

# THESIS

BY

TIGRAN SAIDNIA

---

## Emission kernel of parton shower

Emission kernel of parton shower

---



Karlsruhe institute for Technology (KIT)

Institute of theoretical physics

Referents: PD Dr. Stefan Gieseke

Dr. Simon Plätzer

Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, 15. Januar 2019

---

Tigran Saidnia



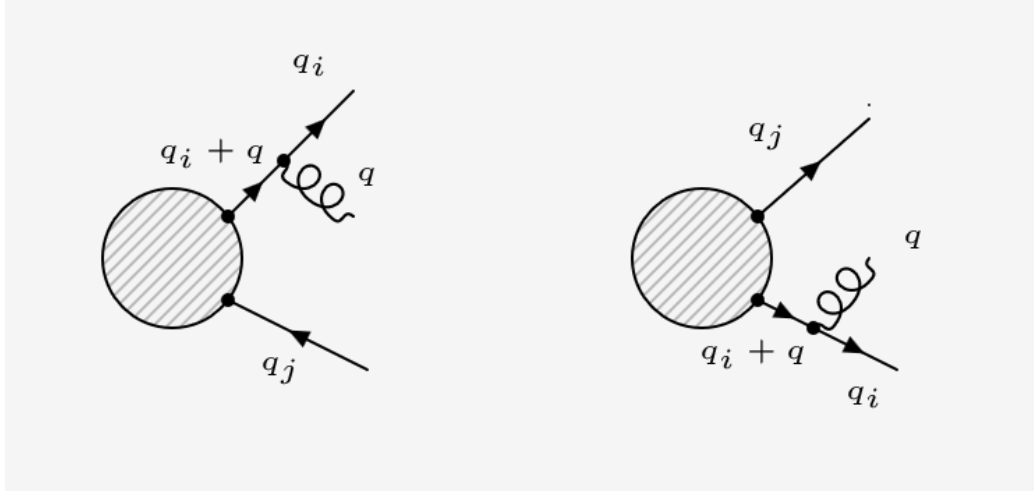
# Inhaltsverzeichnis

<b>Literaturverzeichnis</b>	<b>2</b>
0.1 parametrisation . . . . .	1
0.2 Quark/Antiquark gluon emission kernel . . . . .	2
0.2.1 $qg-\bar{q}$ . . . . .	2
0.2.2 $\bar{q}g-q$ . . . . .	6
0.2.3 $M_1M_2^\dagger$ . . . . .	8
0.2.4 $ M^2 $ . . . . .	10
0.3 Quark/Gluon gluon emission kernel . . . . .	11
0.3.1 $gg-q$ . . . . .	11
0.3.2 $qg-g$ . . . . .	14
0.3.3 $M_1M_2^\dagger$ . . . . .	15
0.3.4 $ M^2 $ . . . . .	18

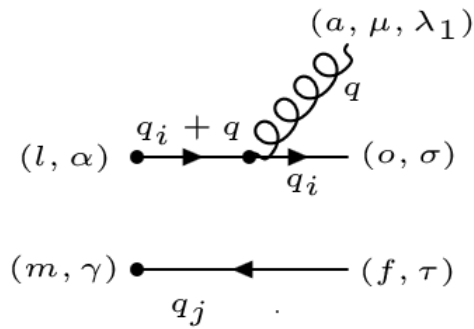
## 0.1 parametrisation

$$\left. \begin{aligned}
 q_i^\mu &= zp_i^\mu + y(1-z)p_j^\mu + \sqrt{zy(1-z)}m_\perp \\
 q^\mu &= (1-z)p_i^\mu + yzp_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_j^\mu &= (1-y)p_j^\mu \\
 y &= \frac{q_i q}{p_i p_j} \\
 q_i + q &= p_i + yp_j \\
 q_j + q &= (1-z)p_i^\mu + (1+yz-y)p_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
 q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
 q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
 \end{aligned} \right\} \text{parametrisation} \quad (1)$$

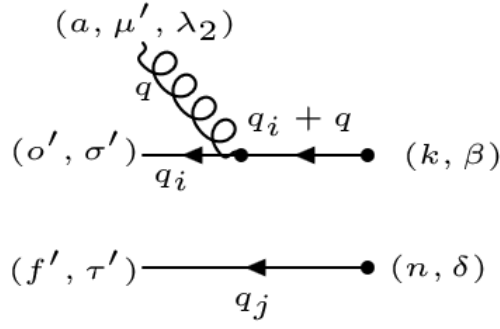
## 0.2 Quark/Antiquark gluon emission kernel



