Karlsruher Institut für Technologie

Thesis

by

Tigran Saidnia

Emission kernel of parton shower

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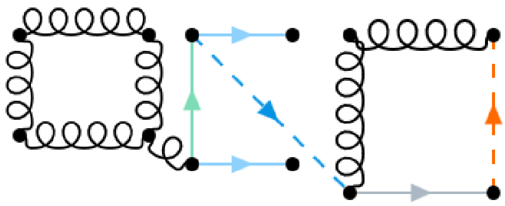

Karlsruhe institute for Technology (KIT)

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statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, February 7, 2019

Tigran Saidnia

Contents

0.1 Old parametrisation

$$\left.\begin{aligned}
{q_i}^{\mu} &= z{p_i}^{\mu} + y(1-z){p_j}^{\mu} + \sqrt{zy(1-z)}m_{\perp} \\
q^{\mu} &= (1-z){p_i}^{\mu} + yz{p_j}^{\mu} - \sqrt{zy(1-z)}m_{\perp} \\
{q_j}^{\mu} &= (1-y){p_j}^{\mu} \\
y &= \frac{q_i q}{p_i p_j} \\
q_i + q &= p_i + y p_j \\
q_j + q &= (1-z){p_i}^{\mu} + (1+yz-y){p_j}^{\mu} - \sqrt{zy(1-z)}m_{\perp} \\
q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
\end{aligned}\right\} \text{parametrisation} \tag{1}$$

0.2 new kinematic

$$\begin{aligned}
{k_l}^{\mu} &= \alpha_l \alpha {\Lambda^{\mu}}_{\nu} {p_i}^{\nu} + y\beta n^{\mu} + \sqrt{y\alpha_l\beta_l}{n^{\mu}}_{\perp,l} \qquad l = 1, ..., m \\
{q_i}^{\mu} &= (1 - \sum_{l=1}^{m} \alpha_l)\alpha {\Lambda^{\mu}}_{\nu} {p_i}^{\nu} + y(1 - \sum_{l=1}^{m} \beta_l) n^{\mu} - \sqrt{y\alpha_l\beta_l}{n^{\mu}}_{\perp,l} \\
{q_k}^{\mu} &= \alpha {\Lambda^{\mu}}_{\nu} {p_k}^{\nu} \qquad k = 1, ..., n \qquad k \neq i
\end{aligned} \tag{2}$$

0.2.1 useful relations

$$\begin{aligned}
{q_i}^2 = {p_i}^2 = {q_k}^2 = k_l^2 = p_j^2 = p_k^2 = n^2 = 0 &\quad \text{All hard momenta are on-shell} \\
Q^{\mu} = {q_i}^{\mu} + \sum_{l=1}^{m} k_l^{\mu} + \sum_{k=1}^{m} q_k^{\mu} = {p_i}^{\mu} + \sum_{k=1}^{m} p_k^{\mu} &\quad \text{total momentum} \\
n^{\mu} = Q^{\mu} - \frac{Q^2}{2p_i \cdot Q} p_i^{\mu} &\quad n^{\mu} \text{ is the recoil} \\
{q_i}^{\mu} + \sum_{l=1}^{m} k_l^{\mu} &= \alpha {\Lambda^{\mu}}_{\nu} {p_i}^{\nu} + y n^{\mu} \\
\alpha {\Lambda^{\mu}}_{\nu} Q^{\nu} &= Q^{\mu} - y n^{\mu} \\
{n^{\mu}}_{\perp,l} {\Lambda^{\mu}}_{\nu} {p_i}^{\nu} &= n_{\perp,l} \cdot n = n_{\perp,l} \cdot Q = 0 \\
{n^{\mu}}_{\perp,l} \cdot p_k &\neq 0 \\
n_{\perp,l}^2 &= -2\alpha {\Lambda^{\mu}}_{\nu} {p_i}^{\nu} n_{\mu} \\
n_{\perp,1}^2 &= -2p_i \cdot Q \\
\alpha_1 &= 1 - \beta_1 \\
\alpha &= \sqrt{1-y}
\end{aligned} \tag{3}$$

Lorenz trafo

$$\alpha\Lambda^{\mu}{}_{\nu} = p_i{}^{\mu} p_{i\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + p_i{}^{\mu} Q_{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu} \frac{(y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu} \tag{4}$$

$$\hat{p_i}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu} p_i{}^{\nu} = p_i{}^{\mu} p_{i\nu} p_i{}^{\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + p_i{}^{\mu} Q_{\nu} p_i{}^{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu} p_i{}^{\nu} \frac{(y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu} p_i{}^{\nu} \tag{5}$$

$$\begin{aligned} \hat{p_i}^{\mu} &= p_i{}^{\mu}(Q\cdot p_i) \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}p_i{}^{\mu} \\ &= p_i{}^{\mu}[\frac{y(1+\sqrt{1-y})}{(2+2\sqrt{1-y}-y)} + \sqrt{1-y}] = p_i{}^{\mu} \end{aligned} \tag{6}$$

$$\boxed{\hat{p_i}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu} p_i{}^{\nu} = p_i{}^{\mu}} \tag{7}$$

$$\hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu} p_k{}^{\nu} = p_i{}^{\mu} p_{i\nu} p_k{}^{\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + p_i{}^{\mu} Q_{\nu} p_k{}^{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu} p_k{}^{\nu} \frac{(y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu} p_k{}^{\nu} \tag{8}$$

$$\begin{aligned} \hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu} p_k{}^{\nu} = p_i{}^{\mu}[&\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\ &+ Q^{\mu}[\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}p_k{}^{\mu} \end{aligned} \tag{9}$$

$$\begin{aligned} &\hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu} p_k{}^{\nu} = p_i{}^{\mu}[\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\ &\quad + Q^{\mu}[\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}p_k{}^{\mu} \\ &\text{with} \\ &A_1 \equiv \frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} \\ &A_2 \equiv \frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} \end{aligned} \tag{10}$$

$$\boxed{\hat{p_k}^{\mu} = A_1\, {p_i}^{\mu} + A_2\, Q^{\mu} + \sqrt{1-y}{p_k}^{\mu}} \tag{11}$$

$$\begin{aligned}
\hat{Q}^{\mu} &= \alpha \Lambda^{\mu}{}_{\nu} Q^{\nu} = {p_i}^{\mu}[\frac{-y^2 Q^2 (p_i\cdot Q)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})Q^2}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\
&+ Q^{\mu}[\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot Q)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}Q^{\mu} \\
&\text{with} \\
S_1 &\equiv \frac{Q^2}{2p_i\cdot Q}[\frac{-y^2}{2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})}{(1+\sqrt{1-y}-\frac{y}{2})}] = \frac{Q^2}{2p_i\cdot Q}y \\
S_2 &\equiv \frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y} = 1-y
\end{aligned} \tag{12}$$

$$\boxed{\hat{Q}^{\mu} = \tfrac{Q^2}{2p_i\cdot Q} y\, {p_i}^{\mu} + (1-y)\, Q^{\mu}} \tag{13}$$

0.3 Single emission part

$$\begin{aligned}
{k_1}^{\mu} &= (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q})){p_i}^{\mu} + y\beta_1 Q^{\mu} + \sqrt{y\alpha_1\beta_1}{n^{\mu}}_{\perp,1} \\
{q_i}^{\mu} &= (\beta_1 - \alpha_1 y(\frac{Q^2}{2p_i\cdot Q})){p_i}^{\mu} + y\alpha_1 n^{\mu} - \sqrt{y\alpha_1\beta_1}{n^{\mu}}_{\perp,l} \\
{q_k}^{\mu} &= \alpha\Lambda^{\mu}{}_{\nu}{p_k}^{\nu} \qquad k=1,...,n \qquad k\neq i
\end{aligned} \tag{14}$$

$$\begin{aligned}
{k_1}^{\mu} &= \zeta_1 {p_i}^{\mu} + \lambda_1 Q^{\mu} + \sqrt{y\alpha_1\beta_1}{n^{\mu}}_{\perp,1} \\
{q_i}^{\mu} &= \zeta_q {p_i}^{\mu} + \lambda_q Q^{\mu} - \sqrt{y\alpha_1\beta_1}{n^{\mu}}_{\perp,l} \\
{q_k}^{\mu} &= A_1 {p_i}^{\mu} + A_2 Q^{\mu} + \sqrt{1-y}p^{\mu}_{k\perp,l}
\end{aligned}$$

$$
\begin{aligned}
\zeta_1\zeta_1 &= ({\alpha_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})^2) \\
\zeta_1\lambda_1 &= (y\alpha_1\beta_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})) \\
\zeta_1\zeta_q &= (\alpha_1\beta_1 - y({\alpha_1}^2+{\beta_1}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2) \\
\zeta_1\lambda_q &= (y{\alpha_1}^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i\cdot Q})) \\
\zeta_q\zeta_q &= ({\beta_1}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q})^2) \\
\zeta_q\lambda_1 &= (y{\beta_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})) \\
\zeta_q\zeta_1 &= (\beta_1\alpha_1 - y({\beta_1}^2+{\alpha_1}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2) \\
\zeta_q\lambda_q &= (y\beta_1\alpha_1 - y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q})) \\
\lambda_1\lambda_1 &= y^2{\beta_1}^2 \\
\lambda_1\zeta_q &= (y{\beta_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})) \\
\lambda_1\lambda_q &= y^2\beta_1\alpha_1 \\
\lambda_1\zeta_1 &= (y\beta_1\alpha_1 - y^2{\beta_1}^2(\frac{Q^2}{2p_i\cdot Q})) \\
\lambda_q\lambda_q &= y^2{\alpha_1}^2 \\
\lambda_q\lambda_1 &= y^2\alpha_1\beta_1 \\
\lambda_q\zeta_q &= (y\alpha_1\beta_1 - y^2{\alpha_1}^2(\frac{Q^2}{2p_i\cdot Q})) \\
\lambda_q\zeta_1 &= (y{\alpha_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))
\end{aligned}
\tag{15}
$$

