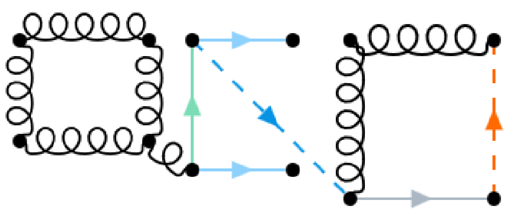
THESIS

BY

TIGRAN SAIDNIA

Emission kernel of parton shower

Emission kernel of parton shower



Karlsruhe institute for Technology (KIT)

Institute of theoretical physics

Referents: PD Dr. Stefan Gieseke

Dr. Simon Plätzer

Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, 21. Januar 2019

Tigran Saidnia

Inhaltsverzeichnis

0.1 parametrisation

$$\left.\begin{aligned}
{q_i}^\mu &= z{p_i}^\mu + y(1-z){p_j}^\mu + \sqrt{zy(1-z)}m_\perp \\
q^\mu &= (1-z){p_i}^\mu + yz{p_j}^\mu - \sqrt{zy(1-z)}m_\perp \\
{q_j}^\mu &= (1-y){p_j}^\mu \\
y &= \frac{q_i q}{p_i p_j} \\
q_i + q &= p_i + yp_j \\
q_j + q &= (1-z){p_i}^\mu + (1+yz-y){p_j}^\mu - \sqrt{zy(1-z)}m_\perp \\
q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
\end{aligned}\right\} \text{parametrisation} \tag{1}$$

0.2 Altarelli-Parisi splitting functions

$$\left.\begin{aligned}
\langle \hat{P_{qq}} \rangle &= C_F[\frac{1+z^2}{1-z} - \varepsilon(1-z)] \\
\langle \hat{P_{gq}} \rangle &= T_R[1 - \frac{2z(1-z)}{1-\varepsilon}] \\
\langle \hat{P_{qg}} \rangle &= C_F[\frac{1+(1-z)^2}{z} - \varepsilon z] \\
\langle \hat{P_{gg}} \rangle &= 2C_A[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)]
\end{aligned}\right\} \text{splitting functions} \tag{2}$$

0.3 Colour factor calculation

fundamental representation in $SU(2)$ and $SU(3)$

$$\begin{aligned} T^a = \tau^a \equiv \frac{\sigma^2}{2} \qquad &\textit{with Pauli matrices } \sigma^a \\ T^a = \vartheta^a \equiv \frac{\lambda^2}{2} \qquad &\textit{with Gell}-\textit{Mann matrices } \lambda^a \end{aligned} \tag{3}$$

$$\begin{aligned} \lambda^1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix} \\ \lambda^5 = \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned} \tag{4}$$

As we can see, λ^3 and λ^8 are diagonal. These generators satisfy:

$$[T^a, T^b] = i\epsilon^{abc}T^c \tag{5}$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \tag{6}$$

The main relation we will use later for SU(N):

$$tr(T^a T^b) = T_{ij}{}^a T_{ji}{}^b = T_F \delta^{ab} \tag{7}$$

$$\sum_a (T^a T^a) = C_F \delta^{ij} \tag{8}$$

$$f^{acd} f^{bcd} = C_A \delta^{ab} \tag{9}$$

With $T_F = \frac{1}{2}$, $C_A = N$ and $C_F = \frac{N^2-1}{2N}$.

$$f^{abc} = -2itr(T^a[T^b, T^c]) \tag{10}$$

$$d^{abc} = 2tr(T^a T^b, T^c) \tag{11}$$

$$T^a T^b = \frac{1}{2}(\frac{1}{N}\delta_{ab} + (d^{abc} + if^{abc})T^c) \tag{12}$$

$$tr(T^a T^b T^c) = \frac{1}{4}(d^{abc} + if^{abc}) \tag{13}$$

$$tr(T^a T^b T^a T^c) = \frac{-1}{4N}\delta_{bc} \tag{14}$$

$$f^{acd}f^{bcd} = N\delta^{ab} \tag{15}$$

$$f^{acd}d^{bcd} = 0 \tag{16}$$

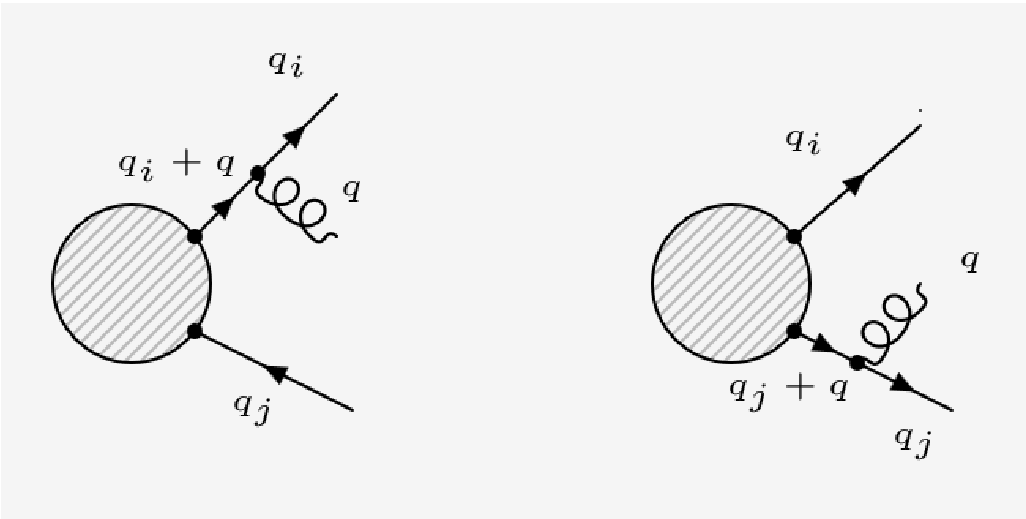
$$f^{ade}f^{bef}f^{cfd} = \frac{N}{2}f^{abc} \tag{17}$$

Fierz identity:

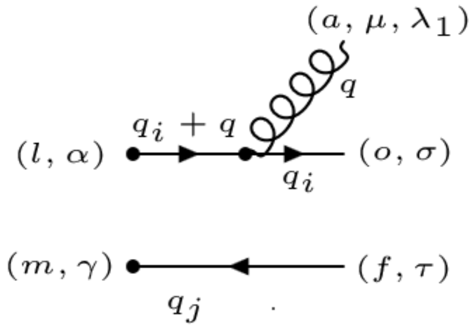
$$\sum_a T_{ij}{}^a T_{kl}{}^a = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}) \tag{18}$$