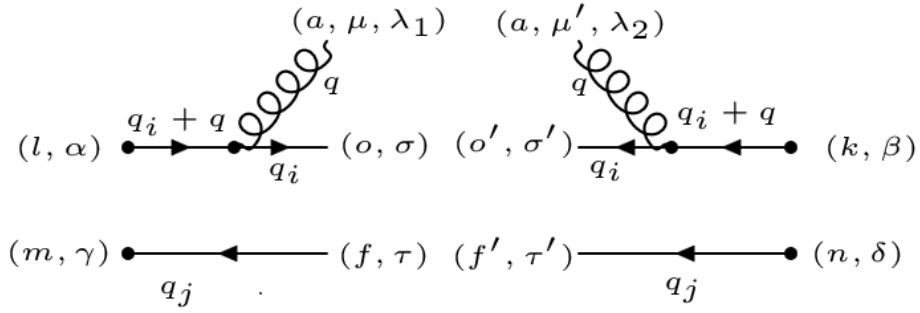
### 0.2.1 $qg\text{-}\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)] \quad (2)$$



$$M_1^\dagger = \left[ \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) \right] [\bar{v}_{\tau'}(q_j)] \quad (3)$$



$$|M_1|^2 = M_1 M_1^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q) [v_\tau(q_j)] \left[ \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) [\bar{v}_{\tau'}(q_j)] \right] \quad (4)$$

$$|M_1|^2 = \left[ \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q) \times (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \right] [\bar{v}_{\tau'}(q_j) v_\tau(q_j)] \quad (5)$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i, \quad \sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \quad (6)$$

and sum over polarization index  $(\lambda_1, \lambda_2)$  :

$$\sum_{\mu, \mu'} \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_{\mu}(q) = -g_{\mu\mu'} \quad (7)$$

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i g_{\mu'\mu} \gamma^{\mu} (\not{q}_i + q)] [\not{q}_j] \quad (8)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i \gamma_{\mu'} (\not{q}_i + q)] [\not{q}_j] \quad (9)$$

In other words we expect the tree level diagram from LO and a number: Which means:

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1: Two shaded circles connected by two horizontal lines. The top line is labeled } P_i \text{ and the bottom line is labeled } P_j. \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{Diagram 2: A triangle loop with vertices labeled } q_i, q_i, \text{ and } q_i + q. \end{array} \right|^2$$

*contribution from LO* *a complex number*

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [P_i][P_j] \otimes (\text{a complex number}) \quad (10)$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N &=: \gamma^{\mu'} \not{q}_i \gamma_{\mu'} = q_{i\sigma} \gamma^{\mu'} \gamma^{\sigma} \gamma_{\mu'} \\ &= q_{i\sigma} (\{\gamma^{\mu'}, \gamma^{\sigma}\} - \gamma^{\sigma} \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} 2g^{\mu'\sigma} \gamma_{\mu'} - d \gamma^{\sigma} \\ &= (2 - d) \not{q}_i \end{aligned} \quad (11)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \not{q}_i (\not{q}_i + q)] [\not{q}_j] \quad (12)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [\not{q}_i \not{q}_i \not{q}_i + \not{q}_i \not{q}_i \not{q} + \not{q} \not{q}_i \not{q}_i + \not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (13)$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \not{q}_i &= q_i^2 = m_i^2 \\ \not{q} \not{q} &= q^2 = m^2 \\ \not{q}_j \not{q}_j &= q_j^2 = m_j^2 \end{aligned} \quad (14)$$

we can first neglect the mass of patrons and ignore each term with  $\not{q}_i \not{q}_i$  and  $\not{q} \not{q}$  as well.

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)(2q_i q)} [\not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (15)$$

$$\begin{aligned} L &= \not{q} \not{q}_i \not{q} = \not{q} [q_{i\sigma} q_\mu (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu)] \\ &\quad \not{q} [2q_i^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu \left[ \frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2} \right] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\ &= \not{q} (2q_i q) - q^2 \not{q}_i \\ &= \not{q} (2q_i q) \end{aligned} \quad (16)$$

After inserting the last result of  $L$  and simplify the term  $(2q_i q)$  from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{q}] [\not{q}_j] \quad (17)$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$|M_1|^2 = (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [(1-z) \not{p}_i + zy \not{p}_j - \sqrt{zy(1-z)} \not{m}_\perp] [(1-y) \not{p}_j] \quad (18)$$

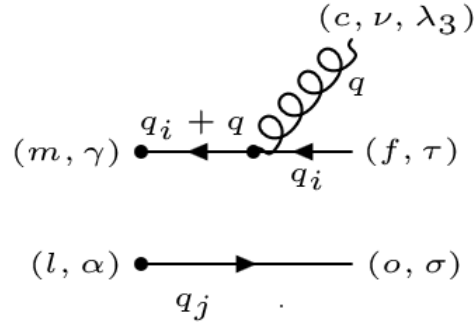
Multiplying the both sides

$$\begin{aligned} |M_1|^2 &= (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [(1-z)(1-y) \not{p}_i \not{p}_j \\ &\quad + zy(1-y) \not{p}_j \not{p}_j + (1-y)\sqrt{zy(1-z)} \not{m}_\perp \not{p}_j] \end{aligned} \quad (19)$$

