{E2}, k_{I2} and k_{R2} can be found to obtain the second step in equations (14-17) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$
\begin{aligned}
k_{S2} = hf_1(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S1}, E_i + \frac{1}{2}k_{E1}, I_i + \frac{1}{2}k_{I1}, R_i + \frac{1}{2}k_{R1}) \\
= h\left(\frac{-\beta}{N_{i2}}\left(I_i + \frac{1}{2}k_{I1}\right)(S_i + \frac{1}{2}k_{S1}) - \mu\left(S_i + \frac{1}{2}k_{S1}\right) + r N_{i2}\right. \\
\left. + \delta\left(R_i + \frac{1}{2}k_{R1}\right)\right)
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
k_{E2} = hf_2(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S1}, E_i + \frac{1}{2}k_{E1}, I_i + \frac{1}{2}k_{I1}, R_i + \frac{1}{2}k_{R1}) \\
= h\left(\frac{\beta}{N_{i2}}\left(I_i + \frac{1}{2}k_{I1}\right)(S_i + \frac{1}{2}k_{S1}) - (\mu + \sigma + \kappa)\left(E_i + \frac{1}{2}k_{E1}\right)\right)
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
k_{I2} = hf_3(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S1}, E_i + \frac{1}{2}k_{E1}, I_i + \frac{1}{2}k_{I1}, R_i + \frac{1}{2}k_{R1}) \\
= h\left(\sigma\left(E_i + \frac{1}{2}k_{E1}\right) - (\mu + \alpha + \gamma)\left(I_i + \frac{1}{2}k_{I1}\right)\right)
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
k_{R2} = hf_4(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S1}, E_i + \frac{1}{2}k_{E1}, I_i + \frac{1}{2}k_{I1}, R_i + \frac{1}{2}k_{R1}) \\
= h\left(\kappa\left(E_i + \frac{1}{2}k_{E1}\right) + \gamma\left(I_i + \frac{1}{2}k_{I1}\right) - \mu\left(R_i + \frac{1}{2}k_{R1}\right)\right. \\
\left. - \delta\left(R_i + \frac{1}{2}k_{R1}\right)\right)
\end{aligned}
\tag{17}
$$

where $i = 0,1, \dots, n-1$.

In the third stage, try to find k_{S3}, k_{E3}, k_{I3} and k_{R3} by substituting in the system (1) as below in equations (18-21) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$k_{S3} = hf_1\left(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S2}, E_i + \frac{1}{2}k_{E2}, I_i + \frac{1}{2}k_{I2}, R_i + \frac{1}{2}k_{R2}\right)$$

$$= h\left(\frac{-\beta}{N_{i3}}\left(I_i + \frac{1}{2}k_{I2}\right)\left(S_i + \frac{1}{2}k_{S2}\right) - \mu\left(S_i + \frac{1}{2}k_{S2}\right) + rN_{i3}\right. \tag{18}$$

$$\left. + \delta\left(R_i + \frac{1}{2}k_{R2}\right)\right)$$

$$k_{E3} = hf_2\left(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S2}, E_i + \frac{1}{2}k_{E2}, I_i + \frac{1}{2}k_{I2}, R_i + \frac{1}{2}k_{R2}\right)$$

$$= h\left(\frac{\beta}{N_{i3}}\left(I_i + \frac{1}{2}k_{I2}\right)\left(S_i + \frac{1}{2}k_{S2}\right) - (\mu + \sigma + \kappa)\left(E_i + \frac{1}{2}k_{E2}\right)\right) \tag{19}$$

$$k_{I3} = hf_3\left(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S2}, E_i + \frac{1}{2}k_{E2}, I_i + \frac{1}{2}k_{I2}, R_i + \frac{1}{2}k_{R2}\right)$$

$$= h\left(\sigma\left(E_i + \frac{1}{2}k_{E2}\right) - (\mu + \alpha + \gamma)\left(I_i + \frac{1}{2}k_{I2}\right)\right) \tag{20}$$

$$k_{R3} = hf_4\left(t_i + \frac{h}{2}, S_i + \frac{1}{2}k_{S2}, E_i + \frac{1}{2}k_{E2}, I_i + \frac{1}{2}k_{I2}, R_i + \frac{1}{2}k_{R2}\right)$$

$$= h\left(\kappa\left(E_i + \frac{1}{2}k_{E2}\right) + \gamma\left(I_i + \frac{1}{2}k_{I2}\right) - \mu\left(R_i + \frac{1}{2}k_{R2}\right)\right.$$

$$\left. - \delta\left(R_i + \frac{1}{2}k_{R2}\right)\right) \tag{21}$$

where $i = 0, 1, \dots, n - 1$.

