Math 322 Office Hours Recap

September 23, 2024

Today there were just a few clarifications of definitions, examples and homework as well as looking at some exercises. None went in to great detail. o I will just share an exercise, as well as some useful practice materials.

Proving a Sufficiently Small Group is Abelian
(This is a problem from Dummit and Foote but I dont like that book, a nearly identical problem is in Lang; Lang pdf free online with ubc) I don't recommend using Lang as a primary reference at this point because it is quite advanced, but it has some interesting topics and exercises.

1. Show that every group of order 5 is abelian

   Solution. $\exists a, b \in G$, such that $ab \neq ba$. This implies a, b distinct and not 'powers' of one another, furthermore a, b cannot both have degree 2, else $ab = ba$ so that we have $a, b, a^{-1}, ab, ba, 1$ are six distinct elements.

Exercises for Students (maybe we can discuss these on Monday or Wednesday)
The previous exercise from Lang is quite simple with Lagranges theorem, here are a few exercises to prove it in a more insightful way

1. Let G be a group, H a subgroup. Prove that the orbits of H (i.e. cosets) partition G, i.e. show $x \in xH$, and $y \in xH \implies xH = yH$.

2. show that for each coset xH, $\#xH = \#H$.

3. Use the result of the previous two exercises to prove Lagrange's theorem: $\#H | \#G$

4. Apply Lagranges theorem to show that every group of prime order p is abelian, and hence every group of order 5 is abelian. How many non-isomorphic groups of order 5 are there? What about order p?

Here are some additional textbook exercises I can personally suggest:

1. Lang, Algebra, Exercises 1.1-1.4 (Yes 1.1 is just the question from earlier)

2. Herstein, Topics in Algebra, Chapters 2.3-2.5 all exercises