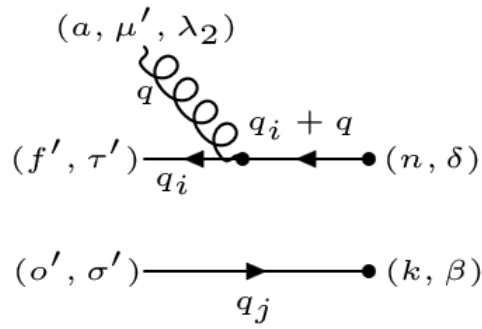
and under consideration of the fact that  $p_i$  and  $p_j$  are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with  $\not{p}_i \not{p}_i$  and  $\not{p}_j \not{p}_j$ . The  $p_i \cdot m_\perp$  and  $p_j \cdot m_\perp$  are always 0 because the  $p_i$  and  $p_j$  are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$|M_1|^2 = (d-2)(1-z)(1-y) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \quad (20)$$

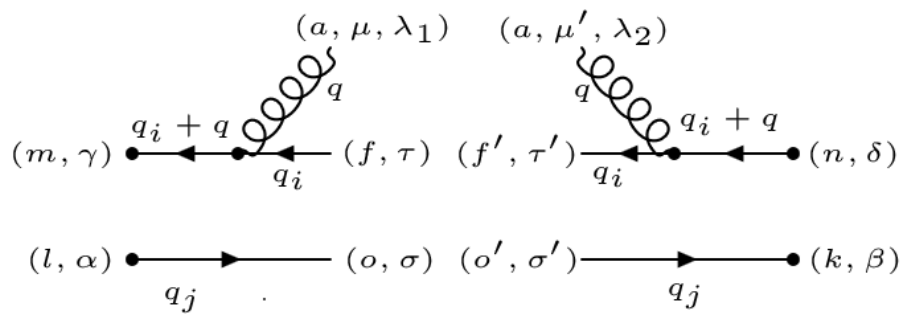


0.2.2  $\bar{q}g$ -q


$$M_2 = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) \right] [u_\sigma(q_i)] \quad (21)$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q)] [\bar{u}_{\sigma'}(q_i)] \quad (22)$$



$$|M_2|^2 = M_2 M_2^\dagger = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_i)] \right. \\ \left. [\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_i)] \right] \quad (23)$$

$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu v_\tau(q_j) \bar{v}_{\tau'}(q_j) \varepsilon^{\lambda_3}_\nu(q) \varepsilon^{\lambda_4}_{\nu'}(q) \gamma^{\nu'} (\not{q}_j + \not{q})] \\ [u_\sigma(q_i)] [\bar{u}_{\sigma'}(q_i)] \quad (24)$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i, \\ \sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \quad (25)$$

and sum over polarization index  $(\lambda_3, \lambda_4)$  :

$$\sum_{\nu, \nu'} \varepsilon^{\lambda_4*}_{\nu'}(q) \varepsilon^{\lambda_3}_\nu(q) = -g_{\nu\nu'} \quad (26)$$

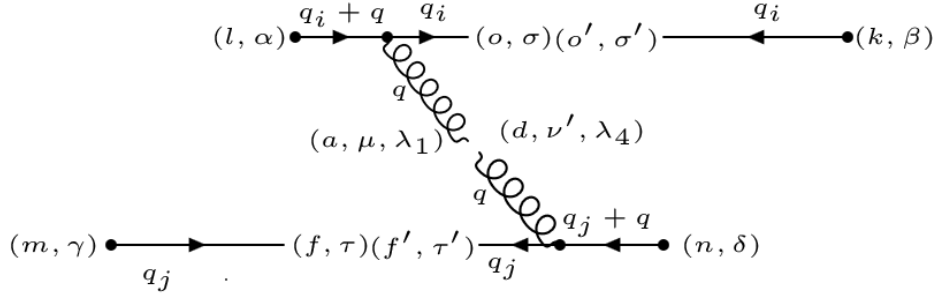
$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu \not{q}_j (-g_{\nu\nu'}) \gamma^{\nu'} (\not{q}_j + \not{q})] [\not{q}_i] \quad (27)$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d - 2) \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(2qq_j)} [\not{q}] [\not{q}_i] \quad (28)$$

finally:

$$|M_2|^2 = -(d - 2) y z^2 \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{2(1 - z)(1 - y)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \quad (29)$$

0.2.3  $M_1 M_2^\dagger$ 

$$M_1 M_2^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)]$$

$$[\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q)] [u_{\sigma'}(q_i)] \quad (30)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_4}_{\nu'}(q)$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (31)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] - g_{\mu\nu'}$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (32)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j \gamma_\mu (\not{q}_j + \not{q})] \quad (33)$$

Expectation:

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1: Two shaded circles connected by two horizontal lines labeled } P_i \text{ and } P_j. \\ \text{Diagram 2: A shaded circle connected to a wavy line.} \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{Diagram 3: A shaded circle connected to a wavy line.} \end{array} \right|^2$$