The fourth stage needs to find k_{S4}, k_{E4}, k_{I4} and k_{R4} as below, in equations (22-25) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$k_{S4} = hf_1\left(t_i + h, S_i + k_{S3}, E_i + k_{E3}, I_i + k_{I3}, R_i + k_{R3}\right)$$

$$= h\left(\frac{-\beta}{N_{i4}}(I_i + k_{I3})(S_i + k_{S3}) - \mu(S_i + k_{S3}) + rN_{i4} + \delta(R_i + k_{R3})\right) \tag{22}$$

$$k_{E4} = hf_2\left(t_i + h, S_i + k_{S3}, E_i + k_{E3}, I_i + k_{I3}, R_i + k_{R3}\right)$$

$$= h\left(\frac{\beta}{N_{i4}}(I_i + k_{I3})(S_i + k_{S3}) - (\mu + \sigma + \kappa)(E_i + k_{E3})\right) \tag{23}$$

$$k_{I4} = hf_3\left(t_i + h, S_i + k_{S3}, E_i + k_{E3}, I_i + k_{I3}, R_i + k_{R3}\right)$$

$$= h\left(\sigma(E_i + k_{E3}) - (\mu + \alpha + \gamma)(I_i + k_{I3})\right) \tag{24}$$

$$k_{R4} = hf_4\left(t_i + h, S_i + k_{S3}, E_i + k_{E3}, I_i + k_{I3}, R_i + k_{R3}\right)$$

$$= h\left(\kappa(E_i + k_{E3}) + \gamma(I_i + k_{I3}) - \mu(R_i + k_{R3}) - \delta(R_i + k_{R3})\right) \tag{25}$$

where $i = 0, 1, \dots, n - 1$.

3.2 Runge-Kutta of Order 45 (RK_{45}) Method

RK_{45} method is a reliable and accurate iteration numerical method that is used to solve the nonlinear system of the system (1). This work needs six steps to find the final form, [22]. It was developed by a German mathematician Erwin Fehlberg in 1969. It is a built-in method from Runge-Kutta family.

The general formula of y_{i+1} for the RK_{45} method [23] is:

$$y_{i+1} = y_i + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5 \tag{26}$$

In order to solve the system of the system (1) using RK_{45} to obtain numerical results, first write the form of RK_{45} as below, in equations (27-30), the general formula of $S_{i+1}, E_{i+1}, I_{i+1}$ and R_{i+1} for the RK_4 method is:

$$S_{i+1} = S_i + \frac{25}{216} k_{S1} + \frac{1408}{2565} k_{S3} + \frac{2197}{4104} k_{S4} - \frac{1}{5} k_{S5} \tag{27}$$

$$E_{i+1} = E_i + \frac{25}{216} k_{E1} + \frac{1408}{2565} k_{E3} + \frac{2197}{4104} k_{E4} - \frac{1}{5} k_{E5} \tag{28}$$

$$I_{i+1} = I_i + \frac{25}{216} k_{E1} + \frac{1408}{2565} k_{E3} + \frac{2197}{4104} k_{E4} - \frac{1}{5} k_{I5} \tag{29}$$

$$R_{i+1} = R_i + \frac{25}{216} k_{R1} + \frac{1408}{2565} k_{R3} + \frac{2197}{4104} k_{R4} - \frac{1}{5} k_{R5} \tag{30}$$

where $i = 0, 1, \ldots, n-1$.