Kapitel 1

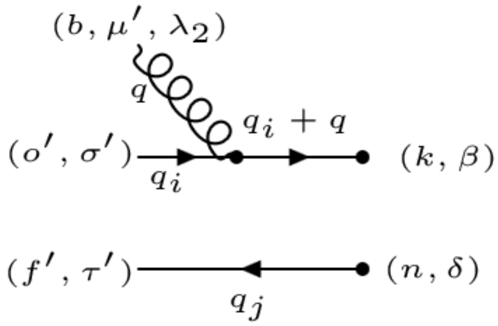
Quark antiquark gluon emission kernel



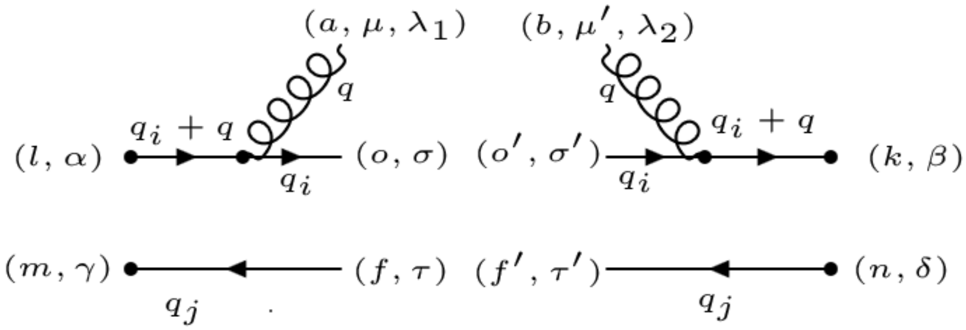
1.1 qg-$\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{1.1}$$



$$M_1{}^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{1.2}$$



$$|M_1|^2 = M_1\, M_1{}^\dagger = [\bar{u}_\sigma(q_i)\, (-ig_s\gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\; \varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\, (ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)][\bar{v}_{\tau'}(q_j)] \tag{1.3}$$

$$|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\; (ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\, \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2 *}{}_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q)$$
$$\times(-ig_s\gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j)v_\tau(q_j)] \tag{1.4}$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i) = \not{q}_i \delta^{oo'},$$
$$\sum_{\tau,\tau'} \bar{v}_\tau(q_j)\, v_{\tau'}(q_j) = \not{q}_j \delta^{ff'} \tag{1.5}$$

and sum over polarization index (λ_1, λ_2) :

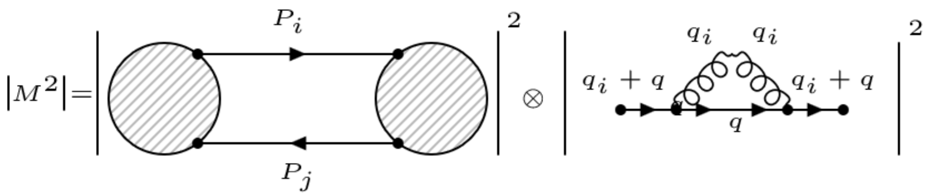
$$\sum_{\mu,\mu'} \varepsilon^{\lambda_2 *}{}_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q) = -g_{\mu\mu'}\delta^{ab} \tag{1.6}$$

$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\; \not{q}_i\, g_{\mu'\mu}\gamma^\mu\, (\not{q}_i + q)][\not{q}_j] \tag{1.7}$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\; \not{q}_i\, \gamma_{\mu'}\, (\not{q}_i + q)][\not{q}_j] \tag{1.8}$$

In other words we expect the tree level diagram from LO and a number: Which means:



contribution from LO *a complex number*

$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[\not{P}_i][\not{P}_j] \; \otimes \; \textit{(a complex number)} \tag{1.9}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'} \; \not{q}_i \, \gamma_{\mu'} &= q_{i\sigma} \, \gamma^{\mu'}\gamma^{\sigma} \; \gamma_{\mu'} \\ &= q_{i\sigma} \, (\{\gamma^{\mu'}, \gamma^{\sigma}\} \; - \gamma^{\sigma}\gamma^{\mu'})\gamma_{\mu'} \\ &= q_{i\sigma} \, 2g^{\mu'\sigma} \, \gamma_{\mu'} \; - \; d \, \gamma^{\sigma} \\ &= (2-d) \; \not{q}_i \end{aligned} \tag{1.10}$$

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q}) \quad \not{q}_i \; (\not{q}_i + q)][\not{q}_j] \tag{1.11}$$

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[\not{q}_i \; \not{q}_i \; \not{q}_i + \; \not{q}_i \; \not{q}_i \; \not{q} + \; \not{q} \; \not{q}_i \; \not{q}_i + \; \not{q} \; \not{q}_i \; \not{q}][\not{q}_j] \tag{1.12}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \; \not{q}_i &= {q_i}^2 = {m_i}^2 \\ \not{q} \; \not{q} &= q^2 = m^2 \\ \not{q}_j \; \not{q}_j &= {q_j}^2 = {m_j}^2 \end{aligned} \tag{1.13}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \; \not{q}_i$ and $\not{q} \; \not{q}$ as well.

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(2q_i q)(2q_i q)}[\not{q} \; \not{q}_i \; \not{q}][\not{q}_j] \tag{1.14}$$

$$
\begin{aligned}
L = \not{q}\ \not{q}_i\ \not{q} = \not{q}[q_{i\sigma} q_\mu\left(\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma\gamma^\mu\right)] \\
\not{q}[2{q_i}^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu\gamma^\mu\gamma^\sigma] \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\frac{\gamma^\mu\gamma^\mu}{2} + \frac{\gamma^\mu\gamma^\mu}{2}]\gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}]\gamma^\sigma \\
= \not{q}(2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
= \not{q}(2q_i q) - q^2\ \not{q}_i \\
= \not{q}(2q_i q)
\end{aligned}
\tag{1.15}
$$

After inserting the last result of L and simplify the term $(2q_i q)$ from the denominator and nominator because , we get:

$$
|M_1|^2 = -(2-d)\,\frac{g_s^2 [T^a]_o{}^k\,[T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)}[\not{q}][\not{q}_j]
\tag{1.16}
$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$
|M_1|^2 = (d-2)\,\frac{g_s^2 [T^a]_o{}^k\,[T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)}[(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][(1-y)\ \not{p}_j]
\tag{1.17}
$$

