1. We first check that f_* is well defined, by definition of being a strong deformation retract we have $f_* : A \to A$ as the identity map on objects, furthermore if γ is a path, then $f(\gamma, 1) \subset Y$, so that we only need check that f_* is well defined on equivalence classes of paths to see that f_* is a map from $\Pi(X, A)$ to $\Pi(Y, A)$. Let γ and γ' be homotopic paths (with respect to A), then since f_* is a strong deformation retract onto Y, we have $f_*(\gamma) \sim_A \gamma$ and $f_*(\gamma') \sim_A \gamma'$ are homotopic. It follows that by transitivity

$$f_*(\gamma) \sim_A \gamma \sim_A \gamma' \sim_A f_*(\gamma')$$

are homotopic with respect to A.

To show f_* is an isomorphism of groupoids it will suffice to provide an inverse. Define g as the embedding of Y into X, it is clear that g_* is identity on objects and well defined on paths. $f_*g_* = 1_{\Pi(Y,A)}$ since both are identity on A, and if γ is a path in Y, then both g and f fix γ, hence $f_*g_*([\gamma]) = [\gamma]$. Now considering g_*f_*, we once again have both being identity on A. Now if γ is a path in X, f being a strong deformation retract implies that γ is homotopic in X to $f(\gamma, 1)$ with respect to A, but then $g(f(\gamma, 1), 1) = f(\gamma, 1)$, so that $g(f(\gamma, 1), 1) \sim_A \gamma$. This implies that $g_*f_*([\gamma]) = [\gamma]$.

2.

3. **(a)** Consider $F = \langle a, b \rangle$ to be the free group on 2 generators. Then we can take the group homomorphism defined on generators, $\varphi : F \to G,\ a \mapsto xy, b \mapsto yxy$. To check that this is onto, we need only check $x, y \in \varphi(F)$, but this is straightforward, since

$$\varphi(ba^{-1}) = \varphi(b)\varphi(a)^{-1} = yxyy^{-1}x^{-1} = y \qquad \varphi(a^2b^{-1}) = \varphi(a)\varphi(ba^{-1})^{-1} = xyy^{-1} = x$$

By definition of G, we have $\ker\varphi = \{\alpha \in F | \varphi(\alpha) \in \langle xyxy^{-1}x^{-1}y^{-1} \rangle\}$, where $xyxy^{-1}x^{-1}y^{-1} = \varphi(a^3b^{-2})$, so that $\varphi(\alpha) \in \langle \varphi(a^3b^{-2}) \rangle$ exactly when $\alpha \in \langle a^3b^{-2} \rangle$. Hence we have $\ker\varphi = \langle a^3b^{-2} \rangle$, so that by the first isomorphism theorem

$$H \simeq F/\langle a^3b^{-2} \rangle = F/\ker\varphi \simeq G$$

(b) We have the relation $xy^2x^{-1} = y^3$, then writing conjugation by x as ϕ, we have in general $\phi(y^{2n}) = \phi(y^2)^n = y^{3n}$. Applying two conjugations it is easy to see that $x^2y^4x^{-2} = xy^6x^{-1} = y^9$. A little harder is

$$x^3y^4x^{-3} = (x^3y)y^4(x^3y)^{-1} = (yx^2)y^4(yx^2)^{-1} = y(x^2y^4x^{-2})y^{-1} = y(y^9)y^{-1} = y^9 = x^2y^4x^{-2}$$

This implies that $y^6 = xy^4x^{-1} = y^4$ and hence $y^2 = 1$. Our original relations then give us $x = yx$ implying $y = 1$ which implies that $x^2 = x^3$, so that $x = 1$ as well.

4.