0.4 Common scalar products

$$
\begin{aligned}
k_1\cdot q_i &= (\zeta_1\lambda_q + \lambda_1\zeta_q)p_i\cdot Q + \lambda_1\lambda_q Q^2 - y\alpha_1\beta_1 {n^2}_{\perp,1} \\
&= [(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q}))y\alpha_1 + y\beta_1(1-\alpha_1-\alpha_1(\frac{Q^2}{2p_i\cdot Q}))]\, p_i\cdot Q \\
&\quad y^2\beta_1\alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_iQ
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\Rightarrow k_1\cdot q_i &= [y{\alpha_1}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y\beta_1 - y\alpha_1\beta_1 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})]\, p_i\cdot Q \\
&\quad y^2\beta_1\alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_iQ
\end{aligned}
$$

$$
\boxed{k_1\cdot q_i = y(\alpha_1+\beta_1)\, p_i\cdot Q} \tag{17}
$$

$$\begin{aligned}k_1\cdot q_k &= (\zeta_1 A_2+\lambda_1 A_1)p_i\cdot Q+\zeta_1\sqrt{1-y}\,p_i\cdot p_k+\lambda_1 A_2\,Q^2+\lambda_1\sqrt{1-y}\,Q\cdot p_k\\ &+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}\\ &=\{[(\alpha_1-y\beta_1(\frac{Q^2}{2p_i\cdot Q}))\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]\\ &+y\beta_1[\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})}+\frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]\}\,p_i\cdot Q\\ &+(\alpha_1-y\beta_1(\frac{Q^2}{2p_i\cdot Q}))\sqrt{1-y}\,p_i\cdot p_k+y\beta_1\frac{(y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}Q^2\\ &+y\beta_1\sqrt{1-y}Q\cdot p_k+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}\end{aligned} \tag{18}$$

$$\begin{aligned}k_1\cdot q_k &= \alpha_1\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)-y\beta_1(\frac{Q^2}{2p_i\cdot Q})\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)\\ &+y\beta_1\frac{-y^2Q^2}{4(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)+y\beta_1\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}\,Q\cdot p_k\\ &+\alpha_1\sqrt{1-y}\,p_i\cdot p_k-y\beta_1(\frac{Q^2}{2p_i\cdot Q})\sqrt{1-y}\,p_i\cdot p_k\\ &+y\beta_1(\frac{Q^2}{2p_i\cdot Q})\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i\cdot p_k)+y\beta_1\sqrt{1-y}(Q\cdot p_k)\\ &+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}\end{aligned} \tag{19}$$

$$\begin{aligned}k_1\cdot q_k &= [\alpha_1\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+y\beta_1\frac{-y^2Q^2}{4(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}+\alpha_1\sqrt{1-y}\\ &-y\beta_1(\frac{Q^2}{2p_i\cdot Q})\sqrt{1-y}]\,p_i\cdot p_k+[y\beta_1\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+y\beta_1\sqrt{1-y}](Q\cdot p_k)\\ &+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}\end{aligned} \tag{20}$$

$$\begin{aligned}k_1\cdot q_k &= \{\alpha_1[\frac{(y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+\sqrt{1-y}]\\ &+y\beta_1(\frac{Q^2}{p_i\cdot Q})[\frac{-y^2}{4(1+\sqrt{1-y}-\frac{y}{2})}-\sqrt{1-y}]\}\,p_i\cdot p_k\\ &+y\beta_1[\frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}+\sqrt{1-y}](Q\cdot p_k)\\ &+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}\end{aligned} \tag{21}$$

$$\boxed{k_1\cdot q_k=[\alpha_1(1-y)+y\beta_1(\tfrac{Q^2}{2p_i\cdot Q})(y-2)]\,p_i\cdot p_k+y\beta_1\,Q\cdot p_k+\sqrt{\alpha_1\beta_1 y(1-y)}p_k\cdot n_{\perp,1}} \tag{22}$$

0.5 Altarelli-Parisi splitting functions

$$\left.\begin{aligned}
\langle \hat{P_{qq}}\rangle &= C_F[\frac{1+z^2}{1-z} - \varepsilon(1-z)] \\
\langle \hat{P_{gq}}\rangle &= T_R[1-\frac{2z(1-z)}{1-\varepsilon}] \\
\langle \hat{P_{qg}}\rangle &= C_F[\frac{1+(1-z)^2}{z} - \varepsilon z] \\
\langle \hat{P_{gg}}\rangle &= 2C_A[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)]
\end{aligned}\right\} \text{splitting functions} \tag{23}$$

0.6 Colour factor calculation

fundamental representation in $SU(2)$ and $SU(3)$

$$\begin{aligned} T^a = \tau^a \equiv \frac{\sigma^2}{2} \qquad & \textit{with Pauli matrices } \sigma^a \\ T^a = \vartheta^a \equiv \frac{\lambda^2}{2} \qquad & \textit{with Gell-Mann matrices } \lambda^a \end{aligned} \tag{24}$$

$$\begin{gathered} \lambda^1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix} \\ \lambda^5 = \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{gathered} \tag{25}$$

As we can see, λ^3 and λ^8 are diagonal. These generators satisfy:

$$[T^a, T^b] = i\epsilon^{abc} T^c \tag{26}$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \tag{27}$$

The main relation we will use later for SU(N):

$$tr(T^a T^b) = T_{ij}{}^a T_{ji}{}^b = T_F \delta^{ab} \tag{28}$$

$$\sum_a (T^a T^a) = C_F \delta^{ij} \tag{29}$$

$$f^{acd} f^{bcd} = C_A \delta^{ab} \tag{30}$$

With $T_F = \frac{1}{2}$, $C_A = N$ and $C_F = \frac{N^2-1}{2N}$.

$$f^{abc} = -2itr(T^a[T^b, T^c]) \tag{31}$$

$$d^{abc} = 2tr(T^a T^b, T^c) \tag{32}$$

$$T^a T^b = \frac{1}{2}(\frac{1}{N}\delta_{ab} + (d^{abc} + if^{abc})T^c) \tag{33}$$

$$tr(T^a T^b T^c) = \frac{1}{4}(d^{abc} + if^{abc}) \tag{34}$$

$$tr(T^aT^bT^aT^c) = \frac{-1}{4N}\delta_{bc} \tag{35}$$

$$f^{acd}f^{bcd} = N\delta^{ab} \tag{36}$$

$$f^{acd}d^{bcd} = 0 \tag{37}$$

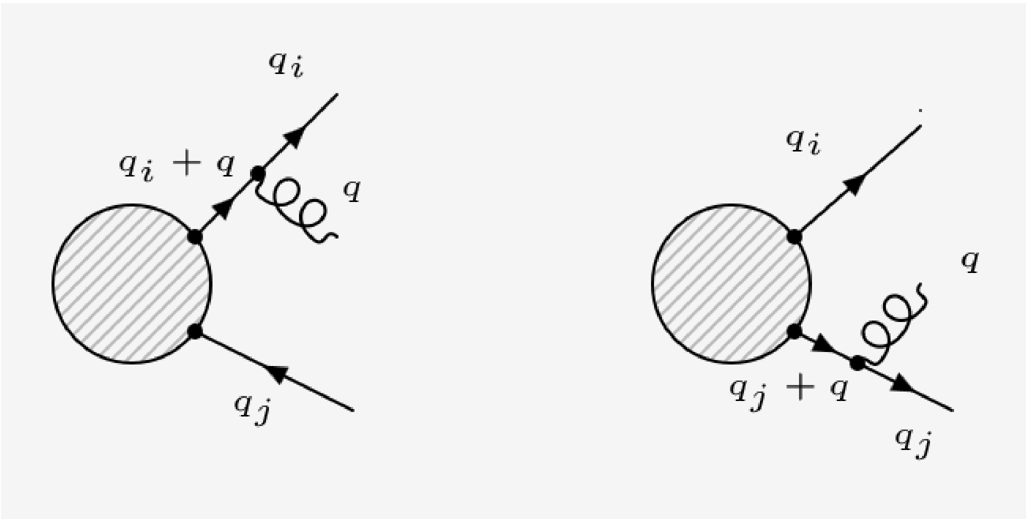
$$f^{ade}f^{bef}f^{cfd} = \frac{N}{2}f^{abc} \tag{38}$$

Fierz identity:

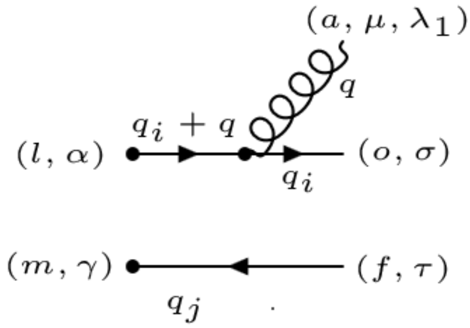
$$\sum_a T_{ij}{}^a T_{kl}{}^a = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}) \tag{39}$$

Chapter 1

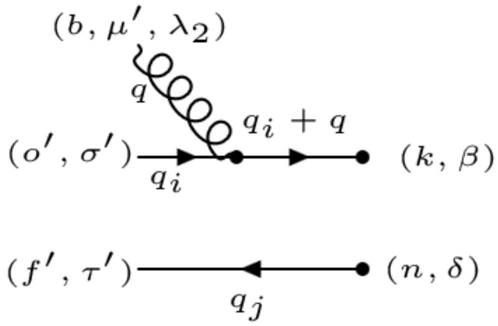
Quark antiquark gluon emission kernel



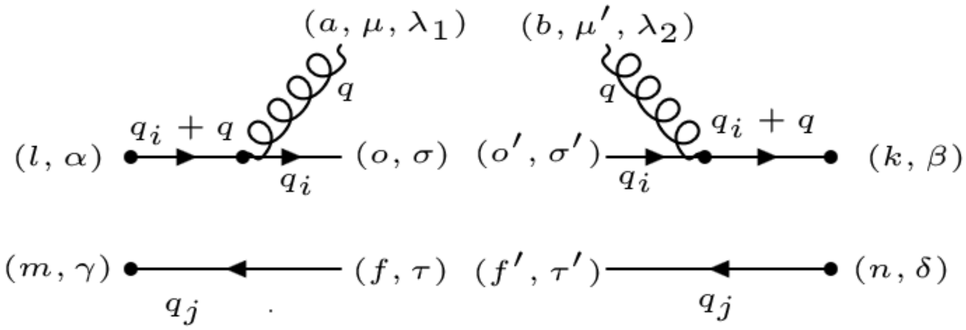
1.1 qg-$\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{1.1}$$



$$M_1{}^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{1.2}$$



$$|M_1|^2 = M_1\, M_1{}^\dagger = [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\;\varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)][\bar{v}_{\tau'}(q_j)] \tag{1.3}$$

$$|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\;(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q)$$
$$\times(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j)v_\tau(q_j)] \tag{1.4}$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) = \not{q}_i\delta^{oo'},$$
$$\sum_{\tau,\tau'} \bar{v}_\tau(q_j)\,v_{\tau'}(q_j) = \not{q}_j\delta^{ff'} \tag{1.5}$$

and sum over polarization index (λ_1, λ_2) :