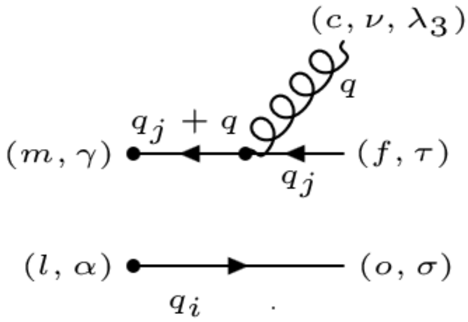
Multiplying the both sides

$$
\begin{aligned}
|M_1|^2 = (d-2)\,\frac{g_s^2 [T^a]_o{}^k\,[T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)}[(1-z)(1-y)\ \not{p}_i\ \not{p}_j \\
+zy(1-y)\ \not{p}_j\ \not{p}_j + (1-y)\sqrt{zy(1-z)}\ \not{m}_\perp\ \not{p}_j]
\end{aligned}
\tag{1.18}
$$

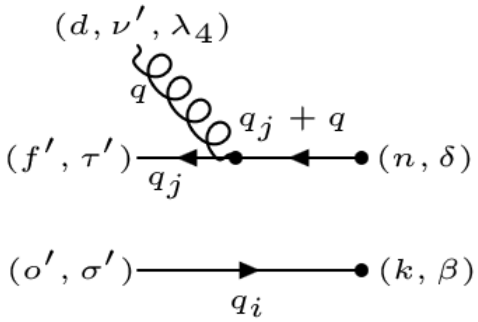
and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\ \not{p}_i$ and $\not{p}_j\ \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$
|M_1|^2 = (d-2)(1-z)(1-y)\,\frac{g_s^2 [T^a]_o{}^k\,[T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j]
\tag{1.19}
$$

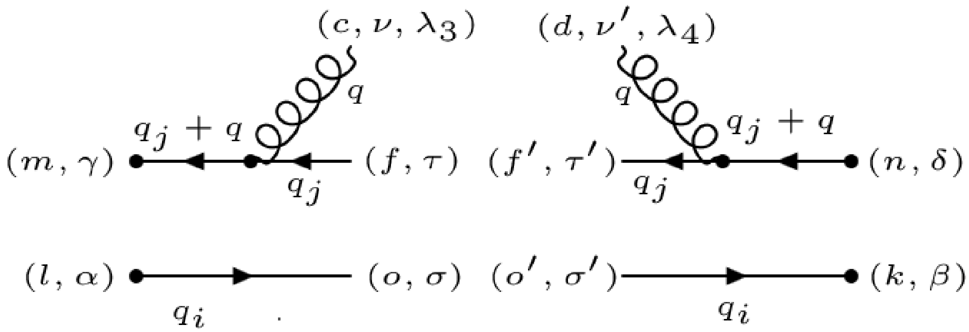
1.2 $\bar{q}$g-q



$$M_2 = [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^\nu \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \tag{1.20}$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \tag{1.21}$$



$$\begin{aligned} |M_2|^2 = M_2\, M_2{}^\dagger = [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^\nu \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \\ [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \end{aligned} \tag{1.22}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu\, v_\tau(q_j)\bar{v}_{\tau'}(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_j + \not{q})] \\ [u_\sigma(q_i)]\,[\bar{u}_{\sigma'}(q_i)] \tag{1.23}$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\begin{aligned} \sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i) &= \not{q}_i \delta^{oo'}, \\ \sum_{\tau,\tau'} \bar{v}_\tau(q_j)\, v_{\tau'}(q_j) &= \not{q}_j \delta^{ff'} \end{aligned} \tag{1.24}$$

and sum over polarization index (λ_3, λ_4) :

$$\sum_{\nu,\nu'} \varepsilon^{\lambda_4}{}^*_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'}\delta^{cd} \tag{1.25}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu\ \ \not{q}_j\, (-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_j + \not{q})]\,[\not{q}_i] \tag{1.26}$$

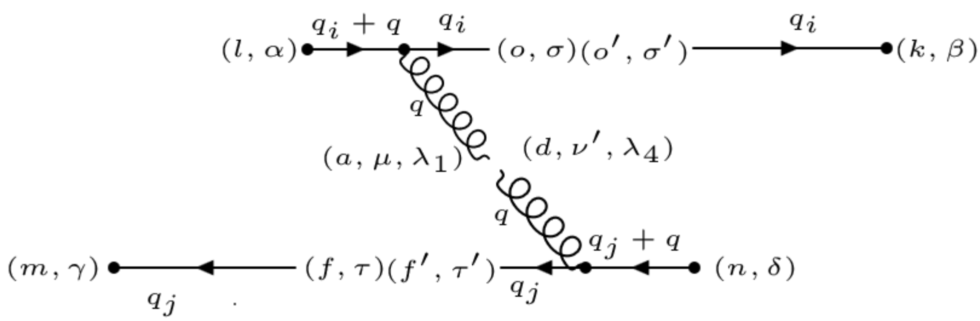
After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{(2qq_j)}[\not{q}]\,[\not{q}_i] \tag{1.27}$$

finally:

$$|M_2|^2 = -(d-2)yz^2\, \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2(1-z)(1-y)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.28}$$

1.3 $M_1 M_2^\dagger$



$$M_1\, M_2{}^\dagger = [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i+\not{q})}{(q_i+q)^2}\,\varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\bar{v}_{\tau'}(q_j)\,(ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\,\frac{-i(\not{q}_j+\not{q})}{(q_j+q)^2}\,\varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[u_{\sigma'}(q_i)] \tag{1.29}$$

$$M_1\, M_2{}^\dagger = \frac{g_s^2[T^a]_o{}^l\,[T^d]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i+\not{q})\,]\varepsilon^{\lambda_1}{}_\mu(q)\,\varepsilon^{\lambda_4}{}_{\nu'}(q)$$
$$[\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j+\not{q})] \tag{1.30}$$