*contribution from LO* *a complex number*

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [(\not{A}_i + \not{A}) \gamma^\mu \not{A}_i] [(\not{A}_j + \not{A}) \gamma_\mu \not{A}_j] \quad (34)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [-(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu + 2(\not{A}_i + \not{A}) q_i^\mu] \quad (35)$$

$$[-(\not{A}_j + \not{A}) \not{A}_j \gamma_\mu + 2(\not{A}_j + \not{A}) q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (36)$$

$$[(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu] [(\not{A}_j + \not{A}) \not{A}_j \gamma_\mu]$$

$$-2[(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu] [(\not{A}_j + \not{A}) q_{j\mu}]$$

$$-2[(\not{A}_i + \not{A}) q_i^\mu] [(\not{A}_j + \not{A}) \not{A}_j \gamma_\mu]$$

$$+4[(\not{A}_i + \not{A}) q_i^\mu] [(\not{A}_j + \not{A}) q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (37)$$

$$[\not{A} \not{A}_i \gamma^\mu] [\not{A} \not{A}_j \gamma_\mu]$$

$$-2[\not{A} \not{A}_i \gamma^\mu] [(\not{A} + \not{A}_j) q_{j\mu}]$$

$$-2[(\not{A} + \not{A}) q_i^\mu] [\not{A} \not{A}_j \gamma_\mu]$$

$$+4[(\not{A} + \not{A}) q_i^\mu] [(\not{A} + \not{A}_j) q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \quad (38)$$

$$[y(1-2z+z^2) \not{p}_i \not{p}_j \gamma^\mu] [(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu]$$

$$+4(q_i^\mu \cdot q_{j\mu}) [(\not{A}_i + \not{A})] [(\not{A}_j + \not{A})]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \quad (39)$$

$$[y(1-2z+z^2) \not{p}_i \not{p}_j \gamma^\mu] [(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu]$$

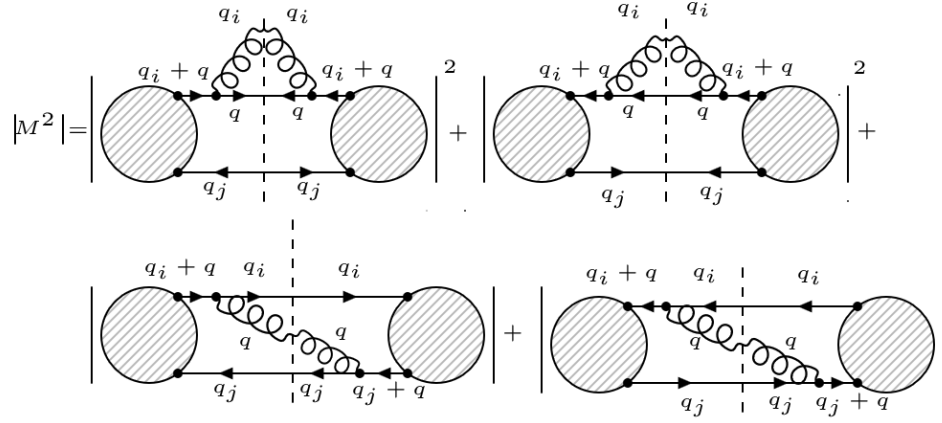
$$+4(p_i \cdot p_j) [(\not{p}_i + y \not{p}_j)] [(1-z) \not{p}_i + (1+yz-y) \not{p}_j - \sqrt{zy(1-z)} \not{n}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} z(1-y) [\not{p}_i] [\not{p}_j] \quad (40)$$

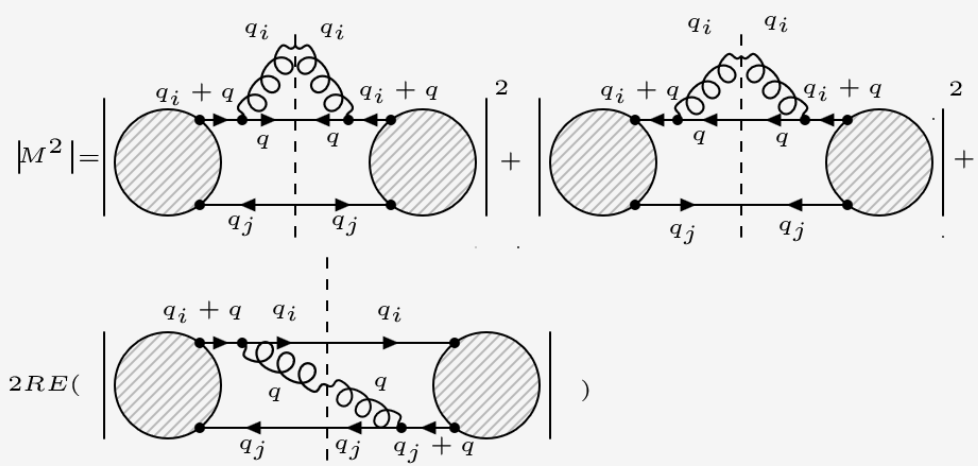
$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(1-z)y(1-2z+z^2)(p_i \cdot p_j)} z [\not{p}_i] [\not{p}_j] \quad (41)$$

