Thesis

by

Tigran Saidnia

Emission kernel of parton shower

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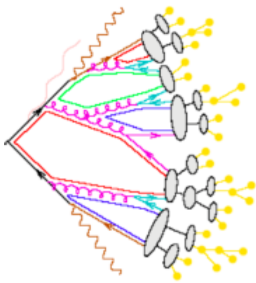

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statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, April 2, 2019

Tigran Saidnia

Abstract

Identification of air-shower induced radio signals in the AERA antenna stations

For a deeper understanding of the origin of high-energy cosmic rays detailed information on the energy, the direction and the mass are necessary. There has been growing interest in the detection of high-energy cosmic rays via the radion emission produced in extensive air showers, i.e. the cascade of secondary particles induced by cosmic rays. An important component of the extensive air showers is the pair of electrons and protons which produce radio emission due to electrodynamic effects, like by Geomagnetic- and Askaryan effects, which are subsequently discussed in detail.
Auger Engineering Radio Array (AERA) which is located at the Pierre Auger observatory in Argentina, aims to detect high-energy cosmic rays through the light of radio signals received by the AERA's antennas. The received signals are not free of noise. Thus the separation of true signals from false-positive signals (caused by noise) is one of the major problems in the radio signal processing, thereby reconstructing the air showers.
The aim of this thesis is to investigate selective cuts on observable describing the true radio signals, in order to separate them from false-positive signals with high efficiency and purity.
The main observables of the selection criteria are signal-to-noise ratio, signal time, and the angle between the measured and expected electric field vectors.
By optimizing the cuts on the observables, we obtained radio signal selection efficiency and purity of 88 % and 99.6 %, respectively.

Contents

Introduction

Introduction

Chapter 1

Theoretical Basics

1.1 Brief history of particle physics

Knowledge is a human need. For thousands of years we have been trying to understand the secrets of the universe. Such riddles fascinated even Johann Wolfgang von Goethe, as he wrote in his book Faust chapter 4 ; eine Tragedie, "What holds the world together in its innermost." Almost 400 years before Christ, an ancient Greek philosopher, Democritus, and his teacher Leukipp claimed that matter cannot be divided at will. Rather, there must be an Atomos (Greek: indivisible) that could no longer be subdivided. Democritus was of the opinion that there were infinitely many atoms with different geometric forms that were in contact in a certain way. He pointed out that a thing has a color, taste or even soul, based on the apparent effect of the composition of these small grains. Wilhelm Capelle: Die Vorsokratiker, Leipzig 1935, S. 399.
This statement of Democritus was first laughed at by the renowned philosopher aristotiles. It took about 2000 years for a chemist named John Dalton to deal with the subject. Based on various test series, he summarized his conclusion in his book A New System of Chemical Philosophy, that all substances consist of spherical indivisible atoms. The atoms of different elements have different masses and volumes. This was exactly the most striking difference to Democritus's atomic world. A New System of Chemical Philosophy, Band 1, Teil 1, Manchester, London 1808,
The discovery of the periodic system by D. Mendeleev and P. Meyer enabled us to arrange the atoms according to their mass in such a way that their properties occur in a certain order.
In 1897 Joseph Thompson was able to obtain a stream of particles by heating metals and deflecting them by a magnetic field. This electron beam was 200 times lighter than the lightest atom, hydrogen. His conclusion was that atoms cannot be indivisible. He suggested that each atom consists of an electrically positively charged sphere in which electrically negatively charged electrons are stored - like raisins in a cake.
furthermore, renowned scientists as well as Marie and Pierre Curie have contributed much to the development of atomic theory by discovering radioactivity, Boltzmann by kinetic gas theory and M. Plank, the founder of quantum physics. However, one of the most important steps in the atomic model was taken by the British physicist E. Rutherford.

He bombarded a thin aluminium foil with a radioactive sample. If Thompson's cake model were correct, only a few alpha particles would be detected behind the aluminium foil. Surprisingly, many particles were visible, which could only be explained by the assumption that the majority of atoms consisted of empty spaces. Another miracle was that some particles could be seen above or below the target sample. Since we knew that the alpha particles were positively charged, we could assume the electric repulsive force of two positive charges. In 1911, RUTHERFORD created the planetary model of the atom, which was developed a year later by his pupil NIELS BOHR (1885-1962) into a model known as the Bohr atom model. At first, however, it remained unclear what this core should consist of. In 1912, the Austrian physicist Victor Hess discovered during his balloon flights that the ionization rate of the Earth's atmosphere increases with altitude. This result was not expected because until then the Earth's radioactivity was known as the only source of air ionization. Therefore, he postulated this new type of radiation as cosmic radiation, which must originate outside the Earth's atmosphere [**?**].
Further investigations two years later confirmed the thesis of a cosmic background of such radiation. After this new discovery, it was discovered that the radiation consists of charged particles. In 1932, the American physicist Carl David Anderson was able to prove the postulated particle of Dirac, the positron, as a component of an air shower through his cloud chamber. For a long time, cosmic rays were the only way to analyse such exotic particles. This changed when particle accelerators were able to generate particles in collisions. But even today, cosmic rays are the only way to study particles of the highest energies, since these energies cannot be reached by today's particle accelerators, such as the LHC. The LHC, the world's largest accelerator at CERN, produces particles with centre-of-mass energy equivalent to a cosmic particle of nearly $10^{17}eV$, with the energy spectrum of cosmic particles reaching up to $10^{20}eV$. However, we can only analyse such exotic particles in detail by increasing the luminosity and procession of the particle accelerators at the nucleus. The discovery of the neutron by Chadwick (1932) showed that atomic nuclei are made up of protons and neutrons. It was also clear that, in addition to gravitation and the electromagnetic force,there should exist two short-range forces in nature: a strong force which binds the nucleons together and a weak force which is responsible for radioactive. In the meantime it was agreed that a new theory was needed for the classification and grouping of this particle zoo. This is how the current standard model came into being. The SLAC experiments showed that the electrons were scattering quasi-free point-like constituents inside the proton which were soon identified with quarks. This was the first time that quarks were shown to be dynamical entities, instead of bookkeeping devices to classify the hadrons (Gell-Mann's eightfold way).

1.2 Standard model

1.3 Quantum chromo dynamics

Nowadays, we know there are four types of interactions, see below:

Interaction	Energy scale	Range [m]	Mediators
Strong	~ 1	10^{-15}	g
Electromagnetic	$\sim 10^{-2}$	∞	γ
Weak	$\sim 10^{-6}$	10^{-18}	$W^{\pm}, Z$
Gravity	$\sim 10^{-38}$	∞	maybe graviton

Otherwise, it's clear meanwhile that nucleons are made up of quark and gluons. Whereby, the gluons are the exchange bosons for this short interaction. To explain the short range

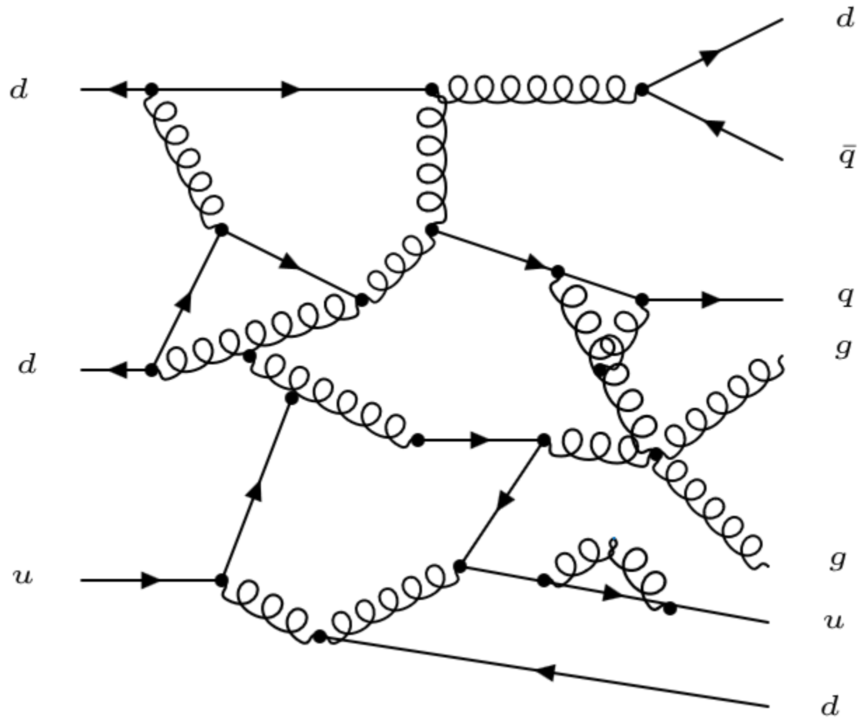


Figure 1.1: That's a schematic picture of neutron structure. at the left side of the diagram is the low-resolution to see. The 3 quarks picture allows us to interpretate the quantum numbers of the neutron in the valence band. We also obtain a high-resolution picture for a large Q^2. Here we have a lot of gluons (gluon sea) and quarks pair.
The interesting thing is, it doesn't matter in which energy scale we observe the quantum number of a neutron, because it is always the same.

of strong interaction Yukawa (1934) postulated mesons as a mediator for this force by the exchange of this massive field quanta. Three years later a candidate (π meson) was found in cosmic rays. Later on it was shown that Massive gauge field quanta break the gauge symmetry though so that the mediator must consequently be massless. But if they are based on the SU(3) gauge symmetry of the QCD[1] Lagrangian massless how can the strong sector be short range? Another question came from a series experiments at SLAC. Through high-energy electron-proton scattering could make evidence of existence

[1]The quantum field theory which describes this area is called Quantum chromo dynamics short QCD.

of quarks and their behaviour like free particles despite the energetically bound inner proton. The solution to these question was explained by Gross, Politzer and Wilczek through asymptotic freedom. This effect can be proved by the running coupling and anti screening in QCD. For the calculation the propagator loop correction in QCD we have to consider both quark loops (negative contribution $\rightarrow$ screening) and gluon loops (positive contribution $\rightarrow$ anti screening).

Figure 1.2: Running coupling compared for QED, with a positive and QCD with a negative beta function

The one loop running coupling in QCD is:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0 \alpha_s(\mu^2) ln(\frac{Q^2}{\mu^2})} \tag{1.1}$$

Where $\beta_0 = \dfrac{11N_c - 2n_f}{12\pi}$, n_f comes from the first diagram and causes screening. n_f is the number of quarks and N_c the number of colours and comes from the second diagram (anti screening).

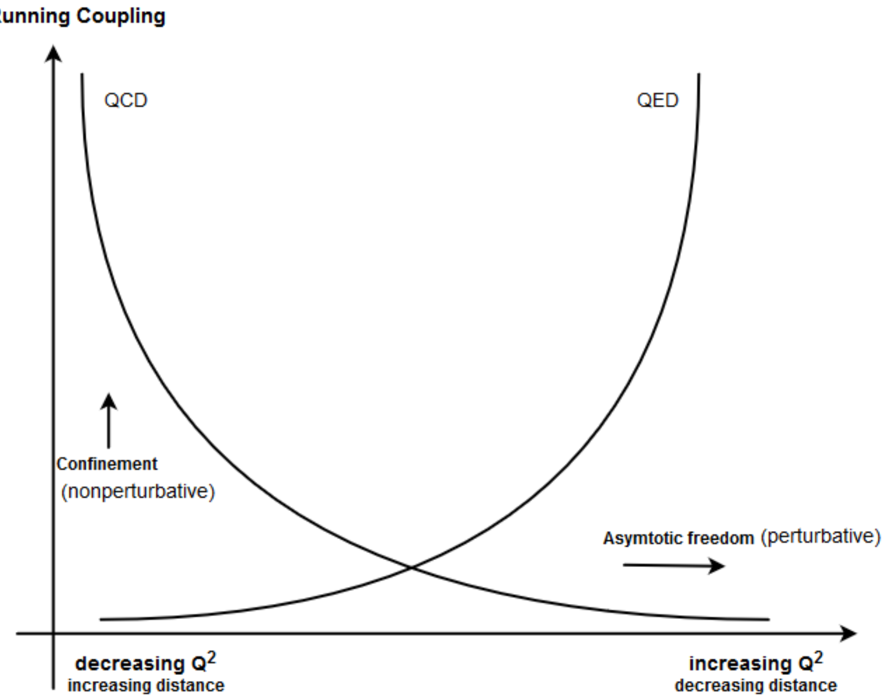
Obviously, with $n_f = 6$ and $N_c = 3$ in standard model we will get $\beta_0 > 0$. The Beta function is defined as:

$$\beta(\alpha) = -(\beta_0 \alpha^2 + \beta_1 \alpha^3 + \beta_2 \alpha^4 + ...) = \frac{d\, \alpha(Q^2)}{d\, ln(Q^2)} \tag{1.2}$$

e.g. $-(\beta_0 \alpha^2) < 0$ will be negative, which is actually the opposite of QED with $\beta_0 = -\dfrac{\pi}{3} \rightarrow -(\beta_0 \alpha^2) > 0$! That means coupling constant in QCD will increase with decreasing Q^2 (increasing distance), In QED vice versa.

Asymptotic freedom allows us to use perturbation theory [2]. Quarks have not yet been observed as free particles. With increasing separation it will be easier to produce quark-antiquark pair than to isolate quark because the coupling between them too strong is. This mechanism is called confinement. Confinement It has been confirmed in Lattice QCD, but not yet mathematically. It belongs to nonperturbative theory. Quarks prefer to bind into hadrons what can be classified to baryons with three quarks state and mesons with a quark-antiquark state. As we know, the wave function of fermions must be antisymmetric according to Pauli exclusion principle under the exchange of two quarks. Interestingly, there are resonance states with spin $\frac{3}{2}$ like Δ^{++}. The spins of the three up quarks are parallel to each other, have the same flavour and orbital angular momentum L=0. This means that an exchange of flavour, spin and space (orbital angular momentum) does not lead to any change. This problem is solved with the additional degree of freedom, the

[2] Actually there is need of two more things, if we want to make the connection between theory and experiment: either infrared safety or factorisation. That will be discussed in the next chapter



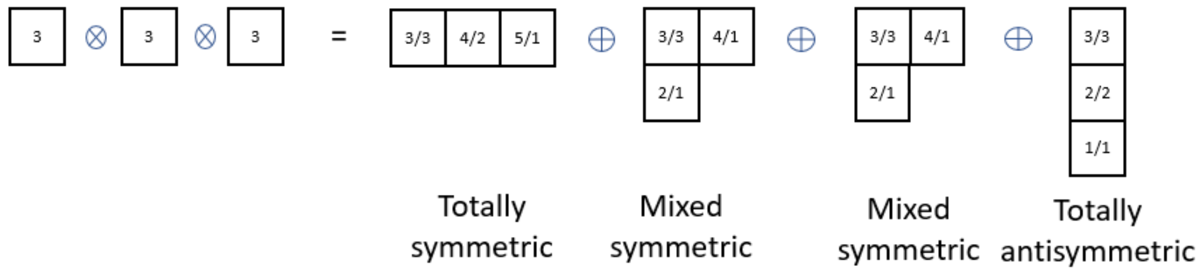
so-called color charge. Each quark comes in one of three colours red, green or blue and also anticolour $\bar{r}, \bar{b}, \bar{g}$ for antiquarks. The hadrons are colour singlets in regard with the hypothesis, , they are invariant under rotations in colour space. The colour hypothesis describes the existence of mesons with $q\bar{q}$ and baryons with qqq. because if the wave function is odd in color, we have solved the spin statistical problem. The total wave function for each particle can be expressed in terms of:

$$\begin{aligned} \Psi_{3q} = \psi_{space} \times \chi_{spin} \times \theta_{colour} \times \phi_{flavour} \\ O(3) \quad SU(2) \quad SU(3) \quad SU(6) \end{aligned} \tag{1.3}$$

Now we can compute all possible States in regard to colour With Young Tableaux. One uses group theory methods, for instance the Young Tableaux technique, to decompose products of irreducible representations into sums.
After using The same procedure for SU(2) and SU(6) for spins and flavours of the three quarks we will get:

$$\begin{aligned} 2 \otimes 2 \otimes 2 &= 4 \oplus 2 \oplus 2 \oplus 0 \\ 6 \otimes 6 \otimes 6 &= 56 \oplus 70 \oplus 70 \oplus 20 \end{aligned} \tag{1.4}$$

As we can see, the total wave function is most complicated in the QCD area. That's the reason why the Lagrangian of QCD is always given in the short form. I'll get to the bottom of Lagrangian in QCD later. Before the QCD is formulated as a gauge



$$= 10 \oplus 8 \oplus 8 \oplus 1$$

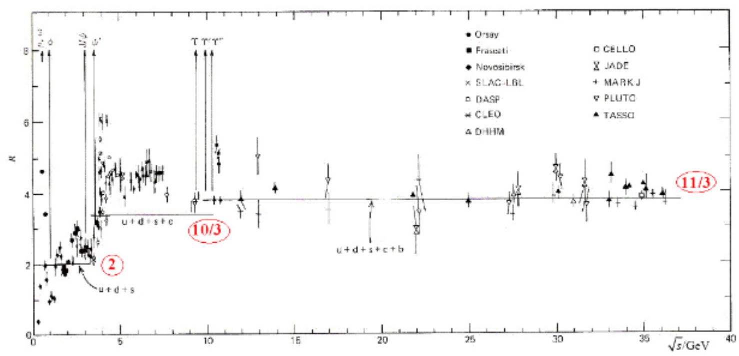
theory, an experiment should be pointed out which makes clear why there is an additional degree of freedom in the QCD and why there is no U(1)-symmetry here. Looking at the electron-positron scattering again, it is important to realize that not only $\mu^+\mu^-$, but also e^+e^-,$\tau^+\tau^-$ and also $q\bar{q}$ can arise, where the quark pairs fragment into hadrons. For the ratio:

$$R = \frac{\sigma(e^+e^- \to Hadronnen)}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1.5}$$

one would expect, due to the fact that the coupling takes place over the charge, that only the sum over the square of the quark charges (because $e^2{}_\mu = 1$) contributes. However, there is an additional factor N_C that can be determined experimentally

$$R = N_C \sum_q e_q^2 \tag{1.6}$$

Without this factor one would expect for $u, d, s,$ u, d, s, c and u, d, s, c, b Respectively $\frac{2}{3}, \frac{10}{9}, \frac{11}{10}$ The experiment showed a third of the respective results though (i.e. $N_C = 3$):



1.4 QCD Lagrangian

QCD like QED and the weak interaction theory is described by representations of a symmetry group. From the condition that the Lagrangian must be invariant under arbitrary global and local symmetry transformations (Noether's theorem) follows the interactions terms. The Lagrangian of QCD is invariant under $U(3) = U(1) \times SU(3)$ global trafo. I'm just going to look into $SU(3)$. We can replace the three Pauli matrices from $SU(2)$ in the Yang-Mills theory by the eight Gell-Mann matrices λ^a with following relation:

$$\begin{aligned} &T^a = \frac{1}{2}\lambda^a \\ &[T^a, T^b] = i f^{abc} T^c \qquad \text{fundamental representation} \\ &(T^a{}_{adj})_b c = -i f^{abc} \qquad \text{adjoint representation} \end{aligned} \tag{1.7}$$

To quantize QCD theory is usually used the Faddeev-Popov in the path integral to fix a gauge and define a gluon propagator. The Lagrangian is given:

$$\begin{aligned} &\mathscr{L} = \mathscr{L}_{free} + \mathscr{L}_{int} \\ &\mathscr{L} = \sum_f \bar{\psi}_{if}(i\gamma^\mu \partial_\mu - m_f)\psi^{if} - \frac{1}{4} F_a{}^{\mu\nu} F^a{}_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^a{}_\mu)(\partial^\nu A^a{}_\nu) + (\partial^\mu \chi^{a*})(\partial_\mu \chi^a) \\ &-g_s \bar{\psi}_i T^a{}_{ij} \psi_j \gamma^\mu A^a{}_\mu - \frac{g_s}{2} f^{abc}(\partial_\mu A^a{}_\nu - \partial_{nu} A^a{}_\mu A_b{}^\mu A_c{}^\nu) - \frac{{g_s}^2}{4} f^{abc}(A_b{}^\mu A_c{}^\nu f^{ade}(A^d{}_\mu A^e{}_\nu) \\ &-g_s f^{abc}(\partial^\mu \chi^{a*})\chi^b A^c{}_\mu \end{aligned} \tag{1.8}$$

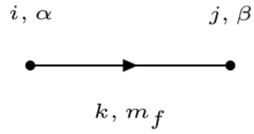
Here i,j are color indices in the fundamental representation, a color index in the adjoint representation of SU(3) respectively. f labels the six flavours of quarks. g_s describes the strong coupling constant and $A^a{}_\mu$ is the gluon field and it corresponds to anon-abelian gauge theory with structure constants f^{abc}. χ^a is a scalar field under Lorenz group, but anti commuting. With The field-strength tensor for QCD by:

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - g_s f_{abc} A^b{}_\mu A^c{}_\nu \tag{1.9}$$

It can be shown that the above Lagrangian is invariant under the following $SU(3)$ gauge transformations:

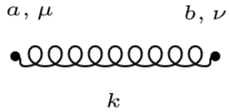
$$\begin{aligned} &\psi'(x) \to exp(i\,\eta_a(x)\,T^a)\,\psi(x) \\ &D' \to \partial_\mu + i g_s T_a\, A'^a_\mu \\ &A'^a_\mu \to A^a{}_\mu - \frac{1}{g_s}\partial_\mu \eta^a(x) + f^{abc}\eta_b(x) A_{c\nu}(x) \end{aligned} \tag{1.10}$$

$$L_{q,free} = \sum_f \bar{\psi}_{if}(i\gamma^\mu\partial_\mu - m_f)\delta_{ij}\psi^{jf} \Rightarrow$$



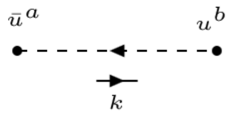
$$= (\frac{i}{\not{k} - m_f})_{\alpha\beta}\,\delta_{ij}$$

$$L_{g,free} = -\frac{1}{4}F_a{}^{\mu\nu}F^a{}_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^a{}_\mu)(\partial^\nu A^a{}_\nu) \Rightarrow$$



$$= \frac{-i}{k^2}(-g_{\mu\nu} - (1-\zeta)\frac{k_\mu k_\nu}{k^2})\delta^{ab}$$

$$L_{ghost,free} = (\partial^\mu\chi^{a*})(\partial_\mu\chi^a) \Rightarrow$$



$$= \frac{i}{k^2}\delta^{ab}$$

1.5 Colour factor calculation

In this section we will calculate the Casimir operators of the respective diagrams for later goals. Fundamental representation in $SU(3)$ are given by:

$$T^a = \vartheta^a \equiv \frac{\lambda^2}{2} \qquad \textit{with Gell} - \textit{Mann matrices } \lambda^a \tag{1.11}$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix},\ \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix},\ \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix},\ \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix}$$
$$\lambda^5 = \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix},\ \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix},\ \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix},\ \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \tag{1.12}$$

As we can see, λ^3 and λ^8 are diagonal. These generators satisfy:

Or in the adjoint representation:

$$[T^a, T^b] = if^{abc}T^c \Rightarrow$$

$$T^aT^b \quad - \quad T^bT^a \quad = \quad if^{abc}T^c$$

$$[F^a, F^b] = i f^{abc} F^c \Rightarrow$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \tag{1.13}$$

One of the most important equation for the colour factor calculation is the Jaccobi-Identity:

$$[T^a, [T^b, T^c]] + [T^c, [T^a, T^b]] + [T^b, [T^c, T^a]] = 0 \tag{1.14}$$

If we write this in terms of the structure constant, we'll get:

$$f^{axd} f^{bcx} + f^{cxd} f^{abx} + f^{bxd} f^{cax} = 0 \tag{1.15}$$

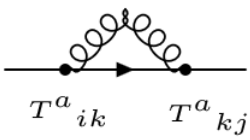
So we are able to compute:

$$f^{abc} = -2i\, tr(T^a[T^b, T^c]) \tag{1.16}$$

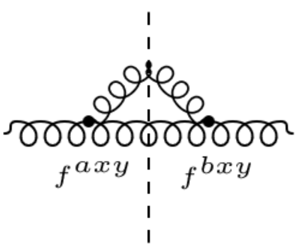
generalize to:

$$f^{abc} f^{xcd} = 4i\, tr(T^a[T^b, [T^c, T^d]]) \tag{1.17}$$

With this relations we can calculate all Casimir operators:



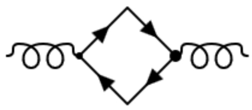
$$(T^a T^a)_{ij} = C_F \delta_{ij} = \frac{N_c^2 - 1}{2N_c} = C_F \sim \frac{N_c}{2}$$



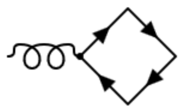
$$\sum_{xy} f^{axy} f^{bxy} = C_A \delta^{ab}$$

Which means the charge of gluon is twice a quark because:

$$C_A = N_c = 2C_F \sim 2(\frac{N_C}{2}) \tag{1.18}$$



$$T_f\,\delta^{ab}$$

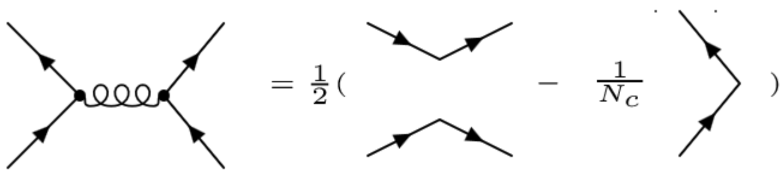


$$tr(T^a) = 0$$

One of the most important relation in this case is the Fierz identity. It shows the difference between QED and QCD!

$$\sum_a T_{ij}{}^a T_{kl}{}^a = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}) \tag{1.19}$$

Graphically it means: The charge transfer in QED takes place along the Fermion line



because photons cannot transport charges. On the other hand, the gluons can transfer color charges because they have color charges themselves.
The main relation we will use later for SU(N):

$$tr(T^aT^b) = T_{ij}{}^a T_{ji}{}^b = T_F\delta^{ab} \tag{1.20}$$

$$\sum_a (T^aT^a) = C_F\delta^{ij} \tag{1.21}$$

$$f^{acd}f^{bcd} = C_A\delta^{ab} \tag{1.22}$$

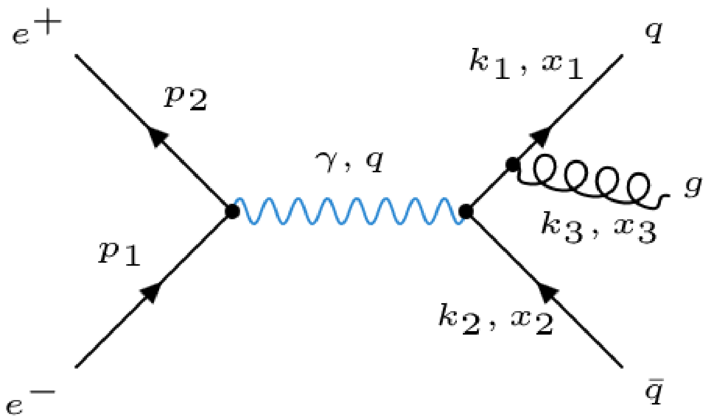
With $T_F = \frac{1}{2}$, $C_A = N$ and $C_F = \frac{N^2-1}{2N}$.