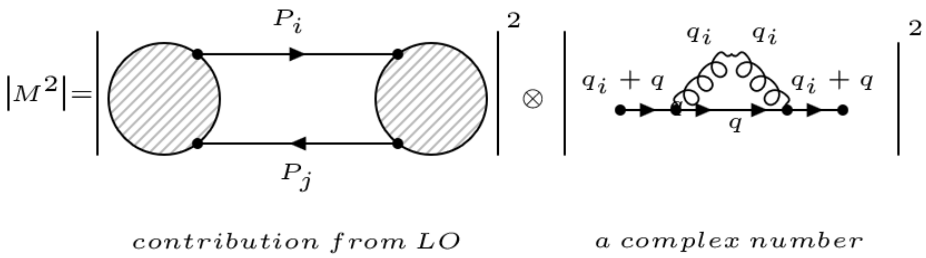
$$\sum_{\mu,\mu'} \varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q) = -g_{\mu\mu'}\delta^{ab} \tag{1.6}$$

$$|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,g_{\mu'\mu}\gamma^\mu\,(\not{q}_i + q)][\not{q}_j] \tag{1.7}$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'}\,(\not{q}_i + q)][\not{q}_j] \tag{1.8}$$

In other words we expect the tree level diagram from LO and a number: Which means:



$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2 (q_i+q)^2} [\not P_i][\not P_j] \ \otimes \ (a\ complex\ number) \tag{1.9}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'} \ \not q_i \, \gamma_{\mu'} &= q_{i\sigma} \, \gamma^{\mu'} \gamma^\sigma \ \gamma_{\mu'} \\ &= q_{i\sigma} \, (\{\gamma^{\mu'}, \gamma^\sigma\} \ - \gamma^\sigma \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} \, 2 g^{\mu' \sigma} \, \gamma_{\mu'} \ - \ d \, \gamma^\sigma \\ &= (2-d) \ \not q_i \end{aligned} \tag{1.10}$$

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2 (q_i+q)^2} [(\not q_i + \not q) \quad \not q_i \ \ (\not q_i + q)][\not q_j] \tag{1.11}$$

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2 (q_i+q)^2} [\not q_i \ \not q_i \ \not q_i + \ \not q_i \ \not q_i \ \not q + \ \not q \ \not q_i \ \not q_i + \ \not q \ \not q_i \ \not q][\not q_j] \tag{1.12}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not q_i \ \not q_i &= {q_i}^2 = {m_i}^2 \\ \not q \ \not q &= q^2 = m^2 \\ \not q_j \ \not q_j &= {q_j}^2 = {m_j}^2 \end{aligned} \tag{1.13}$$

we can first neglect the mass of patrons and ignore each term with $\not q_i \ \not q_i$ and $\not q \ \not q$ as well.

$$|M_1|^2 = -(2-d) \ \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(2 q_i q)(2 q_i q)} [\not q \ \not q_i \ \not q][\not q_j] \tag{1.14}$$

$$
\begin{aligned}
L = \not{q}\ \not{q}_i\ \not{q} = & \not{q}[q_{i\sigma} q_\mu\ (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu)] \\
& \not{q}[2{q_i}^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma \\
& = \not{q}(2 q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
& = \not{q}(2 q_i q) - q_\mu q_{i\sigma} q_\mu [\frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2}] \gamma^\sigma \\
& = \not{q}(2 q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\
& = \not{q}(2 q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
& = \not{q}(2 q_i q) - q^2\ \not{q}_i \\
& = \not{q}(2 q_i q)
\end{aligned}
\tag{1.15}
$$

After inserting the last result of L and simplify the term $(2q_iq)$ from the denominator and nominator because , we get:

$$
|M_1|^2 = -(2-d)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{q}][\not{q}_j]
\tag{1.16}
$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$
|M_1|^2 = (d-2)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][(1-y)\ \not{p}_j]
\tag{1.17}
$$

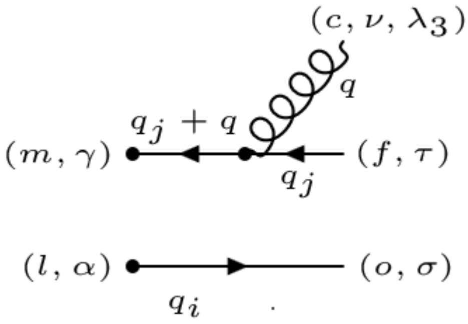
Multiplying the both sides

$$
\begin{aligned}
|M_1|^2 = (d-2)\ & \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \\
& + zy(1-y)\ \not{p}_j\ \not{p}_j + (1-y)\sqrt{zy(1-z)}\ \not{m}_\perp\ \not{p}_j]
\end{aligned}
\tag{1.18}
$$

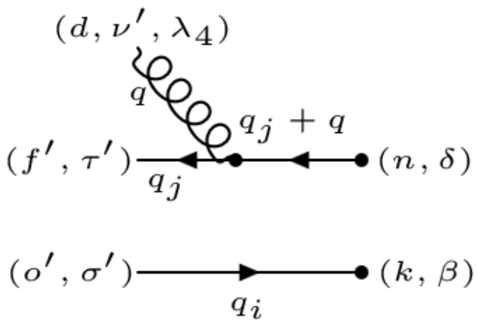
and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\ \not{p}_i$ and $\not{p}_j\ \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$
|M_1|^2 = (d-2)(1-z)(1-y)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]
\tag{1.19}
$$

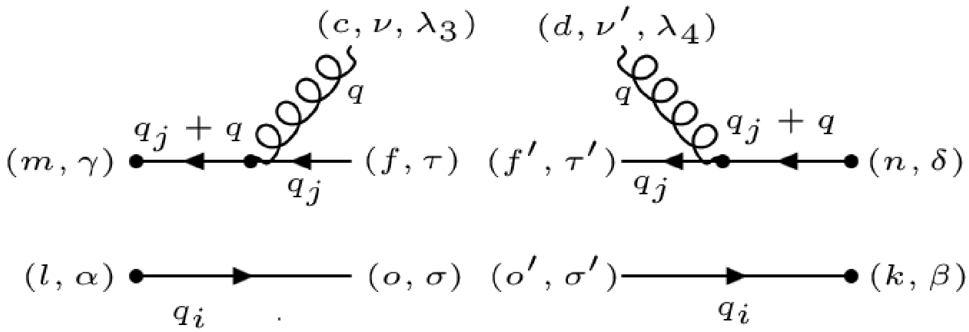
1.2 $\bar{q}$g-q



$$M_2 = [\frac{i(\not{q}_j + \not{q})}{(q_j + q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \tag{1.20}$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \tag{1.21}$$



$$\begin{aligned} |M_2|^2 = M_2\, M_2{}^\dagger = &[\frac{i(\not{q}_j + \not{q})}{(q_j + q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \\ &[\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \end{aligned} \tag{1.22}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu\, v_\tau(q_j)\bar{v}_{\tau'}(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_j + \not{q})] \\ [u_\sigma(q_i)]\,[\bar{u}_{\sigma'}(q_i)] \tag{1.23}$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\begin{aligned} \sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i) &= \not{q}_i \delta^{oo'}, \\ \sum_{\tau,\tau'} \bar{v}_\tau(q_j)\, v_{\tau'}(q_j) &= \not{q}_j \delta^{ff'} \end{aligned} \tag{1.24}$$

and sum over polarization index (λ_3, λ_4) :

$$\sum_{\nu,\nu'} \varepsilon^{\lambda_4 *}{}_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'}\delta^{cd} \tag{1.25}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\gamma^\nu \;\; \not{q}_j\, (-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_j + \not{q})]\,[\not{q}_i] \tag{1.26}$$

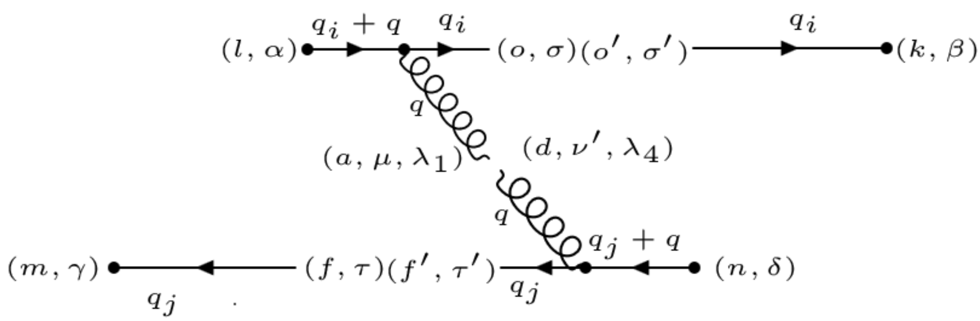
After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{(2qq_j)}[\not{q}]\,[\not{q}_i] \tag{1.27}$$

finally:

$$|M_2|^2 = -(d-2)yz^2\, \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2(1-z)(1-y)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.28}$$

1.3 $M_1 M_2{}^\dagger$



$$M_1\, M_2{}^\dagger = [\bar{u}_\sigma(q_i)\, (-ig_s\gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\, \varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [u_{\sigma'}(q_i)] \tag{1.29}$$

$$M_1\, M_2{}^\dagger = \frac{g_s^2 [T^a]_o{}^l\, [T^d]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,]\varepsilon^{\lambda_1}{}_\mu(q)\, \varepsilon^{\lambda_4}{}_{\nu'}(q)$$
$$[\not{q}_j\, \gamma^{\nu'}\, (\not{q}_j + \not{q})] \tag{1.30}$$

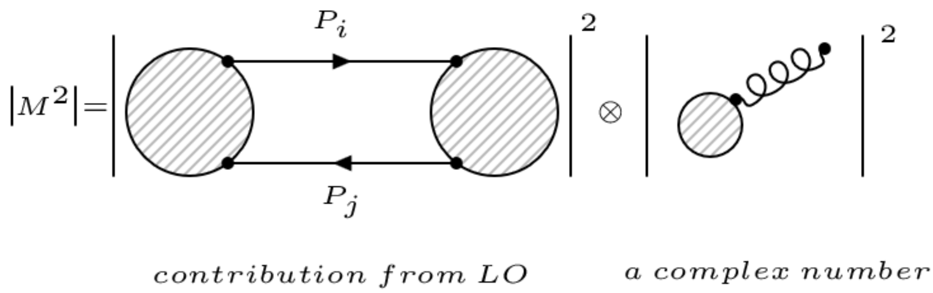
$$M_1\, M_2{}^\dagger = \frac{g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,] - g_{\mu\nu'}$$
$$[\not{q}_j\, \gamma^{\nu'}\, (\not{q}_j + \not{q})] \tag{1.31}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [\not{q}_i\, \gamma^\mu\, (\not{q}_i + \not{q})\,]\, [\not{q}_j\, \gamma_\mu\, (\not{q}_j + \not{q})] \tag{1.32}$$