0.2.4  $|M^2|$ 

$$|M|^2 = |M_1|^2 + |M_2|^2 + M_1 M_2^\dagger + M_1^\dagger M_2 \quad (42)$$

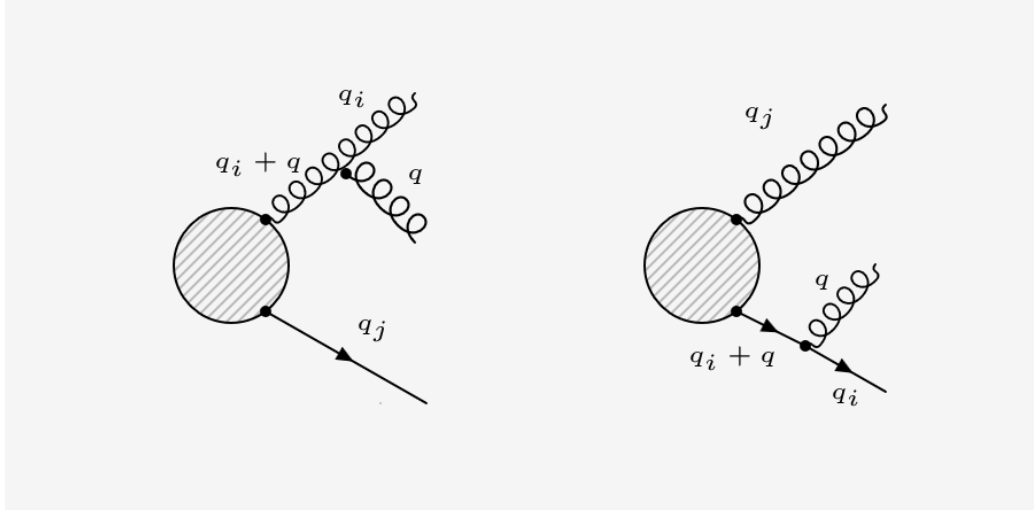


$$|M|^2 = |M_1|^2 + |M_2|^2 + 2RE(M_1 M_2^\dagger) \quad (43)$$

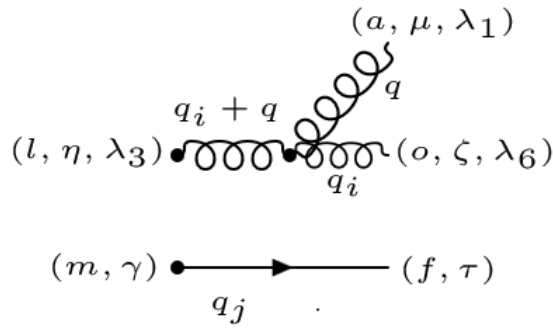


$$\begin{aligned}
 |M|^2 = & (d-2)(1-z)(1-y) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & - (d-2)yz^2 \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & + 2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]\right)
 \end{aligned} \quad (44)$$