The general formula of $k_1, k_2, k_3, k_4, k_5, k_6$ for the RK_{45} method [22] is:

$$k_1 = h f_1(t_i, y_i) \tag{31}$$

$$k_2 = h f_2(t_i + \frac{h}{4}, y_i + \frac{k_1}{4}) \tag{32}$$

$$k_3 = h f_3(t_i + \frac{3h}{8}, y_i + \frac{3k_1}{32} + \frac{9k_2}{32}) \tag{33}$$

$$k_4 = h f_4(t_i + \frac{12h}{13}, y_i + \frac{1932k_1}{2197} - \frac{7200k_2}{2197} + \frac{7296k_3}{2197}) \tag{34}$$

$$k_5 = h f_5(t_i + h, y_i + \frac{439k_1}{219} - 8k_2 + \frac{3680k_3}{513} - \frac{845k_4}{4104}) \tag{35}$$

$$k_6 = h f_6(t_i + \frac{h}{2}, y_i - \frac{8k_1}{27} + 2k_2 - \frac{3544k_3}{2565} + \frac{1859k_4}{4104} - \frac{11k_5}{40}) \tag{36}$$

where $f_1, f_2, f_3, f_4, f_5, f_6$ are unknown functions, t is a time and h is a step size.

Now, To find k_{S1}, k_{E1}, k_{I1} and k_{R1} following the next steps in equations (37-40) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$k_{S1} = h f_1(t_i, S_i, E_i, I_i, R_i) = h \left(\frac{-\beta}{N_{i1}} I_i S_i - \mu S_i + r N_{i1} + \delta R_i \right) \tag{37}$$

$$k_{E1} = h f_2(t_i, S_i, E_i, I_i, R_i) = h \left(\frac{\beta}{N_{i1}} I_i S_i - (\mu + \sigma + \kappa) E_i \right) \tag{38}$$

$$k_{I1} = h f_3(t_i, S_i, E_i, I_i, R_i) = h \left(\sigma E_i - (\mu + \alpha + \gamma) I_i \right) \tag{39}$$

$$k_{R1} = h f_4(t_i, S_i, E_i, I_i, R_i) = h \left(\kappa E_i + \gamma I_i - \mu R_i - \delta R_i \right) \tag{40}$$

where $i = 0, 1, \ldots, n-1$.

In the same way, k_{S2}, k_{E2}, k_{I2} and k_{R2} can be found to obtain the second step in equations (41-44) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$\begin{aligned} k_{S2} = h f_1(t_i + \frac{h}{4}, S_i + \frac{1}{4} k_{S1}, E_i + \frac{1}{4} k_{E1}, I_i + \frac{1}{4} k_{I1}, R_i + \frac{1}{4} k_{R1}) \\ = h \left(\frac{-\beta}{N_{i2}} \left(I_i + \frac{1}{4} k_{I1} \right) \left(S_i + \frac{1}{4} k_{S1} \right) - \mu \left(S_i + \frac{1}{4} k_{S1} \right) + r N_{i2} \right. \\ \left. + \delta \left(R_i + \frac{1}{4} k_{R1} \right) \right) \end{aligned} \tag{41}$$

$$k_{E2} = hf_2 \left(t_i + \frac{h}{4}, S_i + \frac{1}{4}k_{S1}, E_i + \frac{1}{4}k_{E1}, I_i + \frac{1}{4}k_{I1}, R_i + \frac{1}{4}k_{R1}\right.$$
$$= h\left(\frac{\beta}{N_{i2}}\left(I_i + \frac{1}{4}k_{I1}\right)\left(S_i + \frac{1}{4}k_{S1}\right) - (\mu + \sigma + \kappa)\left(E_i + \frac{1}{4}k_{E1}\right)\right) \tag{42}$$

$$k_{I2} = hf_3 \left(t_i + \frac{h}{4}, S_i + \frac{1}{4}k_{S1}, E_i + \frac{1}{4}k_{E1}, I_i + \frac{1}{4}k_{I1}, R_i + \frac{1}{4}k_{R1}\right)$$
$$= h\left(\sigma\left(E_i + \frac{1}{4}k_{E1}\right) - (\mu + \alpha + \gamma)\left(I_i + \frac{1}{4}k_{I1}\right)\right) \tag{43}$$

$$k_{R2} = hf_4 \left(t_i + \frac{h}{4}, S_i + \frac{1}{4}k_{S1}, E_i + \frac{1}{4}k_{E1}, I_i + \frac{1}{4}k_{I1}, R_i + \frac{1}{4}k_{R1}\right)$$
$$= h\left(\kappa\left(E_i + \frac{1}{4}k_{E1}\right) + \gamma\left(I_i + \frac{1}{4}k_{I1}\right) - \mu\left(R_i + \frac{1}{4}k_{R1}\right)\right.$$
$$\left. - \delta\left(R_i + \frac{1}{4}k_{R1}\right)\right) \tag{44}$$

where $i = 0,1, \ldots, n - 1$.