Expectation:

$$M_1\, M_2{}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [(\not{q}_i + \not{q})\, \gamma^\mu \quad \not{q}_i\,]\, [(\not{q}_j + \not{q})\, \gamma_\mu \quad \not{q}_j] \tag{1.33}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} [-(\not{q}_i + \not{q}) \quad \not{q}_i\, \gamma^\mu \; + 2(\not{q}_i + \not{q})\, q_i{}^\mu]$$
$$[-(\not{q}_j + \not{q}) \quad \not{q}_j\, \gamma_\mu \; + 2(\not{q}_j + \not{q}) q_{j\mu}] \tag{1.34}$$



$$
\begin{aligned}
M_1\, M_2{}^\dagger = &\frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} \\
&[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q})\ \not{q}_j \gamma_\mu] \\
&-2[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q}) q_{j\mu}] \\
&-2[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q})\ \not{q}_j\, \gamma_\mu] \\
&+4[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q}) q_{j\mu}]
\end{aligned}
\tag{1.35}
$$

$$
\begin{aligned}
M_1\, M_2{}^\dagger = &\frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} \\
&[\not{q}\ \not{q}_i\, \gamma^\mu]\, [\not{q}\ \not{q}_j \gamma_\mu] \\
&-2[\not{q}\ \not{q}_i\, \gamma^\mu]\, [(\not{q} + \not{q}_j)\ q_{j\mu}] \\
&-2[(\not{q}_i + \not{q})\, q_i{}^\mu][\not{q}\ \not{q}_j\, \gamma_\mu] \\
&+4[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q}) q_{j\mu}]
\end{aligned}
\tag{1.36}
$$

$$
\begin{aligned}
M_1\, M_2{}^\dagger = &\frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \\
&[y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\, \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu] \\
&+4(q_i{}^\mu \cdot q_{j\mu})[(\not{q}_i + \not{q})][(\not{q}_j + \not{q})]
\end{aligned}
\tag{1.37}
$$

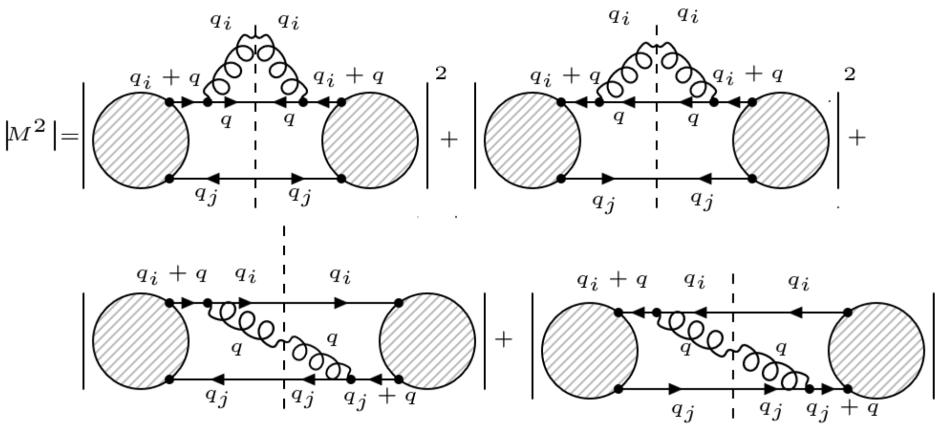
$$
\begin{aligned}
M_1\, M_2{}^\dagger = &\frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \\
&[y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\, \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu] \\
&+4(p_i \cdot p_j)[(\not{p}_i + y\ \not{p}_j)][(1-z)\ \not{p}_i + (1+yz-y)\ \not{p}_j - \sqrt{zy(1-z)}\ \not{n}]
\end{aligned}
\tag{1.38}
$$

$$
M_1\, M_2{}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)} z(1-y)[\not{p}_i][\not{p}_j]
\tag{1.39}
$$

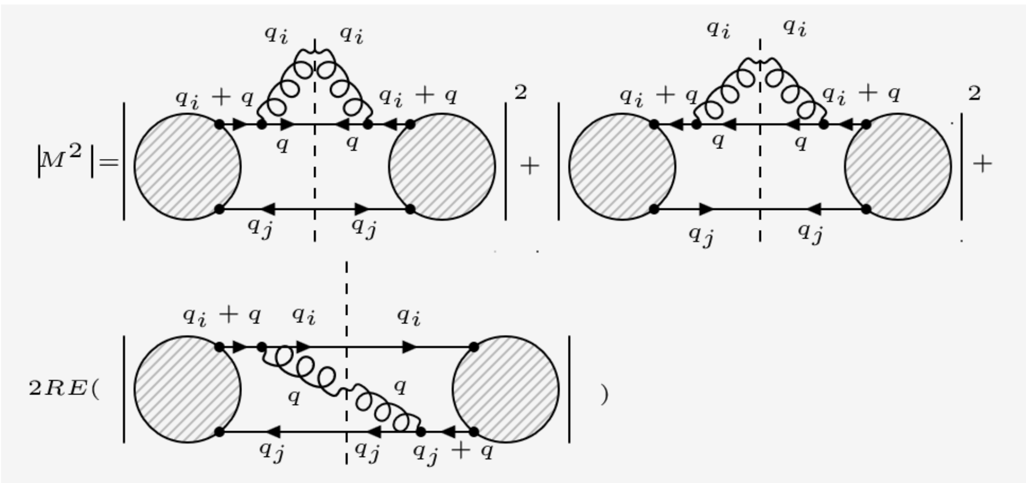
$$
M_1\, M_2{}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)y(1-2z+2z^2)(p_i \cdot p_j)} z[\not{p}_i][\not{p}_j]
\tag{1.40}
$$

1.4 $|M^2|$

$$| M|^2 = | M_1|^2 + | M_2|^2 + M_1\, M_2{}^{\dagger} + M_1{}^{\dagger} M_2 \tag{1.41}$$



$$| M|^2 = | M_1|^2 + | M_2|^2 + 2RE(M_1\, M_2{}^{\dagger}) \tag{1.42}$$



$$\begin{aligned}
| M|^2 = (d-2)(1-z)(1-y)\, &\frac{g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
-(d-2)yz^2\, &\frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
+2RE((\frac{-2z}{z-1})&\frac{g_s^2\; [T^a]_o{}^l\, [T^a]_f{}^n}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j])
\end{aligned} \tag{1.43}$$

$$T^a{}_{o\,k}\, T^a{}_{l\,o} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F\delta_{lk} \tag{1.44}$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a{}_{o\,k}\,T^a{}_{l\,o} = C_F\delta_{lk} = \frac{1}{N}\sum_{l=1}^{N}\delta_{lk}C_F = C_F \tag{1.45}$$

The same calculation for $T^c{}_{m\,f}\,T^c{}_{f\,n}$ and $T^a{}_{o\,l}\,T^a{}_{f\,n}$ turns C_F out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, wich means, if:

$$y \longrightarrow 0 \tag{1.46}$$

$$\begin{aligned}
|\,M|^2 = (d-2)(1-z)(1-y)\,&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
-(d-2)yz^2\,&\frac{g_s^2\,C_F}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
+2RE((\frac{-2z}{z-1})&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j]
\end{aligned} \tag{1.47}$$

$$|\,M|^2 = C_F((d-2)(1-z) - \frac{4z}{z-1})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.48}$$

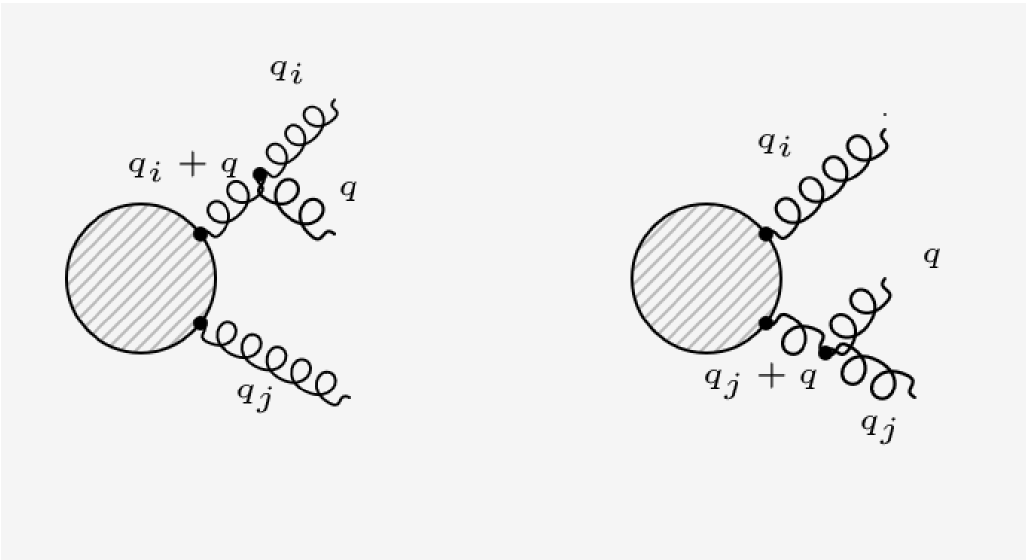
for

$$d = 4-2\epsilon \tag{1.49}$$

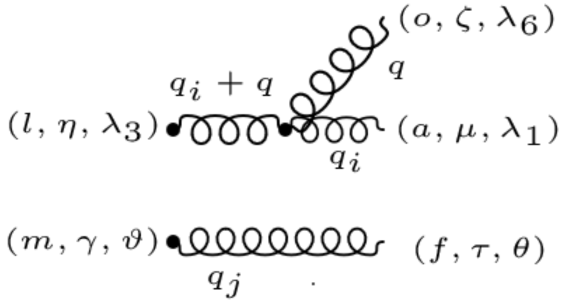
$$\begin{aligned}
|\,M|^2 = C_F((4-2\epsilon-2)(1-z)+\frac{4z}{1-z})\,&\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
= C_F(\frac{2(1-\epsilon)(1-z)^2+4z}{1-z})\,&\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
C_F(\frac{2-4z+2z^2-\epsilon(1-z)^2+4z}{1-z})\,&\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
= C_F(\frac{(1+z^2)}{1-z}-\epsilon(1-z))\,&\frac{g_s^2}{y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
= \langle\,\hat{P}_{qq}\rangle\,&\frac{g_s^2}{q_i\cdot q}[\not{p}_i][\not{p}_j]
\end{aligned} \tag{1.50}$$