Chapter 2

Catani Seymour

2.1 IR and Collinar Divergences

Beyond the LO (Leading order) diagrams it happens singularities. To discuss about these consider first the process $e^- e^+ \to q\bar{q}g$



In order to calculate the cross section of this diagram, we have to consider the gluon emission from the antiquark. Since the calculation is quite long, we concentrate on the final result:

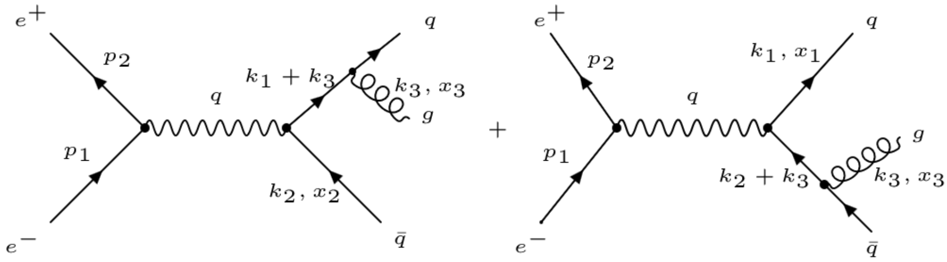


Figure 2.1: Left diagram $e^- e^+ \rightarrow qg\bar{q}$ and right $e^- e^+ \rightarrow q\bar{q}g$

$$\begin{aligned} A =& \frac{\bar{u}(k_1)(-ig_s\gamma^\nu \times T^a)[-i(\not{k}_1 + \not{k}_3)](-iee_q\gamma^\mu)v(k_2)\epsilon_\mu{}^{\lambda_1}\epsilon_\nu{}^{\lambda_2 *}}{(k_1+k_3)^2} \\ -& \frac{\bar{u}(k_1)(-iee_q\gamma^\mu)[i(\not{k}_2 + \not{k}_3)](-ig_s\gamma^\nu \times T^a)v(k_2)\epsilon_\mu{}^{\lambda_1}\epsilon_\nu{}^{\lambda_2 *}}{(k_1+k_3)^2} \\ \Rightarrow A =& -g_s T^a[\frac{\bar{u}\ \not{\epsilon}\,(\not{k}_1 + \not{k}_3)\,\Gamma\, v}{(k_1+k_3)^2} - \frac{\bar{u}\,\Gamma\,(\not{k}_2 + \not{k}_3)\ \not{\epsilon}\, v}{(k_2+k_3)^2}] \quad \text{with } \Gamma = (-iee_q\gamma^\mu)\epsilon_\mu{}^{\lambda_1} \end{aligned} \tag{2.1}$$

Under consideration that the partons are on-shell, we get:

$$A = -g_s T^a[\frac{\bar{u}\ \not{\epsilon}\,(\not{k}_1 + \not{k}_3)\,\Gamma\, v}{2k_1 \cdot k_3} - \frac{\bar{u}\,\Gamma\,(\not{k}_2 + \not{k}_3)\ \not{\epsilon}\, v}{2k_2 \cdot k_3}] \tag{2.2}$$

In the soft limit with $k_0 \rightarrow 0$ we can factorize A_{soft} the amplitude in two parts:

$$A = -g_s T^a[\frac{k_1\ \epsilon}{k_1 \cdot k_3} - \frac{k_2\ \epsilon}{k_2 \cdot k_3}]A_{born} \qquad \text{with } A_{born} = \bar{u}\,\Gamma\, v \tag{2.3}$$

Which one contains all information about colour and momenta and A_{born} with all spin information. If one calculates the cross section for it, one gets:

$$\begin{aligned} A =& C_F g_s^2 \sigma^{born} \int \frac{d^3k}{2k_0(2\pi)^3} 2(\frac{k_1 \cdot k_2}{(k_1 \cdot k_3)(k_2 \cdot k_3)}) \\ & C_F g_s^2 \sigma^{born} \int dcos\,\theta\, \frac{dk_0}{k_0} \frac{4}{(1 - cos\,\theta)(1 + cos\,\theta)} \end{aligned} \tag{2.4}$$

We define the energy fraction by:

$$x_i = \frac{2E_i}{\sqrt{s}} = \frac{2q\ \cdot\ k_i}{s} \tag{2.5}$$

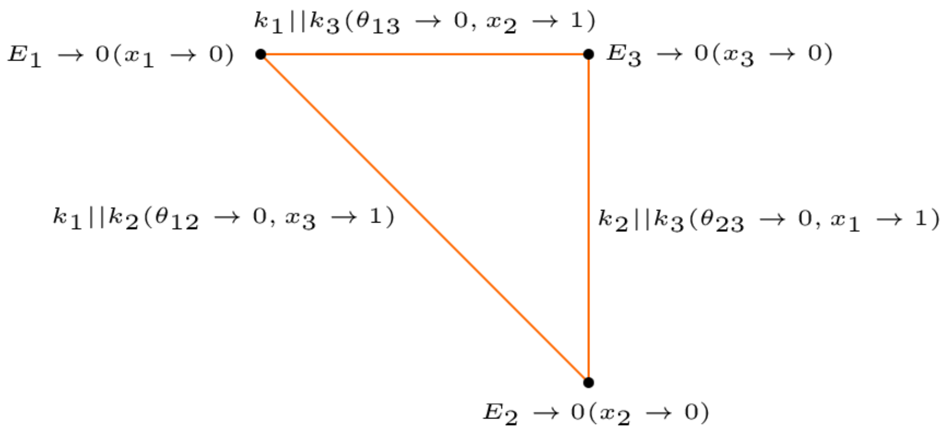
One can show that $\sum x_i = 2$ and thus, that only two of the xi are independent.
The final result is:

$$\frac{d^2\sigma}{dx_1 dx_2} = (\frac{4\pi\alpha}{s}) \sum {e_i}^2 \frac{2\alpha_s}{3\pi} \frac{{x_1}^2 + {x_2}^2}{(1 - x_1)(1 - x_2)} \tag{2.6}$$

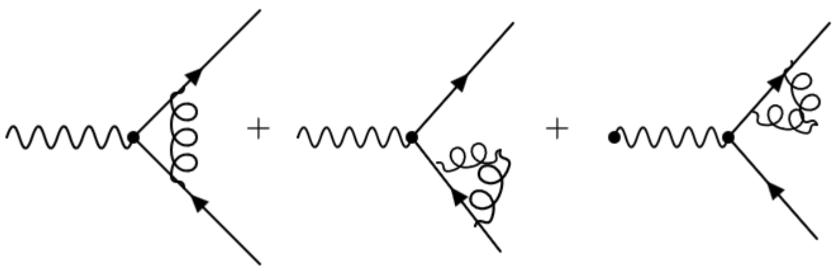
There are three singularities in regard with the final result. If the emitted photon is collinear to the outgoing quark or anti-quark ($x_1 \to 1$ or $x_2 \to 1$) and When the emitted gluon is very soft ($x_1 \to 1$ and $x_2 \to 1$). The singularities come from the quark propagator in each diagram. The denominators contain according Feynmann rules terms with $\sim \frac{1}{(k_i+k_j)^2}$. We can eliminate the quark mass under on-shell condition so that:

$$\frac{1}{(k_i + k_j)^2} = \frac{1}{2k_i \cdot k_j} = \frac{1}{2E_iE_j(1 - cos\theta_{ij})} = \frac{1}{s(1 - x_k)} \tag{2.7}$$

One can show all possibilities for three partons through a triangle:



fortunately, According to KLN-Theorem, IR singularities must cancel when summing the transition rate over all degenerate (initial and
nal) states The sum of the integrals int_R and int_V above is finite. However, this is not true for the individual contributions. The real part contains divergences due to soft and collinear radiation of massless particles. While M_real itself is a tree level amplitude and thus finite, the divergences show up upon integration over the phase space $\int d\phi_3$. In R V , the phase space is the same as for the Born amplitude, but the loop integrals contained in M_virt contain IR singularities. We will use deep inelastic scattering (DIS) to show how



the infra-red singularities are absorbed in the parton distributions.

2.2 Subtraction method

$$|A|^2 = |A^{(0)}{}_m|^2 + |A^{(0)}{}_{m+1}|^2 + 2Re(A^{(0)*}{}_n A^{(1)}{}_m) \tag{2.8}$$

Whereby, $|A^{(0)}{}_m|^2$ is the tree level contribution (Born sector) from Lo and has no divergences, $|A^{(0)}{}_{m+1}|^2 + 2Re(A^{(0)*}{}_m A^{(1)}{}_m)$ comes from the NLO and they are separately divergent. The problem in this case is that Integrations cannot be combined due to different phase space dimensions:

$$\sigma^{NLO} = \int_{m+1} \partial\sigma^R + \int_m \partial\sigma^V \tag{2.9}$$

The real and virtual contributions are both IR divergent and need to regularize in $d = 4 - 2\epsilon$ dim. To tackle this problem one can use the subtraction method in that way one add and subtract a local counter term $\partial\sigma^A$ with same singularity structure as terms $\partial\sigma^R$ to the integral. $\partial\sigma^A$ approximates soft and collinear singularities of $\partial\sigma^R$.

$$\sigma^{NLO} = \int_{m+1} [\partial\sigma^R - \partial\sigma^A] + \int_m [\partial\sigma^V + \int_1 \partial\sigma^A] \tag{2.10}$$

In this case, we can safely set $\epsilon \to 0$ for $\partial\sigma^R|_{\epsilon\to 0} - \partial\sigma^A|_{\epsilon\to 0}$ and run the integral numerically in 4-dim. On the other side, integrate over one-parton PS analytically explicitly cancel poles and then set $\epsilon \to 0$.

$$\sigma^{NLO} = \int_{m+1} [\partial\sigma^R|_{\epsilon\to 0} - \partial\sigma^A|_{\epsilon\to 0}] + \int_m [\partial\sigma^V + \int_1 \partial\sigma^A]_{\epsilon\to 0} \tag{2.11}$$

The virtual contribution must be UV-finite:

$$\int_m \partial\sigma^V = \int_m [\int_{loop} \partial\sigma^V{}_{bare} + \sigma^V{}_{Counter\,term}] \tag{2.12}$$

The addition of $\int_1 \partial\sigma^A$ to the $\int_m \partial\sigma^V$ ensures that IR poles are cancelled. The bare and counter contribution are separately divergent and have also different integral dimensions. One can use the same idea with the subtraction method to solve this problem:

$$\int_m \partial\sigma^V + \int_{loop} \partial\sigma^L - \int_{loop} \partial\sigma^L = \int_m \int_{loop} [\partial\sigma^V{}_{bare} - \partial\sigma^L] + \int_m [\sigma^V{}_{Counter\,term} + \int_{loop} \partial\sigma^L] \tag{2.13}$$

$$\sigma^{NLO} = \int_{m+1} [\partial\sigma^R - \partial\sigma^A] + \int_m \int_{loop} [\partial\sigma^V{}_{bare} - \partial\sigma^L] + \int_m [\sigma^V{}_{Counter\,term} + \int_{loop} \partial\sigma^L + \partial\sigma^A] \tag{2.14}$$

Determination of emission kernels

Now we want to introduce the properties of the counter term $\partial\sigma_A$. This term need to have the same behaviour like $\partial\sigma_R$ in d dimension. This process and specific observable independent term has to be obtained in a way that is independent of the particular jet observable considered. It must be exactly integrable analytically over one-parton phase space in d dim and $\partial\sigma_R - \partial\sigma_A$ has to be integrable via Monte Carlo methods. $\partial\sigma_A$ acts as a local counter-term for $\partial\sigma_B$ At this point, one should derive improved factorization formulae which called dipole formulae. Note that the notation below is symbolic:

$$\partial\sigma_A = \sum_{dipoles} \partial\sigma_B \otimes \partial V_{dipoles} \tag{2.15}$$

Whereby, the sum is over dipoles for all $m+1$ configurations with consideration to given m-parton state. $\partial\sigma_B$ describes the color/spin projection of Born-level exclusive cross section. The symbol $\otimes$ is for phase space convolutions and sums over colour and spin indices. $\partial V_{dipoles}$ will compute just once for all and match the singular property of the real part. So to say, it's universal. The good news is that we are now allowed to use a factorizable mapping from the m + 1-parton phase space to an m-parton subspace. That will be more clearer when we will use the parametrisation in the next chapter. If we integrate over all m+1, we can write:

$$\int_{m+1} \partial\sigma_A = \int_m \partial\sigma_B \otimes \sum_{dipoles} \int_1 \partial V_{dipoles} \tag{2.16}$$

That's exactly the most important thing we got because so is distinguished between the known m-sector from LO and the second the universal factor which contains all ϵ-poles.

Singularity Structure

before we begin with the collinear limit or soft limit respectively we are going to pull up the matrix element from Lo which has this below general form:

$$\mathcal{M}_m{}^{c_1,\ldots,c_m;s_1,\ldots,s_m}(p_1,\ldots,p_m) \tag{2.17}$$

c_i, s_i and p_i denotes respectively the colour, spin indices and the momenta for each m-parton in the tree level matrix element in the final state. A common way at the site is to define a basis in colour+helicity space.

$$\mathcal{M}_m{}^{c_1,\ldots,c_m;s_1,\ldots,s_m}(p_1,\ldots,p_m) \equiv (< c_i,..,c_m| \otimes < s_1,\ldots,s_m|)|1,..,m>_m \tag{2.18}$$

With $< c_i,..,c_m| \otimes < s_1,\ldots,s_m|)|$ as the basis and $|1,..,m>_m$ as a vector in this space. Thus, for the matrix element squared:

$$\begin{aligned} |\mathcal{M}_m|^2 =& (< c_i,..,c_m| \otimes < s_1,\ldots,s_m|)(|c_i,..,c_m> \otimes |s_1,\ldots,s_m>) < 1,..,m|1,..,m> \\ &= \delta_{c_1c_1},\ldots\delta_{c_mc_m} \otimes \delta_{s_1s_1},\ldots\delta_{s_ms_m}\ {}_m< 1,..,m|1,..,m>\ {}_m \end{aligned} \tag{2.19}$$

Define a colour-charge operator T_i with the emission of a gluon from each parton i:

$$T_i = T_i{}^c|c> \tag{2.20}$$

Its action onto the colour space is defined by:

$$< c_1,..,c_i,..c_m,c|T_i|b_1,..,b_i,..b_m >= \delta_{c_1b_1}\ldots T^c{}_{c_ib_i}\ldots\delta_{c_mb_m} \tag{2.21}$$

Where $T^c{}_{c_ib_i}$ the colour-charge matrix in the adjoint representation in the case of gluon emission or colour-charge matrix in the fundamental representation for quark/antiquark emission case. The following properties must be taken into account for that:

$$\begin{aligned} &T_i \cdot T_j = T_j \cdot T_i && \text{if } i \neq j, \text{ commutative property} \\ &T_i^2 = C_i \quad C_i = C_A && \text{for gluon and } C_i = C_F \text{ for (anti)quark} \\ &\sum_{i=1}^{m} T_i|1,...,m>_m = 0 && \text{for single state} \end{aligned} \tag{2.22}$$

Thus, the square of colour-correlated tree-amplitudes for the indices I, J refer either to final-state or initial-state partons will be:

$$|\mathcal{M}^{I,J}{}_{m,a...}|^2 = {}_{m,a...} < 1,...,m; a,...|T_I \cdot T_J|1,...,m; a,... > {}_{m}, a... \tag{2.23}$$

Dipole factorisation

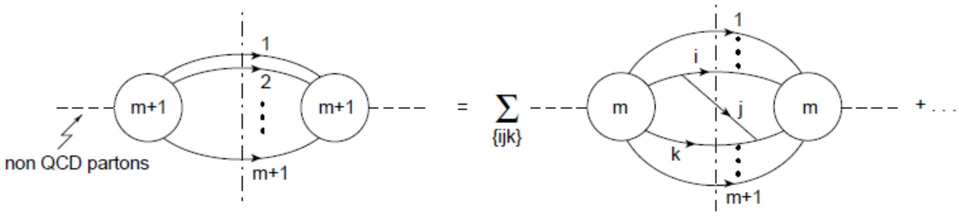
Consider (m+1)-partons with the general matrix element:

$$|\mathcal{M}_{m+1}(Q; p_1,...,p_i,...,p_j,...,p_{m+1})|^2 \tag{2.24}$$

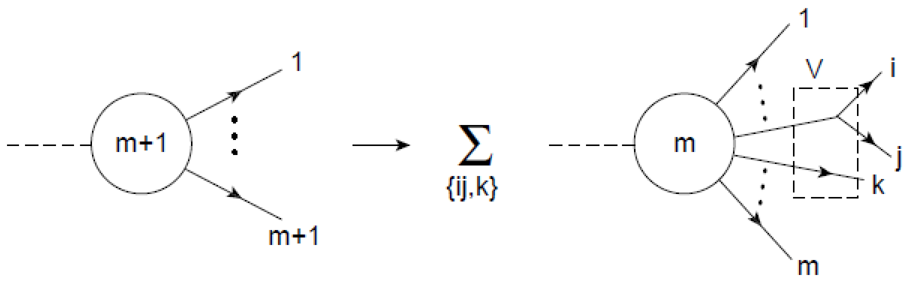
We need to take collinear and soft limits which allow factorisation. In the soft region it is used the kinematic with $p_j \to \lambda q$, $\lambda \to 0$, where q is a arbitrary four vector and λ a scale parameter. The matrix element is characterise with $|\mathcal{M}|^2 \sim \frac{1}{\lambda^2}$. and if p_i and p_j become collinear, we parametrise $p_j = \frac{z}{1-z}p_i$. So the matrix element will be $|\mathcal{M}|^2 \sim \frac{1}{p_i \cdot p_j}$. I am going to go into more details in the next chapter. That's here should be a summery of the behaviour of the matrix elements in different regions. Based on the Catani-Seymour method for (m + 1) partons matrix element, it's possible to factorise out parton k to give $|\mathcal{M}_m|^2$:

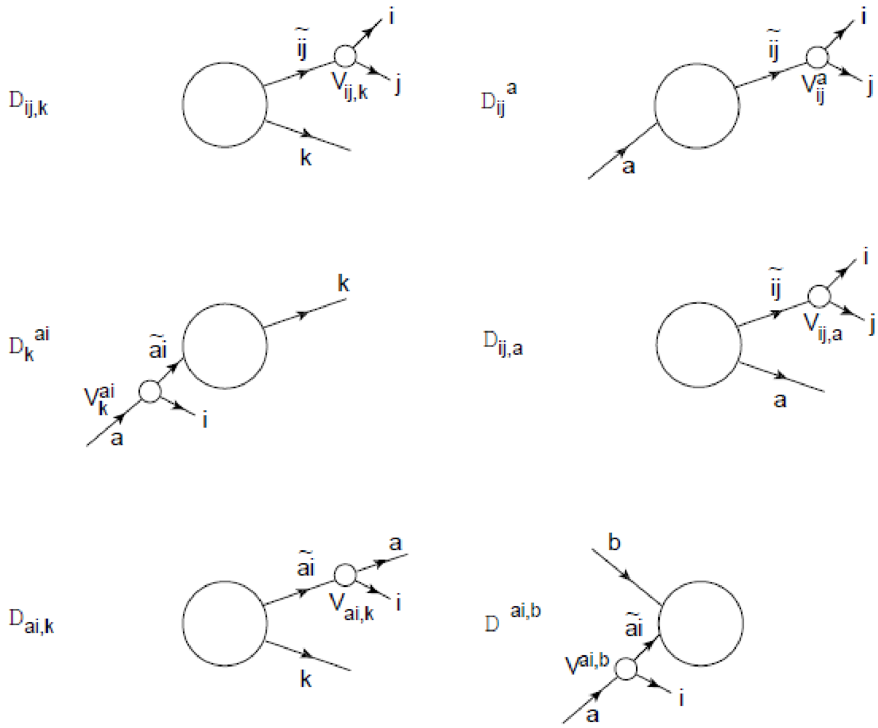
$$|\mathcal{M}_{m+1}|^2 \to \sum |\mathcal{M}_m|^2 \otimes V_{ij,k} \tag{2.25}$$

$V_{ij,k}$ singular factor including parton k and it's interaction with partons i and j from the m parton amplitude this situation can be represented by the bottom diagram.



Here i and k are the emitters and k plays the role of a spectator. The blobs denote the tree-level matrix elements and their complex conjugate. The dots on the right-hand side stand for non-singular terms both in the soft and collinear limits. When the partons i and j become soft and/or collinear, the singularities are factorized into the term $V_{ij,k}$ (the dashed box on the right-hand side) which embodies correlations with a single additional parton k.
In this context the different dipole factorisation for both initial states and final states shall be presented. All these different possibilities can be seen in the diagram below.