In the third stage, try to find k_{S3}, k_{E3}, k_{I3} and k_{R3} by substituting in the system (1) as below in equations (45-48) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$k_{S3} = hf_1 \left(t_i + \frac{3h}{8}, S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}, E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}, I_i + \frac{3}{32}k_{I1}\right.$$
$$+ \frac{9}{32}k_{I2}, R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}\right)$$
$$= h\left(\frac{-\beta}{N_{i3}}\left(I_i + \frac{3}{32}k_{I1} + \frac{9}{32}k_{I2}\right)\left(S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}\right)\right.$$
$$\left. - \mu\left(S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}\right) + rN_{i3} + \delta\left(R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}\right)\right), \tag{45}$$

$$k_{E3} = hf_2 \left(t_i + \frac{3h}{8}, S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}, E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}, I_i + \frac{3}{32}k_{I1}\right.$$
$$+ \frac{9}{32}k_{I2}, R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}$$
$$= h\left(\frac{\beta}{N_{i3}}\left(I_i + \frac{3}{32}k_{I1} + \frac{9}{32}k_{I2}\right)\left(S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}\right)\right.$$
$$\left. - (\mu + \sigma + \kappa)\left(E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}\right)\right) \tag{46}$$

$$k_{I3} = hf_3 \left(t_i + \frac{3h}{8}, S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}, E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}, I_i + \frac{3}{32}k_{I1}\right.$$
$$+ \frac{9}{32}k_{I2}, R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}\right)$$
$$= h\left(\sigma\left(E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}\right) - (\mu + \alpha + \gamma)\left(I_i + \frac{3}{32}k_{I1} + \frac{9}{32}k_{I2}\right)\right) \tag{47}$$

$$
\begin{aligned}
k_{R3} = hf_4 \ (t_i &+ \frac{3h}{8}, S_i + \frac{3}{32}k_{S1} + \frac{9}{32}k_{S2}, E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}, I_i + \frac{3}{32}k_{I1} \\
&+ \frac{9}{32}k_{I2}, R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}) \\
&= h\left(\kappa\left(E_i + \frac{3}{32}k_{E1} + \frac{9}{32}k_{E2}\right) + \gamma\left(I_i + \frac{3}{32}k_{I1} + \frac{9}{32}k_{I2}\right)\right. \\
&\left. -\mu\left(R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}\right) - \delta\left(R_i + \frac{3}{32}k_{R1} + \frac{9}{32}k_{R2}\right)\right)
\end{aligned}
\tag{48}
$$

where $i = 0,1, \dots, n-1$.

The fourth stage needs to find k_{S4}, k_{E4}, k_{I4} and k_{R4} as below, in equations (49-52) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$
\begin{aligned}
k_{S4} = hf_1 \ (t_i &+ \frac{12h}{13}, S_i + \frac{1932}{2197}k_{S1} - \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}, \\
&E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}, I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} \\
&+ \frac{7296}{2197}k_{I3}, R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}) \\
&= h\left(\frac{-\beta}{N_{i4}}\left(I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} + \frac{7296}{2197}k_{I3}\right)(S_i + \frac{1932}{2197}k_{S1}\right. \\
&- \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}\Big) \\
&-\mu\left(S_i + \frac{1932}{2197}k_{S1} - \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}\right) + rN_{i4} \\
&+\delta\left(R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}\right)\Big)
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
k_{E4} = hf_2 \ (t_i &+ \frac{12h}{13}, S_i + \frac{1932}{2197}k_{S1} - \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}, \\
&E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}, I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} \\
&+ \frac{7296}{2197}k_{I3}, R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}) \\
&= h\left(\frac{\beta}{N_{i4}}\left(I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} + \frac{7296}{2197}k_{I3}\right)(S_i + \frac{1932}{2197}k_{S1}\right. \\
&- \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}) - (\mu + \sigma + \kappa)(E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} \\
&+ \frac{7296}{2197}k_{E3}))
\end{aligned}
\tag{50}
$$

