Constructing emission kernels

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What are emission kernels?

- MC event generators - parton shower
- To describe probability of propagating quark/gluon splitting into another quark/gluon

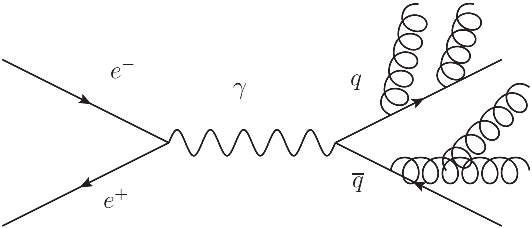


Diagram of an e^+e^- collision and possible parton shower

How do we use them?

- Parton shower algorithm[1] (e.g. Sudakov veto algorithm) with starting scale Q, scale of next emission q, distribution dS_P:

$$\frac{dS_P(\mu, q|Q)}{dq} = \Delta_P(\mu|Q)\delta(q-\mu) + \theta(Q-q)\theta(q-\mu)P(q)\Delta_P(q|Q)$$

- Sudakov form factor: $\Delta_P(q|Q) = exp\Big(-\int_q^Q dk P(k)\Big)$
- Overestimate the emission kernel $R(q) \geq P(q)$
- Evolve to cut-off scale μ

Emission kernels are an input

[1] Plätzer and Sjödahl 2012.

Determination of emission kernels

- Need to take collinear and soft limits which allow factorisation[2]
- Where q_k is momentum of final state parton k:

  a) Soft limit $q_k = \lambda q$, $\lambda \to 0$, $|\mathcal{M}_{m+1,a\ldots}|^2 \propto 1/\lambda^2$
  b) Collinear limit $q_k \to (1-z)q_i/z$,
  $|\mathcal{M}_{m+1,a\ldots}|^2 \propto 1/q_i.q_k$

Dipole factorisation

- Consider $(m+1)$ partons, factorise out parton k to give $|\mathcal{M}_{m,a\ldots}|^2$

$$|\mathcal{M}_{m+1,a\ldots}|^2 \to |\mathcal{M}_{m,a\ldots}|^2 \otimes V_{ik,j}$$

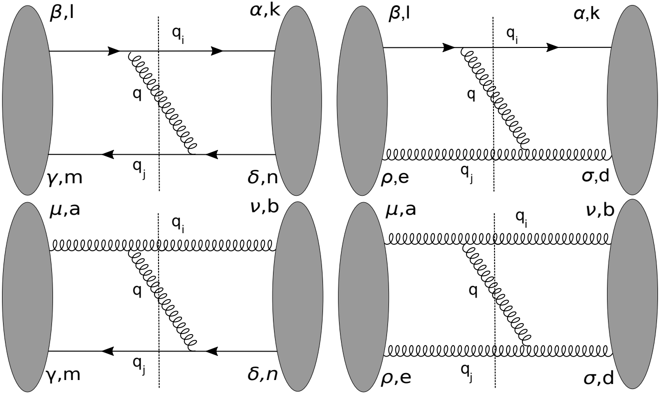
- $V_{ik,j}$ = singular factor including parton k and it's interaction with partons i and j from the m parton amplitude

Non algorithmic method

[2] Catani and Seymour 1997.

Four different dipoles

- Partons i and k are emitters and j is a spectator ($q = q_k$)



Example of 4 possible dipoles: $P_{qq}, P_{qg}, P_{gq}, P_{gg}$

Define emitter and spectator

Emitter p_i and spectator p_j:

$$
\begin{aligned}
p_i^\mu &= q_i^\mu + q^\mu - \frac{y}{1-y} q_j^\mu, \\
p_j^\mu &= \frac{1}{1-y} q_j^\mu, \\
y &= \frac{q_i.q}{q_i.q + q.q_j + q_i.q_j}, \\
z &= \frac{q_i.q_j}{q.q_j + q_i.q_j}.
\end{aligned}
$$

- Mapping 3 partons to 2
- $V_{ik,j}$ depends on y and z
- Momentum conservation: $q_i^\mu + q^\mu + q_j^\mu = p_i^\mu + p_j^\mu$

Altarelli-Parisi splitting functions[3]

- Averaging over polarisations of parton $a \to \langle \hat{P}_{ab} \rangle$

$$\langle \hat{P}_{qq} \rangle = C_F \left[\frac{1+z^2}{1-z} - \epsilon(1-z) \right], \quad \langle \hat{P}_{qg} \rangle = C_F \left[\frac{1+(1-z)^2}{z} - \epsilon z \right],$$
$$\langle \hat{P}_{gq} \rangle = T_R \left[1 - \frac{2z(1-z)}{1-\epsilon} \right], \qquad \langle \hat{P}_{gg} \rangle = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right].$$

- Assume collinear limit, $p_j^\mu \to q_j^\mu, p_i^\mu \to p^\mu$
- Soft singularities for $z \to 0, 1$
- $V_{ik,j} \to 8\pi\mu^{2\epsilon}\alpha_s \hat{P}_{ij}$

[3] Altarelli and Parisi 1977.

Is there a better way to do this?

Problems with current parton showers[4]:

- Catani Seymour method only works for single emission
- No singularity structure for NNLO matching
- Jet substructure studies, full momentum range
- Systematic errors that are becoming limiting

Possible solutions:

- Design new parton shower
- Include more spin and colour interferences
- Use higher-order emission kernels
- More systematic algorithmic method

[4] Dasgupta et al. 2018.