Chapter 2

Gluon gluon gluon emission kernel

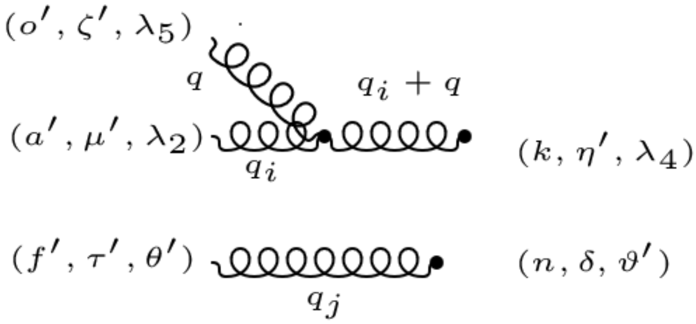


2.1 Gluon-Emitter Bubble



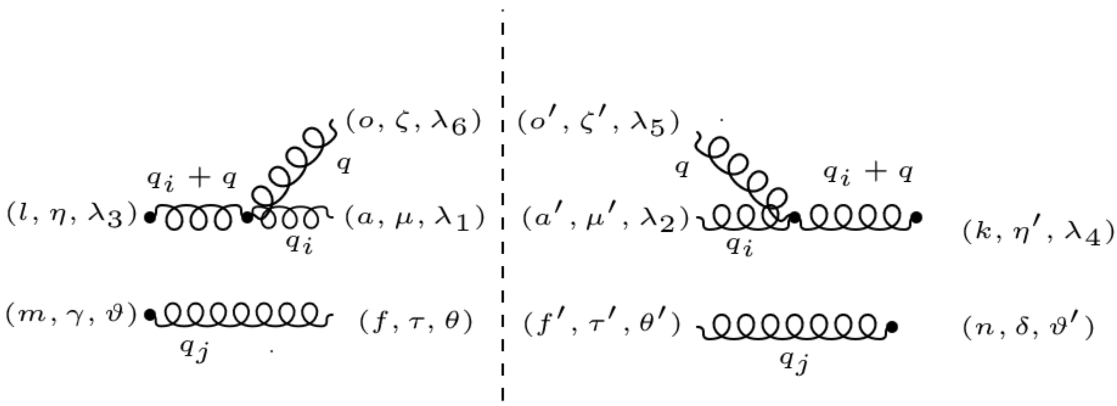
$$M_1 = [\frac{-i}{(q+q_i)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} + g^{\zeta\eta}(-q-(q+q_i))^{\mu} + g^{\eta\mu}(q_i+q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q)\varepsilon^{\lambda_6}{}_{\zeta}(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \tag{2.1}$$

$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q+q_i)^{\mu} + g^{\eta\mu}(2q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \tag{2.2}$$



$$M_1{}^{\dagger} = [\frac{i}{(q_i+q)^2}(-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'}) \\ \varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q_i)\varepsilon^{\lambda_5}{}_{\zeta'}{}^{*}(q)][\varepsilon^{\theta'}{}_{\tau'}{}^{*}(q_j)] \tag{2.3}$$

$$|M_1|^2 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q+q_i)^{\mu} + g^{\eta\mu}(2q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)\,\varepsilon^{\lambda_5}{}_{\zeta'}{}^{*}(q) \\ (-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'})\frac{i}{(q_i+q)^2}][g_{\gamma\delta}] \tag{2.4}$$



$$\begin{aligned} N \equiv g_{\mu\mu'} g_{\zeta\zeta'} [& -g^{\mu\zeta} g^{\mu'\eta'} (q-q_i)^{\eta} (2q_i+q)^{\zeta'} + g^{\mu\zeta} g^{\eta'\zeta'} (q-q_i)^{\eta} (2q+q_i)^{\mu'} \\ & + g^{\mu\zeta} g^{\zeta'\mu'} (q-q_i)^{\eta} (q_i-q)^{\eta'} + g^{\zeta\eta} g^{\mu'\zeta'} (2q+q_i)^{\mu} (2q_i+q)^{\zeta'} \\ & - g^{\zeta\eta} g^{\eta'\zeta'} (2q+q_i)^{\mu} (2q+q_i)^{\mu'} - g^{\zeta\eta} g^{\zeta'\mu'} (2q+q_i)^{\mu} (q_i-q)^{\eta'} \\ & - g^{\eta\mu} g^{\mu'\eta'} (2q_i+q)^{\zeta} (2q_i+q)^{\zeta'} + g^{\eta\mu} g^{\eta'\zeta'} (2q_i+q)^{\zeta} (2q+q_i)^{\mu'} \\ & + g^{\eta\mu} g^{\zeta'\mu'} (2q_i+q)^{\zeta} (q_i-q)^{\eta'}][g_{\gamma\delta}] \end{aligned} \tag{2.5}$$

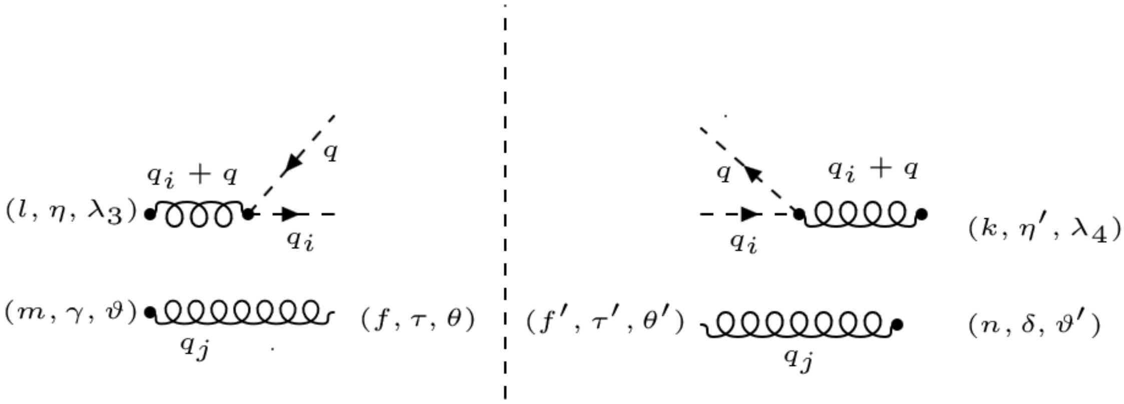
$$\begin{aligned} N \equiv [& -(q-q_i)^{\eta} (2q_i+q)^{\eta'} + (q-q_i)^{\eta} (2q+q_i)^{\eta'} + d(q-q_i)^{\eta} (q_i-q)^{\eta'} \\ & + (2q+q_i)^{\eta'} (2q_i+q)^{\eta} - g^{\eta\eta'} (2q+q_i)^{\mu} (2q+q_i)_{\mu} - (2q+q_i)^{\eta} (q_i-q)^{\eta'} \\ & - g^{\eta\eta'} (2q_i+q)^{\zeta} (2q_i+q)_{\zeta} + (2q_i+q)^{\eta'} (2q+q_i)^{\eta} + (2q_i+q)^{\eta} (q_i-q)^{\eta'}][g_{\gamma\delta}] \end{aligned} \tag{2.6}$$

$$\begin{aligned} N \equiv [& -(q^{\eta} q^{\eta'} + 2q^{\eta} {q_i}^{\eta'} - {q_i}^{\eta} q^{\eta'} - 2{q_i}^{\eta} {q_i}^{\eta'}) + (2q^{\eta} q^{\eta'} + q^{\eta} {q_i}^{\eta'} - 2{q_i}^{\eta} q^{\eta'} - {q_i}^{\eta} {q_i}^{\eta'}) \\ & + (dq^{\eta} {q_i}^{\eta'} - dq^{\eta} q^{\eta'} - d{q_i}^{\eta} {q_i}^{\eta'} + d{q_i}^{\eta} q^{\eta'}) + (4q^{\eta'} {q_i}^{\eta} + 2q^{\eta'} q^{\eta} + 2{q_i}^{\eta'} {q_i}^{\eta} + {q_i}^{\eta'} q^{\eta}) \\ & -(-2q^{\eta} q^{\eta'} + 2q^{\eta} {q_i}^{\eta'} - {q_i}^{\eta} q^{\eta'} + {q_i}^{\eta} {q_i}^{\eta'}) + (2q^{\eta'} q^{\eta} + q^{\eta'} {q_i}^{\eta} + 4{q_i}^{\eta'} q^{\eta} + 2{q_i}^{\eta'} {q_i}^{\eta}) \\ & + (-q^{\eta} q^{\eta'} + q^{\eta} {q_i}^{\eta'} - 2{q_i}^{\eta} q^{\eta'} + 2{q_i}^{\eta} {q_i}^{\eta'}) - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)][g_{\gamma\delta}] \end{aligned} \tag{2.7}$$

$$\begin{aligned} N \equiv [& (6-d) q^{\eta} q^{\eta'} + (d+3) q^{\eta} {q_i}^{\eta'} + (d+3) {q_i}^{\eta} q^{\eta'} + (6-d) {q_i}^{\eta} {q_i}^{\eta'} \\ & - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)][g_{\gamma\delta}] \end{aligned} \tag{2.8}$$

$$\begin{aligned} |M_1|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2 (q_i+q)^2} [& (6-d) q^{\eta} q^{\eta'} + (d+3) q^{\eta} {q_i}^{\eta'} + (d+3) {q_i}^{\eta} q^{\eta'} + (6-d) {q_i}^{\eta} {q_i}^{\eta'} \\ & - g^{\eta\eta'} (5q^2 + 5{q_i}^2 + 8qq_i)][g_{\gamma\delta}] \end{aligned} \tag{2.9}$$

2.1.1 One-loop corrections to the gluon self-energy diagram(Gluon-Emitter Bubble)



$$|M_1|^2_{Ghost\ loop} = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[-{q_i}^{\eta} q^{\eta'} - q^{\eta} {q_i}^{\eta'}][g_{\gamma\delta}] \tag{2.10}$$

$$\begin{aligned} |M_1'|^2 &= |M_1|^2 + |M_1|^2_{Ghost\ loop} \\ &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[(6-d)q^{\eta} q^{\eta'} + (d+3)q^{\eta} {q_i}^{\eta'} \\ +(d+3){q_i}^{\eta} q^{\eta'} &+ (6-d){q_i}^{\eta}{q_i}^{\eta'} - g^{\eta\eta'}(5q^2 + 5{q_i}^2 + 8qq_i) - {q_i}^{\eta} q^{\eta'} - q^{\eta}{q_i}^{\eta'}][g_{\gamma\delta}] \end{aligned} \tag{2.11}$$