$$
\begin{aligned}
k_{I4} = h f_3 \,(& t_i + \frac{12h}{13}, S_i + \frac{1932}{2197}k_{S1} - \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}, \\
& E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}, I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} \\
& + \frac{7296}{2197}k_{I3}, R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}) \\
& = h \,(\sigma\left(E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}\right) - (\mu + \alpha + \gamma)\,(I_i \\
& + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} + \frac{7296}{2197}k_{I3}))
\end{aligned}
\tag{51}
$$

$$
\begin{aligned}
k_{R4} = h f_4 \,(& t_i + \frac{12h}{13}, S_i + \frac{1932}{2197}k_{S1} - \frac{7200}{2197}k_{S2} + \frac{7296}{2197}k_{S3}, \\
& E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}, I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} \\
& + \frac{7296}{2197}k_{I3}, R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3})
\end{aligned}
$$

$$
\begin{aligned}
= h\,\Big(& \kappa\left(E_i + \frac{1932}{2197}k_{E1} - \frac{7200}{2197}k_{E2} + \frac{7296}{2197}k_{E3}\right) \\
& + \gamma\left(I_i + \frac{1932}{2197}k_{I1} - \frac{7200}{2197}k_{I2} + \frac{7296}{2197}k_{I3}\right) \\
& - \mu\left(R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}\right) \\
& - \delta\left(R_i + \frac{1932}{2197}k_{R1} - \frac{7200}{2197}k_{R2} + \frac{7296}{2197}k_{R3}\right)\Big)
\end{aligned}
\tag{52}
$$

where $i = 0,1,\ldots,n-1$.

The fifth stage needs to find k_{S5}, k_{E5}, k_{I5} and k_{R5} As below, in equations (53-56) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size.

$$
\begin{aligned}
k_{S5} = h f_1 \,(& t_i + h, S_i + \frac{439}{216}k_{S1} - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}, \\
& E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4}, I_i + \frac{439}{216}k_{I1} - 8k_{I2} \\
& + \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4}, R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3} \\
& - \frac{845}{4104}k_{R4}) \\
& = h\,(\frac{-\beta}{N_{i5}}\left(I_i + \frac{439}{216}k_{I1} - 8k_{I2} + \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4}\right)(S_i + \frac{439}{216}k_{S1} \\
& - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}\Big) \\
& - \mu\left(S_i + \frac{439}{216}k_{S1} - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}\right) + rN_{i5} \\
& + \delta\left(R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3} - \frac{845}{4104}k_{R4}\right))
\end{aligned}
\tag{53}
$$

$$k_{E5} = hf_2 \left(t_i + h, S_i + \frac{439}{216}k_{S1} - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}, \right.$$

$$E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4}, I_i + \frac{439}{216}k_{I1} - 8k_{I2}$$

$$+ \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4}, R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3}$$

$$\left. - \frac{845}{4104}k_{R4} \right) \tag{54}$$

$$= h \left(\frac{\beta}{N_{i5}} \left(I_i + \frac{439}{216}k_{I1} - 8k_{I2} + \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4} \right) \left(S_i + \frac{439}{216}k_{S1} \right. \right.$$

$$- 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4} \right) - (\mu + \sigma + \kappa) \left(E_i + \frac{439}{216}k_{E1} - 8k_{E2} \right.$$

$$\left. \left. + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4} \right) \right)$$

$$k_{I5} = hf_3 \left(t_i + h, S_i + \frac{439}{216}k_{S1} - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}, \right.$$

$$E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4}, I_i + \frac{439}{216}k_{I1} - 8k_{I2}$$

$$+ \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4}, R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3}$$

$$\left. - \frac{845}{4104}k_{R4} \right) \tag{55}$$

$$= h \left(\sigma \left(E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4} \right) - (\mu + \alpha \right.$$

$$\left. + \gamma) \left(I_i + \frac{439}{216}k_{I1} - 8k_{I2} + \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4} \right) \right)$$

$$k_{R5} = hf_4 \left(t_i + h, S_i + \frac{439}{216}k_{S1} - 8k_{S2} + \frac{3680}{513}k_{S3} - \frac{845}{4104}k_{S4}, \right.$$

$$E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4}, I_i + \frac{439}{216}k_{I1} - 8k_{I2}$$

$$+ \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4}, R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3}$$

$$\left. - \frac{845}{4104}k_{R4} \right) \tag{56}$$

$$= h \left(\kappa \left(E_i + \frac{439}{216}k_{E1} - 8k_{E2} + \frac{3680}{513}k_{E3} - \frac{845}{4104}k_{E4} \right) \right.$$