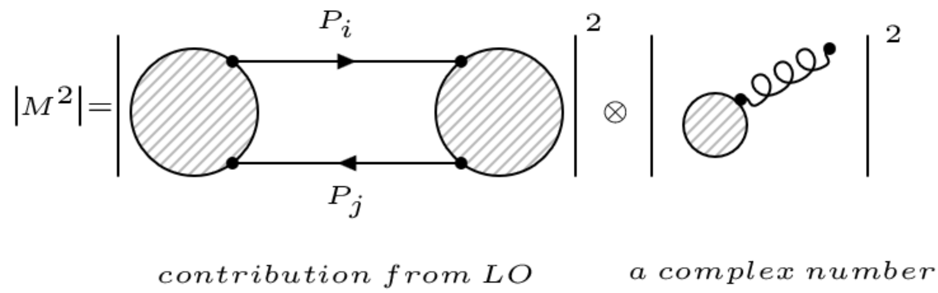
$$M_1\, M_2{}^\dagger = \frac{g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i+\not{q})\,] - g_{\mu\nu'}$$
$$[\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j+\not{q})] \tag{1.31}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i+\not{q})\,]\,[\not{q}_j\,\gamma_\mu\,(\not{q}_j+\not{q})] \tag{1.32}$$

Expectation:

$$M_1\, M_2{}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[(\not{q}_i+\not{q})\,\gamma^\mu\,\;\not{q}_i\,]\,[(\not{q}_j+\not{q})\,\gamma_\mu\,\;\not{q}_j] \tag{1.33}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[-(\not{q}_i+\not{q})\;\;\not{q}_i\,\gamma^\mu\;+2(\not{q}_i+\not{q})\,q_i{}^\mu]$$
$$[-(\not{q}_j+\not{q})\;\;\not{q}_j\,\gamma_\mu\;+2(\not{q}_j+\not{q})q_{j\mu}] \tag{1.34}$$



$$
\begin{aligned}
M_1\,M_2{}^\dagger = {} & \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)} \\
& [(\not{q}_i + \not{q})\;\not{q}_i\,\gamma^\mu]\,[(\not{q}_j + \not{q})\;\not{q}_j\gamma_\mu] \\
& -2[(\not{q}_i + \not{q})\;\not{q}_i\,\gamma^\mu]\,[(\not{q}_j + \not{q})q_{j\mu}] \\
& -2[(\not{q}_i + \not{q})\,q_i{}^\mu][(\not{q}_j + \not{q})\;\not{q}_j\,\gamma_\mu] \\
& +4[(\not{q}_i + \not{q})\,q_i{}^\mu][(\not{q}_j + \not{q})q_{j\mu}]
\end{aligned}
\tag{1.35}
$$

$$
\begin{aligned}
M_1\,M_2{}^\dagger = {} & \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)} \\
& [\not{q}\;\not{q}_i\,\gamma^\mu]\,[\not{q}\;\not{q}_j\gamma_\mu] \\
& -2[\not{q}\;\not{q}_i\,\gamma^\mu]\,[(\not{q} + \not{q}_j)\;q_{j\mu}] \\
& -2[(\not{q}_i + \not{q})\,q_i{}^\mu][\not{q}\;\not{q}_j\,\gamma_\mu] \\
& +4[(\not{q}_i + \not{q})\,q_i{}^\mu][(\not{q}_j + \not{q})q_{j\mu}]
\end{aligned}
\tag{1.36}
$$

$$
\begin{aligned}
M_1\,M_2{}^\dagger = {} & \frac{-g_s^2\;[T^a]_o{}^l\,[T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\
& [y(1-2z+z^2)\;\not{p}_i\;\;\not{p}_j\,\gamma^\mu]\,[(1-z)(1-y)\;\not{p}_i\;\not{p}_j\gamma_\mu] \\
& +4(q_i{}^\mu\cdot q_{j\mu})[(\not{q}_i + \not{q})][(\not{q}_j + \not{q})]
\end{aligned}
\tag{1.37}
$$

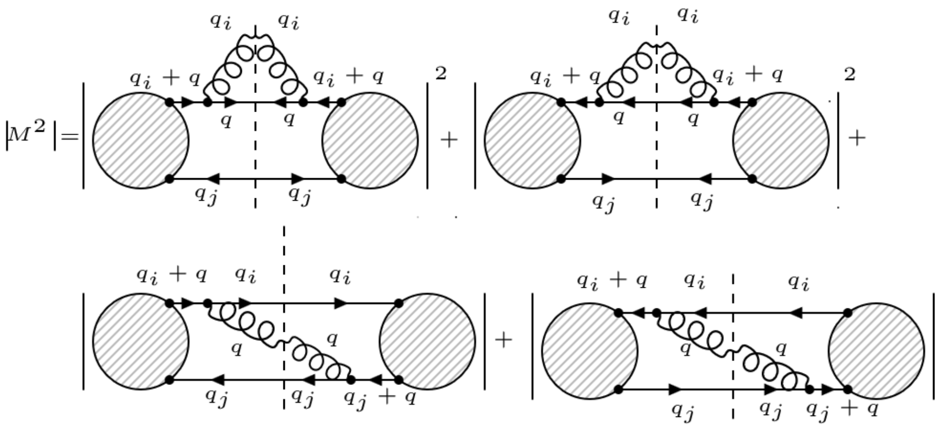
$$
\begin{aligned}
M_1\,M_2{}^\dagger = {} & \frac{-g_s^2\;[T^a]_o{}^l\,[T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\
& [y(1-2z+z^2)\;\not{p}_i\;\;\not{p}_j\,\gamma^\mu]\,[(1-z)(1-y)\;\not{p}_i\;\not{p}_j\gamma_\mu] \\
& +4(p_i\cdot p_j)[(\not{p}_i + y\;\not{p}_j)][(1-z)\;\not{p}_i + (1+yz-y)\;\not{p}_j - \sqrt{zy(1-z)}\;\not{n}]
\end{aligned}
\tag{1.38}
$$

$$
M_1\,M_2{}^\dagger = \frac{-g_s^2\;[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)} z(1-y)[\not{p}_i][\not{p}_j]
\tag{1.39}
$$