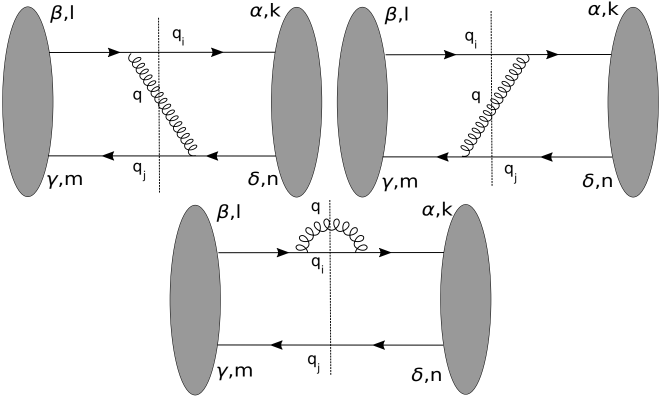
New mapping

Use different mapping without taking explicit limits:

$$
\begin{aligned}
q_i^\mu &= z p_i^\mu + y(1-z) p_j^\mu + \sqrt{zy(1-z)}\, m_\perp, \\
q^\mu &= (1-z) p_i^\mu + yz p_j^\mu - \sqrt{zy(1-z)}\, m_\perp, \\
q_j^\mu &= (1-y) p_j^\mu, \\
y &= \frac{q_i . q}{p_i . p_j}.
\end{aligned}
$$

- Includes soft limit ($z \to 1$) and collinear limit ($y \to 0$)
- $m_\perp$ represents the transverse component
- Even this is not perfect (WIP)...

Example: $q\overline{q}$ emission kernel



Diagrams needed to calculate the $P_{q\overline{q}}$ emission kernel

- Have to consider tree level splittings and self energies

Example: First diagram of $q\overline{q}$

Before mapping:

$$|M_{(i)}|^2 = \frac{-g_s^2(t_i^c)_l^k(t_j^c)_n^m}{4(q_i.q)(q_j.q)}[(\not{q}_i)\gamma^\lambda(\not{q}_i+\not{q})]_{\alpha\beta}[(\not{q}_j+\not{q})\gamma_\lambda(\not{q}_j)]_{\gamma\delta}$$

After mapping:

$$|M_{(i)}|^2 = \frac{g_s^2(t_i^c)_l^k(t_j^c)_n^m}{2(q_i.q)}\frac{2z}{1-z}[\not{p}_i]_{\alpha\beta}[-\not{p}_j]_{\gamma\delta}$$

- Can ignore finite terms, $y(1-z)$, and momenta are on-shell.
- Need to combine with two other diagrams to give emission kernel

Summary

What I've found so far:

- Can replicate the Catani Seymour results in collinear limit
- Also have additional non-leading terms
- Some implementation in Mathematica

What's next?

- Still need to modify mapping, recoil correction for multiple emissions
- Aim is to go to 2-emission case
- Develop diagrammatic algorithmic procedure
- Implementation in Herwig parton shower

Backup slides

Collinear limit

- Use mapping to collinear momentum:

$$q_i^\mu = z p_i^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2pn},$$

$$q^\mu = (1-z) p_i^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2pn},$$

$$2q_i.q = -\frac{k_\perp^2}{z(1-z)}.$$

- p_i^μ is the colllinear momentum
- $k_\perp$ is the transverse componenent. In the collinear limit $k_\perp \to 0$

Equations for MC implementation

Structure function

Sudakov form factor

$$\Delta_i(t) = exp[-\int \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}_{ji}(z)]$$

$$f(x,t) = \Delta(t) f(x,t_0) + \int \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z) f(x/z, t')$$

- Where $f(x, t_0)$ is the initial parton distribution
- Sudakov form factor is probability of evolving from t_0 to t without branching.

MC algorithm

Algorithm steps

- Solve $\frac{\Delta(t_2)}{\Delta(t_1)}(ISR) or \frac{\Delta(t_1)}{\Delta(t_2)}(FSR) = \mathcal{R}$ where $\mathcal{R}$ is a random number between 0 and 1
- t is evolution variable that can be chosen e.g. $p_\perp$
- Check that $t_2 <$ hard subprocess scale(ISR) or IR cut off (FSR), generate momentum fraction $z = x_2/x_1$ with probability distribution proportional to $(\alpha_s/2\pi)P(z)$
- Repeat until cut-off scale is reached

Sudakov Veto Algorithm

Algorithm steps

1. Use $S_R(\mu, x_\mu | q, x | k)$ to generate trial splitting scale and variables
2. Define cut-off scale μ, if $q = \mu$ then there is no emission
3. Accept trial scale with probability $P(q, x)/R(q, x)$ or repeat process with $k = q$
4. Repeat until cut-off scale is reached

DGLAP equation

Definition

$$t\frac{\partial}{\partial t}q(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{d\xi}{\xi}P(\frac{x}{\xi})q(\xi,t)$$

- Where $P = P_{qq}^{(0)}$ which is the first term of P_{qq} when perturbatively expanded in the running coupling

References I

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- QCD and Collider Physics, Ellis, Stirling and Webber. (p.170)