$$+ \gamma \left(I_i + \frac{439}{216}k_{I1} - 8k_{I2} + \frac{3680}{513}k_{I3} - \frac{845}{4104}k_{I4} \right)$$

$$- \mu \left(R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3} - \frac{845}{4104}k_{R4} \right)$$

$$\left. - \delta \left(R_i + \frac{439}{216}k_{R1} - 8k_{R2} + \frac{3680}{513}k_{R3} - \frac{845}{4104}k_{R4} \right) \right)$$

where $i = 0,1, \dots, n-1$.

The sixth stage needs to find k_{S6}, k_{E6}, k_{I6} and k_{R6} as below, in equations (57-60) where f_1, f_2, f_3, f_4 are unknown functions, t is a time and h is a step size:

$$k_{S6} = hf_1\left(t_i + \frac{h}{2}, S_i - \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5},\right.$$

$$E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5}, I_i - \frac{8}{27}k_{I1}$$

$$+ 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}, R_i - \frac{8}{27}k_{R1} + 2k_{R2}$$

$$\left. - \frac{3544}{2565}k_{R3} + \frac{1859}{4104}k_{R4} - \frac{11}{40}k_{R5}\right)$$

$$= h\left(\frac{-\beta}{N_{i6}}\left(I_i - \frac{8}{27}k_{I1} + 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}\right)(S_i\right. \tag{57}$$

$$- \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5}\right)$$

$$- \mu\left(S_i - \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5}\right) + rN_{i6}$$

$$\left. + \delta\left(R_i - \frac{8}{27}k_{R1} + 2k_{R2} - \frac{3544}{2565}k_{R3} + \frac{1859}{4104}k_{R4} - \frac{11}{40}k_{R5}\right)\right)$$

$$k_{E6} = hf_2\left(t_i + \frac{h}{2}, S_i - \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5},\right.$$

$$E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5}, I_i - \frac{8}{27}k_{I1}$$

$$+ 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}, R_i - \frac{8}{27}k_{R1} + 2k_{R2}$$

$$\left. - \frac{3544}{2565}k_{R3} + \frac{1859}{4104}k_{R4} - \frac{11}{40}k_{R5}\right) \tag{58}$$

$$= h\left(\frac{\beta}{N_{i6}}\left(I_i - \frac{8}{27}k_{I1} + 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}\right)(S_i\right.$$

$$- \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5}) - (\mu + \sigma + \kappa)(E_i$$

$$\left. - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5})\right)$$

$$k_{I6} = hf_3\left(t_i + \frac{h}{2}, S_i - \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5},\right.$$

$$E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5}, I_i - \frac{8}{27}k_{I1}$$

$$+ 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}, R_i - \frac{8}{27}k_{R1} + 2k_{R2}$$

$$\left. - \frac{3544}{2565}k_{R3} + \frac{1859}{4104}k_{R4} - \frac{11}{40}k_{R5}\right) \tag{59}$$

$$= h\left(\sigma\left(E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5}\right) - (\mu\right.$$

$$\left. + \alpha + \gamma)\left(I_i - \frac{8}{27}k_{I1} + 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}\right)\right)$$

$$
\begin{aligned}
k_{I6} = hf_3 \, (t_i + \frac{h}{2}, S_i - \frac{8}{27}k_{S1} + 2k_{S2} - \frac{3544}{2565}k_{S3} + \frac{1859}{4104}k_{S4} - \frac{11}{40}k_{S5}, \\
E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5}, I_i - \frac{8}{27}k_{I1} \\
+ 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}, R_i - \frac{8}{27}k_{R1} + 2k_{R2} \\
- \frac{3544}{2565}k_{R3} + \frac{1859}{4104}k_{R4} - \frac{11}{40}k_{R5}) \\
= h \, (\sigma \left(E_i - \frac{8}{27}k_{E1} + 2k_{E2} - \frac{3544}{2565}k_{E3} + \frac{1859}{4104}k_{E4} - \frac{11}{40}k_{E5} \right) - (\mu \\
+ \alpha + \gamma) \, (I_i - \frac{8}{27}k_{I1} + 2k_{I2} - \frac{3544}{2565}k_{I3} + \frac{1859}{4104}k_{I4} - \frac{11}{40}k_{I5}))
\end{aligned}
\tag{60}
$$

where $i = 0,1, \dots, n-1$.