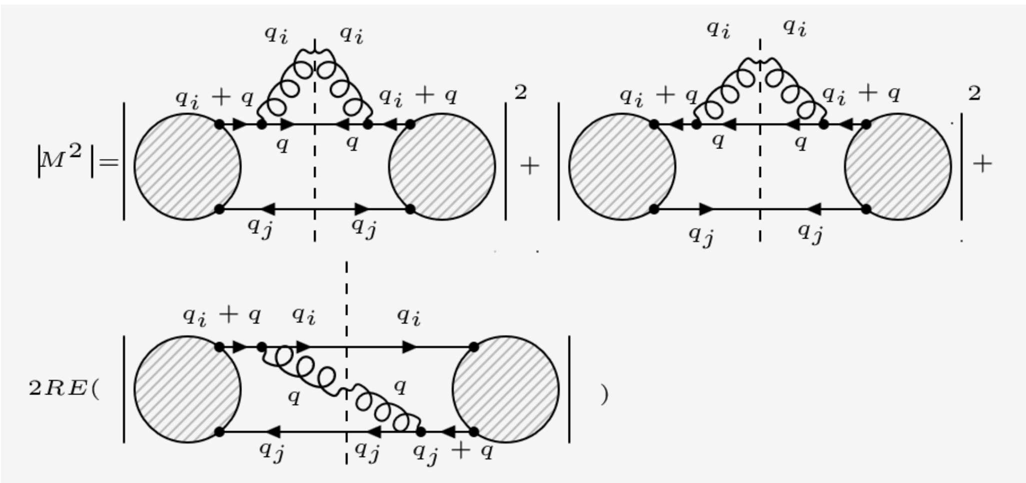
$$
M_1\,M_2{}^\dagger = \frac{-g_s^2\;[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(1-z)y(1-2z+z^2)(p_i\cdot p_j)} z[\not{p}_i][\not{p}_j]
\tag{1.40}
$$

1.4 $|M^2|$

$$| M|^2 = | M_1|^2 + | M_2|^2 + M_1 \, {M_2}^{\dagger} + {M_1}^{\dagger} M_2 \tag{1.41}$$



$$| M|^2 = | M_1|^2 + | M_2|^2 + 2RE(M_1 \, {M_2}^{\dagger}) \tag{1.42}$$



$$\begin{aligned} | M|^2 = {} & (d-2)(1-z)(1-y) \, \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & -(d-2)yz^2 \, \frac{g_s^2 \, [T^c]_f{}^m \, [T^c]_f{}^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & +2RE((\frac{-2z}{z-1}) \frac{g_s^2 \, \, [T^a]_o{}^l \, [T^a]_f{}^n}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]) \end{aligned} \tag{1.43}$$

$$T^a{}_{o\,k} \, T^a{}_{l\,o} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F \delta_{lk} \tag{1.44}$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a{}_{o\,k}\,T^a{}_{l\,o} = C_F \delta_{lk} = \frac{1}{N}\sum_{l=1}^{N} \delta_{lk} C_F = C_F \tag{1.45}$$

The same calculation for $T^c{}_{m\,f}\,T^c{}_{f\,n}$ and $T^a{}_{o\,l}\,T^a{}_{f\,n}$ turns C_F out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, wich means, if:

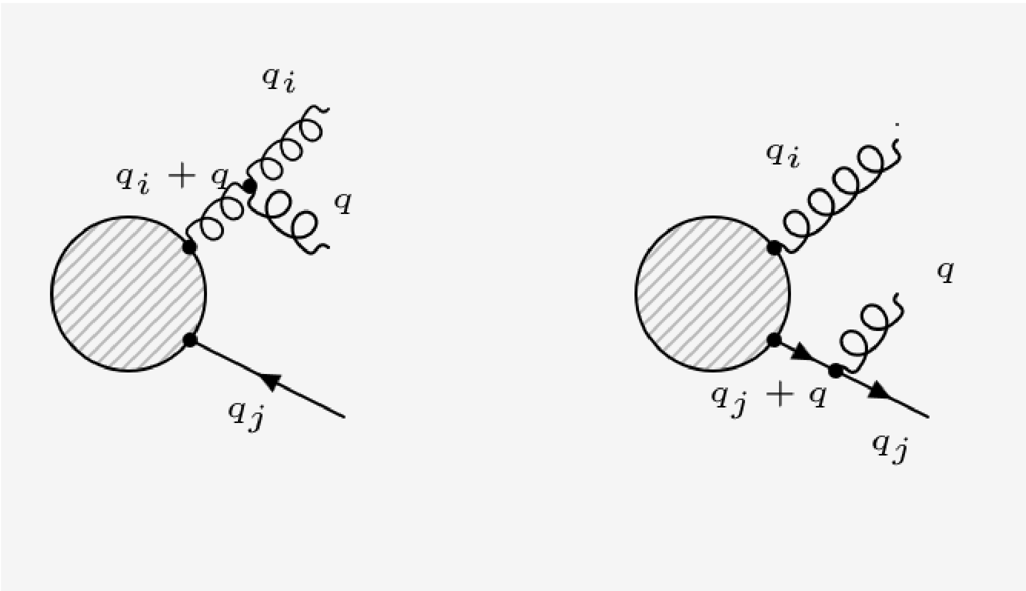
$$y \longrightarrow 0 \tag{1.46}$$

$$\begin{aligned} |\,M|^2 = (d-2)(1-z)(1-y)\,&\frac{g_s^2 C_F}{2y(1-2z+z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \\ -(d-2)yz^2\,&\frac{g_s^2\,C_F}{2(1-z)(1-y)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \\ +2RE((\frac{-2z}{z-1})&\frac{g_s^2 C_F}{2y(1-2z+z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \end{aligned} \tag{1.47}$$

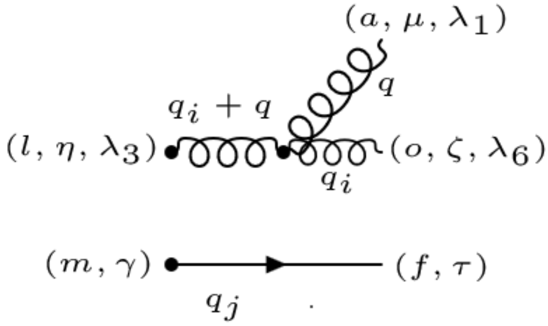
$$|\,M|^2 = C_F((d-2)(1-z) - \frac{4z}{z-1})\,\frac{g_s^2}{2y(1-2z+z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.48}$$

