1. (a) Since $g \geq 0$, 0 is a simple function less than g, hence for any set $E \in \mathcal{M}$,

$$\mu_g(E) = \int_E g d\mu \geq int_E 0 d\mu = 0$$

Proving nonnegativity. Furthermore, we have

$$\mu_g(\emptyset) = \int \chi_\emptyset g d\mu = \int 0 d\mu = 0$$

So it will suffice to sheck for countable additivity. Consider the disjoint collection $\{E_i\}_1^\infty$, and denote $E := \bigcup_1^\infty E_i$. Then $\chi_E = \sum_1^\infty \chi_{E_i}$, so that

$$\begin{aligned}\mu_g(E) = \int_E g d\mu = \int g\chi_E d\mu = \int \sum_1^\infty g\chi_{E_i} d\mu = \int \lim_{n\to\infty} \sum_1^n g\chi_{E_i} d\mu \\ \overset{\text{MCT}}{=} \lim_{n\to\infty} \int \sum_1^n g\chi_{E_i} d\mu = \lim_{n\to\infty} \sum_1^n \int g\chi_{E_i} d\mu = \sum_1^\infty \int g\chi_{E_i} d\mu\end{aligned}$$

The second last equality follows from linearity.

(b)