### 0.3 Quark/Gluon gluon emission kernel

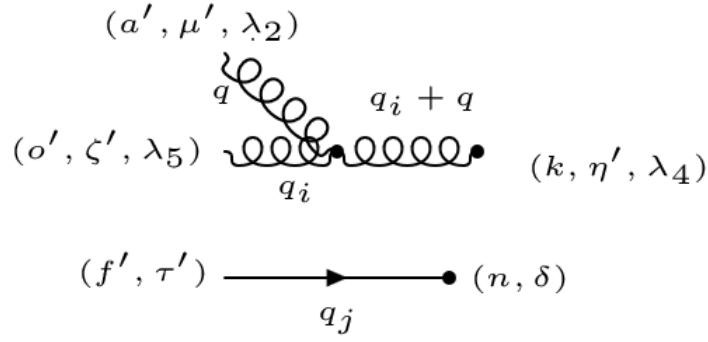


#### 0.3.1 gg-q

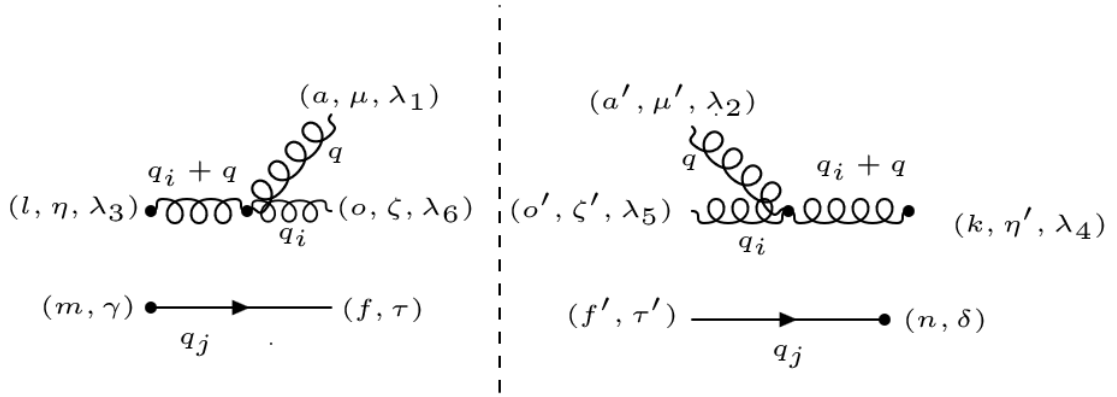


$$M_1 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{aol} (g^{\mu\zeta} (-q_i + q)^\eta + g^{\zeta\eta} (-q - (q_i + q))^\mu + g^{\eta\mu} (q_i + q + q_i)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_6}_\zeta(q_i) \right] [\bar{u}_\tau(q_j)] \quad (45)$$

$$M_1 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{aol} (g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_6}_\zeta(q_i) \right] [\bar{u}_\tau(q_j)] \quad (46)$$



$$M_1^\dagger = \left[ \frac{i}{(q_i + q)^2} (-g_s f^{a' o' k} (g^{\mu' \zeta'} (q - q_i)^{\eta'} - g^{\zeta' \eta'} (2q_i + q)^{\mu'} + g^{\eta' \mu'} (2q_i + q)^{\zeta'}) \right. \\ \left. \varepsilon^{\lambda_2}_{\mu'}(q) \varepsilon^{\lambda_5}_{\zeta'}(q_i) \right] [u_{\tau'}(q_j)] \quad (47)$$



$$|M_1|^2 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q_i + q)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_2}_{\mu'}(q) \varepsilon^{\lambda_6}_\zeta(q_i) \varepsilon^{\lambda_5}_{\zeta'}(q_i) \right. \\ \left. (-g_s f^{a' o' k} (g^{\mu' \zeta'} (q - q_i)^{\eta'} - g^{\zeta' \eta'} (2q_i + q)^{\mu'} + g^{\eta' \mu'} (2q_i + q)^{\zeta'}) \frac{i}{(q_i + q)^2} [\bar{u}_\tau(q_j) u_{\tau'}(q_j)] \right] \quad (48)$$

$$|M_1|^2 = \frac{g_s^2 f^{a o l} f^{a' o' k}}{(q_i + q)^2 (q_i + q)^2} \\ [(g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q_i + q)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \\ g_{\mu \mu'} g^{\zeta \zeta'} (g_{\mu' \zeta'} (q - q_i)_{\eta'} - g_{\zeta' \eta'} (2q_i + q)_{\mu'} + g_{\eta' \mu'} (2q_i + q)_{\zeta'})] [\not{A}_j] \quad (49)$$

$$|M_1|^2 = \frac{g_s^2 f^{a o l} f^{a' o' k}}{(q_i + q)^2 (q_i + q)^2} \quad (50)$$

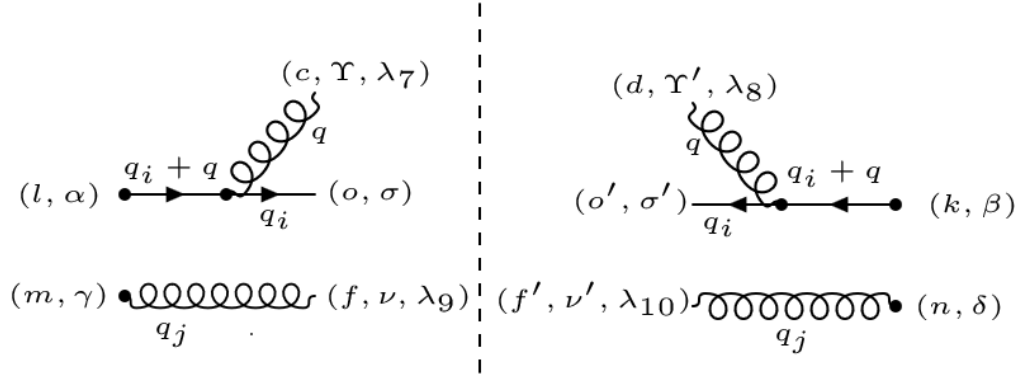
$$[(g^{\mu'\zeta}(q - q_i)^\eta - g^{\zeta\eta}(2q_i + q)_{\mu'} + g^\eta_{\mu'}(2q_i + q)^\zeta) \\ (g^\eta_{\mu'}(q - q_i)_{\eta'} - g^{\zeta\eta}_{\eta'}(2q_i + q)_{\mu'} + g_{\eta'\mu'}(2q_i + q)^\zeta)][\not{A}_j]$$

$$|M_1|^2 = \frac{g_s^2 f^{a o l} f^{a' o' k}}{(q_i + q)^2 (q_i + q)^2} \quad (51)$$