4. Results and Discussion

Numerical solutions for the nonlinear influenza model in Australia in 1919 are discussed and analyzed in this section. Table 3 is to compare the values of variables $S(t), E(t), I(t), R(t)$ between RK_4 and RK_{45} methods. Where step size h in { 0.5, 0.25, 0.04 } and interval [0,70] days. For $E(t)$ has the smallest value with ($h = 0.04$), every 12 hours during 70 days in the RK_{45} compared with the RK_4. It is clear from Table 3 that RK_{45} method is better than RK_4 method. The Matlab program is used to numerically solve the influenza pandemic model.

Figures 1, 2 describes the influenza epidemic in Australia in 1919. In Figure 1 $(a), (b), (c)$ of RK_4, the curve is high then it starts to decrease after 20 days in the proportion of susceptible population $S(t)$. While the curve is low then it starts to increase then it goes down after 40 days in the proportion of exposed population $E(t)$ and the proportion of infective population $I(t)$, the curve is low then it starts to increase after 20 days in the proportion of recovered population $R(t)$, this means that susceptible population will be infected with influenza epidemics in the coming years. This method is convergence with RK_{45} method was in Figure 2 $(a), (b), (c)$ of RK_{45}, the curve is high then it starts to decrease after 20 days in the proportion of susceptible population $S(t)$. while the curve is low then it starts to increase then it goes down after 40 days in the proportion of exposed population $E(t)$ and the proportion of infective population $I(t)$, the curve is low then it starts to increase after 20 days in the proportion of recovered population $R(t)$, this means that susceptible population will be infected with influenza epidemics in the coming years. These methods are convergent in solution because they are iterative numerical methods. These methods agree with previous studies [1].

Table3: Numerical solution and values of influenza mode

Variable	h/day^{-1}	RK_4	RK_{45}
	0.5	0.59720331	0.59720329
$S(t)$	0.25	0.59715737	0.59715736
	0.04	0.59711877	0.59711877
	0.5	0.00224669	0.00224670
$E(t)$	0.25	0.00224599	0.00224599
	0.04	0.00224539	0.00227205

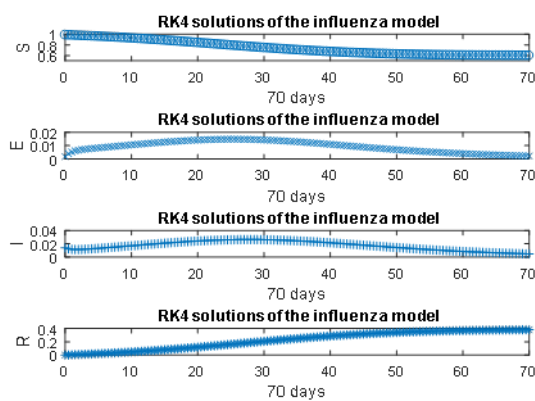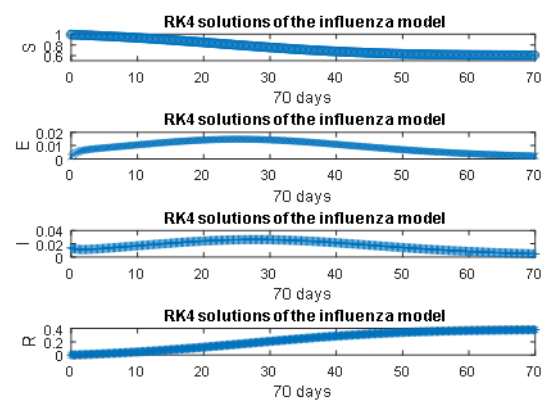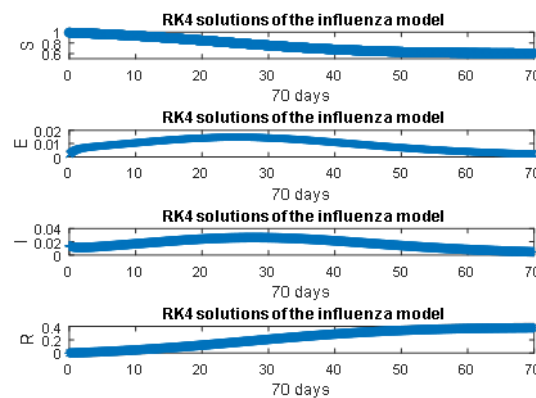|			
$I(t)$	0.5	0.00473296	0.00473297
	0.25	0.00473178	0.00473178
	0.04	0.00473076	0.00473076
$R(t)$	0.5	0.38142439	0.38142440
	0.25	0.38146692	0.38146694
	0.04	0.38150269	0.38150269

