1. **(a)** Let $x \in \{a \in X | \exists U$ open, such that $a \in U \subset A\}$, then $U \cup A^\circ$ is an open subset of A containing A°, hence by maximality $x \in U \cup A^\circ = A^\circ$. If $a \in A^o$, then a is in a open subset contained in A proving the other set inclusion.

let $x \in \{x \in X | \forall U$ open with $x \in U$ and $U \cap A \neq \emptyset\}^c$, then there exists some open $U \subset A^c$ containing x, so that $A \subset U^c$ is closed this implies $\overline{A} \subset U^c$ and hence $x \notin \overline{A}$. Conversely, if $x \in \overline{A}^c$, then $\overline{A}^c$ is an open set disjoint from A containing x, so that $x \in \{x \in X | \forall U$ open with $x \in U$ and $U \cap A \neq \emptyset\}^c$.

(b) U° is open by definition, so $U^\circ = U$ implies U open. If U is open, then U is an open set contained in U, so that $U \subset U^\circ$ and hence $U = U^\circ$.

$\overline{A}$ is closed, hence $A = \overline{A}$ implies A is closed. Now suppose that A is closed, then A is a closed set containing A, hence $A \supset \overline{A}$, which implies $A = \overline{A}$.

(c) The compliment of A° is closed, and $A^\circ \subset A$ implies that $(A^\circ)^c \supset A^c$, implying that $\overline{A^c} \supset (A^\circ)^c$. Conversely, if $x \in \overline{A^c}$, then by part (a), any open set containing x has non-empty intersection with A^c, hence there does not exist an open set U containing x, such that $U \subset A$, applying (a) again ,this means that $x \notin A^\circ$

$\overline{A}^c$ is an open set contained in A^c, hence $\overline{A}^c \subset (A^c)^\circ$. Conversely, if $x \in \overline{A}$, then from (a), any open set containing x has non-trivial intersection with A, hence applying part (a) again we get that $x \notin (A^c)^\circ$, hence $\overline{A} \subset ((A^c)^\circ)^c$, contraposing this gives the desired equality.

2. Consider the collection $\mathcal{I}$ of closed sets in X, which are not finite unions of irreducibles. Every descending chain being eventually constant is equivalent to every descending chain having a lower bound (i.e. If $\cap_i F_i = F_j$, then F_j is a lower bound on the chain). Thus we can apply Zorn's lemma which furnishes a minimal element Z in $\mathcal{I}$, if Z were not irreducible, then it would need to be a union of closed subsets $Z_1 \cup Z_2$, since Z is not a finite union of irreducibles, the same must apply to one of Z_1 or Z_2, but this contradicts the minimality of $Z \in \mathcal{I}$. It follows that $\mathcal{I} = \emptyset$, so that X is a finite union of irreducible elements.