1. Let $\{\lambda_i\}_I \cup \{\lambda_j\}_J$ be a partition of unity subordinate to A^c, B^c, with $i \in I \iff \text{supp}(\lambda_i) \subset A^c$. Then $f = \sum_I \lambda_i$ is smooth (smoothness is a local property and locally it is a finite sum of smooth functions), and $f \equiv 0$ on A by construction. Finally on B we have $1 = f + \sum_J \lambda_j = f + 0 = f$ since λ_j are only supported on B^c. $\square$

2. In order to make sense of the problem, we first should check that for a linear map A, $\text{Graph}(A) \subset \mathbb{R}^{2k}$ is a submanifold. To do so we can take the chart on $(\mathbb{R}^k)^2$ to be $(x, y) \mapsto (x, y - Ax)$, this map is clearly smooth and with smooth inverse (to get the inverse just add Ax), moreover we see that $\text{Graph}(A)$ is a linear subspace by construction on this chart, now that everything makes sense we should forget about ever doing this and just use the standard coordinate chart on $\mathbb{R}^{2k}$.

Now if 1 is not an eigenvalue of A we are done trivially, since $\text{Graph}(A) \cap \Delta = \emptyset$ so that transversality is vacuously true. Now suppose that 1 is an eigenvalue of A with eigenvector v, then since $\text{Graph}(A)$ and Δ are both k dimensional, it suffices to check that the intersection of their tangent spaces is non-zero at (v, v) to see that they are not transverse. Noticing that a path in $\text{Graph}(A)$ is of the form $(\gamma(t), A\gamma(t))$, we can take $\gamma(t) = tv$, to see that $\begin{pmatrix} v \\ v \end{pmatrix} \in T_{(v,v)}\text{Graph}(A)$, by virtually the same argument $\begin{pmatrix} v \\ v \end{pmatrix}$ is also in $T_{(v,v)}\Delta$, so that $T_{(v,v)}\text{Graph}(A) \cap T_{(v,v)}\Delta \supset \text{span}(v)$ has dimension atleast one, from here we are done since

$$\begin{aligned} 2k > 2k - 1 &\geq \dim T_{(v,v)}\text{Graph}(A) + \dim T_{(v,v)}\Delta - \dim T_{(v,v)}(\text{Graph}(A) \cap T_{(v,v)}\Delta) \\ &= \dim(T_{(v,v)}\text{Graph}(A) + T_{(v,v)}\Delta) \end{aligned}$$

$\square$

3. **(a)** The convention chosen effectively fixes our polygon up to rotation and translation. This is because for any fixed rotation, we choose the translate of the polygon with $p_1 = 0$, which is a unique representative for this fixed rotation. Fixing the rotation is fine since the rotation can be chosen independent of translation as the unique representative with $p_n - p_1 \in \mathbb{R}_{>0} \times \{0\}$. $\square$

(b) Define the function