

**1. (1)** It goes without saying what the objects are. As per morphisms if  $0 \rightarrow A_* \rightarrow B_* \rightarrow C_* \rightarrow 0$  and  $0 \rightarrow X_* \rightarrow Y_* \rightarrow Z_* \rightarrow 0$  are two short exact sequences of chain complexes, define a morphism  $f$  to be morphisms of chain complexes  $(f^\alpha)_{n=1}^{A,B,C}$  so that  $f^A : A_* \rightarrow X_*$  and analogously for  $B, C$ . Moreover we require commutativity of the following:

$$\begin{array}{ccccc} A_* & \xrightarrow{\iota} & B_* & \xrightarrow{q} & C_* \\ \downarrow f & & \downarrow f & & \downarrow f \\ X_* & \xrightarrow{\iota} & Y_* & \xrightarrow{q} & Z_* \end{array}$$

**(2)** Since long exact sequences are chain complexes, we can simply view them as a subcategory.

**(3)** As mentioned in the problem statement the functor on objects has been constructed, namely it is the snake lemma. We need to define the action on morphisms and functoriality. Simply take  $f \mapsto f_* : H_n(A) \rightarrow H_n(X)$  for all  $n$ , and likewise for  $H_n(B), H_n(C)$ . Since homology is functorial our construction satisfies identity and inverse properties, but we still need to show that the following commutes

$$\begin{array}{ccccccc} \cdots & H_n(A) & \xrightarrow{\iota_*} & H_n(B) & \xrightarrow{q_*} & H_n(C) & \xrightarrow{\delta} H_{n-1}(A) \cdots \\ & \downarrow f_* & & \downarrow f_* & & \downarrow f_* & \downarrow f_* \\ \cdots & H_n(X) & \xrightarrow{\iota_*} & H_n(Y) & \xrightarrow{q_*} & H_n(Z) & \xrightarrow{\delta} H_{n-1}(X) \end{array}$$

For the squares other than the one with  $\delta$ , this follows directly from the commutativity conditions on  $f$  and functoriality of homology. For the square with the  $\delta$ -s we need to check. Consider  $[c] \in H_n(C)$ , then we use the proof of snake lemma to show that  $[x] = f_*\delta([c])$  is equal to  $\delta(f_*[c])$ .

Recall from the proof that  $\delta([c])$  is defined by  $[a]$  where for (an arbitrary)  $b$  such that  $q(b) = c$  we have  $a = \iota^{-1}(d(b))$ . Now considering such a  $b$ , by