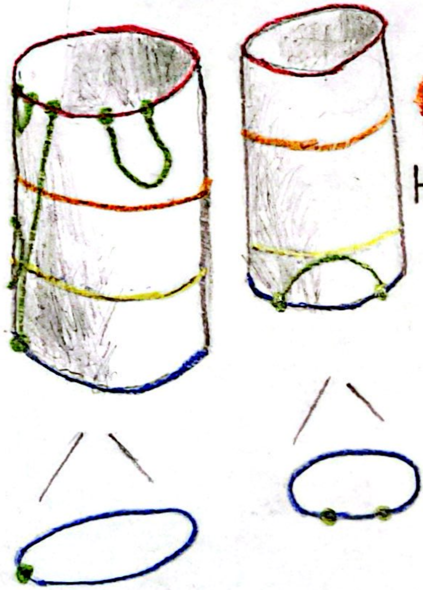
Homotopy Invariance of $I_2(f,Z)$

$f_0 : S^1 \sqcup S^1 \longrightarrow \Sigma_2$

$\Sigma_2 \supset Z$

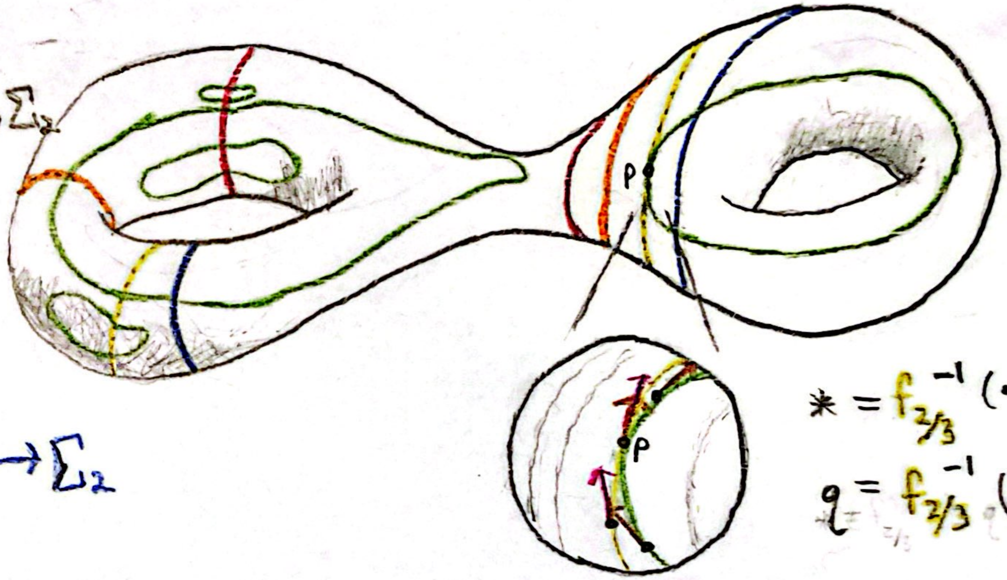


$f_{1/3}$

$H:(S^1 \sqcup S^1)\times I \to \Sigma_2$

$f_{2/3}$

$f_1 : S^1 \sqcup S^1 \longrightarrow \Sigma_2$



$* = f_{2/3}^{-1}(\cdot)$

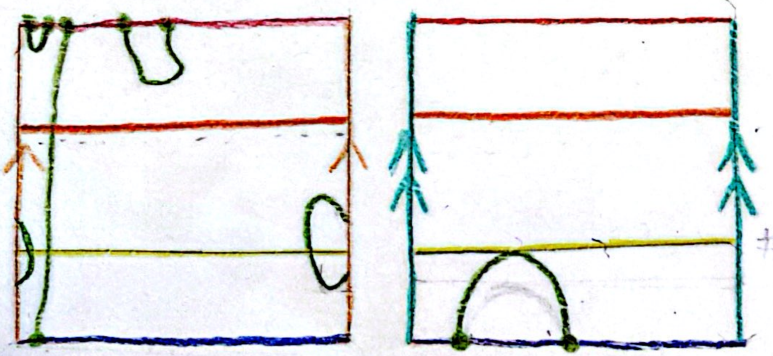
$q = f_{2/3}^{-1}(p)$

$\langle \longrightarrow \rangle = T_{\cdot} Z$

$\langle \longrightarrow \rangle = \mathrm{Im}\, d_* f_{2/3}$

$T_p Z = \mathrm{Im}\, d_q f_{2/3}$, $f_{2/3} \not\pitchfork Z$

This explains $\# f_{2/3}^{-1}(Z) = 4$, with $4 \not\equiv 5 \pmod 2$, $5 = \# f_0^{-1}(Z)$.



$H^{-1}(Z) \cong I \sqcup I \sqcup I \sqcup I \sqcup S^1$

$f_1^{-1}(Z) \sqcup f_0^{-1}(Z) = \partial H^{-1}(Z) \cong \sqcup^8 \{pt.\} = \partial(I \sqcup I \sqcup I \sqcup I \sqcup S^1)$, $f_1 \pitchfork Z$ and $f_0 \pitchfork Z$