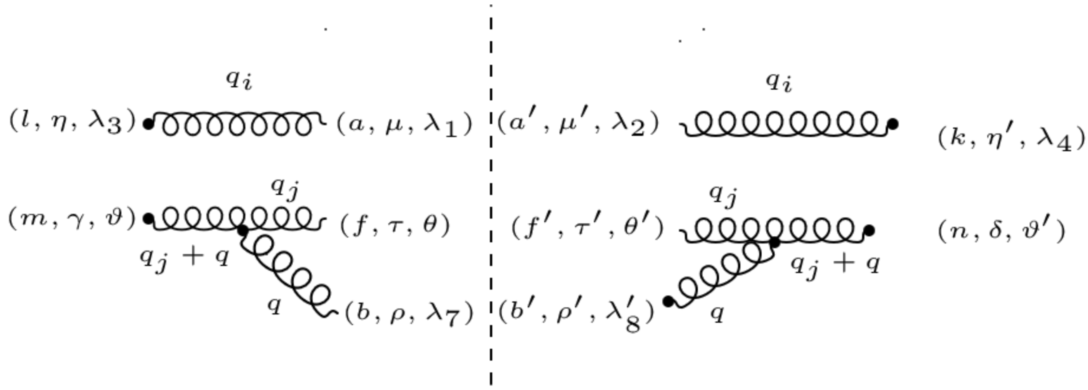
$$\begin{aligned} |M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}&[(6-d)q^{\eta} q^{\eta'} + (d+2)q^{\eta}{q_i}^{\eta'} \\ &+(d+2){q_i}^{\eta} q^{\eta'} + (6-d){q_i}^{\eta}{q_i}^{\eta'} - g^{\eta\eta'}(8qq_i)][g_{\gamma\delta}] \end{aligned} \tag{2.12}$$

$$\begin{aligned} |M_1'|^2 &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{y^2(\alpha_1+\beta_1)^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\ [(6-d)(\zeta_1 {p_i}^{\eta} + \lambda_1 Q^{\eta} + \sqrt{y\alpha_1\beta_1} n^{\eta}{}_{\perp,1})&(\zeta_1 {p_i}^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n^{\eta'}{}_{\perp,1}) \\ +(d+2)(\zeta_1 {p_i}^{\eta} + \lambda_1 Q^{\eta} + \sqrt{y\alpha_1\beta_1} n^{\eta}{}_{\perp,1})&(\zeta_q {p_i}^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n^{\eta'}{}_{\perp,1}) \\ +(d+2)(\zeta_q {p_i}^{\eta} + \lambda_q Q^{\eta} - \sqrt{y\alpha_1\beta_1} n^{\eta}{}_{\perp,1})&(\zeta_1 {p_i}^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n^{\eta'}{}_{\perp,1}) \\ +(6-d)(\zeta_q {p_i}^{\eta} + \lambda_q Q^{\eta} - \sqrt{y\alpha_1\beta_1} n^{\eta}{}_{\perp,1})&(\zeta_q {p_i}^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n^{\eta'}{}_{\perp,1}) \\ &-g^{\eta\eta'}(8y(\alpha_1+\beta_1)\,p_i\cdot Q)][g_{\gamma\delta}] \end{aligned} \tag{2.13}$$

$$
\begin{aligned}
|M_1'|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{y^2(\alpha_1+\beta_1)^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
[(6-d)[\zeta_1\zeta_1 {p_i}^{\eta}{p_i}^{\eta'} + \zeta_1\lambda_1 {p_i}^{\eta}Q^{\eta'} + \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
+\lambda_1\zeta_1 Q^{\eta}{p_i}^{\eta'} + \lambda_1\lambda_1 Q^{\eta}Q^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
+\zeta_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
[(d+2)[\zeta_1\zeta_q {p_i}^{\eta}{p_i}^{\eta'} + \zeta_1\lambda_q {p_i}^{\eta}Q^{\eta'} - \zeta_1\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
+\lambda_1\zeta_q Q^{\eta}{p_i}^{\eta'} + \lambda_1\lambda_q Q^{\eta}Q^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
+\zeta_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} + \lambda_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
[(d+2)[\zeta_q\zeta_1 {p_i}^{\eta}{p_i}^{\eta'} + \zeta_q\lambda_1 {p_i}^{\eta}Q^{\eta'} + \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
+\lambda_q\zeta_1 Q^{\eta}{p_i}^{\eta'} + \lambda_q\lambda_1 Q^{\eta}Q^{\eta'} + \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
-\zeta_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}] \\
[(6-d)[\zeta_q\zeta_q {p_i}^{\eta}{p_i}^{\eta'} + \zeta_q\lambda_q {p_i}^{\eta}Q^{\eta'} - \zeta_q\sqrt{y\alpha_1\beta_1}{p_i}^{\eta}{n^{\eta'}}_{\perp,1} \\
+\lambda_q\zeta_q Q^{\eta}{p_i}^{\eta'} + \lambda_q\lambda_q Q^{\eta}Q^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}Q^{\eta}{n^{\eta'}}_{\perp,1} \\
-\zeta_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{p_i}^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}{n^{\eta}}_{\perp,1}{n^{\eta'}}_{\perp,1}]
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
|M_1'|^2 = {} & \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{y^2(\alpha_1+\beta_1)^2\,(p_i\cdot Q)\,(p_i\cdot Q)} \\
& [(6-d)[(\alpha_1{}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2\beta_1{}^2(\frac{Q^2}{2p_i\cdot Q})^2)p_i{}^\eta p_i{}^{\eta'} \\
& +(y\alpha_1\beta_1 - y^2\beta_1{}^2(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\eta Q^{\eta'} + \zeta_1\sqrt{y\alpha_1\beta_1}p_i{}^\eta n^{\eta'}{}_{\perp,1} \\
& +(y\beta_1\alpha_1 - y^2\beta_1{}^2(\frac{Q^2}{2p_i\cdot Q}))Q^\eta p_i{}^{\eta'} + y^2\beta_1{}^2Q^\eta Q^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}Q^\eta n^{\eta'}{}_{\perp,1} \\
& +\zeta_1\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}p_i{}^{\eta'} + \lambda_1\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}Q^{\eta'} + \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1}] \\
& [(d+2)[(\alpha_1\beta_1 - y(\alpha_1{}^2+\beta_1{}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2)p_i{}^\eta p_i{}^{\eta'} \\
& +(y\alpha_1{}^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\eta Q^{\eta'} - \zeta_1\sqrt{y\alpha_1\beta_1}p_i{}^\eta n^{\eta'}{}_{\perp,1} \\
& +(y\beta_1{}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))Q^\eta p_i{}^{\eta'} + y^2\beta_1\alpha_1Q^\eta Q^{\eta'} \\
& -\lambda_1\sqrt{y\alpha_1\beta_1}Q^\eta n^{\eta'}{}_{\perp,1} + \zeta_q\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}p_i{}^{\eta'} \\
& +\lambda_q\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1}] \\
& [(d+2)[(\beta_1\alpha_1 - y(\beta_1{}^2+\alpha_1{}^2)(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})^2)p_i{}^\eta p_i{}^{\eta'} \\
& +(y\beta_1{}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\eta Q^{\eta'} + \zeta_q\sqrt{y\alpha_1\beta_1}p_i{}^\eta n^{\eta'}{}_{\perp,1} \\
& +(y\alpha_1{}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}))Q^\eta p_i{}^{\eta'} + y^2\alpha_1\beta_1Q^\eta Q^{\eta'} \\
& +\lambda_q\sqrt{y\alpha_1\beta_1}Q^\eta n^{\eta'}{}_{\perp,1} \\
& -\zeta_1\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}p_i{}^{\eta'} - \lambda_1\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}Q^{\eta'} - \sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1}] \\
& [(6-d)[(\beta_1{}^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y^2\alpha_1{}^2(\frac{Q^2}{2p_i\cdot Q})^2)p_i{}^\eta p_i{}^{\eta'} \\
& +(y\beta_1\alpha_1 - y^2\alpha_1{}^2(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\eta Q^{\eta'} - \zeta_q\sqrt{y\alpha_1\beta_1}p_i{}^\eta n^{\eta'}{}_{\perp,1} \\
& +(y\alpha_1\beta_1 - y^2\alpha_1{}^2(\frac{Q^2}{2p_i\cdot Q}))Q^\eta p_i{}^{\eta'} + y^2\alpha_1{}^2Q^\eta Q^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}Q^\eta n^{\eta'}{}_{\perp,1} \\
& -\zeta_q\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}p_i{}^{\eta'} - \lambda_q\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}Q^{\eta'} \\
& +\sqrt{y\alpha_1\beta_1}\sqrt{y\alpha_1\beta_1}n^\eta{}_{\perp,1}n^{\eta'}{}_{\perp,1}]
\end{aligned} \tag{2.15}
$$

2.2 Gluon-Spectator Bubble



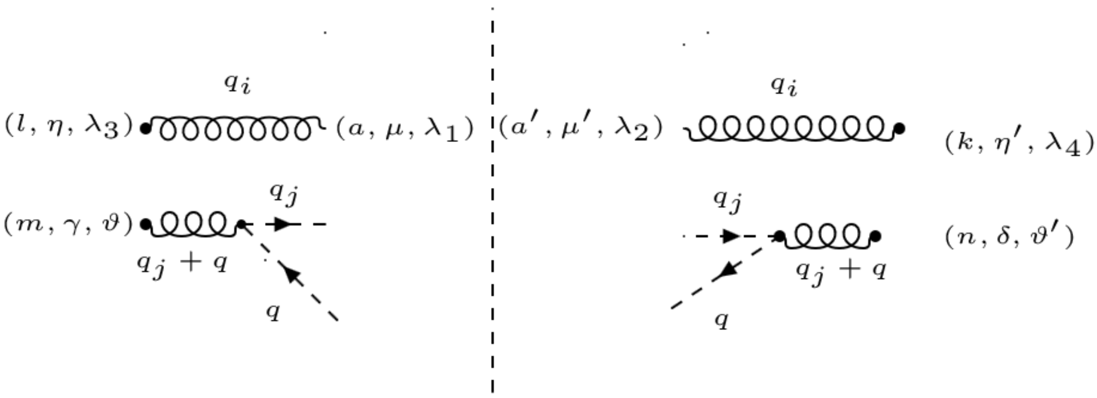
$$|M_2|^2 = [\frac{-i}{(q_j+q)^2}(-g_s f^{\,b\,f\,m}(g^{\tau\gamma}(-2q_j-q)^\rho + g^{\gamma\rho}(2q+q_j)^\tau + g^{\rho\tau}(q_j-q)^\gamma)$$
$$g_{\tau\tau'}g_{\rho\rho'}(-g_s f^{\,b'\,n\,f'}(g^{\rho'\delta}(-2q-q_j)^{\tau'} + g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\tau'\rho'}(q-q_j)^\delta)\frac{i}{(q_j+q)^2}][g_{\eta\eta'}] \tag{2.16}$$