The circle in the center of each sub diagram presents the m-partons matrix element and the tilde labels the collinear splitting process for the initial or final states. For this work, the first upper diagram with final-state singularities without initial-state partons, is completely sufficient and is discussed here in detail with its formula. The matrix element for this is written:

$$|\mathcal{M}_{m+1}|^2 =< 1, ..., m+1||1, ..., m+1 >= \sum_{k\neq i,j} \mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) + \text{finite terms} \tag{2.26}$$

The first term with the sum over dipoles is divergent as $p_i \cdot p_j \to 0$. It needs to drop all finite terms. These dipole terms are explicitly given by:

$$\mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) = \frac{-1}{2p_i \cdot p_j}{}_m < 1, ..., \tilde{ij}, ..., k, ..., m{+}1| \frac{T_k \cdot T_{ij}}{{T_{ij}}^2} V_{ij,k}|1, ..., \tilde{ij}, ..., k, ..., m{+}1 >_m \tag{2.27}$$

Where $T_k \cdot T_{ij}$ are the color charges of spectator and emitter
$V_{ij,k}$ splitting kernel in helicity space of emitter explicit form depends on parton type become proportional to Altarelli-Parisi splitting functions and Eikonal factors in collinear and soft limits.
Using the kinematic:

$$\begin{aligned} \tilde{p}_{ij}^{\mu} &= {p_i}^{\mu} + {p_j}^{\mu} - \frac{y_{ij,k}}{1-y_{ij,k}} {p_k}^{\mu} \\ \tilde{p}_{k}^{\mu} &= \frac{1}{1-y_{ij,k}} {p_k}^{\mu} \\ \text{with} \quad y_{ij,k} &= \frac{p_i \cdot p_j}{p_i \cdot p_j + p_j \cdot p_k + p_k \cdot p_i} \end{aligned} \tag{2.28}$$

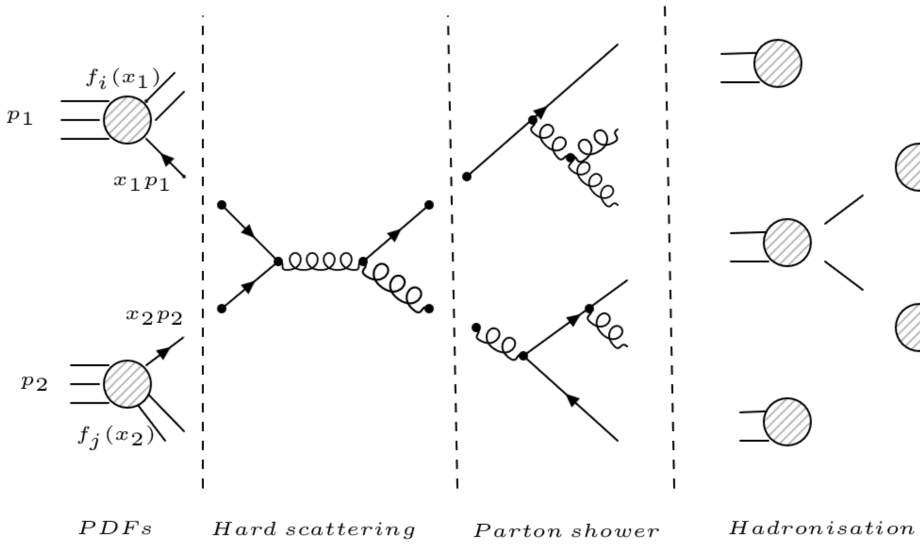
Note, that momenta are on-shell. Due to momenta conservation ${p_i}^{\mu} + {p_j}^{\mu} + {p_k}^{\mu} = \tilde{p}_{k}^{\mu} + \tilde{p}_{ij}^{\mu}$.

2.3 Factorisation

The hadron hadron scattering can be written as:

$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \sigma_{ij}(x_1, x_2, Q^2/\mu^2 ...) \tag{2.29}$$

Here the (arbitrary) factorisation scale μ can be thought of as the scale which separates



the long and short-distance physics. Roughly speaking, a parton with a transverse momentum less than μ is then considered to be part of the hadron structure and is absorbed in the parton distribution. Partons with larger transverse momenta participate in the hard scattering process with a short-distance partonic cross-section. The factorisation theorem also applies to deep inelastic scattering. The DIS cross section can be written as:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}[(1-y)F_2(x, Q^2) + xy^2 F_1(x, Q^2)] \tag{2.30}$$

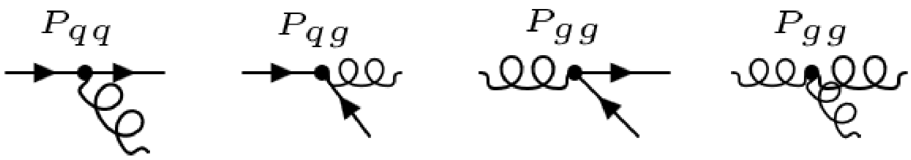
In this case we need to introduce the structure function, is defined as the charge weighted sum of the parton momentum densities, the probability that the parton carries a momentum fraction x. The index i denotes the quark. flavour.

$$F_2{}^{exp}(x) = \sum_i e_i{}^2 x f_i(x) \tag{2.31}$$

The evolution of a quark distribution due to gluon radiation and is called the DGLAP evolution equation.

$$\frac{\partial f(x, \mu^2)}{\partial\, ln\, \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y, \mu^2) P_{qq}(\frac{x}{y} + O(\alpha_s{}^2) \tag{2.32}$$

There three more spiriting possibilities according to below diagrams.



$$\left.\begin{aligned}\langle \hat{P}_{qq} \rangle &= C_F[\frac{1+z^2}{1-z} - \varepsilon(1-z)] \\ \langle \hat{P}_{gq} \rangle &= T_R[1 - \frac{2z(1-z)}{1-\varepsilon}] \\ \langle \hat{P}_{qg} \rangle &= C_F[\frac{1+(1-z)^2}{z} - \varepsilon z] \\ \langle \hat{P}_{gg} \rangle &= 2C_A[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)]\end{aligned}\right\} \text{splitting functions} \tag{2.33}$$

2.4 Mapping 3 partons to 2

As we have already seen in the last section, the dipole factorization is obtained from the square matrix element. Now we want to take a closer look at the four possible parton showers from the previous section. For this goal we first calculate the respective matrix elements and the complex-conjugated one of them in the known 3 parton evaluation. This results in the dipole-term, which contains the soft- and collinear singularities. Then this is parametrized with a certain kinematics in order to separate the finite terms from infinities. First of all, the Emmiter and the spectator are defined. After the substitution of the new momenta we get splitting kernel in helicity space and color charges of spectator and emitter. Then we ignore the finite terms because we are looking for the singular terms. It should be noted that at the beginning of this thesis a parametrisation was used, which unfortunately only works for LO. This was recognized later and therefore a new kinematics was used. This theoretical description will become clearer as we begin to implement the mappings. First of all the kinematics have to be introduced.

2.5 Old mapping

First we start with the old mapping, as promised.

$$\left.\begin{aligned} {q_i}^\mu &= z{p_i}^\mu + y(1-z){p_j}^\mu + \sqrt{zy(1-z)}{m^\mu}_\perp \\ q^\mu &= (1-z){p_i}^\mu + yz{p_j}^\mu - \sqrt{zy(1-z)}{m^\mu}_\perp \\ {q_j}^\mu &= (1-y){p_j}^\mu \end{aligned}\right\} \quad \text{parametrisation} \tag{2.34}$$

Where q is the radiated soft momentum, q_i the momenta of the emitter and q_j the momentum of the spectator is. the dimensionless variable is given by $y = \frac{q_i \cdot q}{p_i \cdot p_j}$. Note that both the emitter and the spectator are on-shell. momentum conservation is implemented exactly:

$${q_i}^\mu + q^\mu + {q_k}^\mu = {p_i}^\mu + {p_j}^\mu + {m^\mu}_\perp \tag{2.35}$$

For this mapping it is useful to calculate some common relation:

$$\begin{aligned} &q_i + q = p_i + yp_j \\ &q_j + q = (1-z){p_i}^\mu + (1+yz-y){p_j}^\mu - \sqrt{zy(1-z)}{m^\mu}_\perp \\ &q_i \cdot q = y(1-2z+2z^2)(p_i \cdot p_j) \\ &q_i \cdot q_j = z(1-y)(p_i \cdot p_j) \\ &q_j \cdot q = (1-z)(1-y)(p_i \cdot p_j) \end{aligned} \tag{2.36}$$

2.6 new kinematic

For the general m emission case it must be defined a new mapping. The parametrisation of the splitting momenta is formalized as:

$$\begin{aligned}
{k_l}^\mu &= \alpha_l \alpha \Lambda^\mu{}_\nu {p_i}^\nu + y\beta n^\mu + \sqrt{y\alpha_l\beta_l} n^\mu{}_{\perp,l} \qquad\qquad l = 1, ..., m \\
{q_i}^\mu &= (1 - \sum_{l=1}^m \alpha_l)\alpha \Lambda^\mu{}_\nu {p_i}^\nu + y(1 - \sum_{l=1}^m \beta_l) n^\mu - \sqrt{y\alpha_l\beta_l} n^\mu{}_{\perp,l} \\
{q_k}^\mu &= \alpha \Lambda^\mu{}_\nu {p_k}^\nu \qquad\quad k = 1, ..., n \qquad k \neq i
\end{aligned} \tag{2.37}$$

$k = 1, ..., n$ labels the emission momenta and is taken to be massless ${k_l}^2 = 0$. Where the label l denotes the count of emissions. In this work we just want to considerate the one-emission kernels. The other important issue here is that all hard momenta are on-shell, ${p_k}^2 = {q_k}^2 = 0$.
n^μ is an auxiliary light-like vector which is necessary to specify the transverse component of $n^\mu{}_{\perp,l}$. To absorb the recoil we define n^μ as:

$$n^\mu = Q^\mu - \frac{Q^2}{2p_i \cdot Q} {p_i}^\mu \tag{2.38}$$

Whereby Q is the total momentum with:

$$Q^\mu = {q_i}^\mu + \sum_{l=1}^m k_l^\mu + \sum_{k=1}^m q_k^\mu = {p_i}^\mu + \sum_{k=1}^m p_k^\mu \tag{2.39}$$

To fulfil the condition that the emission momenta are massless, we need the following condition:

$$\begin{aligned}
n^\mu{}_{\perp,l} \Lambda^\mu{}_\nu {p_i}^\nu &= n_{\perp,l} \cdot n = n_{\perp,l} \cdot Q = 0 \\
n^\mu{}_{\perp,l} \cdot p_k &\neq 0
\end{aligned} \tag{2.40}$$

$n^2_{\perp,l} = -2\alpha\Lambda^\mu{}_\nu {p_i}^\nu n_\mu$ is not on-shell and in terms of single emission case we get $n^2_{\perp,1} = -2p_i \cdot Q$. The parameter y is related to the virtuality of the splitting parton:

$${q_i}^\mu + \sum_{l=1}^m k_l^\mu = \alpha \Lambda^\mu{}_\nu {p_i}^\nu + y n^\mu \tag{2.41}$$

With $\alpha = \sqrt{1 - y}$.

Lorenz transformation of momenta

In order to be able to work with the parametrisation, we have to do the Lorenz transformation of the Emitters, Spectator and total momentum first.

$$\begin{aligned}\alpha\Lambda^{\mu}{}_{\nu} &= {p_i}^{\mu}p_{i\nu}\frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + {p_i}^{\mu}Q_{\nu}\frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}\\ &+ Q^{\mu}p_{i\nu}\frac{(y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu}\end{aligned} \tag{2.42}$$

In the collinear limit of $y \to 0, \alpha \to 1$ this transformation reduces to trivial $\eta^{\mu}{}_{\nu}$. Finally we are going to compute the Lorenz transformation of the Momenta. The detailed calculation of them can be found in Appendix A.

$$\boxed{\hat{p_i}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_i}^{\nu} = {p_i}^{\mu}} \tag{2.43}$$

$$\boxed{\hat{Q}^{\mu} = \tfrac{Q^2}{2p_i\cdot Q}y\, {p_i}^{\mu} + (1-y)\, Q^{\mu}} \tag{2.44}$$

$$\boxed{\hat{p_k}^{\mu} = A_1\, {p_i}^{\mu} + A_2\, Q^{\mu} + \sqrt{1-y}{p_k}^{\mu}} \tag{2.45}$$

with

$$\begin{aligned}A_1 &\equiv \frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}\\ A_2 &\equiv \frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}\end{aligned}$$

2.7 Single emission part

In terms of one emission where $l = 1$ the mapping will be simplified as:

$$\begin{aligned}{k_1}^{\mu} &= (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})){p_i}^{\mu} + y\beta_1 Q^{\mu} + \sqrt{y\alpha_1\beta_1}n^{\mu}{}_{\perp,1}\\ {q_i}^{\mu} &= (\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i\cdot Q})){p_i}^{\mu} + y\alpha_1 Q^{\mu} - \sqrt{y\alpha_1\beta_1}n^{\mu}{}_{\perp,l}\\ {q_k}^{\mu} &= \alpha\Lambda^{\mu}{}_{\nu}{p_k}^{\nu} \qquad k = 1,...,n \qquad k \neq i\end{aligned} \tag{2.46}$$

$$\begin{aligned}{k_1}^{\mu} &= \zeta_1{p_i}^{\mu} + \lambda_1 Q^{\mu} + \sqrt{y\alpha_1\beta_1}n^{\mu}{}_{\perp,1}\\ {q_i}^{\mu} &= \zeta_q{p_i}^{\mu} + \lambda_q Q^{\mu} - \sqrt{y\alpha_1\beta_1}n^{\mu}{}_{\perp,l}\\ {q_k}^{\mu} &= A_1{p_i}^{\mu} + A_2 Q^{\mu} + \sqrt{1-y}p_k^{\mu}\end{aligned}$$

2.8 Common scalar products

To investigate the mapping it is useful to determine the dot products between these four vectors. To understand the often occurring pre-factor products one should look them up in the appendix A.

$$\boxed{k_1 \cdot q_i = y(\alpha_1 + \beta_1)^2 \, p_i \cdot Q = y \, p_i \cdot Q} \tag{2.47}$$

$$\boxed{k_1 \cdot q_k = [\alpha_1(1-y) + y\beta_1(\tfrac{Q^2}{2p_i \cdot Q})] \, p_i \cdot p_k + y\beta_1 \, Q \cdot p_k + \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1}} \tag{2.48}$$

$$\boxed{q_i \cdot q_k = [\beta_1(1-y) + y\alpha_1(\tfrac{Q^2}{2p_i \cdot Q})] \, p_i \cdot p_k + y\alpha_1 \, Q \cdot p_k - \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1}} \tag{2.49}$$

2.9 Recipe for the use of the new parametrisation

In the previous chapter we have discussed that the singularities come from the propagators in each diagram since the denominators contain according Feynmann rules terms with $\sim \frac{1}{2q_a \cdot q_b}$. Whereby a and b here place holder the respective momenta. Since the calculations are sometimes very complicated and confusing, the procedure for eliminating the finite terms is as follows:
In the calculating of the square matrix elements always appear products in the form of $p_a \cdot p_b$ both in the numerator and denominator. The denominator shows which pre-factor causes the singularity. As we know, if , we get zero in the denominator. These terms from the numerator with the same prefix can be omitted from the beginning because they appear in both the denominator and the numerator and are therefore finite. This is explicitly shown below for two common denominators.

2.9.1 Parametrization in terms of $(k_1 \cdot q_i)(k_1 \cdot q_k)$

$$\boxed{(k_1 \cdot q_i)(k_1 \cdot q_k) \approx y(1-\beta_1)(1-y)\,(p_i \cdot p_k)(p_i \cdot Q)} \tag{2.50}$$

Here you can quickly see that this term converges for $y \to 0$ and $\beta_1 \to 1$ towards zero. That means, you could ignore all terms with $y(1-\beta_1)$. However, since the equation becomes rather large quickly if we first use all the momenta products and then drop the terms with the pre-factor out of the denominator, this is already done for the scalar products. And this is exactly the biggest simplification in the calculation. The result looks like this:

$$
\begin{aligned}
{k_1}^\eta {k_1}^{\eta'} &= [(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2]{p_i}^\eta {p_i}^{\eta'} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}){p_i}^\eta Q^{\eta'} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})Q^\eta {p_i}^{\eta'}\\
{k_1}^\eta {q_i}^{\eta'} &= [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y{\beta_1}^2 Q^\eta {p_i}^{\eta'}\\
{q_i}^\eta {k_1}^{\eta'} &= [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y{\beta_1}^2 {p_i}^\eta Q^{\eta'}\\
{q_i}^\eta {q_i}^{\eta'} &= {\beta_1}^2 {p_i}^\eta {p_i}^{\eta'}\\
{k_1}^\eta {q_k}^{\eta'} &= [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_i}^\eta {p_k}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^\eta {p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,{p_i}^\eta Q^{\eta'}\\
&\quad + y\beta_1 A_1\,Q^\eta {p_i}^{\eta'} + y\beta_1 A_2\,Q^\eta Q^{\eta'} + y\beta_1\sqrt{1-y}Q^\eta {p_k}^{\eta'}\\
{q_i}^\eta {q_k}^{\eta'} &= A_1\beta_1 {p_i}^\eta {p_i}^{\eta'} + A_2\beta_1 {p_i}^\eta Q^{\eta'} + \beta_1\sqrt{1-y}{p_i}^\eta {p_k}^{\eta'}\\
{q_k}^\eta {k_1}^{\eta'} &= [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_k}^\eta {p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^\eta {p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,Q^\eta {p_i}^{\eta'}\\
&\quad + y\beta_1 A_1\,{p_i}^\eta Q^{\eta'} + y\beta_1 A_2\,Q^\eta Q^{\eta'} + y\beta_1\sqrt{1-y}{p_k}^\eta Q^{\eta'}\\
{q_k}^\eta {q_i}^{\eta'} &= A_1\beta_1 {p_i}^\eta {p_i}^{\eta'} + A_2\beta_1 Q^\eta {p_i}^{\eta'} + \beta_1\sqrt{1-y}{p_k}^\eta {p_i}^{\eta'}
\end{aligned}
\tag{2.51}
$$

2.9.2 Parametrization in terms of $(k_1 \cdot q_i)(k_1 \cdot q_i)$

$$
\boxed{(k_1\cdot q_i)(k_1\cdot q_i) = y^2(p_i\cdot Q)(p_i\cdot Q)} \tag{2.52}
$$

With the same interpretation from above one could say that this term converges just for $y \to 0$ towards zero. That's why we will remove all product terms with y^2.

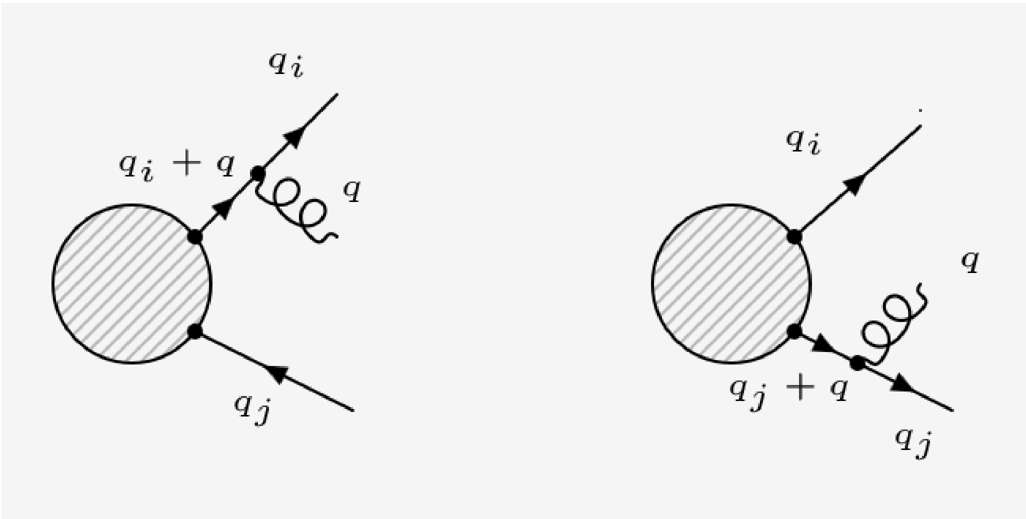
$$
\begin{aligned}
{k_1}^\eta {k_1}^{\eta'} &= [(1-\beta_1)^2 - 2y\beta_1(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y\beta_1(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q}){p_i}^\eta Q^{\eta'} + y\beta_1(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})Q^\eta {p_i}^{\eta'}\\
{k_1}^\eta {q_i}^{\eta'} &= [\beta_1(1-\beta_1) - y(1-\beta_1)^2(\frac{Q^2}{2p_i\cdot Q}) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y(1-\beta_1)^2 Q^\eta {p_i}^{\eta'}\\
{q_i}^\eta {k_1}^{\eta'} &= [\beta_1(1-\beta_1) - y(1-\beta_1)^2(\frac{Q^2}{2p_i\cdot Q}) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y(1-\beta_1)^2 {p_i}^\eta Q^{\eta'}\\
{q_i}^\eta {q_i}^{\eta'} &= [{\beta_1}^2 - 2y\beta_1(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y\beta_1(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q}){p_i}^\eta Q^{\eta'} + y\beta_1(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})Q^\eta {p_i}^{\eta'}\\
{k_1}^\eta {q_k}^{\eta'} &= (1-\beta_1)A_1 {p_i}^\eta {p_i}^{\eta'} + (1-\beta_1)A_2 {p_i}^\eta Q^{\eta'} + (1-\beta_1)\sqrt{1-y}{p_i}^\eta {p_k}^{\eta'}\\
{q_i}^\eta {q_k}^{\eta'} &= A_1\beta_1 {p_i}^\eta {p_i}^{\eta'} + A_2\beta_1 {p_i}^\eta Q^{\eta'} + \beta_1\sqrt{1-y}{p_i}^\eta {p_k}^{\eta'}\\
{q_k}^\eta {k_1}^{\eta'} &= (1-\beta_1)A_1 {p_i}^\eta {p_i}^{\eta'} + (1-\beta_1)A_2 Q^\eta {p_i}^{\eta'} + (1-\beta_1)\sqrt{1-y}{p_k}^\eta {p_i}^{\eta'}\\
{q_k}^\eta {q_i}^{\eta'} &= A_1\beta_1 {p_i}^\eta {p_i}^{\eta'} + A_2\beta_1 Q^\eta {p_i}^{\eta'} + \beta_1\sqrt{1-y}{p_k}^\eta {p_i}^{\eta'}
\end{aligned}
\tag{2.53}
$$

This is how we go about it

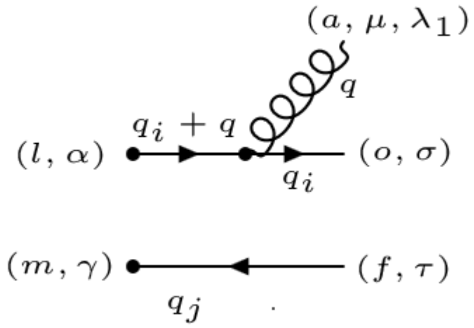
- First of all, let's take a look at a possible spliting. For this we have to make sure that all possible meaningful diagrams have been considered.
- The square matrix element is a complex number and therefore:
- we have to keep all indices as they are defined at M_1 and M_2. This is very important for the calculation of the interference term in order to get a meaningful result.
- In the case of the gluon loop, you have to add the corresponding ghost diagram.
- for all parts of the matrix element, use the parametrisation and the recipe for sending the kinematics from the last section. After the addition of all terms we get all singularities (soft and collinear).
- To determine whether everything was calculated correctly, one goes to the collinear limits, so to speak for y, because here one has to get the Alterali-Parisi splitting function known in the parton shower

Chapter 3

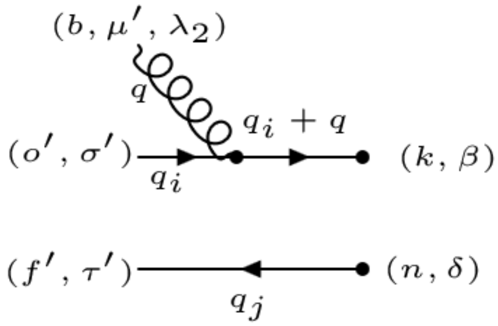
Quark antiquark gluon emission kernel



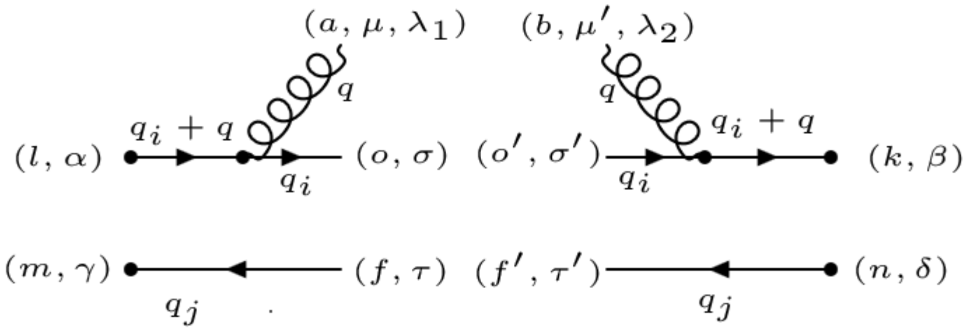
3.1 qg-$\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{3.1}$$



$$M_1{}^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{3.2}$$



$$|M_1|^2 = M_1\, M_1^\dagger = [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i+\not{q})}{(q_i+q)^2}\,\varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\frac{-i(\not{q}_i+\not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'}\times[T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)][\bar{v}_{\tau'}(q_j)] \quad (3.3)$$

$$|M_1|^2 = [\frac{-i(\not{q}_i+\not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'}\times[T^b]_{o'}{}^k)\,\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q)$$
$$\times(-ig_s\gamma^\mu\times[T^a]_o{}^l)\,\frac{i(\not{q}_i+\not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j)v_\tau(q_j)] \quad (3.4)$$

and after sum over the lorenz index (σ,σ') and (τ,τ') and unsing the spin addition relation:

$$\sum_{\sigma,\sigma'}\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) = \not{q}_i\delta^{oo'},$$
$$\sum_{\tau,\tau'}\bar{v}_\tau(q_j)\,v_{\tau'}(q_j) = \not{q}_j\delta^{ff'} \quad (3.5)$$

and sum over polarization index (λ_1,λ_2) :