Kapitel 2

Gluon quark gluon emission kernel

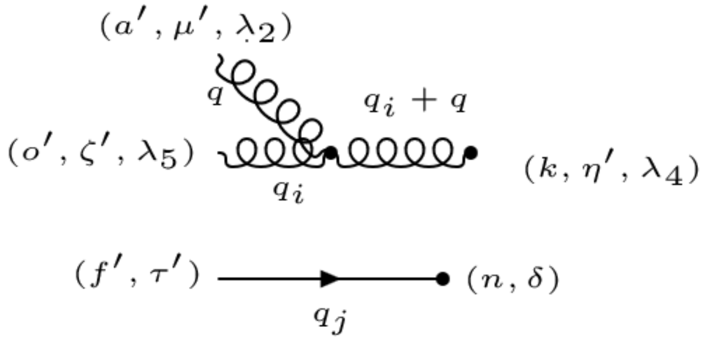


2.1 gg-q

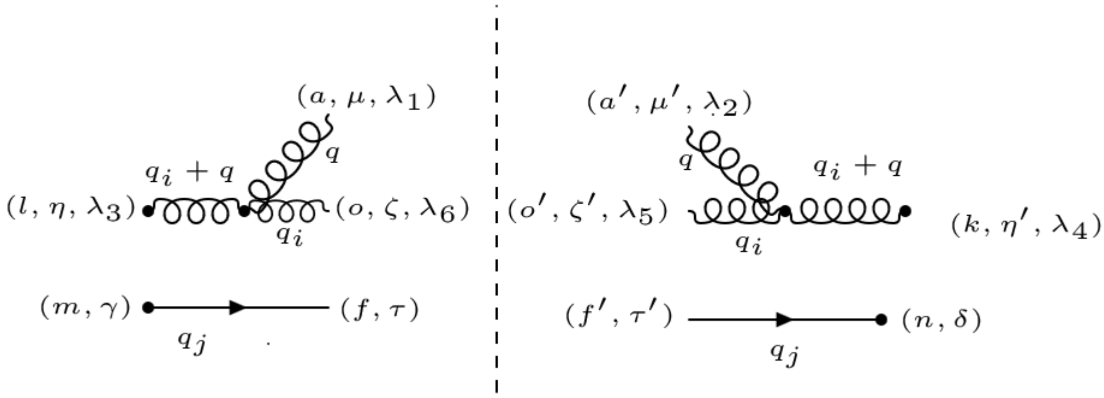


$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(-q_i+q)^\eta + g^{\zeta\eta}(-q-(q_i+q))^\mu + g^{\eta\mu}(q_i+q+q_i)^\zeta) \\ \varepsilon^{\lambda_1}{}_\mu(q)\varepsilon^{\lambda_6}{}_\zeta(q_i)][\bar{u}_\tau(q_j)] \tag{2.1}$$

$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\ \varepsilon^{\lambda_1}{}_\mu(q)\varepsilon^{\lambda_6}{}_\zeta(q_i)][\bar{u}_\tau(q_j)] \tag{2.2}$$



$$M_1{}^\dagger = [\frac{i}{(q_i+q)^2}(-g_s f^{\,a'\,o'\,k}(g^{\mu'\zeta'}(q-q_i)^{\eta'} - g^{\zeta'\eta'}(2q_i+q)^{\mu'} + g^{\eta'\mu'}(2q_i+q)^{\zeta'}) \\ \varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)\varepsilon^{\lambda_5}{}_{\zeta'}{}^*(q_i)][u_{\tau'}(q_j)] \tag{2.3}$$



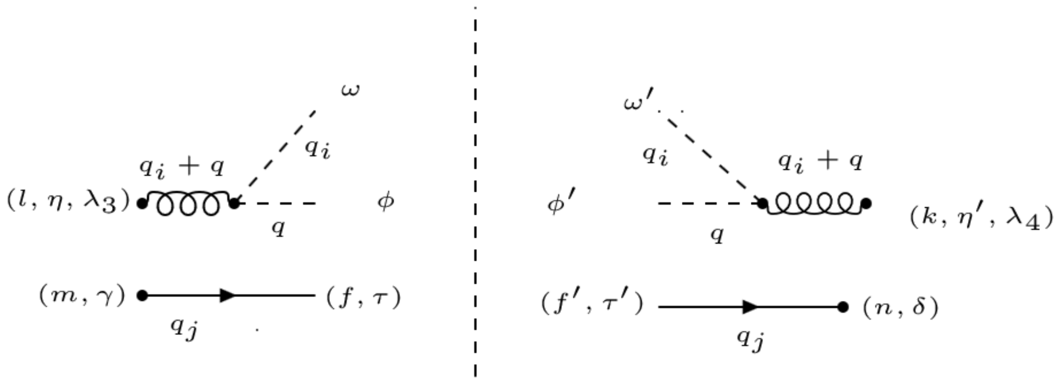
$$\begin{aligned}|M_1|^2 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta)\\ \varepsilon^{\lambda_1}{}_\mu(q)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)\varepsilon^{\lambda_6}{}_\zeta(q_i)\,\varepsilon^{\lambda_5}{}_{\zeta'}{}^*(q_i)\\ (-g_s f^{\,a'\,o'\,k}(g^{\mu'\zeta'}(q-q_i)^{\eta'} - g^{\zeta'\eta'}(2q_i+q)^{\mu'} + g^{\eta'\mu'}(2q_i+q)^{\zeta'})\frac{i}{(q_i+q)^2}][\bar{u}_\tau(q_j)u_{\tau'}(q_j)]\end{aligned} \tag{2.4}$$

$$\begin{aligned}|M_1|^2 = \frac{{g_s}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2}\\ [(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta)\\ g_{\mu\mu'}g^{\zeta\zeta'}(g_{\mu'\zeta'}(q-q_i)_{\eta'} - g_{\zeta'\eta'}(2q_i+q)_{\mu'} + g_{\eta'\mu'}(2q_i+q)_{\zeta'})][\not{q}_j]\end{aligned} \tag{2.5}$$