$$[g^{\mu'\zeta}(q - q_i)^\eta g^\eta_{\mu'}(q - q_i)_{\eta'} \\ - g^{\mu'\zeta}(q - q_i)^\eta g^{\zeta\eta}_{\eta'}(2q_i + q)_{\mu'} \\ + g^{\mu'\zeta}(q - q_i)^\eta g_{\eta'\mu'}(2q_i + q)^\zeta \\ - g^{\zeta\eta}(2q_i + q)_{\mu'} g^\eta_{\mu'}(q - q_i)_{\eta'} \\ + g^{\zeta\eta}(2q_i + q)_{\mu'} g^{\zeta\eta}_{\eta'}(2q_i + q)_{\mu'} \\ - g^{\zeta\eta}(2q_i + q)_{\mu'} g_{\eta'\mu'}(2q_i + q)^\zeta \\ + g^\eta_{\mu'}(2q_i + q)^\zeta g^\eta_{\mu'}(q - q_i)_{\eta'} \\ - g^\eta_{\mu'}(2q_i + q)^\zeta g^{\zeta\eta}_{\eta'}(2q_i + q)_{\mu'} \\ + g^\eta_{\mu'}(2q_i + q)^\zeta g_{\eta'\mu'}(2q_i + q)^\zeta)][\not{A}_j]$$



## 0.3.2 qg-g



$$|M_2|^2 = M_2 M_2^\dagger = [\bar{u}_\sigma(q_j) (-ig_s \gamma^\Gamma \times [T^c]_o^l) \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_7 \Gamma}(q)] [\varepsilon^{\lambda_9 \nu}(q_i)]$$

$$[\frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} (ig_s \gamma^{\Gamma'} \times [T^d]_{o'}^k) u_{\sigma'}(q_j) \varepsilon^{\lambda_8 \Gamma'}(q)] [\varepsilon^{\lambda_{10} \nu'}(q_i)] \quad (52)$$

$$|M_2|^2 = [\frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\Gamma \times [T^c]_o^l) \bar{u}_\sigma(q_j) u_{\sigma'}(q_j) \varepsilon^{\lambda_7 \Gamma}(q) \varepsilon^{\lambda_8 \Gamma'}(q)]$$

$$\times (ig_s \gamma^{\Gamma'} \times [T^d]_{o'}^k) \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} [\varepsilon^{\lambda_{10} \nu'}(q_i) \varepsilon^{\lambda_9 \nu}(q_i)] \quad (53)$$

$$|M_2|^2 = \frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\Gamma \not{q}_j (-g_{\Gamma \Gamma'})$$

$$\gamma^{\Gamma'} (\not{q}_j + \not{q})] [-g_{\nu' \nu}] \quad (54)$$

Expect:

$$|M_2|^2 = \left| \text{diagram} \right|^2 \otimes \left| \text{diagram} \right|^2$$

*contribution from LO*                      *a complex number*

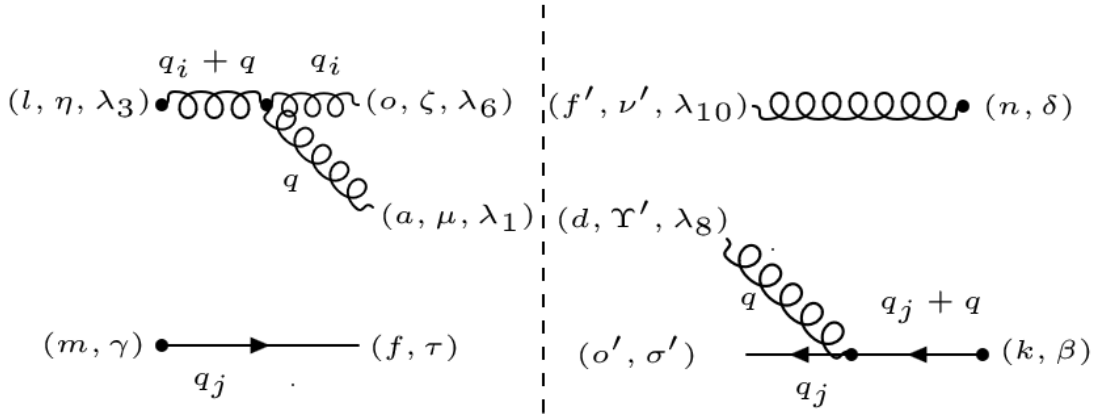
$$|M_2|^2 = -\frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\Upsilon \not{q}_j \gamma^\Upsilon (\not{q}_j + \not{q})] [-g_{\nu'\nu}] \quad (55)$$

$$|M_2|^2 = (d-2) \frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{(2qq_j)} [\not{q}] [-g_{\nu'\nu}] \quad (56)$$

$$|M_2|^2 = (d-2) \frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{2(1-z)(1-y)(p_i \cdot p_j)} [(1-z) \not{p}_i + zy \not{p}_j - \sqrt{zy(1-z)} \not{m}_\perp] [-g_{\nu'\nu}] \quad (57)$$

$$|M_2|^2 = (d-2)zy \frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_j] [-g_{\nu'\nu}] \quad (58)$$