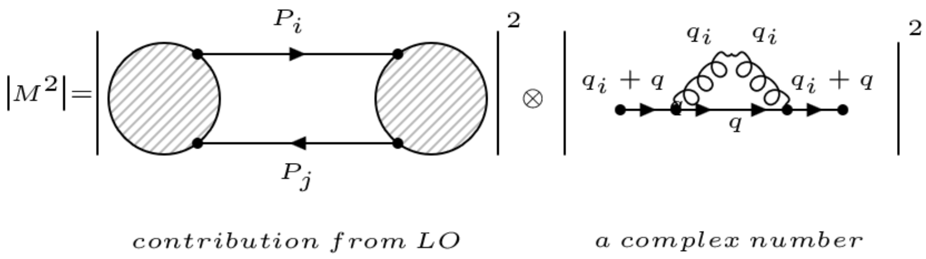
$$\sum_{\mu,\mu'}\varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q) = -g_{\mu\mu'}\delta^{ab} \quad (3.6)$$

$$|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,g_{\mu'\mu}\gamma^\mu\,(\not{q}_i+q)][\not{q}_j] \quad (3.7)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'}\,(\not{q}_i+q)][\not{q}_j] \quad (3.8)$$

In other words we expect the tree level diagram from LO and a number: Which means:



$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i + q)^2 (q_i + q)^2} [\not{P}_i][\not{P}_j] \otimes \ (a \ complex \ number) \tag{3.9}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'} \ \not{q}_i \, \gamma_{\mu'} &= q_{i\sigma} \, \gamma^{\mu'} \gamma^{\sigma} \ \gamma_{\mu'} \\ &= q_{i\sigma} \, (\{\gamma^{\mu'}, \gamma^{\sigma}\} \ - \gamma^{\sigma}\gamma^{\mu'})\gamma_{\mu'} \\ &= q_{i\sigma} \, 2g^{\mu'\sigma} \, \gamma_{\mu'} \ - \ d \, \gamma^{\sigma} \\ &= (2-d) \ \not{q}_i \end{aligned} \tag{3.10}$$

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \ \ \not{q}_i \ \ (\not{q}_i + q)][\not{q}_j] \tag{3.11}$$

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i + q)^2 (q_i + q)^2} [\not{q}_i \ \not{q}_i \ \not{q}_i + \ \not{q}_i \ \not{q}_i \ \not{q} + \ \not{q} \ \not{q}_i \ \not{q}_i + \ \not{q} \ \not{q}_i \ \not{q}][\not{q}_j] \tag{3.12}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \ \not{q}_i &= {q_i}^2 = {m_i}^2 \\ \not{q} \ \not{q} &= q^2 = m^2 \\ \not{q}_j \ \not{q}_j &= {q_j}^2 = {m_j}^2 \end{aligned} \tag{3.13}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \ \not{q}_i$ and $\not{q} \ \not{q}$ as well.

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(2q_i q)(2q_i q)} [\not{q} \ \not{q}_i \ \not{q}][\not{q}_j] \tag{3.14}$$

$$
\begin{aligned}
L = \not{q}\ \not{q}_i\ \not{q} = \not{q}[q_{i\sigma} q_\mu\ (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma\gamma^\mu)] \\
\not{q}[2{q_i}^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\frac{\gamma^\mu\gamma^\mu}{2} + \frac{\gamma^\mu\gamma^\mu}{2}]\gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}]\gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
= \not{q}(2q_i q) - q^2\ \not{q}_i \\
= \not{q}(2q_i q)
\end{aligned}
\tag{3.15}
$$

After inserting the last result of L and simplify the term $(2q_i q)$ from the denominator and nominator because , we get:

$$
|M_1|^2 = -(2-d)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{q}][\not{q}_j]
\tag{3.16}
$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$
|M_1|^2 = (d-2)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][(1-y)\ \not{p}_j]
\tag{3.17}
$$

Multiplying the both sides

$$
\begin{aligned}
|M_1|^2 = (d-2)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \\
+ zy(1-y)\ \not{p}_j\ \not{p}_j + (1-y)\sqrt{zy(1-z)}\ \not{m}_\perp\ \not{p}_j]
\end{aligned}
\tag{3.18}
$$

and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\ \not{p}_i$ and $\not{p}_j\ \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$
|M_1|^2 = (d-2)(1-z)(1-y)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]
\tag{3.19}
$$

with the new parametrisation

$$|M_1|^2 = (d-2)\,\frac{g_s^2 C_F}{(2k_1 \cdot q_i)}[\not{k}_1][\not{q}_k] \tag{3.20}$$

$$\begin{aligned}|M_1|^2 = (d-2)\,\frac{g_s^2\,C_F}{2y\,p_i \cdot Q}[(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\ \not{p}_i + y\beta_1\ \not{Q} + \sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,1}]\\ [A_1\ \not{p}_i + A_2\ \not{Q} + \sqrt{1-y}\ \not{p}_k]\end{aligned} \tag{3.21}$$

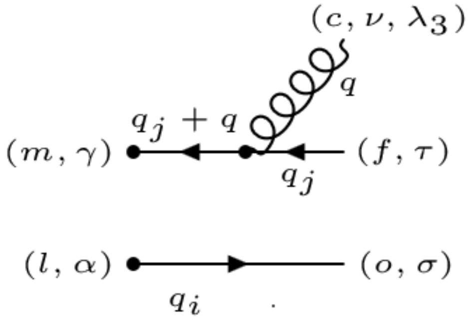
$$\begin{aligned}|M_1|^2 = (d-2)\,\frac{g_s^2\,C_F}{2y\,p_i \cdot Q}[(A_2(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q})) + A_1 y\beta_1)p_i \cdot Q\\ + (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\sqrt{1-y}p_i \cdot p_k + A_2 y\beta_1 Q^2 + \sqrt{1-y}\sqrt{y\alpha_1\beta_1}n_{\perp,1} \cdot p_k]\end{aligned} \tag{3.22}$$

For the collinearity $y \to 0$ we'll get:

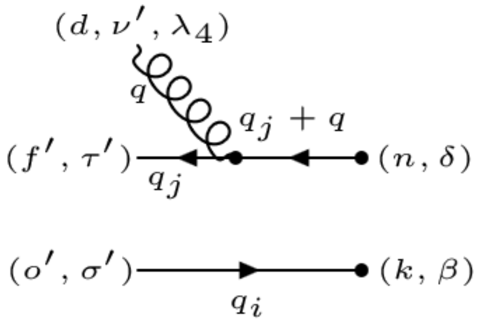
$$\begin{aligned}|M_1|^2 = (d-2)\,\frac{g_s^2\,C_F}{2y\,p_i \cdot Q}[(A_2(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q})) + A_1 y\beta_1)\ \not{p}_i\ \not{Q}\\ + (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\sqrt{1-y}\ \not{p}_i\ \not{p}_k + A_2 y\beta_1 Q^2 + \sqrt{1-y}\sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,1}\ \not{p}_k]\end{aligned} \tag{3.23}$$

$$|M_1|^2 = (d-2)(1-\beta_1)\sqrt{1-y}\ \frac{g_s^2\,C_F}{2y\,p_i \cdot Q}[\not{p}_i\ \not{p}_k] \tag{3.24}$$

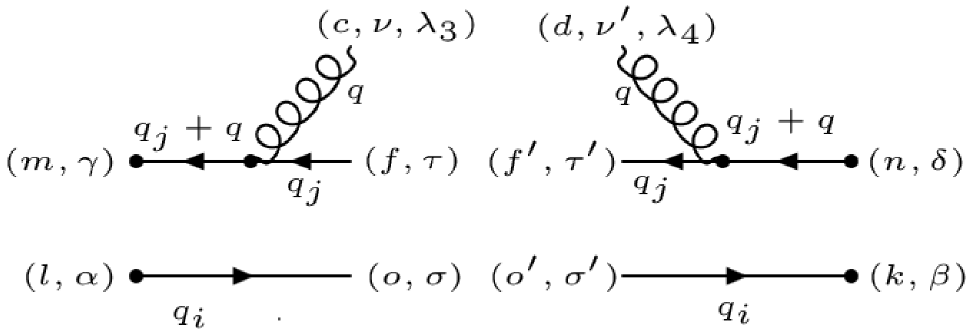
3.2 $\bar{q}$g-q



$$M_2 = [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \tag{3.25}$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \tag{3.26}$$



$$\begin{aligned}|M_2|^2 = M_2\, M_2{}^\dagger = {} & [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \\ & [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)]\end{aligned} \tag{3.27}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu\, v_\tau(q_j)\bar{v}_{\tau'}(q_j)\,\varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_j + \not{q})] \\ [u_\sigma(q_i)]\,[\bar{u}_{\sigma'}(q_i)] \tag{3.28}$$

and after sum over the lorenz index (σ,σ') and (τ,τ') and unsing the spin addition relation:

$$\begin{aligned} \sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) &= \not{q}_i\delta^{oo'}, \\ \sum_{\tau,\tau'} \bar{v}_\tau(q_j)\,v_{\tau'}(q_j) &= \not{q}_j\delta^{ff'} \end{aligned} \tag{3.29}$$

and sum over polarization index (λ_3, λ_4) :

$$\sum_{\nu,\nu'} \varepsilon^{\lambda_4}{}^*_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'}\delta^{cd} \tag{3.30}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu\ \ \not{q}_j\,(-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_j + \not{q})]\,[\not{q}_i] \tag{3.31}$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{(2qq_j)}[\not{q}]\,[\not{q}_i] \tag{3.32}$$

finally:

$$|M_2|^2 = -(d-2)yz^2\,\frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \tag{3.33}$$

with the new kinematic

$$|M_2|^2 = (d-2)\frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2k_1 \cdot q_k}[\not{k}_1]\,[\not{q}_i] \tag{3.34}$$

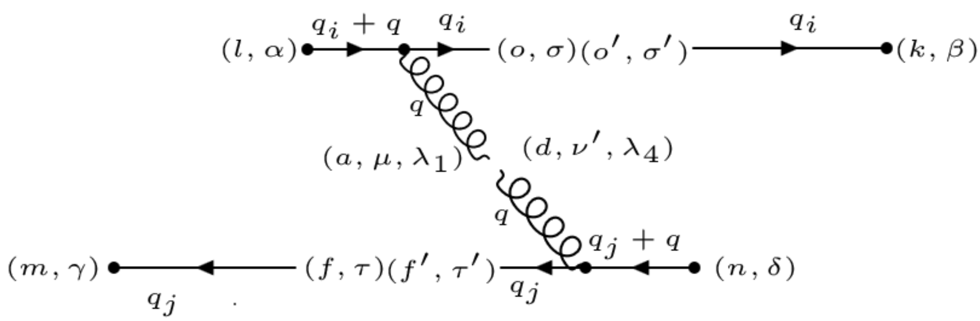
$$\begin{aligned}
&|M_2|^2 = (d-2)\frac{g_s^2\, C_F}{2k_1 \cdot q_k}[(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\ \not{p}_i + y\beta_1\ \not{Q} + \sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,1}]\\
&[(\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q}))\ \not{p}_i + y\alpha_1\ \not{Q} - \sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,l}]
\end{aligned} \tag{3.35}$$

$$\begin{aligned}
&|M_2|^2 = (d-2)\frac{g_s^2\, C_F}{2k_1 \cdot q_k}[y\alpha_1(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\ \not{p}_i\ \not{Q} + y\beta_1(\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q}))]\ \not{Q}\ \not{p}_i\\
&+ y^2\alpha_1\beta_1 Q^2 - y\beta_1\sqrt{y\alpha_1\beta_1}\ \not{Q}\ \not{n}_{\perp,1} + y\beta_1\sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,1}\ \not{Q} - y\alpha_1\beta_1\, n_{\perp,l}{}^2\\
&+ (\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q})\sqrt{y\alpha_1\beta_1}\ \not{n}_{\perp,1}\ \not{p}_i - (\alpha_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q})\sqrt{y\alpha_1\beta_1}\ \not{p}_i\ \not{n}_{\perp,1}]
\end{aligned} \tag{3.36}$$

Which means:

$$\begin{aligned}
|M_2|^2 &\sim (d-2)\frac{g_s^2\, C_F}{2k_1 \cdot q_k}y[...]\\
|M_2|^2 &\to 0 \quad \text{for} \quad y \to 0
\end{aligned} \tag{3.37}$$

3.3 $M_1 M_2{}^\dagger$



$$\begin{aligned} M_1\, M_2{}^\dagger = {} & [\bar{u}_\sigma(q_i)\,(-i g_s \gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\; \varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)] \\ & [\bar{v}_{\tau'}(q_j)\,(i g_s \gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\; \varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[u_{\sigma'}(q_i)] \end{aligned} \tag{3.38}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{g_s^2 [T^a]_o{}^l\, [T^d]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,]\varepsilon^{\lambda_1}{}_\mu(q)\, \varepsilon^{\lambda_4}{}_{\nu'}(q) \\ & [\not{q}_j\, \gamma^{\nu'}\, (\not{q}_j + \not{q})] \end{aligned} \tag{3.39}$$

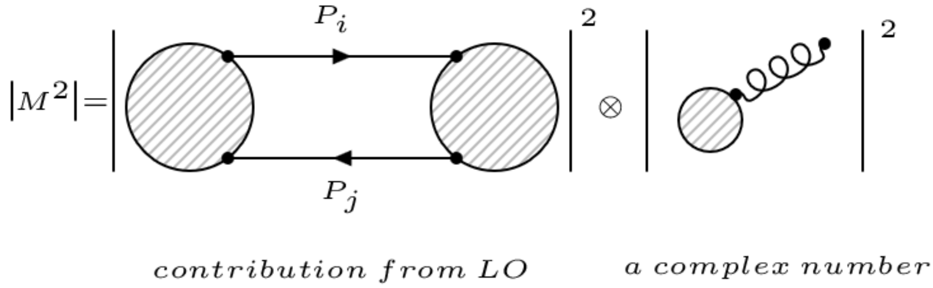
$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,] - g_{\mu\nu'} \\ & [\not{q}_j\, \gamma^{\nu'}\, (\not{q}_j + \not{q})] \end{aligned} \tag{3.40}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,]\, [\not{q}_j\, \gamma_\mu\, (\not{q}_j + \not{q})] \tag{3.41}$$

Expectation:

$$M_1\, M_2{}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [(\not{q}_i + \not{q})\, \gamma^\mu\, \not{q}_i\,]\, [(\not{q}_j + \not{q})\, \gamma_\mu\, \not{q}_j] \tag{3.42}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [-(\not{q}_i + \not{q})\;\; \not{q}_i\, \gamma^\mu\, + 2(\not{q}_i + \not{q})\, q_i{}^\mu] \\ & [-(\not{q}_j + \not{q})\;\; \not{q}_j\, \gamma_\mu\, + 2(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \tag{3.43}$$



$$\begin{aligned}M_1\, {M_2}^\dagger = &\frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)}[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q})\ \not{q}_j \gamma_\mu]\\ &- 2[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q}) q_{j\mu}] - 2[(\not{q}_i + \not{q})\, {q_i}^\mu][(\not{q}_j + \not{q})\ \not{q}_j\, \gamma_\mu]\\ &+ 4[(\not{q}_i + \not{q})\, {q_i}^\mu][(\not{q}_j + \not{q}) q_{j\mu}]\end{aligned} \tag{3.44}$$

$$\begin{aligned}M_1\, {M_2}^\dagger = &\frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)}[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q})\ \not{q}_j \gamma_\mu]\\ &- 2[(\not{q}_i + \not{q})\ \not{q}_i\ \not{q}_j]\, [\not{q}_j + \not{q}] - 2[\not{q}_i + \not{q}\,][(\not{q}_j + \not{q})\ \not{q}_j\ \not{q}_i]\\ &+ 4[(\not{q}_i + \not{q})\, {q_i}^\mu][(\not{q}_j + \not{q}) q_{j\mu}]\end{aligned} \tag{3.45}$$

$$\begin{aligned}M_1\, {M_2}^\dagger = &\frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)}[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q})\ \not{q}_j \gamma_\mu]\\ &+ 4[(\not{q}_i + \not{q})\, {q_i}^\mu][(\not{q}_j + \not{q}) q_{j\mu}]\end{aligned} \tag{3.46}$$

$$\begin{aligned}M_1\, {M_2}^\dagger = &\frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i\cdot p_j)(p_i\cdot p_j)}\\ &[y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\ \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu]\\ &+4({q_i}^\mu \cdot q_{j\mu})[(\not{q}_i + \not{q})][(\not{q}_j + \not{q})]\end{aligned} \tag{3.47}$$

$$\begin{aligned}M_1\, {M_2}^\dagger = &\frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i\cdot p_j)(p_i\cdot p_j)}\\ &[y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\ \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu]\\ &+4(p_i\cdot p_j)[(\not{p}_i + y\ \not{p}_j)][(1-z)\ \not{p}_i + (1+yz-y)\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}]\end{aligned} \tag{3.48}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)(1-y)y(1-2z+2z^2)(p_i\cdot p_j)} z(1-y)[\not{p}_i][\not{p}_j] \tag{3.49}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)y(1-2z+2z^2)(p_i\cdot p_j)} z[\not{p}_i][\not{p}_j] \tag{3.50}$$

With the new kinematic

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i k_1)(2q_k k_1)} [(\not{q}_i + \not{k}_1)\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_k + \not{k}_1)\ \not{q}_k \gamma_\mu] \\ + 4[(\not{q}_i + \not{k}_1)\, {q_i}^\mu][(\not{q}_k + \not{k}_1) q_{k\mu}] \tag{3.51}$$

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{4y(1-\beta_1)(1-y)\, (p_i \cdot p_k)(p_i \cdot Q)} \\ [(\not{q}_i\ \not{q}_i + \not{k}_1\ \not{q}_i)\, \gamma^\mu][(\not{q}_k\ \not{q}_k + \not{k}_1\ \not{q}_k)\gamma_\mu] + 4({q_i}^\mu q_{k\mu})[\not{q}_i + \not{k}_1\,][\not{q}_k + \not{k}_1] \tag{3.52}$$

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{4y(1-\beta_1)(1-y)\, (p_i \cdot p_k)(p_i \cdot Q)} \\ [\not{k}_1\ \not{q}_i\, \gamma^\mu][\not{k}_1\ \not{q}_k \gamma_\mu] + 4(q_i \cdot q_k)[\not{q}_i\ \not{q}_k + \not{k}_1\ \not{q}_k + \not{q}_i\ \not{k}_1] \tag{3.53}$$

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{4y(1-\beta_1)(1-y)\, (p_i \cdot p_k)(p_i \cdot Q)} \\ 4(A_1\beta_1 p_i \cdot p_i + A_2\beta_1 p_i \cdot Q + \beta_1\sqrt{1-y} p_i \cdot p_k) \\ [A_1\beta_1\ \not{p}_i\ \not{p}_i + A_2\beta_1\ \not{p}_i\ \not{Q} + \beta_1\sqrt{1-y}\ \not{p}_i\ \not{p}_k \\ + [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})]\sqrt{1-y}\ \not{p}_i\ \not{p}_k - y\beta_1(\frac{Q^2}{2p_i \cdot Q})A_1\ \ \not{p}_i\ \not{p}_i \\ - y\beta_1(\frac{Q^2}{2p_i \cdot Q})A_2\ \ \not{p}_i\ \not{Q} + y\beta_1 A_1\ \ \not{Q}\ \not{p}_i + y\beta_1 A_2\ \ \not{Q}\ \not{Q} + y\beta_1\sqrt{1-y}\ \not{Q}\ \not{p}_k \\ + [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i \cdot Q})]\ \not{p}_i\ \not{p}_i + y{\beta_1}^2\ \not{p}_i\ \not{Q}] \tag{3.54}$$

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{4y(1-\beta_1)(1-y)\, (p_i \cdot p_k)(p_i \cdot Q)} \\ 4(A_2\beta_1 p_i \cdot Q + \beta_1\sqrt{1-y} p_i \cdot p_k)[A_2\beta_1\ \not{p}_i\ \not{Q} + \beta_1\sqrt{1-y}\ \not{p}_i\ \not{p}_k \\ + [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})]\sqrt{1-y}\ \not{p}_i\ \not{p}_k - y\beta_1(\frac{Q^2}{2p_i \cdot Q})A_2\ \ \not{p}_i\ \not{Q} \\ + y\beta_1 A_1\ \ \not{Q}\ \not{p}_i + y\beta_1 A_2\ \ \not{Q}\ \not{Q} + y\beta_1\sqrt{1-y}\ \not{Q}\ \not{p}_k + y{\beta_1}^2\ \not{p}_i\ \not{Q}] \tag{3.55}$$

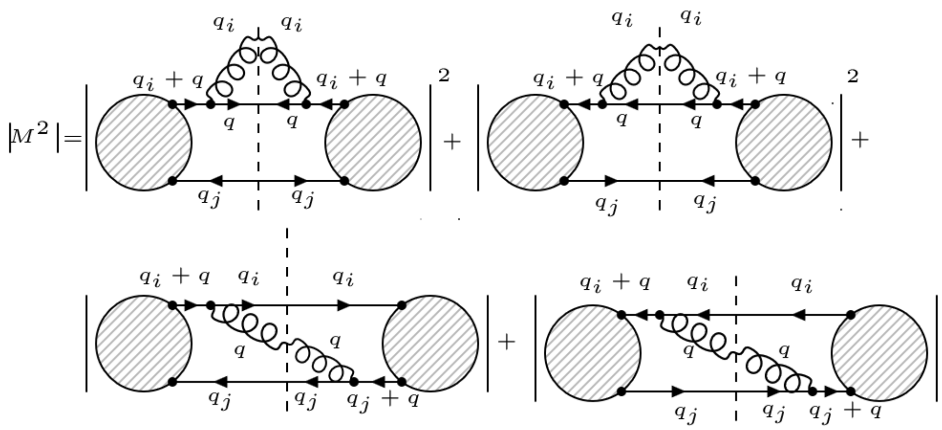
$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{4y(1-\beta_1)(1-y)\, (p_i \cdot p_k)(p_i \cdot Q)} \\ 4(\beta_1\sqrt{1-y} p_i \cdot p_k)[\beta_1\sqrt{1-y}\ \not{p}_i\ \not{p}_k + (1-\beta_1)\sqrt{1-y}\ \not{p}_i\ \not{p}_k] \tag{3.56}$$

$$M_1\, {M_2}^{\dagger} = \frac{-g_s^2\, C_F}{y(1-\beta_1)\, (p_i \cdot p_k)(p_i \cdot Q)} \beta_1 (p_i \cdot p_k)[\beta_1\ \not{p}_i\ \not{p}_k + (1-\beta_1)\ \not{p}_i\ \not{p}_k] \tag{3.57}$$