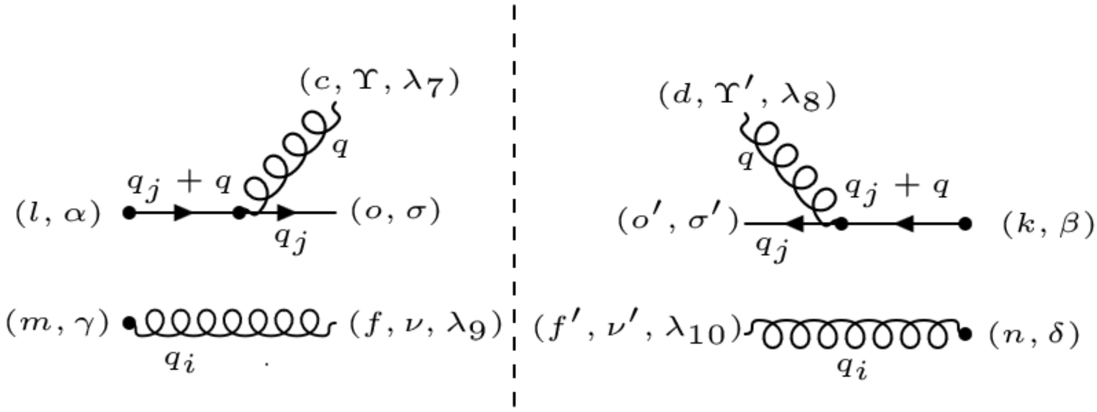
$$\begin{aligned}|M_1|^2 = \frac{{g_s}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2}\\ [(g^{\mu'\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)_{\mu'} + g^{\eta}{}_{\mu'}(2q_i+q)^\zeta)\\ (g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} + g_{\eta'\mu'}(2q_i+q)^\zeta)][\not{q}_j]\end{aligned} \tag{2.6}$$

$$\begin{aligned}|M_1|^2 = \frac{{g_s}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2}\\ [g^{\mu'\zeta}(q-q_i)^\eta\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\mu'\zeta}(q-q_i)^\eta\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'}\\ +g^{\mu'\zeta}(q-q_i)^\eta\, g_{\eta'\mu'}(2q_i+q)^\zeta) - g^{\zeta\eta}(2q_i+q)_{\mu'}\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'}\\ +g^{\zeta\eta}(2q_i+q)_{\mu'}\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} - g^{\zeta\eta}(2q_i+q)_{\mu'}\, g_{\eta'\mu'}(2q_i+q)^\zeta)\\ +g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'}\\ +g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g_{\eta'\mu'}(2q_i+q)^\zeta)][\not{q}_j]\end{aligned} \tag{2.7}$$

2.1.1 One-loop corrections to the gluon self-energy diagram



2.2 qg-g

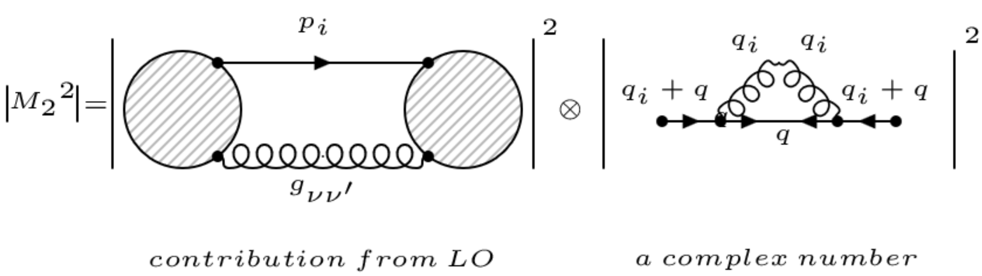


$$\begin{aligned}|M_2|^2 = M_2\, M_2{}^{\dagger} = {} & [\bar{u}_\sigma(q_j)\,(-ig_s\gamma^{\Upsilon} \times [T^c]_o{}^l)\,\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}\;\varepsilon^{\lambda_7}{}_{\Upsilon}(q)][\varepsilon^{\lambda_9}{}_{\nu}(q_i)] \\ & [\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\,(ig_s\gamma^{\Upsilon'} \times [T^d]_{o'}{}^k)\,u_{\sigma'}(q_j)\,\varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q)][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)]\end{aligned} \tag{2.8}$$

$$\begin{aligned}|M_2|^2 = {} & [\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\;(-ig_s\gamma^{\Upsilon} \times [T^c]_o{}^l)\,\bar{u}_\sigma(q_j)\,u_{\sigma'}(q_j)\,\varepsilon^{\lambda_7}{}_{\Upsilon}(q)\varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q) \\ & \times(ig_s\gamma^{\Upsilon'} \times [T^d]_{o'}{}^k)\,\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)\varepsilon^{\lambda_9}{}_{\nu}(q_i)]\end{aligned} \tag{2.9}$$

$$\begin{aligned}|M_2|^2 = {} & \frac{g_s{}^2[T^c]_o{}^l[T^d]_{o'}{}^k}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\;\gamma^{\Upsilon}\;\;\not{q}_j\,(-g_{\Upsilon\Upsilon'}) \\ & \gamma^{\Upsilon'}\,(\not{q}_j + \not{q})][-g_{\nu'\nu}]\end{aligned} \tag{2.10}$$

Expect:



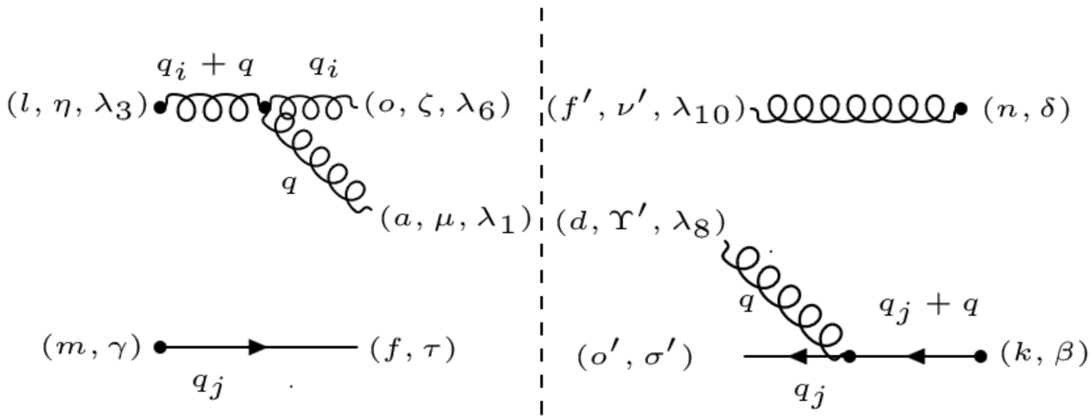
$$|M_2|^2 = -\frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{(q_j+q)^2 (q_j+q)^2} [(\not{q}_j + \not{q})\ \gamma^\Upsilon\ \not{q}_j\ \gamma_\Upsilon\ (\not{q}_j + \not{q})][-g_{\nu'\nu}] \tag{2.11}$$

$$|M_2|^2 = (d-2) \frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{(2qq_j)} [\not{q}][-g_{\nu'\nu}] \tag{2.12}$$

$$|M_2|^2 = (d-2) \frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{2(1-z)(1-y)(p_i \cdot p_j)} [(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][-g_{\nu'\nu}] \tag{2.13}$$

$$|M_2|^2 = (d-2) zy \frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_j][-g_{\nu'\nu}] \tag{2.14}$$