### 0.3.3 $M_1 M_2^\dagger$



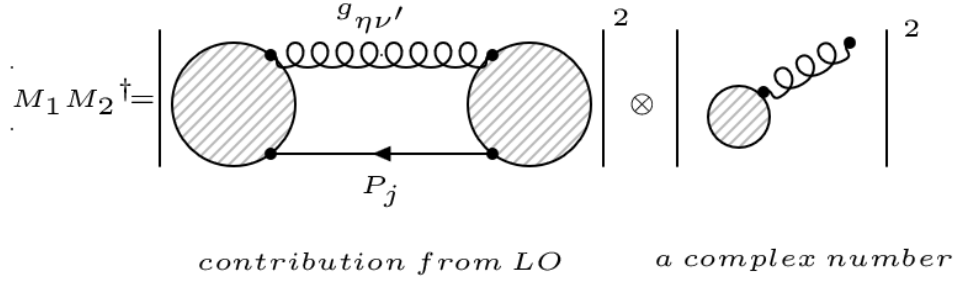
$$M_1 M_2^\dagger = \left[ \frac{-ig_\eta \zeta}{(q_i + q)^2} (-g_s f^{aol} (g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q) [\bar{u}_\tau(q_j)] \left[ \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} (ig_s \gamma^{\mu'} \times [T^d]_{o'}^k) u_{\sigma'}(q_j) \varepsilon^{\lambda_8}_{\Upsilon'}(q) \right] [\varepsilon^{\lambda_{10}}_{\nu'}(q_i)] \right] \quad (59)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(qq_i)(qq_j)} [(g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)^\zeta)] \\ (-g_{\zeta\nu'}) (-g_{\mu\Upsilon'}) [(\not{q}_j + \not{q}) \gamma^{\Upsilon'} \not{q}_j] \quad (60)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(qq_i)(qq_j)} [(g^{\mu\nu'} (q - q_i)^\eta - g^{\eta\nu'} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)_{\nu'})] \\ [(\not{q}_j + \not{q}) \gamma_\mu \not{q}_j] \quad (61)$$

Expect:





$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(qq_i)(qq_j)} [-g^{\eta}_{\nu'}(2q_i + q)^\mu] [-(\not{A}_j + \not{A}) \not{A}_j \gamma_\mu + 2(\not{A}_j + \not{A}) q_{j\mu}] \quad (62)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(qq_i)(qq_j)} [-g^{\eta}_{\nu'}(2q_i + q)^\mu] [-(qq_j) \gamma_\mu + 2(\not{A}_j + \not{A}) q_{j\mu}] \quad (63)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(qq_i)(qq_j)} [(qq_j) g^{\eta}_{\nu'}(2 \not{A}_i + \not{A}) - 2((2q_i + q) \cdot q_j) g^{\eta}_{\nu'}(\not{A}_j + \not{A})] \quad (64)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)(p_i \cdot p_j)} [g^{\eta}_{\nu'}][(qq_j)(2 \not{A}_i + \not{A}) - 2((2q_i + q) \cdot q_j) (\not{A}_j + \not{A})] \quad (65)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)(p_i \cdot p_j)} [g^{\eta}_{\nu'}][(1-z)(1-y)(p_i \cdot p_j)(2 \not{A}_i + \not{A}) - (4q_i \cdot q_j + 2q \cdot q_j) (\not{A}_j + \not{A})] \quad (66)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)(p_i \cdot p_j)} [g^{\eta}_{\nu'}][(1-z)(1-y)(p_i \cdot p_j)(2 \not{A}_i + \not{A}) - (4(z(1-y)(p_i \cdot p_j)) + 2((1-z)(1-y)(p_i \cdot p_j))) (\not{A}_j + \not{A})] \quad (67)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} [g^{\eta}_{\nu'}][(1-z)(1-y)(2 \not{A}_i + \not{A}) - (4(z(1-y)) + 2((1-z)(1-y))) (\not{A}_j + \not{A})] \quad (68)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} [g^\eta_{\nu'}] [-(4z(1-y)((1-z) \not{p}_i + (1+yz-y) \not{p}_j - \sqrt{zy(1-z)} \not{n}_\perp)] \quad (69)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{4(1-z)(1-y)y(1-2z+z^2)(p_i \cdot p_j)} [g^\eta_{\nu'}] [-(4z(1-y)(1+yz-y) \not{p}_j)] \quad (70)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{(1-z)y(1-2z+z^2)(p_i \cdot p_j)} [g^\eta_{\nu'}] [-z(1+yz-y) \not{p}_j] \quad (71)$$

$$M_1 M_2^\dagger = \frac{ig_s^2 f^{aol} [T^d]_{o'}^k}{(1-z)y(1-2z+z^2)(p_i \cdot p_j)} [g^\eta_{\nu'}] [-z \not{p}_j] \quad (72)$$

### 0.3.4 $|M^2|$

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right|^2 + 2 \operatorname{Re} \left( \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \right)$$