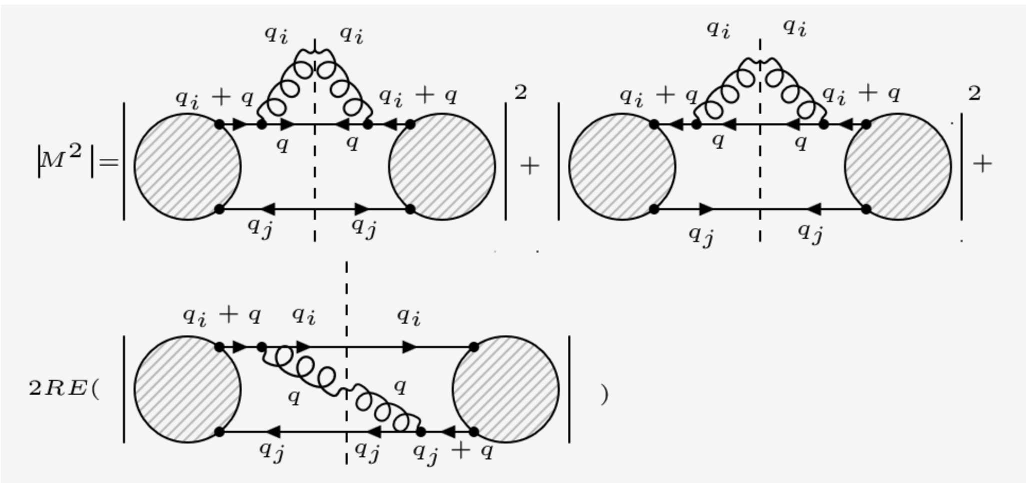
$$M_1\, {M_2}^{\dagger} = \frac{\beta_1}{(1-\beta_1)} \frac{-g_s^2\, C_F}{y\, (p_i \cdot Q)} [\not{p}_i\ \not{p}_k] \tag{3.58}$$

3.4 $|M^2|$

$$| M|^2 = | M_1|^2 + | M_2|^2 + M_1\, M_2{}^\dagger + M_1{}^\dagger M_2 \tag{3.59}$$



$$| M|^2 = | M_1|^2 + | M_2|^2 + {\color{red}2RE(M_1\, M_2{}^\dagger)} \tag{3.60}$$



$$\begin{aligned} | M|^2 = (d-2)(1-z)(1-y)\, & \frac{g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \\ -(d-2)yz^2\, & \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2(1-z)(1-y)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \\ +2RE((\frac{-2z}{z-1}) & \frac{g_s^2\;\, [T^a]_o{}^l\, [T^a]_f{}^n}{2y(1-2z+2z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j]) \end{aligned} \tag{3.61}$$

$$T^a{}_{o\,k}\, T^a{}_{l\,o} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F \delta_{lk} \tag{3.62}$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a{}_{o\,k}\,T^a{}_{l\,o} = C_F\delta_{lk} = \frac{1}{N}\sum_{l=1}^{N}\delta_{lk}C_F = C_F \tag{3.63}$$

The same calculation for $T^c{}_{m\,f}\,T^c{}_{f\,n}$ and $T^a{}_{o\,l}\,T^a{}_{f\,n}$ turns C_F out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, wich means, if:

$$y \longrightarrow 0 \tag{3.64}$$

$$\begin{aligned}
|\,M|^2 = (d-2)(1-z)(1-y)\,&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
-(d-2)yz^2\,&\frac{g_s^2\,C_F}{2(1-z)(1-y)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
+2RE((\frac{-2z}{z-1})&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j]
\end{aligned} \tag{3.65}$$

$$|\,M|^2 = C_F((d-2)(1-z) - \frac{4z}{z-1})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \tag{3.66}$$

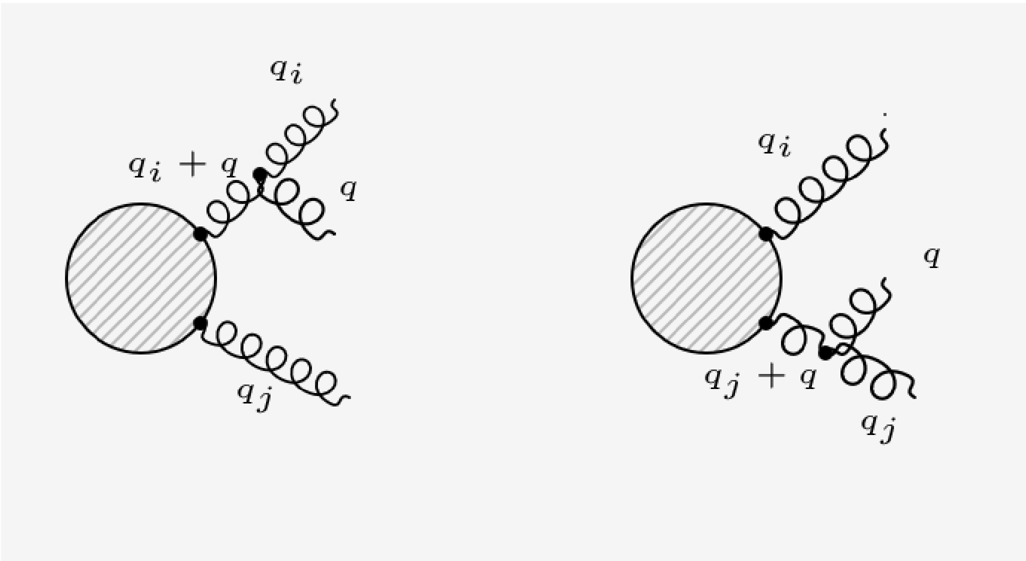
for

$$d = 4-2\epsilon \tag{3.67}$$

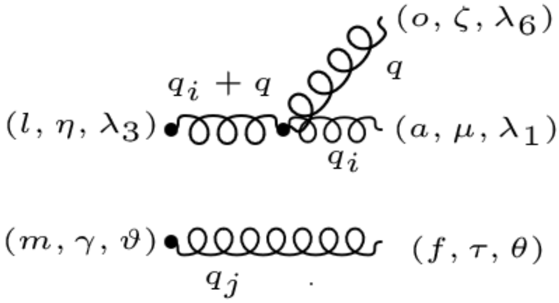
$$\begin{aligned}
|\,M|^2 &= C_F((4-2\epsilon-2)(1-z) + \frac{4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
&= C_F(\frac{2(1-\epsilon)(1-z)^2+4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
&C_F(\frac{2-4z+2z^2-\epsilon(1-z)^2+4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
&= C_F(\frac{(1+z^2)}{1-z} - \epsilon(1-z))\,\frac{g_s^2}{y(1-2z+2z^2)(p_i\cdot p_j)}[\not p_i][\not p_j] \\
&= \langle\,\hat{P_{qq}}\rangle\,\frac{g_s^2}{q_i\cdot q}[\not p_i][\not p_j]
\end{aligned} \tag{3.68}$$

Chapter 4

Gluon gluon gluon emission kernel

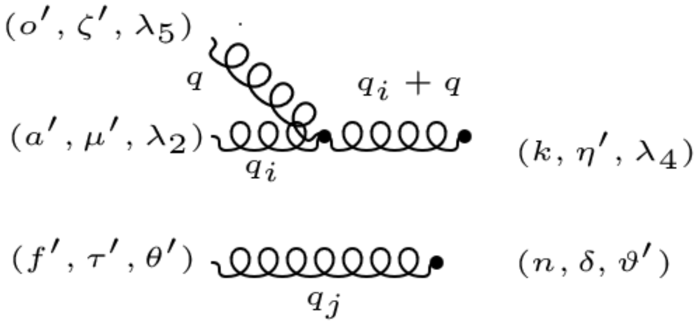


4.1 Gluon-Emitter Bubble



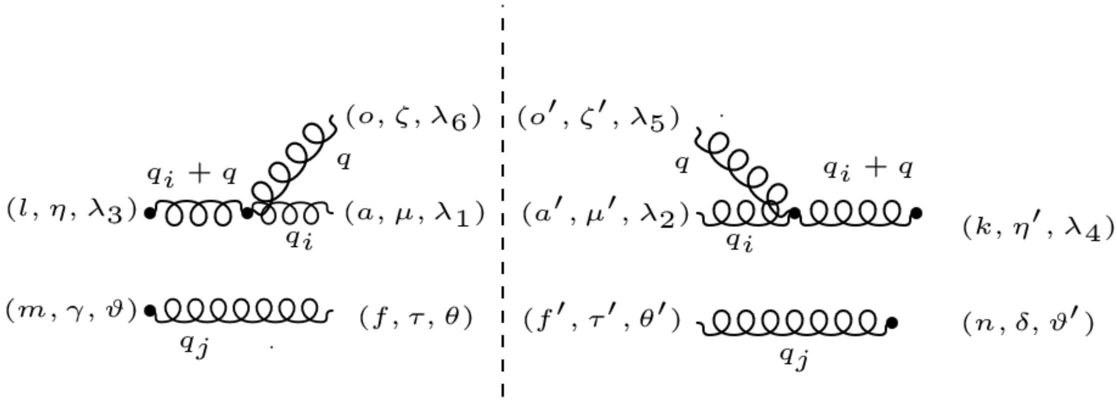
$$M_1 = [\frac{-i}{(q+q_i)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} + g^{\zeta\eta}(-q-(q+q_i))^{\mu} + g^{\eta\mu}(q_i+q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q)\varepsilon^{\lambda_6}{}_{\zeta}(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \tag{4.1}$$

$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q+q_i)^{\mu} + g^{\eta\mu}(2q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \tag{4.2}$$



$$M_1{}^{\dagger} = [\frac{i}{(q_i+q)^2}(-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'}) \\ \varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q_i)\varepsilon^{\lambda_5}{}_{\zeta'}{}^{*}(q)][\varepsilon^{\theta'}{}_{\tau'}{}^{*}(q_j)] \tag{4.3}$$

$$|M_1|^2 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q+q_i)^{\mu} + g^{\eta\mu}(2q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)\,\varepsilon^{\lambda_5}{}_{\zeta'}{}^{*}(q) \\ (-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'})\frac{i}{(q_i+q)^2}][g^{\gamma\delta}] \tag{4.4}$$



$$\begin{aligned} N \equiv g_{\mu\mu'} g_{\zeta\zeta'} [& -g^{\mu\zeta} g^{\mu'\eta'} (q - q_i)^{\eta} (2q_i + q)^{\zeta'} + g^{\mu\zeta} g^{\eta'\zeta'} (q - q_i)^{\eta} (2q + q_i)^{\mu'} \\ & + g^{\mu\zeta} g^{\zeta'\mu'} (q - q_i)^{\eta} (q_i - q)^{\eta'} + g^{\zeta\eta} g^{\mu'\zeta'} (2q + q_i)^{\mu} (2q_i + q)^{\zeta'} \\ & - g^{\zeta\eta} g^{\eta'\zeta'} (2q + q_i)^{\mu} (2q + q_i)^{\mu'} - g^{\zeta\eta} g^{\zeta'\mu'} (2q + q_i)^{\mu} (q_i - q)^{\eta'} \\ & - g^{\eta\mu} g^{\mu'\eta'} (2q_i + q)^{\zeta} (2q_i + q)^{\zeta'} + g^{\eta\mu} g^{\eta'\zeta'} (2q_i + q)^{\zeta} (2q + q_i)^{\mu'} \\ & + g^{\eta\mu} g^{\zeta'\mu'} (2q_i + q)^{\zeta} (q_i - q)^{\eta'}] [g^{\gamma\delta}] \end{aligned} \tag{4.5}$$

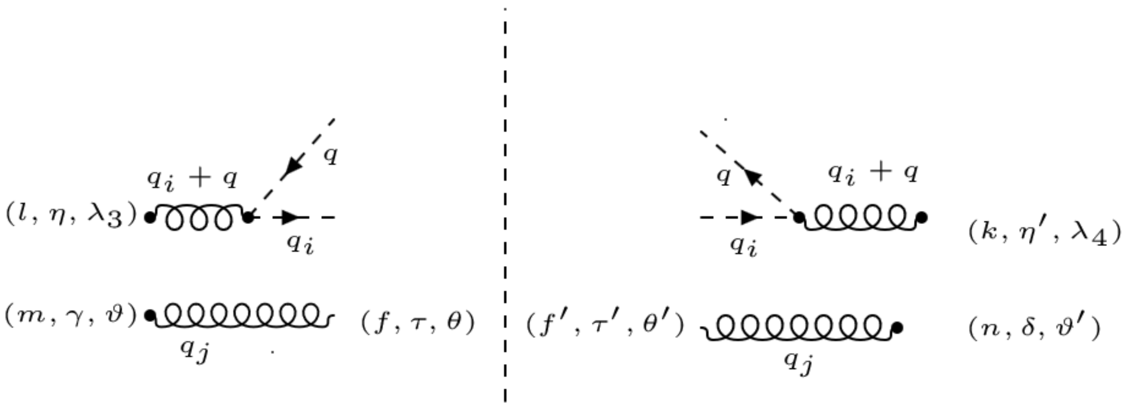
$$\begin{aligned} N \equiv [& -(q - q_i)^{\eta} (2q_i + q)^{\eta'} + (q - q_i)^{\eta} (2q + q_i)^{\eta'} + d(q - q_i)^{\eta} (q_i - q)^{\eta'} \\ & + (2q + q_i)^{\eta'} (2q_i + q)^{\eta} - g^{\eta\eta'} (2q + q_i)^{\mu} (2q + q_i)_{\mu} - (2q + q_i)^{\eta} (q_i - q)^{\eta'} \\ & - g^{\eta\eta'} (2q_i + q)^{\zeta} (2q_i + q)_{\zeta} + (2q_i + q)^{\eta'} (2q + q_i)^{\eta} + (2q_i + q)^{\eta} (q_i - q)^{\eta'}] [g^{\gamma\delta}] \end{aligned} \tag{4.6}$$

$$\begin{aligned} N \equiv [& -(q^{\eta} q^{\eta'} + 2q^{\eta} {q_i}^{\eta'} - {q_i}^{\eta} q^{\eta'} - 2{q_i}^{\eta} {q_i}^{\eta'}) + (2q^{\eta} q^{\eta'} + q^{\eta} {q_i}^{\eta'} - 2{q_i}^{\eta} q^{\eta'} - {q_i}^{\eta} {q_i}^{\eta'}) \\ & + (dq^{\eta} {q_i}^{\eta'} - dq^{\eta} q^{\eta'} - d{q_i}^{\eta} {q_i}^{\eta'} + d{q_i}^{\eta} q^{\eta'}) + (4q^{\eta'} {q_i}^{\eta} + 2q^{\eta'} q^{\eta} + 2{q_i}^{\eta'} {q_i}^{\eta} + {q_i}^{\eta'} q^{\eta}) \\ & - (-2q^{\eta} q^{\eta'} + 2q^{\eta} {q_i}^{\eta'} - {q_i}^{\eta} q^{\eta'} + {q_i}^{\eta} {q_i}^{\eta'}) + (2q^{\eta'} q^{\eta} + q^{\eta'} {q_i}^{\eta} + 4{q_i}^{\eta'} q^{\eta} + 2{q_i}^{\eta'} {q_i}^{\eta}) \\ & + (-q^{\eta} q^{\eta'} + q^{\eta} {q_i}^{\eta'} - 2{q_i}^{\eta} q^{\eta'} + 2{q_i}^{\eta} {q_i}^{\eta'}) - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)] [g^{\gamma\delta}] \end{aligned} \tag{4.7}$$

$$\begin{aligned} N \equiv [& (6 - d) q^{\eta} q^{\eta'} + (d + 3) q^{\eta} {q_i}^{\eta'} + (d + 3) {q_i}^{\eta} q^{\eta'} + (6 - d) {q_i}^{\eta} {q_i}^{\eta'} \\ & - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)] [g^{\gamma\delta}] \end{aligned} \tag{4.8}$$

$$\begin{aligned} |M_1|^2 = \frac{g_s^2 \, f^{\,a\,o\,l} \, f^{\,a\,k\,o}}{(q_i + q)^2 (q_i + q)^2} [& (6 - d) q^{\eta} q^{\eta'} + (d + 3) q^{\eta} {q_i}^{\eta'} + (d + 3) {q_i}^{\eta} q^{\eta'} + (6 - d) {q_i}^{\eta} {q_i}^{\eta'} \\ & - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)] [g^{\gamma\delta}] \end{aligned} \tag{4.9}$$

4.1.1 One-loop corrections to the gluon self-energy diagram(Gluon-Emitter Bubble)



$$|M_1|^2_{Ghost\ loop} = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[-q_i{}^{\eta}q^{\eta'} - q^{\eta}q_i{}^{\eta'}][g^{\gamma\delta}] \tag{4.10}$$

$$\begin{aligned} |M_1'|^2 &= |M_1|^2 + |M_1|^2_{Ghost\ loop} \\ &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[(6-d)q^{\eta}q^{\eta'} + (d+3)q^{\eta}q_i{}^{\eta'} \\ &+(d+3)q_i{}^{\eta}q^{\eta'} + (6-d)q_i{}^{\eta}q_i{}^{\eta'} - g^{\eta\eta'}(5q^2+5q_i{}^2+8qq_i) - q_i{}^{\eta}q^{\eta'} - q^{\eta}q_i{}^{\eta'}][g^{\gamma\delta}] \end{aligned} \tag{4.11}$$

$$\begin{aligned} |M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}&[(6-d)q^{\eta}q^{\eta'} + (d+2)q^{\eta}q_i{}^{\eta'} \\ &+(d+2)q_i{}^{\eta}q^{\eta'} + (6-d)q_i{}^{\eta}q_i{}^{\eta'} - g^{\eta\eta'}(8qq_i)][g^{\gamma\delta}] \end{aligned} \tag{4.12}$$

$$\begin{aligned} |M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y^2(\alpha_1+\beta_1)^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\ &[(6-d)(\zeta_1 p_i{}^{\eta} + \lambda_1 Q^{\eta} + \sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1})(\zeta_1 p_i{}^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1}n^{\eta'}{}_{\perp,1}) \\ &+(d+2)(\zeta_1 p_i{}^{\eta} + \lambda_1 Q^{\eta} + \sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1})(\zeta_q p_i{}^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1}n^{\eta'}{}_{\perp,1}) \\ &+(d+2)(\zeta_q p_i{}^{\eta} + \lambda_q Q^{\eta} - \sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1})(\zeta_1 p_i{}^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1}n^{\eta'}{}_{\perp,1}) \\ &+(6-d)(\zeta_q p_i{}^{\eta} + \lambda_q Q^{\eta} - \sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1})(\zeta_q p_i{}^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1}n^{\eta'}{}_{\perp,1}) \\ &-8g^{\eta\eta'}[(\alpha_1{}^2+\beta_1{}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}]][g^{\gamma\delta}] \end{aligned} \tag{4.13}$$

$$
\begin{aligned}
|M_1'|^2 = & \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{y^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
& [(6-d)[\zeta_1\zeta_1 {p_i}^{\eta}{p_i}^{\eta'} + \zeta_1\lambda_1 {p_i}^{\eta}Q^{\eta'} + \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\lambda_1\zeta_1 Q^{\eta}{p_i}^{\eta'} + \lambda_1\lambda_1 Q^{\eta}Q^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\zeta_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(d+2)[\zeta_1\zeta_q {p_i}^{\eta}{p_i}^{\eta'} + \zeta_1\lambda_q {p_i}^{\eta}Q^{\eta'} - \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\lambda_1\zeta_q Q^{\eta}{p_i}^{\eta'} + \lambda_1\lambda_q Q^{\eta}Q^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\zeta_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} + \lambda_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(d+2)[\zeta_q\zeta_1 {p_i}^{\eta}{p_i}^{\eta'} + \zeta_q\lambda_1 {p_i}^{\eta}Q^{\eta'} + \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\lambda_q\zeta_1 Q^{\eta}{p_i}^{\eta'} + \lambda_q\lambda_1 Q^{\eta}Q^{\eta'} + \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
& -\zeta_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(6-d)[\zeta_q\zeta_q {p_i}^{\eta}{p_i}^{\eta'} + \zeta_q\lambda_q {p_i}^{\eta}Q^{\eta'} - \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
& +\lambda_q\zeta_q Q^{\eta}{p_i}^{\eta'} + \lambda_q\lambda_q Q^{\eta}Q^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
& -\zeta_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} \\
& -8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}]][g^{\gamma\delta}]
\end{aligned} \tag{4.14}
$$

$$
\begin{aligned}
|M_1'|^2 = & \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
& [(6-d)[({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2){p_i}^\eta {p_i}^{\eta'} \\
& +(y\alpha_1\beta_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})){p_i}^\eta Q^{\eta'} + \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^\eta {n^{\eta'}}_{\perp,1} \\
& +(y\beta_1\alpha_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}))Q^\eta {p_i}^{\eta'} + y^2{\beta_1}^2 Q^\eta Q^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}Q^\eta {n^{\eta'}}_{\perp,1} \\
& +\zeta_1\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{p_i}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(d+2)[(\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2){p_i}^\eta {p_i}^{\eta'} \\
& +(y{\alpha_1}^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i\cdot Q})){p_i}^\eta Q^{\eta'} - \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^\eta {n^{\eta'}}_{\perp,1} \\
& +(y{\beta_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))Q^\eta {p_i}^{\eta'} + y^2\beta_1\alpha_1 Q^\eta Q^{\eta'} \\
& -\lambda_1\sqrt{y\alpha_1\beta_1}Q^\eta {n^{\eta'}}_{\perp,1} + \zeta_q\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{p_i}^{\eta'} \\
& +\lambda_q\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(d+2)[(\beta_1\alpha_1 - y({\beta_1}^2+{\alpha_1}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2){p_i}^\eta {p_i}^{\eta'} \\
& +(y{\beta_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})){p_i}^\eta Q^{\eta'} + \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^\eta {n^{\eta'}}_{\perp,1} \\
& +(y{\alpha_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))Q^\eta {p_i}^{\eta'} + y^2\alpha_1\beta_1 Q^\eta Q^{\eta'} \\
& +\lambda_q\sqrt{y\alpha_1\beta_1}Q^\eta {n^{\eta'}}_{\perp,1} \\
& -\zeta_1\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{p_i}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
& [(6-d)[({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q})^2){p_i}^\eta {p_i}^{\eta'} \\
& +(y\beta_1\alpha_1 - y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q})){p_i}^\eta Q^{\eta'} - \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^\eta {n^{\eta'}}_{\perp,1} \\
& +(y\alpha_1\beta_1 - y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q}))Q^\eta {p_i}^{\eta'} + y^2{\alpha_1}^2 Q^\eta Q^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}Q^\eta {n^{\eta'}}_{\perp,1} \\
& -\zeta_q\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{p_i}^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}Q^{\eta'} \\
& +\sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^\eta}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}]][g^{\gamma\delta}]
\end{aligned}
\tag{4.15}
$$

$$
\begin{aligned}
|M_1'|^2 = {} & \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
& [(6-d)\{({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y\alpha_1\beta_1 p_i{}^{\eta}Q^{\eta'} + \zeta_1\sqrt{y\alpha_1\beta_1}p_i{}^{\eta}n^{\eta'}{}_{\perp,1} \\
& +y\beta_1\alpha_1 Q^{\eta}p_i{}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}n^{\eta'}{}_{\perp,1} + \zeta_1\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}p_i{}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}Q^{\eta'} \\
& +y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} + (d+2)\{(\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y{\alpha_1}^2 p_i{}^{\eta}Q^{\eta'} \\
& -\zeta_1\sqrt{y\alpha_1\beta_1}p_i{}^{\eta}n^{\eta'}{}_{\perp,1} + y{\beta_1}^2 Q^{\eta}p_i{}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}n^{\eta'}{}_{\perp,1} + \zeta_q\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}p_i{}^{\eta'} \\
& +\lambda_q\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}Q^{\eta'} - y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} \\
& +(d+2)\{(\beta_1\alpha_1 - y({\beta_1}^2+{\alpha_1}^2)(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y{\beta_1}^2 p_i{}^{\eta}Q^{\eta'} + \zeta_q\sqrt{y\alpha_1\beta_1}p_i{}^{\eta}n^{\eta'}{}_{\perp,1} \\
& +y{\alpha_1}^2 Q^{\eta}p_i{}^{\eta'} + \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}n^{\eta'}{}_{\perp,1} - \zeta_1\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}p_i{}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}Q^{\eta'} \\
& -y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} + (6-d)\{({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y\beta_1\alpha_1 p_i{}^{\eta}Q^{\eta'} \\
& -\zeta_q\sqrt{y\alpha_1\beta_1}p_i{}^{\eta}n^{\eta'}{}_{\perp,1} + y\alpha_1\beta_1 Q^{\eta}p_i{}^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}n^{\eta'}{}_{\perp,1} - \zeta_q\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}p_i{}^{\eta'} \\
& -\lambda_q\sqrt{y\alpha_1\beta_1}n^{\eta}{}_{\perp,1}Q^{\eta'} + y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}]][g^{\gamma\delta}]
\end{aligned}
\tag{4.16}
$$