$(a)\ h = 0.5, m = 140$ $(b)\ h = 0.25, m = 280$

$(c)\ h = 0.04, m = 1750$

Figure 1. Numerical solution of the influenza model around the value of parameters using RK_4 of $S(t), E(t), I(t), R(t)$ in 1919 when step size $h = 0.5, 0.25, 0.04$, iterations $m = 140, 280, 1750$.

$(a)\ h = 0.5, m = 140$

$(b)\ h = 0.25, m = 280$

$(c)\ h = 0.04, m = 1750$

Figure 2. Numerical solution of the influenza model around the value of parameters using RK_{45} of $S(t), E(t), I(t), R(t)$ in 1919 when step size $h = 0.5, 0.25, 0.04$, iteration $m = 140, 280, 1750$.
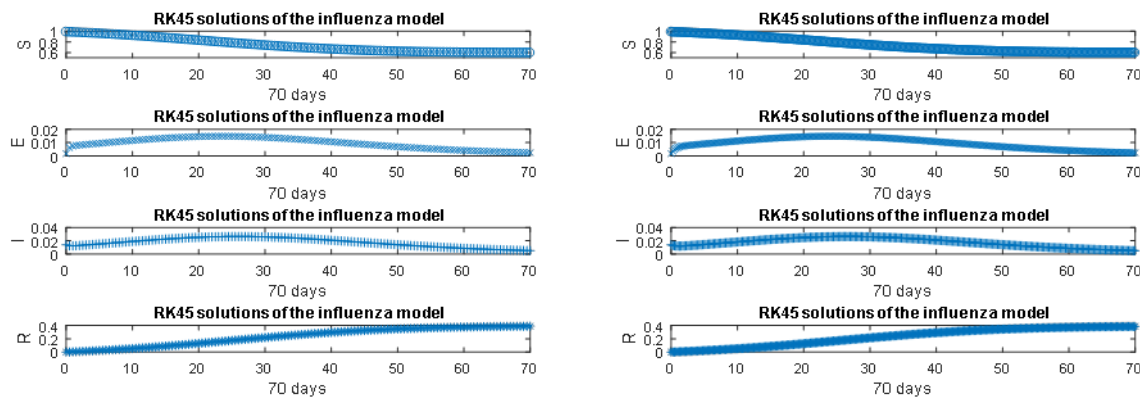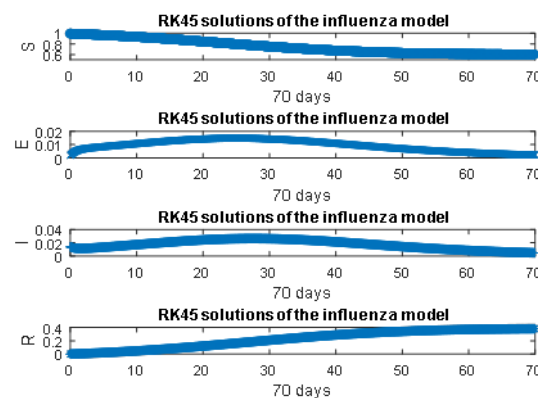
Conclusion

In the current study, some reliable, accurate, and approximate methods are used for solving a nonlinear system of epidemic models for ordinary differential equations of the first order. There is a convergence in the results of the numerical methods which are Runge-Kutta of order 4 and of order 45 as shown in Table 3. The numerical methods RK_4 and RK_{45} help us to analyze the spread of the epidemic in the influenza model. The results obtained showed that individual $S(t)$ of the proportion of the susceptible population, individual $E(t)$ of the proportion of the exposed population, and individual $I(t)$ of the proportion of the infective population are gradually decreased to 70 days. While individual $R(t)$ the proportion of the recovered population gradually increased until. A MATLAB program has been helped to solve this problem.

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