$$|M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b'\,n\,f'}\delta^{aa'}\delta^{ff'}\delta^{bb'}}{(q_j+q)^2(q_j+q)^2}[g_{\tau\tau'}g_{\rho\rho'}(g^{\tau\gamma}(2q_j+q)^\rho g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$-g^{\tau\gamma}(2q_j+q)^\rho g^{\delta\tau'}(2q_j+q)^{\rho'} - g^{\tau\gamma}(2q_j+q)^\rho g^{\tau'\rho'}(q-q_j)^\delta - g^{\gamma\rho}(2q+q_j)^\tau g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$+g^{\gamma\rho}(2q+q_j)^\tau g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\gamma\rho}(2q+q_j)^\tau g^{\tau'\rho'}(q-q_j)^\delta - g^{\rho\tau}(q_j-q)^\gamma g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$+g^{\rho\tau}(q_j-q)^\gamma g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\rho\tau}(q_j-q)^\gamma g^{\tau'\rho'}(q-q_j)^\delta][g_{\eta\eta'}] \tag{2.17}$$

$$\begin{aligned} |M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}&[(2q+q_j)^\gamma(2q_j+q)^\delta \\ -g^{\delta\gamma}(2q_j+q)^\rho(2q_j+q)_\rho - (2q_j+q)^\gamma(q-q_j)^\delta &- g^{\delta\gamma}(2q+q_j)^\tau(2q+q_j)_\tau \\ +(2q_j+q)^\gamma(2q+q_j)^\delta + (2q+q_j)^\gamma(q-q_j)^\delta &- (q_j-q)^\gamma(2q+q_j)^\delta \\ +(q_j-q)^\gamma(2q_j+q)^\delta &+ d(q_j-q)^\gamma(q-q_j)^\delta][g_{\eta\eta'}] \end{aligned} \tag{2.18}$$

$$\begin{aligned} |M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}&[(3+d)q^\gamma {q_j}^\delta + (6-d)q^\gamma q^\delta \\ +(6-d){q_j}^\gamma {q_j}^\delta + (3+d){q_j}^\gamma q^\delta &- g^{\delta\gamma}(5{q_j}^2 + 5q^2 + 8qq_j) \\ &][g_{\eta\eta'}] \end{aligned} \tag{2.19}$$

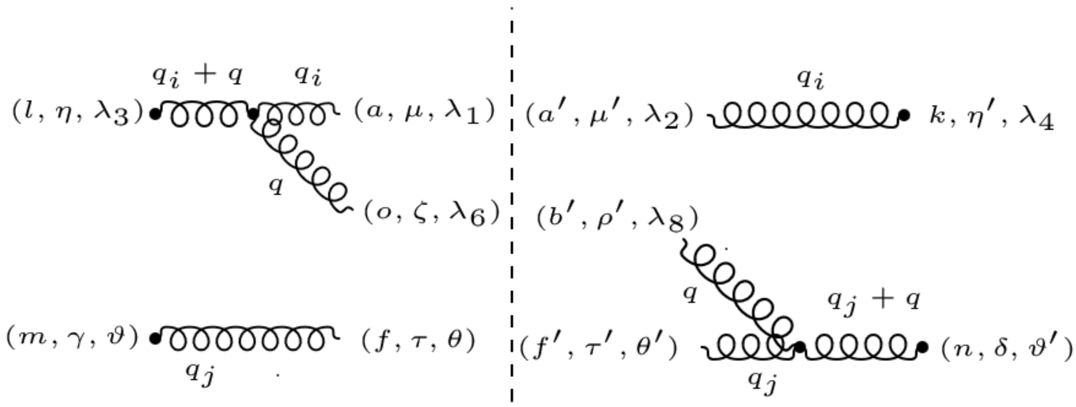
2.2.1 One-loop corrections to the gluon self-energy diagram (Gluon-Spectator Bubble)



$$|M_1|^2_{Ghost\ loop} = \frac{g_s^2 f^{bfm} f^{bnf}}{(q_j+q)^2(q_j+q)^2}[-q_j{}^\gamma q^\delta - q^\delta q_j{}^\gamma][g_{\gamma\delta}] \tag{2.20}$$

$$\begin{aligned}|M_2'|^2 = \frac{g_s^2 f^{bfm} f^{bnf}}{(q_j+q)^2(q_j+q)^2}[&(2+d)q^\gamma q_j{}^\delta + (6-d)q^\gamma q^\delta \\ &+(6-d)q_j{}^\gamma q_j{}^\delta + (2+d)q_j{}^\gamma q^\delta - g^{\delta\gamma}(8qq_j) \\ &][g_{\eta\eta'}]\end{aligned} \tag{2.21}$$

2.3 Interference term $M_1 M_2{}^\dagger$



$$
\begin{aligned}
M_1 M_2{}^{\dagger} = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,l\,o}(-g^{\mu\eta}(2q_i+q)^{\zeta}+g^{\eta\zeta}(2q+q_i)^{\mu}+g^{\zeta\mu}(q_i-q)^{\eta})\\
\varepsilon^{\lambda_1}{}_{\mu}(q_i)\varepsilon^{\lambda_6}{}_{\zeta}(q)][\varepsilon^{\theta}{}_{\tau}{}^{*}(q_j)]\\
[\frac{i}{(q+q_j)^2}(-g_s f^{\,f'\,n\,b'}(g^{\tau'\delta}(2q_j+q)^{\rho'}-g^{\delta\rho'}(2q+q_j)^{\tau'}+g^{\rho'\tau'}(q-q_j)^{\delta})\\
\varepsilon^{\theta'}{}_{\tau'}{}^{*}(q_j)\varepsilon^{\lambda_8}{}_{\rho'}{}^{*}(q)][\varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q_i)]
\end{aligned}
\tag{2.22}
$$

$$
\begin{aligned}
M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,a\,l\,o} f^{\,f'\,n\,b'}\delta^{aa'}\delta^{ob'}\delta^{ff'}}{(q_i+q)^2(q_j+q)^2}[g_{\mu\mu'}g_{\tau\tau'}\\
(-g^{\mu\eta}(2q_i+q)^{\zeta}+g^{\eta\zeta}(2q+q_i)^{\mu}+g^{\zeta\mu}(q_i-q)^{\eta})\\
g_{\zeta\rho'}(g^{\tau'\delta}(2q_j+q)^{\rho'}-g^{\delta\rho'}(2q+q_j)^{\tau'}+g^{\rho'\tau'}(q-q_j)^{\delta})]
\end{aligned}
\tag{2.23}
$$

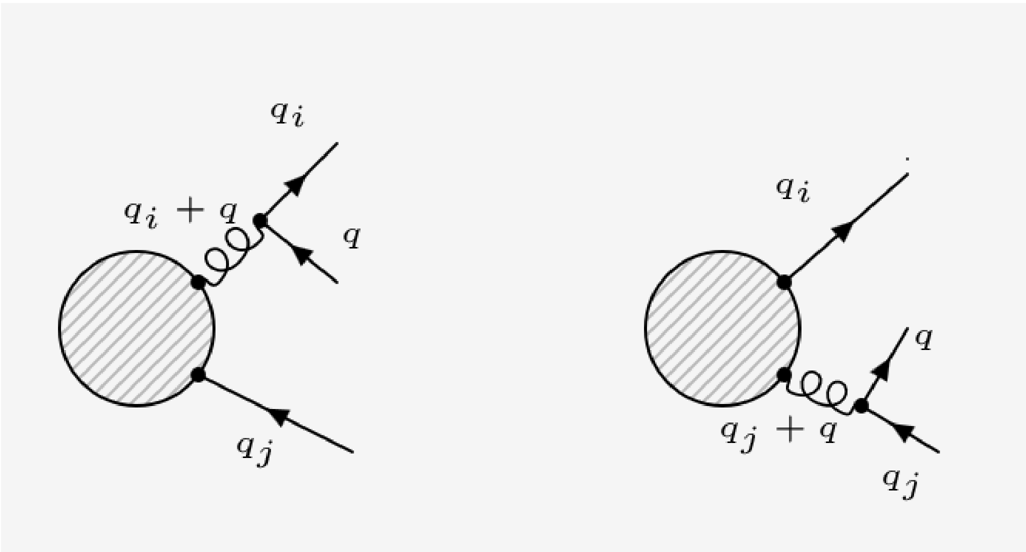
$$
\begin{aligned}
M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,a\,l\,o} f^{\,f\,n\,o}}{(q_i+q)^2(q_j+q)^2}[g_{\mu\eta'}g_{\gamma\tau'}\\
(-g^{\mu\eta}g_{\zeta\rho'}g^{\tau'\delta}(2q_i+q)^{\zeta}(2q_j+q)^{\rho'}+g^{\mu\eta}g_{\zeta\rho'}g^{\delta\rho'}(2q_i+q)^{\zeta}(2q+q_j)^{\tau'}\\
-g^{\mu\eta}g_{\zeta\rho'}g^{\rho'\tau'}(2q_i+q)^{\zeta}(q_j-q)^{\delta}+g^{\eta\zeta}g_{\zeta\rho'}g^{\tau'\delta}(2q+q_i)^{\mu}(2q_j+q)^{\rho'}\\
-g^{\eta\zeta}g_{\zeta\rho'}g^{\delta\rho'}(2q+q_i)^{\mu}(2q+q_j)^{\tau'}+g^{\eta\zeta}g_{\zeta\rho'}g^{\rho'\tau'}(2q+q_i)^{\mu}(q_j-q)^{\delta}\\
+g^{\zeta\mu}g_{\zeta\rho'}g^{\tau'\delta}(q_i-q)^{\eta}(2q_j+q)^{\rho'}-g^{\zeta\mu}g_{\zeta\rho'}g^{\delta\rho'}(q_i-q)^{\eta}(2q+q_j)^{\tau'}\\
+g^{\zeta\mu}g_{\zeta\rho'}g^{\rho'\tau'}(q_i-q)^{\eta}(q_j-q)^{\delta})]
\end{aligned}
\tag{2.24}
$$

$$
\begin{aligned}
M_1 M_2{}^{\dagger} = \frac{g_s^2 f^{\,a\,l\,o} f^{\,f\,n\,o}}{(q_i+q)^2(q_j+q)^2}[g_{\mu\eta'}g_{\gamma\tau'}\\
(-g^{\mu\eta}g^{\tau'\delta}(2q_i+q)^{\zeta}(2q_j+q)_{\zeta}+g^{\mu\eta}(2q+q_j)^{\tau'}(2q_i+q)^{\delta}-g^{\mu\eta}(2q_i+q)^{\tau'}(q-q_j)^{\delta}\\
+g^{\tau'\delta}(2q+q_i)^{\mu}(2q_j+q)^{\eta}-g^{\eta\delta}(2q+q_i)^{\mu}(2q+q_j)^{\tau'}+g^{\eta\tau'}(2q+q_i)^{\mu}(q_j-q)^{\delta}\\
+g^{\tau'\delta}(2q_j+q)^{\mu}(q_i-q)^{\eta}-g^{\mu\delta}(q_i-q)^{\eta}(2q+q_j)^{\tau'}+g^{\mu\tau'}(q_i-q)^{\eta}(q_j-q)^{\delta})]
\end{aligned}
\tag{2.25}
$$