$$
\begin{aligned}
|M_1'|^2 = {} & \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k}}{4y^2\,(p_i\cdot Q)\,(p_i} \\
& [(6-d)\{({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y\alpha_1\beta_1 p_i{}^{\eta}Q^{\eta'} + y\beta_1\alpha_1 Q \\
& +y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'} \\
& +(d+2)\{(\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y{\alpha_1}^2 p_i{}^{\eta}Q^{\eta'} + y{\beta_1}^2 Q \\
& -y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} + (d+2)\{(\beta_1\alpha_1 - y({\beta_1}^2+{\alpha_1}^2)(\frac{Q^2}{2p_i\cdot Q}))p_i{}^{\eta}p_i{}^{\eta'} + y{\beta_1}^2 p_i \\
& +y{\alpha_1}^2 Q^{\eta}p_i{}^{\eta'} - y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} + (6-d)\{({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i \\
& +y\beta_1\alpha_1 p_i{}^{\eta}Q^{\eta'} + y\alpha_1\beta_1 Q^{\eta}p_i{}^{\eta'} + y\alpha_1\beta_1 n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1}\} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}]]
\end{aligned}
\tag{4.17}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[(6-d)({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q})) \\
&+(6-d)({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))]{p_i}^{\eta}{p_i}^{\eta'} \\
&+[2(6-d)y\alpha_1\beta_1 + (d+2)y({\alpha_1}^2+{\beta_1}^2)]{p_i}^{\eta}Q^{\eta'} \\
&+[2(6-d)y\beta_1\alpha_1 + (d+2)y({\alpha_1}^2+{\beta_1}^2)]Q^{\eta}{p_i}^{\eta'} \\
&+[2(6-d)-2(d+2)]y\alpha_1\beta_1 {n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}][g^{\gamma\delta}]]
\end{aligned}
\tag{4.18}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[(6-d)({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q})) \\
&+(6-d)({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))]{p_i}^{\eta}{p_i}^{\eta'} \\
&+y[(4d-8){\alpha_1}^2 + (8-4d)\alpha_1 + (d+2)]{p_i}^{\eta}Q^{\eta'} \\
&+y[(4d-8){\alpha_1}^2 + (8-4d)\alpha_1 + (d+2)]Q^{\eta}{p_i}^{\eta'} \\
&+y[8-4d](\alpha_1-{\alpha_1}^2){n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}][g^{\gamma\delta}]]
\end{aligned}
\tag{4.19}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[[8-4d]\beta_1(1-\beta_1){n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))n_{\perp,1}\cdot n_{\perp,1}][g^{\gamma\delta}]]
\end{aligned}
\tag{4.20}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[8[\epsilon-1]\beta_1(1-\beta_1){n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q - (\beta_1(1-\beta_1))(-2p_i\cdot Q)][g^{\gamma\delta}]]
\end{aligned}
\tag{4.21}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[8[\epsilon-1]\beta_1(1-\beta_1){n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2+{\beta_1}^2)p_i\cdot Q + 2\alpha_1\beta_1 p_i\cdot Q)][g^{\gamma\delta}]]
\end{aligned}
\tag{4.22}
$$

$$
\begin{aligned}
|M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
&[8[\epsilon-1]\beta_1(1-\beta_1){n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1} - 8g^{\eta\eta'}[(\alpha_1+\beta_1)^2 p_i\cdot Q)][g^{\gamma\delta}]]
\end{aligned}
\tag{4.23}
$$

$$|M_1'|^2 = \frac{g_s^2 \, f^{\,a\,o\,l} \, f^{\,a\,k\,o}}{y\,(p_i \cdot Q)}[2[\epsilon - 1]\beta_1(1-\beta_1)n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1} - 2g^{\eta\eta'}][g^{\gamma\delta}]] \tag{4.24}$$

Another way:

$$
\begin{aligned}
{k_1}^{\eta}{k_1}^{\eta'} &= ({\alpha_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1{p_i}^{\eta}Q^{\eta'} + y\alpha_1\beta_1 Q^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
{k_1}^{\eta}{q_i}^{\eta'} &= (\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y{\alpha_1}^2{p_i}^{\eta}Q^{\eta'} + y{\beta_1}^2 Q^{\eta}{p_i}^{\eta'} - y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
{q_i}^{\eta}{k_1}^{\eta'} &= (\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y{\beta_1}^2{p_i}^{\eta}Q^{\eta'} + y{\alpha_1}^2 Q^{\eta}{p_i}^{\eta'} - y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
{q_i}^{\eta}{q_i}^{\eta'} &= ({\beta_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1{p_i}^{\eta}Q^{\eta'} + y\alpha_1\beta_1 Q^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1}
\end{aligned}
\tag{4.25}
$$

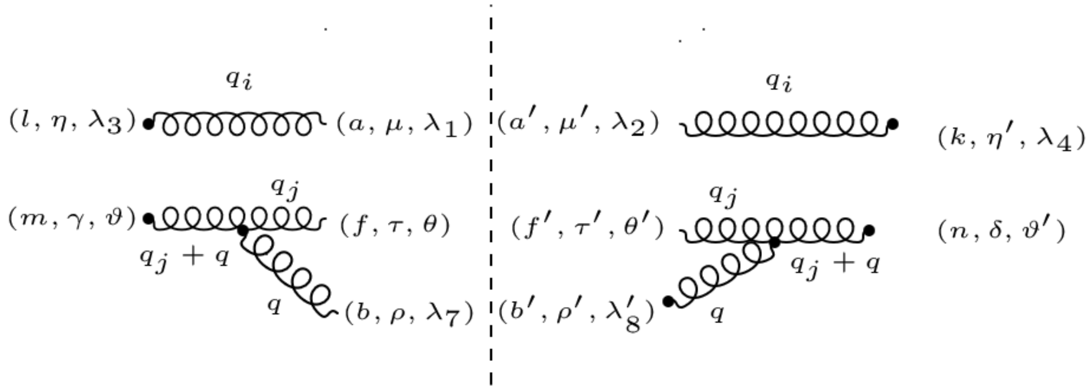
$$
\begin{aligned}
N \equiv &(6-d)({\alpha_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1{p_i}^{\eta}Q^{\eta'} + y\alpha_1\beta_1 Q^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
&+ (d+2)(\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y{\alpha_1}^2{p_i}^{\eta}Q^{\eta'} + y{\beta_1}^2 Q^{\eta}{p_i}^{\eta'} - y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
&+ (d+2)(\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y{\beta_1}^2{p_i}^{\eta}Q^{\eta'} + y{\alpha_1}^2 Q^{\eta}{p_i}^{\eta'} - y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
&+ (6-d)({\beta_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})){p_i}^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1{p_i}^{\eta}Q^{\eta'} + y\alpha_1\beta_1 Q^{\eta}{p_i}^{\eta'} + y\alpha_1\beta_1 n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
&- 8g^{\eta\eta'}[({\alpha_1}^2 + {\beta_1}^2)p_i \cdot Q - (\beta_1(1-\beta_1))n_{\perp,1} \cdot n_{\perp,1}]
\end{aligned}
\tag{4.26}
$$

$$
\begin{aligned}
N \equiv &[(6-d)({\alpha_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})) + (d+2)(\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})) \\
&+ (d+2)(\alpha_1\beta_1 - y({\alpha_1}^2 + {\beta_1}^2)(\frac{Q^2}{2p_i \cdot Q})) + (6-d)({\beta_1}^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))]{p_i}^{\eta}{p_i}^{\eta'} \\
&+ [(6-d)y\alpha_1\beta_1 + (d+2)y{\alpha_1}^2 + (d+2)y{\beta_1}^2 + (6-d)y\alpha_1\beta_1]{p_i}^{\eta}Q^{\eta'} \\
&+ [(6-d)y\alpha_1\beta_1 + (d+2)y{\beta_1}^2 + (d+2)y{\alpha_1}^2 + (6-d)y\alpha_1\beta_1]Q^{\eta}{p_i}^{\eta'} \\
&+ [(6-d)y\alpha_1\beta_1 - (d+2)y\alpha_1\beta_1 - (d+2)y\alpha_1\beta_1 + (6-d)y\alpha_1\beta_1]n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} \\
&- 8g^{\eta\eta'}[({\alpha_1}^2 + {\beta_1}^2)p_i \cdot Q - (\beta_1(1-\beta_1))n_{\perp,1} \cdot n_{\perp,1}]
\end{aligned}
\tag{4.27}
$$

$$
\begin{aligned}
&|M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i \cdot Q)^2}[(12-2d)y\alpha_1\beta_1 - 2(d+2)y\alpha_1\beta_1]n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} - 8yg^{\eta\eta'}p_i \cdot Q][g_{\gamma\delta}] \\
&\Rightarrow |M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i \cdot Q)^2}[(12-2d)\alpha_1\beta_1 - 2(d+2)\alpha_1\beta_1]n^{\eta}_{\perp,1}n^{\eta'}_{\perp,1} - 8g^{\eta\eta'}({\alpha_1}^2 + {\beta_1}^2)p_i \cdot Q][g_{\gamma\delta}] \\
&|M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{4y\,(p_i \cdot Q)\,(p_i \cdot Q)} \\
&[8[\epsilon - 1]\beta_1(1-\beta_1)n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1} - 8g^{\eta\eta'}[({\alpha_1}^2 + {\beta_1}^2)p_i \cdot Q - \beta_1\alpha_1(-2p_i \cdot Q)]][g_{\gamma\delta}]
\end{aligned}
\tag{4.28}
$$

$$|M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{y\,(p_i \cdot Q)}[2[\epsilon - 1]\beta_1(1-\beta_1)n^{\eta}{}_{\perp,1}n^{\eta'}{}_{\perp,1} - 2g^{\eta\eta'}][g^{\gamma\delta}] \tag{4.29}$$

4.2 Gluon-Spectator Bubble



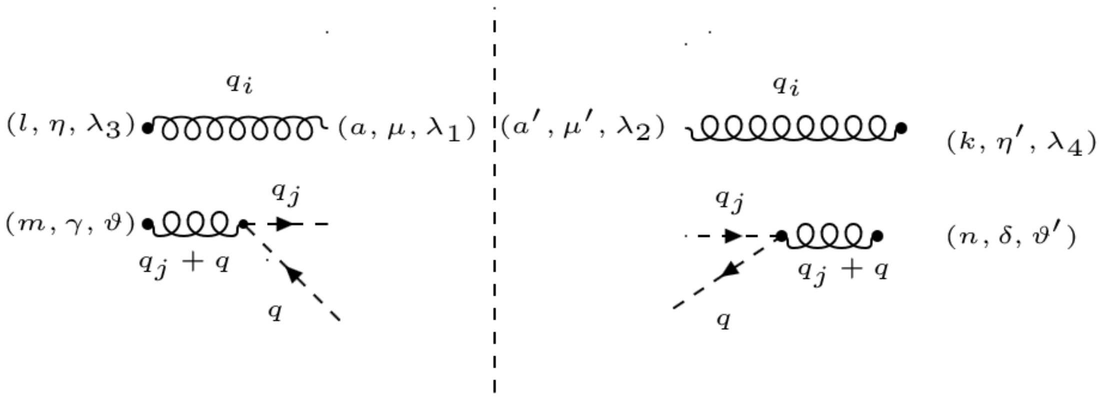
$$
\begin{aligned}
|M_2|^2 = [\frac{-i}{(q_j+q)^2}(-g_s f^{\,b\,f\,m}(g^{\tau\gamma}(-2q_j-q)^\rho + g^{\gamma\rho}(2q+q_j)^\tau + g^{\rho\tau}(q_j-q)^\gamma)\\
g_{\tau\tau'}g_{\rho\rho'}(-g_s f^{\,b'\,n\,f'}(g^{\rho'\delta}(-2q-q_j)^{\tau'} + g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\tau'\rho'}(q-q_j)^\delta)\frac{i}{(q_j+q)^2}][g^{\eta\eta'}]
\end{aligned} \tag{4.30}
$$

$$
\begin{aligned}
|M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b'\,n\,f'}\delta^{aa'}\delta^{ff'}\delta^{bb'}}{(q_j+q)^2(q_j+q)^2}[g_{\tau\tau'}g_{\rho\rho'}(g^{\tau\gamma}(2q_j+q)^\rho g^{\rho'\delta}(2q+q_j)^{\tau'}\\
-g^{\tau\gamma}(2q_j+q)^\rho g^{\delta\tau'}(2q_j+q)^{\rho'} - g^{\tau\gamma}(2q_j+q)^\rho g^{\tau'\rho'}(q-q_j)^\delta - g^{\gamma\rho}(2q+q_j)^\tau g^{\rho'\delta}(2q+q_j)^{\tau'}\\
+g^{\gamma\rho}(2q+q_j)^\tau g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\gamma\rho}(2q+q_j)^\tau g^{\tau'\rho'}(q-q_j)^\delta - g^{\rho\tau}(q_j-q)^\gamma g^{\rho'\delta}(2q+q_j)^{\tau'}\\
+g^{\rho\tau}(q_j-q)^\gamma g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\rho\tau}(q_j-q)^\gamma g^{\tau'\rho'}(q-q_j)^\delta][g^{\eta\eta'}]
\end{aligned} \tag{4.31}
$$

$$
\begin{aligned}
|M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[(2q+q_j)^\gamma(2q_j+q)^\delta\\
-g^{\delta\gamma}(2q_j+q)^\rho(2q_j+q)_\rho - (2q_j+q)^\gamma(q-q_j)^\delta - g^{\delta\gamma}(2q+q_j)^\tau(2q+q_j)_\tau\\
+(2q_j+q)^\gamma(2q+q_j)^\delta + (2q+q_j)^\gamma(q-q_j)^\delta - (q_j-q)^\gamma(2q+q_j)^\delta\\
+(q_j-q)^\gamma(2q_j+q)^\delta + d(q_j-q)^\gamma(q-q_j)^\delta][g^{\eta\eta'}]
\end{aligned} \tag{4.32}
$$

$$
\begin{aligned}
|M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[(3+d)q^\gamma {q_j}^\delta + (6-d)q^\gamma q^\delta\\
+(6-d){q_j}^\gamma {q_j}^\delta + (3+d){q_j}^\gamma q^\delta - g^{\delta\gamma}(5{q_j}^2 + 5q^2 + 8qq_j)\\
][g^{\eta\eta'}]
\end{aligned} \tag{4.33}
$$

4.2.1 One-loop corrections to the gluon self-energy diagram (Gluon-Spectator Bubble)



$$|M_2|^2_{Ghost\,loop} = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[-{q_j}^{\gamma} q^{\delta} - q^{\delta}{q_j}^{\gamma}][g^{\eta\eta'}] \tag{4.34}$$

$$\begin{aligned}|M_2'|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}&[(2+d)q^{\gamma}{q_j}^{\delta} + (6-d)q^{\gamma}q^{\delta} \\ &+(6-d){q_j}^{\gamma}{q_j}^{\delta} + (2+d){q_j}^{\gamma}q^{\delta} - g^{\delta\gamma}(8qq_j)][g^{\eta\eta'}]\end{aligned} \tag{4.35}$$

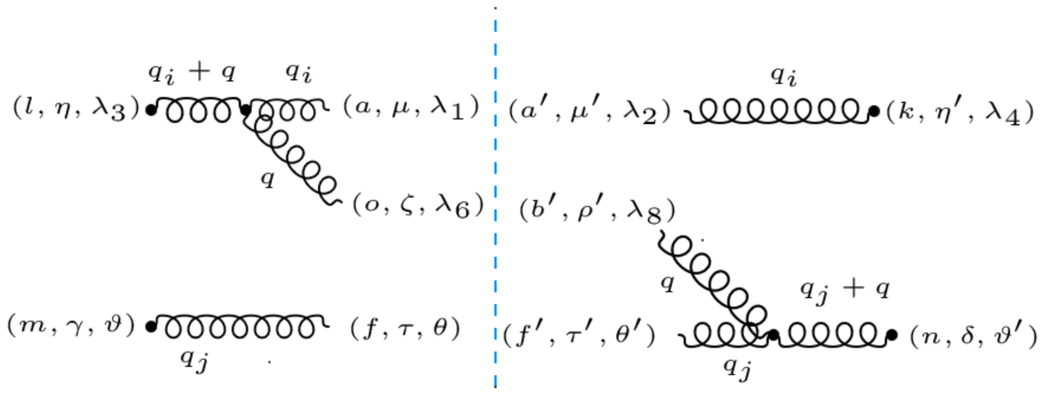
$$|M_2'|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{4(q_j\cdot q)(q_j\cdot q)}[-8g^{\delta\gamma}(q\cdot q_j)][g^{\eta\eta'}] \tag{4.36}$$

$$|M_2'|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j\cdot q)}[-2g^{\delta\gamma}][g^{\eta\eta'}] \tag{4.37}$$

$$|M_2'|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(1-\beta_1)(1-y)\,(p_i\cdot p_k)}[-2g^{\delta\gamma}][g^{\eta\eta'}] \tag{4.38}$$

4.3 Interference term $M_1{M_2}^{\dagger}$

$$\begin{aligned}&M_1{M_2}^{\dagger} = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,l\,a\,o}(g^{\eta\mu}(2q_i+q)^{\zeta} + g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q+q_i)^{\mu})\varepsilon^{\lambda_1}{}_{\mu}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)] \\ &[{\varepsilon^{\theta}{}_{\tau}}^{*}(q_j)] \\ &[\frac{i}{(q+q_j)^2}(-g_s f^{\,f'\,b'\,n}(g^{\tau'\rho'}(q_j-q)^{\delta} + g^{\rho'\delta}(2q+q_j)^{\tau'} - g^{\delta\tau'}(2q_j+q)^{\rho'}){\varepsilon^{\theta'}{}_{\tau'}}^{*}(q_j){\varepsilon^{\lambda_8}{}_{\rho'}}^{*}(q)] \\ &[{\varepsilon^{\lambda_2}{}_{\mu'}}^{*}(q_i)]\end{aligned} \tag{4.39}$$



$$
\begin{aligned}
M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,l\,a\,o} f^{\,f'\,b'\,n} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i+q)^2 (q_j+q)^2} & [g_\mu{}^{\eta'} g_{\tau\tau'} (g^{\eta\mu}(2q_i+q)^\zeta + g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q+q_i)^\mu) \\
g_{\zeta\rho'} (g^{\tau'\rho'}(q_j-q)^\delta + g^{\rho'\delta}(2q+q_j)^{\tau'} - g^{\delta\tau'}(2q_j+q)^{\rho'}]
\end{aligned}
\tag{4.40}
$$

$$
\begin{aligned}
& M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,l\,a\,o} f^{\,f'\,b'\,n} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i+q)^2 (q_j+q)^2} \\
& [g^{\eta\eta'}(2q_i+q)^\gamma (q_j-q)^\delta + g^{\eta\eta'}(2q+q_j)^\gamma (2q_i+q)^\delta - g^{\eta\eta'} g^{\gamma\delta}(2q_i+q)\cdot(2q_j+q) \\
& + g^{\gamma\eta'}(q-q_i)^\eta (q_j-q)^\delta + g^{\eta'\delta}(q-q_i)^\eta (2q+q_j)^\gamma - g^{\gamma\delta}(q-q_i)^\eta (2q_j+q)^{\eta'} \\
& - g^{\gamma\eta}(2q+q_i)^{\eta'}(q_j-q)^\delta - g^{\eta\delta}(2q+q_i)^{\eta'}(2q+q_j)^\gamma + g^{\gamma\delta}(2q_j+q)^\eta (2q+q_i)^{\eta'}]
\end{aligned}
\tag{4.41}
$$

$$
\begin{aligned}
& M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,l\,a\,o} f^{\,f\,o\,n}}{4(q\cdot q_i)(q\cdot q_j)} \\
& \{g^{\eta\eta'}[2q_i{}^\gamma q_j{}^\delta + 2q_i{}^\gamma q^\delta + q^\gamma q_j{}^\delta + q^\gamma q^\delta + 4q^\gamma q_i{}^\delta + 2q^\gamma q^\delta + 2q_j{}^\gamma q_i{}^\delta + q_j{}^\gamma q^\delta] \\
& - g^{\eta\eta'} g^{\gamma\delta}(2q\cdot q_j + q\cdot q + 4q_i\cdot q_j + 2q_i\cdot q) + g^{\gamma\eta'}[q^\eta q_j{}^\delta - q^\eta q^\delta - q_i{}^\eta q_j{}^\delta + q_i{}^\eta q^\delta] \\
& + g^{\eta'\delta}[2q^\eta q^\gamma + q^\eta q_j{}^\gamma + q_i{}^\eta q^\gamma + q_i{}^\eta q_j{}^\gamma] - g^{\gamma\delta}[2q^\eta q_j{}^{\eta'} + q^\eta q^{\eta'} - 2q_i{}^\eta q_j{}^{\eta'} - q_i{}^\eta q^{\eta'}] \\
& - g^{\gamma\eta}[2q^{\eta'} q_j{}^\delta - 2q^{\eta'} q^\delta + q_i{}^{\eta'} q_j{}^\delta - q_i{}^{\eta'} q^\delta] - g^{\eta\delta}[4q^{\eta'} q^\gamma + 2q^{\eta'} q_j{}^\gamma + 2q_i{}^{\eta'} q^\gamma + q_i{}^{\eta'} q_j{}^\gamma] \\
& + g^{\gamma\delta}[4q_j{}^\eta q^{\eta'} + 2q_j{}^\eta q_i{}^{\eta'} + q^\eta q^{\eta'} + q^\eta q_i{}^{\eta'}]\}
\end{aligned}
\tag{4.42}
$$

$$
\begin{aligned}
{k_1}^{\eta}{k_1}^{\eta'} &= [(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2]{p_i}^{\eta}{p_i}^{\eta'} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}){p_i}^{\eta}Q^{\eta'} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})Q^{\eta}{p_i}^{\eta'}\\
{k_1}^{\eta}{q_i}^{\eta'} &= [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^{\eta}{p_i}^{\eta'} + y{\beta_1}^2Q^{\eta}{p_i}^{\eta'}\\
{q_i}^{\eta}{k_1}^{\eta'} &= [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^{\eta}{p_i}^{\eta'} + y{\beta_1}^2{p_i}^{\eta}Q^{\eta'}\\
{q_i}^{\eta}{q_i}^{\eta'} &= {\beta_1}^2{p_i}^{\eta}{p_i}^{\eta'}\\
{k_1}^{\eta}{q_k}^{\eta'} &= [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_i}^{\eta}{p_k}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^{\eta}{p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,{p_i}^{\eta}Q^{\eta'}\\
&\quad + y\beta_1A_1\,Q^{\eta}{p_i}^{\eta'} + y\beta_1A_2\,Q^{\eta}Q^{\eta'} + y\beta_1\sqrt{1-y}Q^{\eta}{p_k}^{\eta'}\\
{q_i}^{\eta}{q_k}^{\eta'} &= A_1\beta_1{p_i}^{\eta}{p_i}^{\eta'} + A_2\beta_1{p_i}^{\eta}Q^{\eta'} + \beta_1\sqrt{1-y}{p_i}^{\eta}{p_k}^{\eta'}\\
{q_k}^{\eta}{k_1}^{\eta'} &= [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_k}^{\eta}{p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^{\eta}{p_i}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,Q^{\eta}{p_i}^{\eta'}\\
&\quad + y\beta_1A_1\,{p_i}^{\eta}Q^{\eta'} + y\beta_1A_2\,Q^{\eta}Q^{\eta'} + y\beta_1\sqrt{1-y}{p_k}^{\eta}Q^{\eta'}\\
{q_k}^{\eta}{q_i}^{\eta'} &= A_1\beta_1{p_i}^{\eta}{p_i}^{\eta'} + A_2\beta_1Q^{\eta}{p_i}^{\eta'} + \beta_1\sqrt{1-y}{p_k}^{\eta}{p_i}^{\eta'}
\end{aligned}
\tag{4.43}
$$

