1. a Let $x \in A_1^\circ \cup A_2^\circ$, then without loss of generality it will suffice to show $A_1^\circ \subset (A_1 \cup A_2)^\circ$. We have that $A_1^\circ \subset A_1 \subset A_1 \cup A_2$, which is open, then

$$(A_1 \cup A_2)^\circ \subset (A_1 \cup A_2)^\circ \cup A_1^\circ \subset A_1 \cup A_2$$

is open, and since $(A_1 \cup A_2)^\circ$ is the largest open set contained in $A_1 \cup A_2$, it must be the case that

$$(A_1 \cup A_2)^\circ = (A_1 \cup A_2)^\circ \cup A_1^\circ$$

so that $A_1^\circ \subset (A_1 \cup A_2)^\circ$. To show that equality need not hold, consider the usual topology on $\mathbb{R}$, then

(b) Once again, it will suffice to show that $\overline{A_1} \supset \overline{A_1 \cap A_2}$, then $\overline{A_2} \supset \overline{A_1 \cap A_2} \implies \overline{A_1} \cap \overline{A_2} \supset \overline{A_1 \cap A_2}$ will follow by symmetry. Note that $\overline{A_1} \supset A_1 \supset A_1 \cap A_2$ is closed, so that in particular

$$\overline{A_1 \cap A_2} \supset \overline{A_1} \cap \overline{A_1 \cap A_2} \supset A_1 \cap (A_1 \cap A_2) = A_1 \cap A_2$$

is closed, implying that since $\overline{A_1 \cap A_2}$ is smallest closed set containing $A_1 \cap A_2$, we must have $\overline{A_1 \cap A_2} = \overline{A_1} \cap \overline{A_1 \cap A_2}$ which, in particular, implies that $\overline{A_1 \cap A_2} \subset \overline{A_1}$.