2.3 $M_1 {M_2}^\dagger$

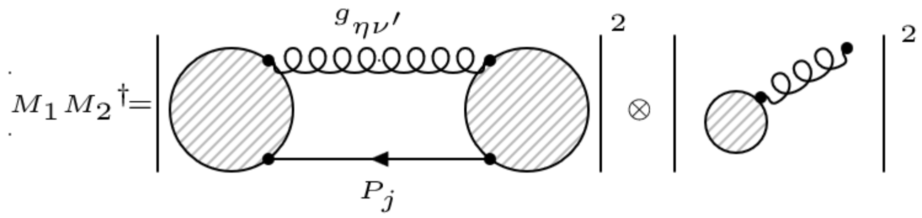


$$\begin{aligned} M_1\ {M_2}^\dagger &= [\frac{-i g_{\eta\zeta}}{(q_i+q)^2} (-g_s f^{\,a\,o\,l} (g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\ &\varepsilon^{\lambda_1}{}_\mu(q_i) \varepsilon^{\lambda_6}{}_\zeta(q)][\bar{u}_\tau(q_j)][\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\ (i g_s \gamma^{\mu'} \times [T^d]_{o'}{}^k)\ u_{\sigma'}(q_j)\ \varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q)][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)] \end{aligned} \tag{2.15}$$

$$\begin{aligned} M_1\ {M_2}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\ [T^d]_{o'}{}^k}{4(qq_i)(qq_j)} &[(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta)] \\ &(-g_{\zeta\nu'})(-g_{\mu\Upsilon'})[(\not{q}_j + \not{q})\gamma^{\Upsilon'}\ \not{q}_j] \end{aligned} \tag{2.16}$$

$$\begin{aligned} M_1\ {M_2}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\ [T^d]_{o'}{}^k}{4(qq_i)(qq_j)} &[(g^\mu{}_{\nu'}(q-q_i)^\eta - g^\eta{}_{\nu'}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)_{\nu'})] \\ &[(\not{q}_j + \not{q})\gamma_\mu\ \not{q}_j] \end{aligned} \tag{2.17}$$

Expect:



contribution from LO *a complex number*

$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[-g^{\eta}{}_{\nu'}(2q_i+q)^{\mu}][-(\not{q}_j+\not{q})\,\not{q}_j\gamma_\mu + 2(\not{q}_j+\not{q})q_{j\mu}] \tag{2.18}$$

$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[-g^{\eta}{}_{\nu'}(2q_i+q)^{\mu}][-(qq_j)\gamma_\mu + 2(\not{q}_j+\not{q})q_{j\mu}] \tag{2.19}$$

$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[(qq_j)g^{\eta}{}_{\nu'}(2\,\not{q}_i+\not{q}) - 2((2q_i+q)\cdot q_j)\,g^{\eta}{}_{\nu'}(\not{q}_j+\not{q})] \tag{2.20}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(qq_j)(2\,\not{q}_i+\not{q}) - 2((2q_i+q)\cdot q_j)\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.21}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(p_i\cdot p_j)(2\,\not{q}_i+\not{q}) - (4q_i\cdot q_j + 2q\cdot q_j)\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.22}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(p_i\cdot p_j)(2\,\not{q}_i+\not{q}) \\ & -(4(z(1-y)(p_i\cdot p_j)) + 2((1-z)(1-y)(p_i\cdot p_j)))\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.23}$$

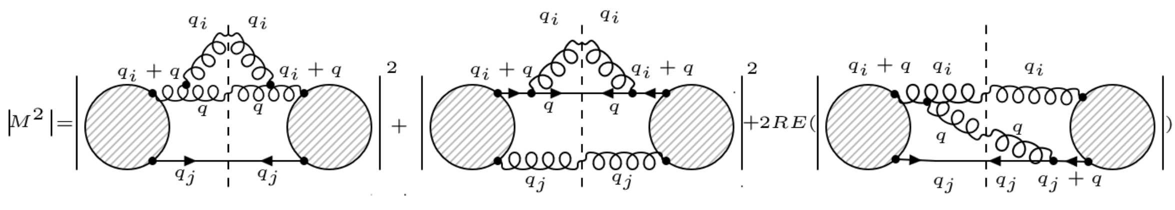
$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(2\,\not{q}_i+\not{q}) \\ & -(4(z(1-y)) + 2((1-z)(1-y)))\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.24}$$

$$
\begin{aligned}
M_1\, {M_2}^{\dagger} = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} \\
& [g^{\eta}{}_{\nu'}][(1-z)(1-y)(2y(1-z)\ \not{p}_j + yz\ \not{p}_j) \\
& -2(1+z)(1-y)\ ((1+yz-z)\ \not{p}_j)]
\end{aligned}
\tag{2.25}
$$

$$
\begin{aligned}
M_1\, {M_2}^{\dagger} = & \frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} \\
& [g^{\eta}{}_{\nu'}][-2(1+z)(1-y)\ (1+yz-z)\ \not{p}_j]
\end{aligned}
\tag{2.26}
$$

$$
M_1\, {M_2}^{\dagger} = -2(1+z)\,(1+yz-z)\frac{i g_s^2 f^{\,a\,o\,l}\, [T^d]_{o'}{}^{k}}{4(1-z)y(1-2z+z^2)(p_i \cdot p_j)}[g^{\eta}{}_{\nu'}][\not{p}_j]
\tag{2.27}
$$

2.4 $|M^2|$


$|M^2|=$
q_i
q_i
$q_i + q$
$q_i + q$
q
q
q_j
q_j
2
$+$
q_i
q_i
$q_i + q$
$q_i + q$
q
q
q_j
q_j
2
$+2RE($
$q_i + q$
q_i
q_i
q
q
q_j
q_j
$q_j + q$
$)$