Calculation of the first Term

$$
\begin{aligned}
&g^{\eta\eta'}[2\{A_1\beta_1{p_i}^{\gamma}{p_i}^{\delta} + A_2\beta_1{p_i}^{\gamma}Q^{\delta} + \beta_1\sqrt{1-y}{p_i}^{\gamma}{p_k}^{\delta}\}\\
&+2\{[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^{\gamma}{p_i}^{\delta} + y{\beta_1}^2{p_i}^{\gamma}Q^{\delta}\}\\
&+\{[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_i}^{\gamma}{p_k}^{\delta} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^{\gamma}{p_i}^{\delta} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,{p_i}^{\gamma}Q^{\delta}\\
&+y\beta_1A_1\,Q^{\gamma}{p_i}^{\delta} + y\beta_1A_2\,Q^{\gamma}Q^{\delta} + y\beta_1\sqrt{1-y}Q^{\gamma}{p_k}^{\delta}\}\\
&+3\{[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2]{p_i}^{\gamma}{p_i}^{\delta} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}){p_i}^{\gamma}Q^{\delta} - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})Q^{\gamma}{p_i}^{\delta}\}\\
&+4\{[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^{\gamma}{p_i}^{\delta} + y{\beta_1}^2Q^{\gamma}{p_i}^{\delta}\}\\
&+2\{A_1\beta_1{p_i}^{\gamma}{p_i}^{\delta} + A_2\beta_1Q^{\gamma}{p_i}^{\delta} + \beta_1\sqrt{1-y}{p_k}^{\gamma}{p_i}^{\delta}\}\\
&+\{[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_k}^{\gamma}{p_i}^{\delta} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\,{p_i}^{\gamma}{p_i}^{\delta} - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\,Q^{\gamma}{p_i}^{\delta}\\
&+y\beta_1A_1\,{p_i}^{\gamma}Q^{\delta} + y\beta_1A_2\,Q^{\gamma}Q^{\delta} + y\beta_1\sqrt{1-y}{p_k}^{\gamma}Q^{\delta}\}]
\end{aligned}
\tag{4.44}
$$

$$
\begin{aligned}
& g^{\eta\eta'}\{[2A_1\beta_1 + 2[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})] \\
& + 4[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})] + 3[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2] \\
& + 2A_1\beta_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 \ - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\ ]{p_i}^\gamma {p_i}^\delta \\
& + [2A_2\beta_1 + 2y{\beta_1}^2 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \ - 3y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) + y\beta_1 A_1]{p_i}^\gamma Q^\delta \\
& + [2\beta_1 + [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]]\sqrt{1-y}{p_i}^\gamma {p_k}^\delta \\
& + [y\beta_1 A_1 + 4y{\beta_1}^2 + 2A_2\beta_1 - 3y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\ ]\ Q^\gamma {p_i}^\delta \\
& + [y\beta_1 A_2 + y\beta_1 A_2]\ Q^\gamma Q^\delta + y\beta_1\sqrt{1-y}Q^\gamma {p_k}^\delta \\
& + [2\beta_1 + [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]]\sqrt{1-y}{p_k}^\gamma {p_i}^\delta + y\beta_1\sqrt{1-y}{p_k}^\gamma Q^\delta\}
\end{aligned}
\tag{4.45}
$$

Calculation of the second term

$$
-g^{\eta\eta'}g^{\gamma\delta}(2q\cdot q_j + q\cdot q + 4q_i\cdot q_j + 2q_i\cdot q) \tag{4.46}
$$

$$
\begin{aligned}
& -g^{\eta\eta'}g^{\gamma\delta}[2([\alpha_1(1-y) + y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\ p_i\cdot p_k + y\beta_1\ Q\cdot p_k + \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}) \\
& 4([\beta_1(1-y) + y\alpha_1(\frac{Q^2}{2p_i\cdot Q})]\ p_i\cdot p_k + y\alpha_1\ Q\cdot p_k - \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}) \\
& \ + 2(y\ p_i\cdot Q)]
\end{aligned}
\tag{4.47}
$$

Calculation of the third term

$$
\begin{aligned}
& + g^{\gamma\eta'}\{[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\sqrt{1-y}{p_i}^\eta {p_k}^\delta - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1\ {p_i}^\eta {p_i}^\delta - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2\ {p_i}^\eta Q^{\eta'} \\
& + y\beta_1 A_1\ Q^\eta {p_i}^\delta + y\beta_1 A_2\ Q^\eta Q^\delta + y\beta_1\sqrt{1-y}Q^\eta {p_k}^\delta \\
& - [[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2]{p_i}^\eta {p_i}^\delta - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}){p_i}^\eta Q^\delta - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})Q^\eta {p_i}^\delta] \\
& - [A_1\beta_1 {p_i}^\eta {p_i}^\delta + A_2\beta_1 {p_i}^\eta Q^\delta + \beta_1\sqrt{1-y}{p_i}^\eta {p_k}^\delta] \\
& + [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]{p_i}^\eta {p_i}^{\eta'} + y{\beta_1}^2{p_i}^\eta Q^{\eta'}\}
\end{aligned}
\tag{4.48}
$$

Calculation of the fourth term

$$
\begin{aligned}
&+ g^{\eta'\delta}\{[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q}) - \beta_1]\sqrt{1-y}{p_i}^{\eta}{p_k}^{\gamma} \\
&+ [2[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2] - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 + A_1\beta_1 + \\
&[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]]{p_i}^{\eta}{p_i}^{\gamma} \\
&+ [-2y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \; + A_2\beta_1 + y{\beta_1}^2]{p_i}^{\eta}Q^{\gamma} \\
&+ [y\beta_1 A_1 \; + 2y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]Q^{\eta}{p_i}^{\gamma} + y\beta_1 A_2\, Q^{\eta}Q^{\gamma} + y\beta_1\sqrt{1-y}Q^{\eta}{p_k}^{\gamma}\}
\end{aligned}
\tag{4.49}
$$

Calculation of the fifth term

$$
\begin{aligned}
&- g^{\gamma\delta}\{[2[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})] - 2\beta_1]\sqrt{1-y}{p_i}^{\eta}{p_k}^{\eta'} \\
&[-2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 + [(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2] - 2A_1\beta_1 \\
&- [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]]\; {p_i}^{\eta}{p_i}^{\eta'} \\
&[-2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \; - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) - y{\beta_1}^2 - 2A_2\beta_1]{p_i}^{\eta}Q^{\eta'} \\
&+ [2y\beta_1 A_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]\; Q^{\eta}{p_i}^{\eta'} + 2y\beta_1 A_2\, Q^{\eta}Q^{\eta'} + 2y\beta_1\sqrt{1-y}Q^{\eta}{p_k}^{\eta'}\}
\end{aligned}
\tag{4.50}
$$

Calculation of the sixth term

$$
\begin{aligned}
&- g^{\gamma\eta}\{[2[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})] + \beta_1]\sqrt{1-y}{p_i}^{\eta'}{p_k}^{\delta} \\
&[-2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 - 2[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2] \\
&- [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})] + A_1\beta_1]\; {p_i}^{\eta'}{p_i}^{\delta} \\
&[-2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \; + 2y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) + A_2\beta_1 - y{\beta_1}^2]{p_i}^{\eta'}Q^{\delta} \\
&+ [2y\beta_1 A_1 + 2y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]\; Q^{\eta'}{p_i}^{\delta} + 2y\beta_1 A_2\, Q^{\eta'}Q^{\delta} + 2y\beta_1\sqrt{1-y}Q^{\eta'}{p_k}^{\delta}\}
\end{aligned}
\tag{4.51}
$$

Calculation of the seventh term

$$
\begin{aligned}
&- g^{\eta\delta}\{[2[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})] + \beta_1]\sqrt{1-y}{p_i}^{\eta'}{p_k}^{\gamma} \\
&[4[(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2] - 2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 + A_1\beta_1 \\
&+ 2[\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]]{p_i}^{\eta'}{p_i}^{\gamma} \\
&+ [-4y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) - 2y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \; + 2y{\beta_1}^2 + A_2\beta_1]{p_i}^{\eta'}Q^{\gamma} \\
&+ [-4y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) + 2y\beta_1A_1]Q^{\eta}{p_i}^{\eta'} + 2y\beta_1A_2\,Q^{\eta}Q^{\eta'} + 2y\beta_1\sqrt{1-y}Q^{\eta'}{p_k}^{\gamma}\}
\end{aligned}
\tag{4.52}
$$

Calculation of the eighth term

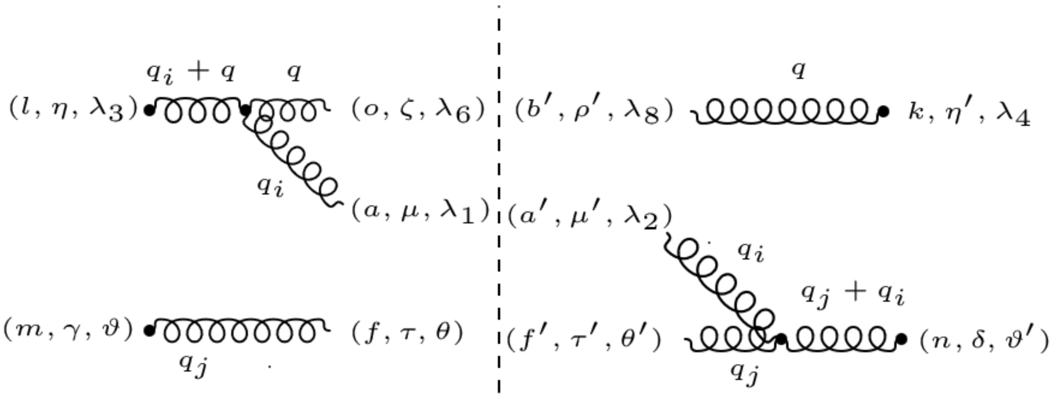
$$
\begin{aligned}
&+ g^{\gamma\delta}\{[4[(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i\cdot Q})] + 2\beta_1]\sqrt{1-y}{p_k}^{\eta}{p_i}^{\eta'} \\
&+ [-4y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_1 + 2A_1\beta_1 + [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})] \\
&+ [(1-\beta_1)^2 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2]]\,{p_i}^{\eta}{p_i}^{\eta'} \\
&+ [4y\beta_1A_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})]\,{p_i}^{\eta}Q^{\eta'} + 4y\beta_1A_2\,Q^{\eta}Q^{\eta'} + 4y\beta_1\sqrt{1-y}{p_k}^{\eta}Q^{\eta'} \\
&+ [2A_2\beta_1 - 4y\beta_1(\frac{Q^2}{2p_i\cdot Q})A_2 \; - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q}) + y{\beta_1}^2]Q^{\eta}{p_i}^{\eta'}\}
\end{aligned}
\tag{4.53}
$$

Final result

$$
\begin{aligned}
&M_1{M_2}^{\dagger} = \frac{g_s^2C_A}{4y(1-\beta_1)(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)}g^{\eta\eta'}g^{\gamma\delta} \\
&[2([\alpha_1(1-y) + y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\beta_1\,Q\cdot p_k + \sqrt{\alpha_1\beta_1y(1-y)}p_k\cdot n_{\perp,1}) \\
&4([\beta_1(1-y) + y\alpha_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\alpha_1\,Q\cdot p_k - \sqrt{\alpha_1\beta_1y(1-y)}p_k\cdot n_{\perp,1}) \\
&+ 2(y\,p_i\cdot Q)]
\end{aligned}
\tag{4.54}
$$

$$
\begin{aligned}
M_1{M_2}^{\dagger} = g_s^2\,C_A\,g^{\eta\eta'}g^{\gamma\delta}[&\frac{1}{2y(p_i\cdot Q)} + \frac{\beta_1(\frac{Q^2}{2p_i\cdot Q})}{2y(1-\beta_1)(1-y)\,(p_i\cdot Q)} \\
+ &\frac{\beta_1\,Q\cdot p_k}{2y(1-\beta_1)(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} + \frac{\beta_1}{y(1-\beta_1)(p_i\cdot Q)} + \frac{1}{2(1-\beta_1)(1-y)(p_i\cdot p_k)}]
\end{aligned}
\tag{4.55}
$$

4.4 Interference term of inverse $M_1 {M_2^\dagger}'$



$$M_1 M_2{}^\dagger = \frac{g_s^2 f^{\,l\,o\,a} f^{\,f'\,a'\,n} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i+q)^2 (q_j+q_i)^2} [g_\zeta{}^{\eta'} g^\gamma{}_{\tau'} (g^{\eta\zeta}(2q+q_i)^\mu + g^{\zeta\mu}(q_i-q)^\eta - g^{\mu\eta}(2q_i+q)^\zeta)$$
$$g_{\mu\mu'}(g^{\tau'\mu'}(q_j-q_i)^\delta + g^{\mu'\delta}(2q_i+q_j)^{\tau'} - g^{\delta\tau'}(2q_j+q_i)^{\mu'}] \tag{4.56}$$

$$\begin{aligned} M_1 M_2{}^\dagger &= \frac{g_s^2 f^{\,l\,o\,a} f^{\,f\,a\,n}}{4(q\cdot q_i)(q_i\cdot q_j)} \\ &[g^{\eta\eta'}(2q+q_i)^\gamma (q_j-q_i)^\delta + g^{\eta\eta'}(2q_i+q_j)^\gamma (2q+q_i)^\delta - g^{\eta\eta'} g^{\gamma\delta}(2q+q_i)\cdot(2q_j+q_i) \\ &+ g^{\gamma\eta'}(q_i-q)^\eta (q_j+q_i)^\delta + g^{\eta'\delta}(q_i-q)^\eta (2q_i+q_j)^\gamma - g^{\gamma\delta}(q_i-q)^\eta (2q_j+q_i)^{\eta'} \\ &- g^{\gamma\eta}(2q_i+q)^{\eta'}(q_j-q_i)^\delta - g^{\eta\delta}(2q_i+q)^{\eta'}(2q_i+q_j)^\gamma + g^{\gamma\delta}(2q_j+q_i)^\eta (2q_i+q)^{\eta'}] \end{aligned} \tag{4.57}$$

4.5 Parametrization in terms of $(k_1\cdot q_i)(q_i\cdot q_k)$

$$\boxed{(k_1\cdot q_i)(k_1\cdot q_k) \approx y\beta_1(1-y)\,(p_i\cdot Q)(p_i\cdot Q)} \tag{4.58}$$

Calculation of the third term

$$-g^{\eta\eta'} g^{\gamma\delta}\{4k_1\cdot q_j + 2k_1\cdot q_i + 2q_i\cdot q_k\} \tag{4.59}$$

$$\begin{aligned} M_1 M_2{}^\dagger &= \frac{g_s^2 C_A}{4y\beta_1(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} g^{\eta\eta'} g^{\gamma\delta} \\ &[4([\alpha_1(1-y) + y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\beta_1\,Q\cdot p_k + \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}) \\ &2([\beta_1(1-y) + y\alpha_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\alpha_1\,Q\cdot p_k - \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}) \\ &+ 2(y\,p_i\cdot Q)] \end{aligned} \tag{4.60}$$

$$
\begin{aligned}
&-g^{\eta\eta'} g^{\gamma\delta}[4([\alpha_1(1-y)+y\beta_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\beta_1\,Q\cdot p_k + \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1})\\
&2([\beta_1(1-y)+y\alpha_1(\frac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\alpha_1\,Q\cdot p_k - \sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1})\\
&+2(y\,p_i\cdot Q)]
\end{aligned}
\tag{4.61}
$$

$$
\begin{aligned}
M_1{M_2}^\dagger = g_s^2\,C_A\,g^{\eta\eta'}g^{\gamma\delta}[&\frac{1-\beta_1}{y\beta_1(p_i\cdot Q)} + \frac{1}{2y(p_i\cdot Q)} + \frac{(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})}{2y\beta_1(1-y)\,(p_i\cdot Q)}\\
&+\frac{(1-\beta_1)\,Q\cdot p_k}{2y\beta_1(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} + \frac{1}{2(1-\beta_1)(1-y)(p_i\cdot p_k)}]
\end{aligned}
\tag{4.62}
$$

4.6 $|M^2|$

$$
\begin{aligned}
&|M|^2 = |M'_2|^2 + |M'_1|^2 + 2RE(M_1{M_2}^\dagger + M_1{M_2}^{\dagger\prime})\\
&|M|^2 = \frac{g_s^2\,C_A}{y\,(p_i\cdot Q)}[2[\epsilon-1]\beta_1(1-\beta_1)n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1} - 2g^{\eta\eta'}][g^{\gamma\delta}]\\
&+\frac{g_s^2\,C_A}{(1-\beta_1)(1-y)\,(p_i\cdot p_k)}[-2g^{\delta\gamma}][g^{\eta\eta'}]\\
&+2Re(g_s^2\,C_A\,g^{\eta\eta'}g^{\gamma\delta}[\frac{1}{2y(p_i\cdot Q)} + \frac{\beta_1(\frac{Q^2}{2p_i\cdot Q})}{2y(1-\beta_1)(1-y)\,(p_i\cdot Q)}\\
&+\frac{\beta_1\,Q\cdot p_k}{2y(1-\beta_1)(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} + \frac{\beta_1}{y(1-\beta_1)(p_i\cdot Q)} + \frac{1}{2(1-\beta_1)(1-y)(p_i\cdot p_k)}]\\
&+g_s^2\,C_A\,g^{\eta\eta'}g^{\gamma\delta}[\frac{1-\beta_1}{y\beta_1(p_i\cdot Q)} + \frac{1}{2y(p_i\cdot Q)} + \frac{(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})}{2y\beta_1(1-y)\,(p_i\cdot Q)}\\
&+\frac{(1-\beta_1)\,Q\cdot p_k}{2y\beta_1(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} + \frac{1}{2(1-\beta_1)(1-y)(p_i\cdot p_k)}])
\end{aligned}
\tag{4.63}
$$

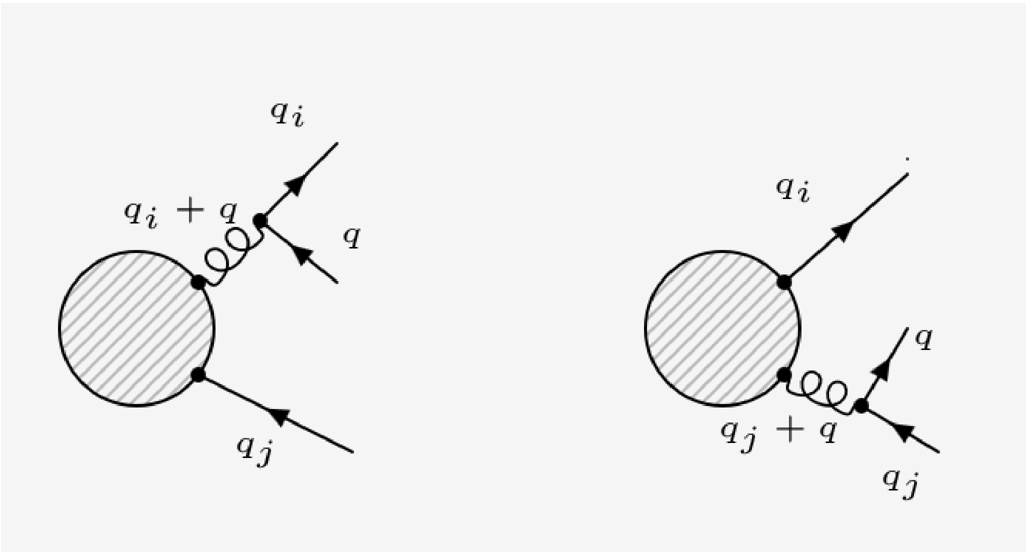
$$
\begin{aligned}
&|M|^2 = |M'_2|^2 + |M'_1|^2 + 2RE(M_1{M_2}^\dagger + M_1{M_2}^{\dagger\prime})\\
&|M|^2 = g_s^2\,C_A\,g^{\eta\eta'}g^{\gamma\delta}[2[\epsilon-1]\beta_1(1-\beta_1)n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1} + \frac{\beta_1(\frac{Q^2}{2p_i\cdot Q})}{y(1-\beta_1)(1-y)\,(p_i\cdot Q)}\\
&+\frac{\beta_1\,Q\cdot p_k}{y(1-\beta_1)(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)} + \frac{2\beta_1}{y(1-\beta_1)(p_i\cdot Q)}\\
&+\frac{2(1-\beta_1)}{y\beta_1(p_i\cdot Q)} + \frac{(1-\beta_1)(\frac{Q^2}{2p_i\cdot Q})}{y\beta_1(1-y)\,(p_i\cdot Q)} + \frac{(1-\beta_1)\,Q\cdot p_k}{y\beta_1(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)}]
\end{aligned}
\tag{4.64}
$$

$$|M|^2 = g_s^2\, C_A\, g^{\eta\eta'} g^{\gamma\delta}[2\beta_1(1-\beta_1) + \frac{2\beta_1}{y(1-\beta_1)(p_i\cdot Q)} + \frac{2(1-\beta_1)}{y\beta_1(p_i\cdot Q)} + \frac{(\frac{Q^2}{2p_i\cdot Q})}{y\beta_1(1-y)\,(p_i\cdot Q)} + \frac{Q\cdot p_k}{y\beta_1(1-y)\,(p_i\cdot p_k)(p_i\cdot Q)}] \tag{4.65}$$

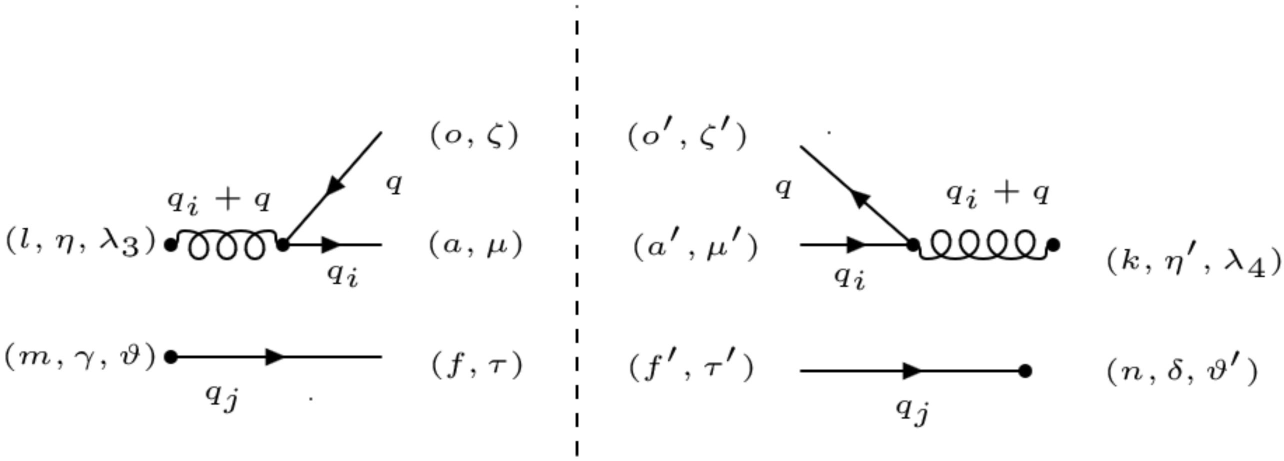
$$|M|^2 = 2\frac{g_s^2\, C_A}{y(p_i\cdot Q)}\, g^{\eta\eta'} g^{\gamma\delta}[\beta_1(1-\beta_1) + \frac{\beta_1}{1-\beta_1} + \frac{1-\beta_1}{\beta_1}] \tag{4.66}$$

Chapter 5

Quark gluon quark emission kernel



5.1 Quark loop



$$|M_1|^2 = [\frac{-i}{(q_i+q)^2} \not{q}_i(-ig_s\gamma^\eta \times [T^l]_a{}^o) \not{q}(ig_s\gamma^{\eta'} \times [T^k]_{o'}{}^{a'})\frac{i}{(q_i+q)^2}][\not{q}_j] \tag{5.1}$$

$$|M_1|^2 = \frac{g_s{}^2[T^l]_a{}^o[T^k]_{o'}{}^{a'}}{4(k_1\cdot q_i)(k_1\cdot q_i)}[\not{q}_i\gamma^\eta \not{k}_1 \gamma^{\eta'}][\not{q}_k] \tag{5.2}$$

$$|M_1|^2 = -\frac{g_s{}^2[T^l]_a{}^o[T^k]_{o'}{}^{a'}}{4y^2(p_i\cdot Q)(p_i\cdot Q)}[\not{q}_i \not{k}_1 \gamma^\eta\gamma^{\eta'}][\not{q}_k] \tag{5.3}$$

$$\begin{aligned}
|M_1|^2 &= -\frac{g_s{}^2[T^l]_a{}^o[T^k]_{o'}{}^{a'}}{4y^2(p_i\cdot Q)(p_i\cdot Q)} \\
&[((\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i\cdot Q})) \not{p}_i + y\alpha_1 \not{Q} - \sqrt{y\alpha_1\beta_1} \not{n}_{\perp,l}) \\
&((\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})) \not{p}_i + y\beta_1 \not{Q} + \sqrt{y\alpha_1\beta_1} \not{n}_{\perp,1}) \gamma^\eta\gamma^{\eta'}] \\
&[A_1 \not{p}_i + A_2 \not{Q} + \sqrt{1-y} \not{p}_k]
\end{aligned} \tag{5.4}$$