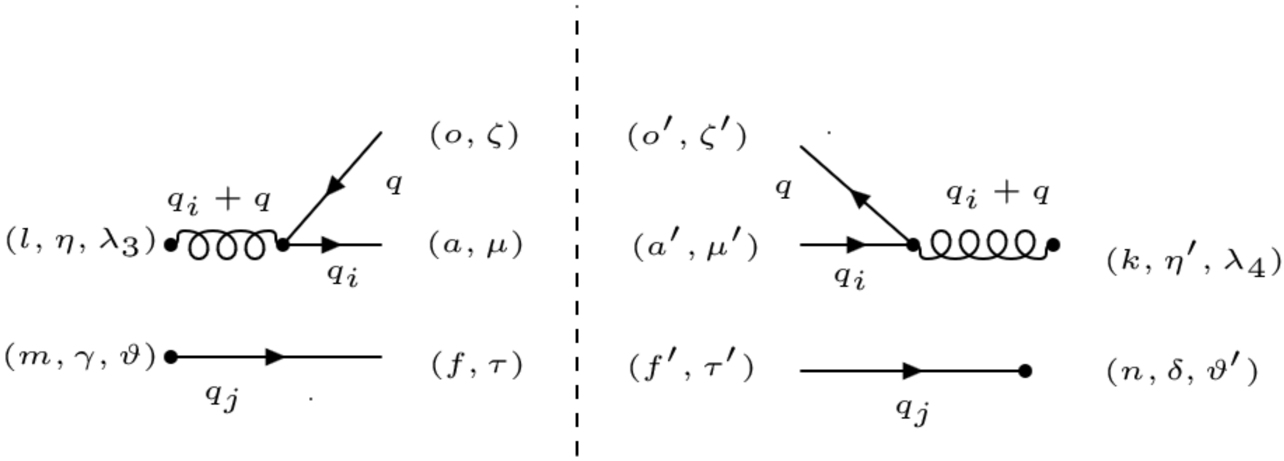
2.4 $|M^2|$

Chapter 3

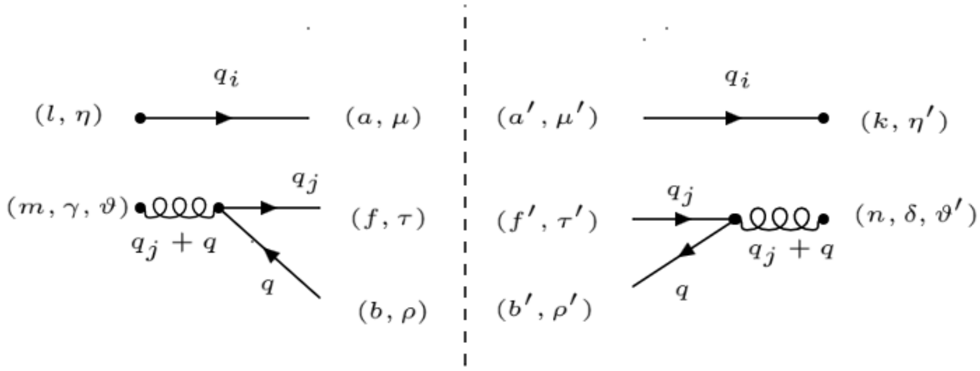
Quark gluon quark emission kernel



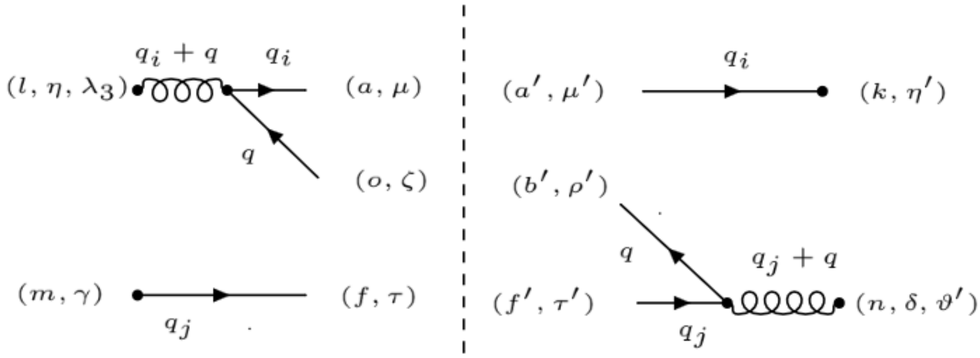
3.1 Gluon-Emitter Quark loop



3.2 Gluon-Spectator Quark loop

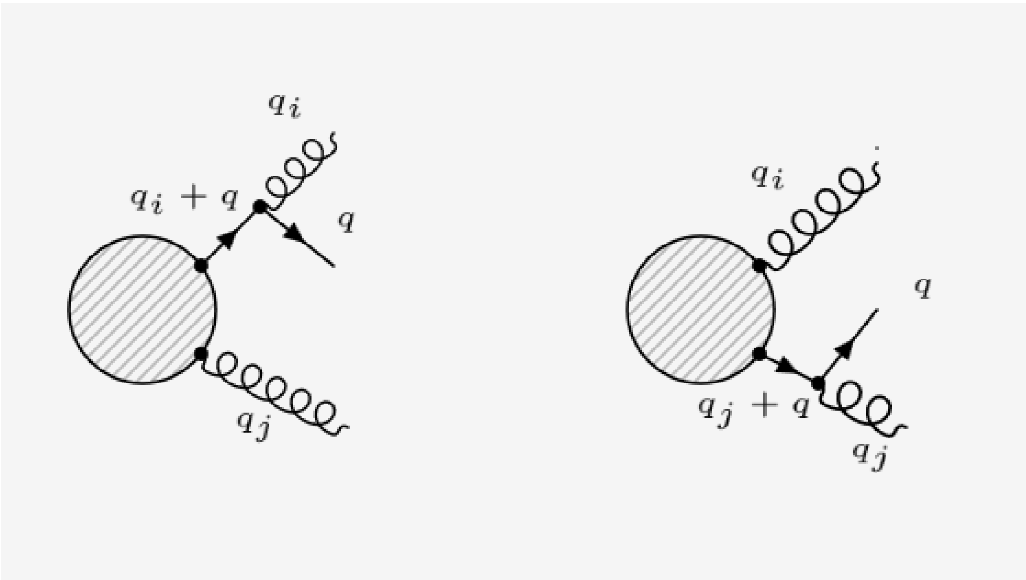


3.3 Gluon-Emitter Quark loop

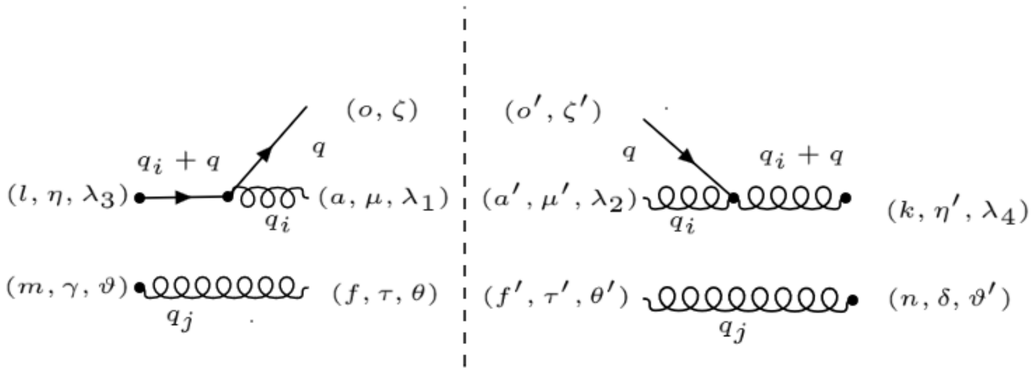


Chapter 4

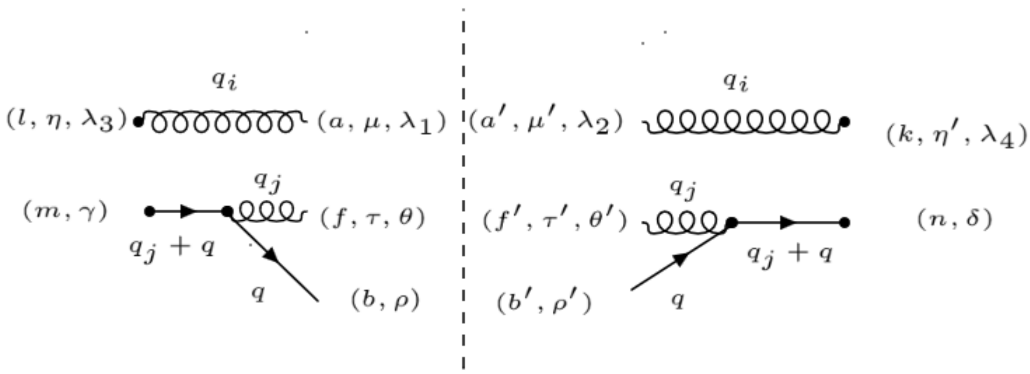
Gluon quark quark emission kernel



4.1 M_1


(o,ζ)
(o',ζ')
q_i+q
q
q
q_i+q
(l,η,λ_3)
(a,μ,λ_1)
(a',μ',λ_2)
(k,η',λ_4)
q_i
q_i
(m,γ,ϑ)
(f,τ,θ)
(f',τ',θ')
(n,δ,ϑ')
q_j
q_j

4.2 M_2


q_i
(l,η,λ_3)
(a,μ,λ_1)
(a',μ',λ_2)
q_i
(k,η',λ_4)
q_j
(m,γ)
(f,τ,θ)
(f',τ',θ')
q_j
(n,δ)
q_j+q
q_j+q
q
(b,ρ)
(b',ρ')
q

4.3 $M1M_2^\dagger$