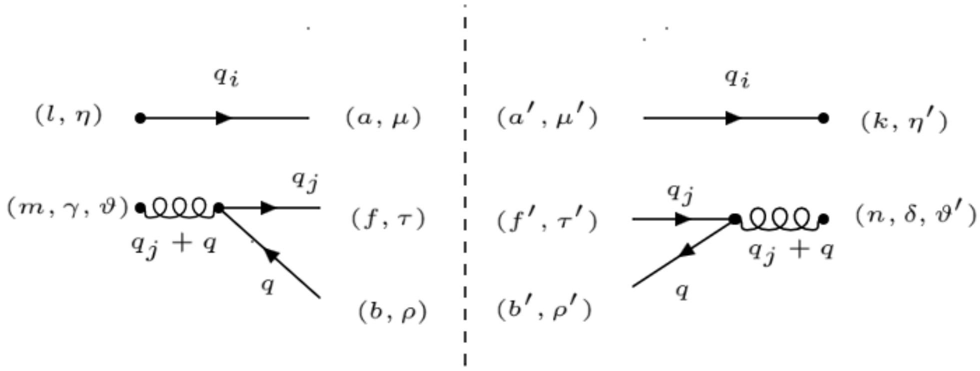
$$\begin{aligned}
|M_1|^2 &= -\frac{g_s{}^2[T^l]_a{}^o[T^k]_{o'}{}^{a'}}{4y^2(p_i\cdot Q)(p_i\cdot Q)} \\
&[(y\beta_1(\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i\cdot Q})) \not{p}_i \not{Q} + y\alpha_1(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})) \not{p}_i \not{Q} + y^2\alpha_1\beta_1 \not{Q} \not{Q})g^{\eta\eta'}] \\
&[A_1 \not{p}_i + A_2 \not{Q} + \sqrt{1-y} \not{p}_k]
\end{aligned} \tag{5.5}$$

$$|M_1|^2 = -\frac{{g_s}^2 [T^l]_a{}^o [T^k]_{o'}{}^{a'}}{4y^2 (p_i \cdot Q)(p_i \cdot Q)} [(y{\beta_1}^2 \ \not{p}_i \ \not{Q} + y{\alpha_1}^2 \ \not{Q} \ \not{p}_i) g^{\eta\eta'}][A_1 \ \not{p}_i + A_2 \ \not{Q} + \sqrt{1-y} \ \not{p}_k] \tag{5.6}$$

$$|M_1|^2 = -\frac{{g_s}^2 [T^l]_a{}^o [T^k]_{o'}{}^{a'}}{4y^2 (p_i \cdot Q)(p_i \cdot Q)} [y({\beta_1}^2 - {\alpha_1}^2) \ \not{p}_i \ \not{Q} g^{\eta\eta'}][A_1 \ \not{p}_i + A_2 \ \not{Q} + \sqrt{1-y} \ \not{p}_k] \tag{5.7}$$

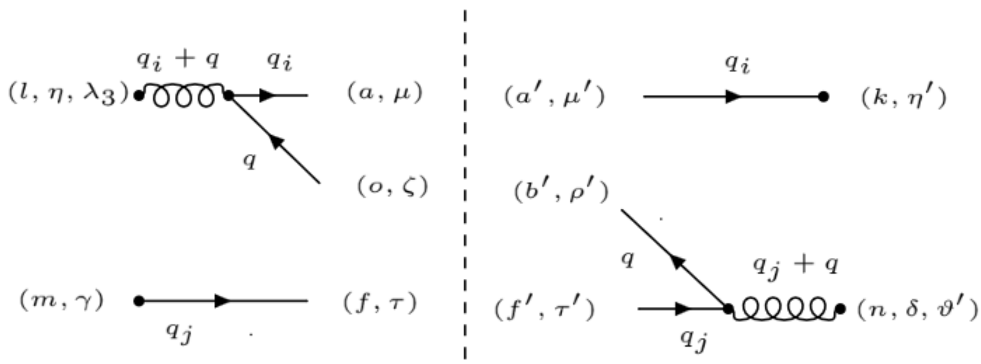
$$|M_1|^2 = -\frac{{g_s}^2 [T^l]_a{}^o [T^k]_{o'}{}^{a'}}{4y (p_i \cdot Q)(p_i \cdot Q)} [g^{\eta\eta'}][\sqrt{1-y}({\beta_1}^2 - {\alpha_1}^2) \ \not{p}_i \ \not{Q} \ \not{p}_k] \tag{5.8}$$

5.2 Spectator Quark loop



$$|M_2|^2 = \frac{{g_s}^2[T^m]_f{}^b[T^n]_f{}^b}{4(k_1 \cdot q_k)(k_1 \cdot q_k)}[\not{q}_k\gamma^\gamma \not{k}_1 \gamma^\delta][\not{q}_i] \tag{5.9}$$

5.3 Interference term



$$M_1\, M_2{}^\dagger = \frac{g_s{}^2 [T^l]_a{}^o [T^n]_f{}^o}{4(qq_i)(qq_j)} [\not{q}_i \gamma^\eta \; \not{q} \; \gamma^\delta \; \not{q}_j] \tag{5.10}$$

$$M_1\, M_2{}^\dagger = -\frac{g_s{}^2 [T^l]_a{}^o [T^n]_f{}^o}{4(k_1 \cdot q_i)(k_1 \cdot q_k)} [\not{q}_i \; \not{k}_1 \quad \not{q}_k][g^{\eta\delta}] \tag{5.11}$$

$$\begin{aligned} M_1\, M_2{}^\dagger &= -\frac{g_s{}^2 [T^l]_a{}^o [T^n]_f{}^o}{4y(1-\beta_1)(1-y)\,(p_i \cdot p_k)(p_i \cdot Q)} [g^{\eta\delta}] \\ &[((\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q}))\; \not{p}_i + y\alpha_1 \; \not{Q} - \sqrt{y\alpha_1\beta_1}\; \not{n}_{\perp,l}) \\ &((\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\; \not{p}_i + y\beta_1 \; \not{Q} + \sqrt{y\alpha_1\beta_1}\; \not{n}_{\perp,1}) \\ &(A_1 \; \not{p}_i + A_2 \; \not{Q} + \sqrt{1-y} \; \not{p}_k)] \end{aligned} \tag{5.12}$$

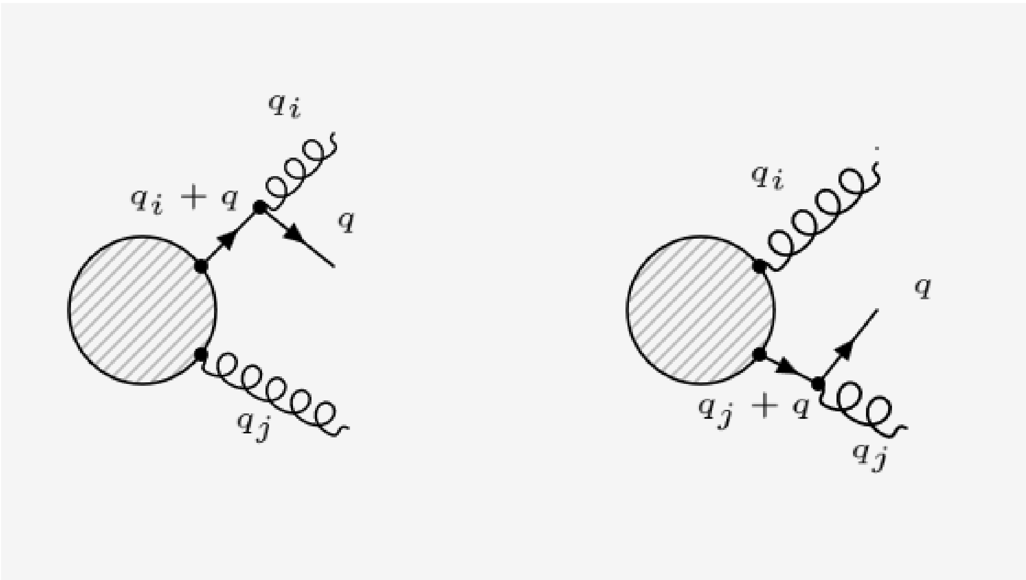
$$M_1\, M_2{}^\dagger = -\frac{g_s{}^2 [T^l]_a{}^o [T^n]_f{}^o}{4y(1-\beta_1)(1-y)\,(p_i \cdot p_k)(p_i \cdot Q)} [g^{\eta\delta}][\beta_1 \sqrt{1-y} \; \not{p}_i \; \not{Q} \; \not{p}_k] \tag{5.13}$$

5.4 $|M^2|$

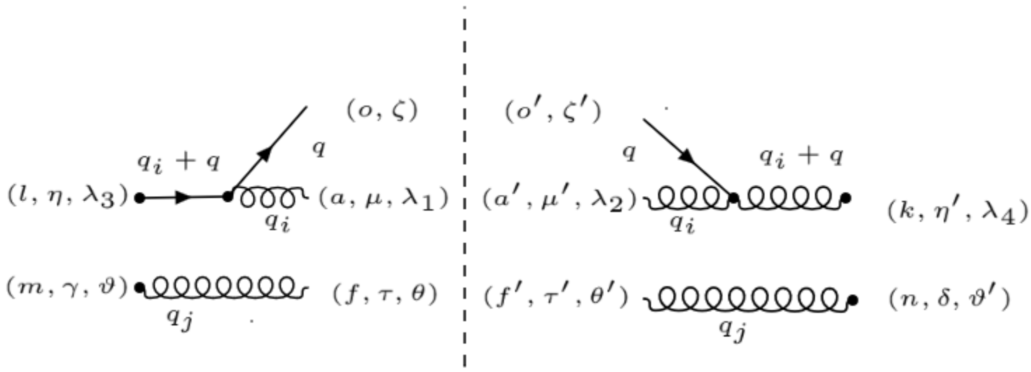
$$\begin{aligned} &|M|^2 = |M_2|^2 + |M_1|^2 + 2RE(M_1 M_2{}^\dagger) \\ &-\frac{g_s{}^2 [T^l]_a{}^o [T^k]_{o'}{}^{a'}}{4y(p_i \cdot Q)(p_i \cdot Q)} [g^{\eta\eta'}][\sqrt{1-y}(\beta_1{}^2 - \alpha_1{}^2) \; \not{p}_i \; \not{Q} \; \not{p}_k] \\ &+ 2RE(-\frac{g_s{}^2 [T^l]_a{}^o [T^n]_f{}^o}{4y(1-\beta_1)(1-y)\,(p_i \cdot p_k)(p_i \cdot Q)} [g^{\eta\delta}][\beta_1 \sqrt{1-y} \; \not{p}_i \; \not{Q} \; \not{p}_k]) \end{aligned} \tag{5.14}$$

Chapter 6

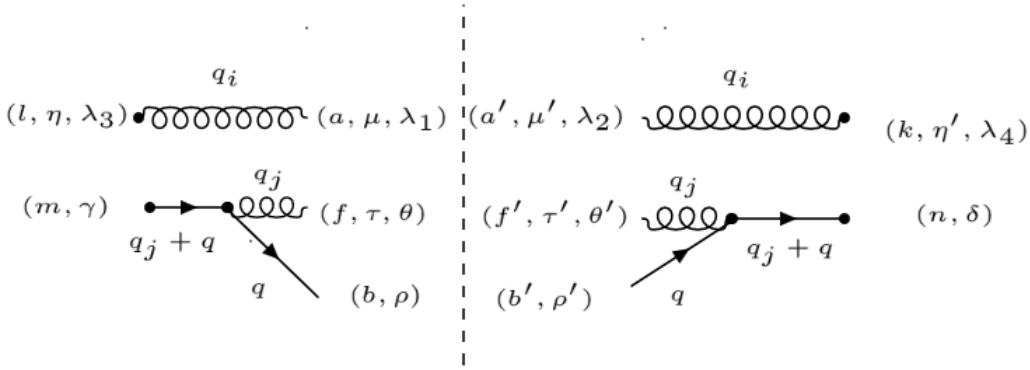
Gluon quark quark emission kernel



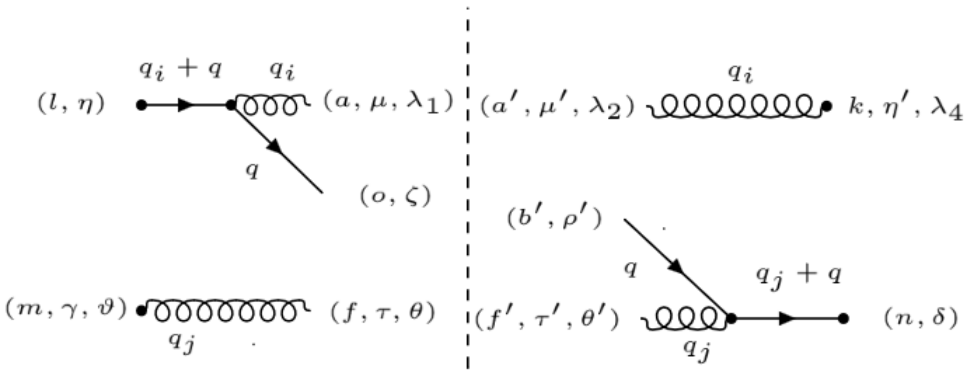
6.1 M_1


(o, ζ)
(o', ζ')
$q_i + q$
q
q
$q_i + q$
(l, η, λ_3)
(a, μ, λ_1)
(a', μ', λ_2)
(k, η', λ_4)
q_i
q_i
(m, γ, ϑ)
(f, τ, θ)
(f', τ', θ')
(n, δ, ϑ')
q_j
q_j

6.2 M_2


q_i
(l, η, λ_3)
(a, μ, λ_1)
(a', μ', λ_2)
q_i
(k, η', λ_4)
q_j
(m, γ)
(f, τ, θ)
$q_j + q$
q
(b, ρ)
(f', τ', θ')
q_j
(n, δ)
$q_j + q$
(b', ρ')
q

6.3 $M1M_2{}^\dagger$



Bibliography

Appendix A

MATHEMATICAL TOOLS

Lorenz transformation of momenta $\hat{p_i}^\mu, \hat{p_k}^\mu$ and $\hat{Q}^\mu$

$$\hat{p_i}^\mu = \alpha\Lambda^\mu{}_\nu p_i{}^\nu = p_i{}^\mu p_{i\nu} p_i{}^\nu \frac{-y^2 Q^2}{4(p_i \cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + p_i{}^\mu Q_\nu p_i{}^\nu \frac{y(1+\sqrt{1-y})}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}$$
$$+ Q^\mu p_{i\nu} p_i{}^\nu \frac{(y^2-y-y\sqrt{1-y})}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^\mu{}_\nu p_i{}^\nu$$

$$\hat{p_i}^\mu = p_i{}^\mu (Q \cdot p_i) \frac{y(1+\sqrt{1-y})}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y} p_i{}^\mu$$
$$= p_i{}^\mu [\frac{y(1+\sqrt{1-y})}{(2+2\sqrt{1-y}-y)} + \sqrt{1-y}] = p_i{}^\mu$$

$$\boxed{\hat{p_i}^\mu = \alpha\Lambda^\mu{}_\nu p_i{}^\nu = p_i{}^\mu} \tag{6.1}$$

$$\hat{p_k}^\mu = \alpha\Lambda^\mu{}_\nu p_k{}^\nu = p_i{}^\mu [\frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]$$
$$+ Q^\mu [\frac{(y^2-y-y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y} p_k{}^\mu$$

$$\hat{p_k}^\mu = \alpha\Lambda^\mu{}_\nu p_k{}^\nu = p_i{}^\mu [\frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]$$
$$+ Q^\mu [\frac{(y^2-y-y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y} p_k{}^\mu$$

with

$$A_1 \equiv \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}$$
$$A_2 \equiv \frac{(y^2-y-y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}$$

$$\boxed{\hat{p_k}^\mu = A_1\, p_i{}^\mu + A_2\, Q^\mu + \sqrt{1-y}p_k{}^\mu} \tag{6.2}$$

$$\hat{Q}^\mu = \alpha\Lambda^\mu{}_\nu Q^\nu = p_i{}^\mu[\frac{-y^2Q^2(p_i\cdot Q)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})Q^2}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]$$
$$+ Q^\mu[\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot Q)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}Q^\mu$$

with

$$S_1 \equiv \frac{Q^2}{2p_i\cdot Q}[\frac{-y^2}{2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})}{(1+\sqrt{1-y}-\frac{y}{2})}] = \frac{Q^2}{2p_i\cdot Q}y$$
$$S_2 \equiv \frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y} = 1-y$$

$$\boxed{\hat{Q}^\mu = \tfrac{Q^2}{2p_i\cdot Q}y\, p_i{}^\mu + (1-y)\, Q^\mu} \tag{6.3}$$

The often occurring pre-factor products

$$\begin{aligned}
\zeta_1\zeta_1 &= (\alpha_1{}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2\beta_1{}^2(\frac{Q^2}{2p_i\cdot Q})^2)\\
\zeta_1\lambda_1 &= (y\alpha_1\beta_1 - y^2\beta_1{}^2(\frac{Q^2}{2p_i\cdot Q}))\\
\zeta_1\zeta_q &= (\alpha_1\beta_1 - y(\alpha_1{}^2+\beta_1{}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2)\\
\zeta_1\lambda_q &= (y\alpha_1{}^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i\cdot Q}))\\
\zeta_q\zeta_q &= (\beta_1{}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1{}^2(\frac{Q^2}{2p_i\cdot Q})^2)\\
\zeta_q\lambda_1 &= (y\beta_1{}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))\\
\zeta_q\lambda_q &= (y\beta_1\alpha_1 - y^2\alpha_1{}^2(\frac{Q^2}{2p_i\cdot Q}))\\
\lambda_1\lambda_1 &= y^2\beta_1{}^2 \qquad \lambda_1\lambda_q = y^2\beta_1\alpha_1 \qquad \lambda_q\lambda_q = y^2\alpha_1{}^2
\end{aligned} \tag{6.4}$$

Common scalar products

$$
\begin{aligned}
k_1 \cdot q_i &= (\zeta_1 \lambda_q + \lambda_1 \zeta_q) p_i \cdot Q + \lambda_1 \lambda_q Q^2 - y \alpha_1 \beta_1 n^2{}_{\perp,1} \\
&= [(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))y\alpha_1 + y\beta_1(\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i \cdot Q}))]\, p_i \cdot Q \\
&\quad y^2 \beta_1 \alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_i Q \\
\Rightarrow k_1 \cdot q_i &= [y{\alpha_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y{\beta_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})]\, p_i \cdot Q \\
&\quad y^2 \beta_1 \alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_i Q
\end{aligned}
\tag{6.5}
$$

$$
\boxed{k_1 \cdot q_i = y(\alpha_1 + \beta_1)^2\, p_i \cdot Q = y\, p_i \cdot Q}
\tag{6.6}
$$

$$
\begin{aligned}
k_1 \cdot q_k &= (\zeta_1 A_2 + \lambda_1 A_1) p_i \cdot Q + \zeta_1 \sqrt{1-y}\, p_i \cdot p_k + \lambda_1 A_2\, Q^2 + \lambda_1 \sqrt{1-y}\, Q \cdot p_k \\
&+ \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1} \\
&= \{[(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})}] \\
&+ y\beta_1[\frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2(1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})}]\}\, p_i \cdot Q \\
&+ (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\sqrt{1-y}\, p_i \cdot p_k + y\beta_1 \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} Q^2 \\
&+ y\beta_1\sqrt{1-y} Q \cdot p_k + \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned}
\tag{6.7}
$$

$$
\begin{aligned}
k_1 \cdot q_k &= \alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})}(p_i \cdot p_k) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})\frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})}(p_i \cdot p_k) \\
&+ y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})}(p_i \cdot p_k) + y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})}\, Q \cdot p_k \\
&+ \alpha_1 \sqrt{1-y}\, p_i \cdot p_k - y\beta_1(\frac{Q^2}{2p_i \cdot Q})\sqrt{1-y}\, p_i \cdot p_k \\
&+ y\beta_1(\frac{Q^2}{2p_i \cdot Q})\frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})}(p_i \cdot p_k) + y\beta_1\sqrt{1-y}(Q \cdot p_k) \\
&+ \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned}
\tag{6.8}
$$

$$
\begin{aligned}
k_1 \cdot q_k &= [\alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \alpha_1 \sqrt{1-y} \\
&- y\beta_1(\frac{Q^2}{2p_i \cdot Q})\sqrt{1-y}]\, p_i \cdot p_k + [y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1\sqrt{1-y}](Q \cdot p_k) \\
&+ \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned}
\tag{6.9}
$$

$$\begin{aligned} k_1\cdot q_k = &\{\alpha_1[\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+\sqrt{1-y}] \\ &+y\beta_1(\frac{Q^2}{p_i\cdot Q})[\frac{-y^2}{4(1+\sqrt{1-y}-\frac{y}{2})}-\sqrt{1-y}]\}\,p_i\cdot p_k \\ &+y\beta_1[\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+\sqrt{1-y}](Q\cdot p_k) \\ &+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1} \end{aligned} \tag{6.10}$$

$$\boxed{k_1\cdot q_k = [\alpha_1(1-y)+y\beta_1(\tfrac{Q^2}{2p_i\cdot Q})]\,p_i\cdot p_k + y\beta_1\,Q\cdot p_k+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}} \tag{6.11}$$

$$\begin{aligned} q_i\cdot q_k =& (\zeta_q A_2+\lambda_q A_1)p_i\cdot Q+\zeta_q\sqrt{1-y}\,p_i\cdot p_k+\lambda_q A_2\,Q^2+\lambda_q\sqrt{1-y}\,Q\cdot p_k \\ &-\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1} \\ =&\{[(\beta_1-y\alpha_1(\frac{Q^2}{2p_i\cdot Q}))\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\ &+y\alpha_1[\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})}+\frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]\}\,p_i\cdot Q \\ &+(\beta_1-y\alpha_1(\frac{Q^2}{2p_i\cdot Q}))\sqrt{1-y}\,p_i\cdot p_k+y\alpha_1\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}Q^2 \\ &+y\alpha_1\sqrt{1-y}Q\cdot p_k-\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1} \end{aligned} \tag{6.12}$$

$$\begin{aligned} q_i\cdot q_k =& \beta_1\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)-y\alpha_1(\frac{Q^2}{2p_i\cdot Q})\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k) \\ &+y\alpha_1\frac{-y^2Q^2}{4(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)+y\alpha_1\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}\,Q\cdot p_k \\ &+\beta_1\sqrt{1-y}\,p_i\cdot p_k-y\alpha_1(\frac{Q^2}{2p_i\cdot Q})\sqrt{1-y}\,p_i\cdot p_k \\ &+y\alpha_1(\frac{Q^2}{2p_i\cdot Q})\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)+y\alpha_1\sqrt{1-y}(Q\cdot p_k) \\ &-\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1} \end{aligned} \tag{6.13}$$

$$\begin{aligned} q_i\cdot q_k =& [\beta_1\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+y\alpha_1\frac{-y^2Q^2}{4(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}+\beta_1\sqrt{1-y} \\ &-y\alpha_1(\frac{Q^2}{2p_i\cdot Q})\sqrt{1-y}]\,p_i\cdot p_k+[y\alpha_1\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+y\alpha_1\sqrt{1-y}](Q\cdot p_k) \\ &-\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1} \end{aligned} \tag{6.14}$$

$$
\begin{aligned}
k_1 \cdot q_k = \{ & \beta_1 [\frac{(y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}] \\
& + y\alpha_1 (\frac{Q^2}{p_i \cdot Q})[\frac{-y^2}{4(1+\sqrt{1-y}-\frac{y}{2})} - \sqrt{1-y}]\}\, p_i \cdot p_k \\
& + y\alpha_1 [\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}](Q \cdot p_k) \\
& - \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned}
\tag{6.15}
$$

$$
\boxed{q_i \cdot q_k = [\beta_1 (1-y) + y\alpha_1 (\tfrac{Q^2}{2p_i \cdot Q})]\, p_i \cdot p_k + y\alpha_1\, Q \cdot p_k - \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}}
\tag{6.16}
$$

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