Thesis

by

Tigran Saidnia

Emission kernel of parton shower

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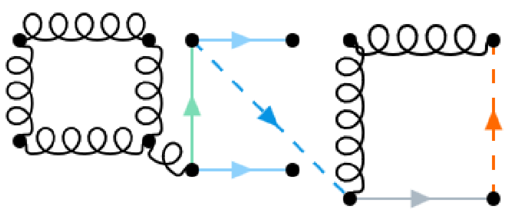

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Institute of theoretical physics

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Dr. Simon Plätzer
Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, 15. Januar 2019

Tigran Saidnia

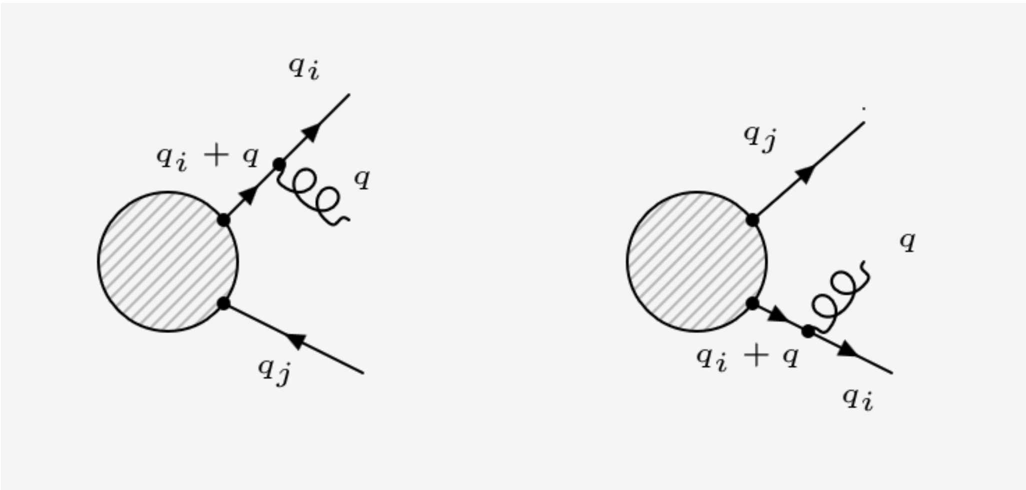
Inhaltsverzeichnis

0.1 parametrisation

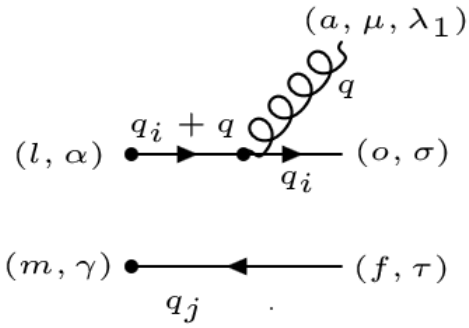
$$
\left.
\begin{aligned}
q_i{}^{\mu} &= z p_i{}^{\mu} + y(1-z) p_j{}^{\mu} + \sqrt{zy(1-z)} m_{\perp} \\
q^{\mu} &= (1-z) p_i{}^{\mu} + yz p_j{}^{\mu} - \sqrt{zy(1-z)} m_{\perp} \\
q_j{}^{\mu} &= (1-y) p_j{}^{\mu} \\
y &= \frac{q_i q}{p_i p_j} \\
q_i + q &= p_i + y p_j \\
q_j + q &= (1-z) p_i{}^{\mu} + (1 + yz - y) p_j{}^{\mu} - \sqrt{zy(1-z)} m_{\perp} \\
q_i \cdot q &= y(1 - 2z + 2z^2)(p_i \cdot p_j) \\
q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
\end{aligned}
\right\} \text{parametrisation} \tag{1}
$$

Kapitel 1

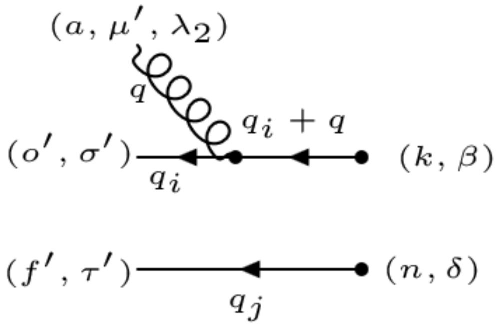
Quark antiquark gluon emission kernel



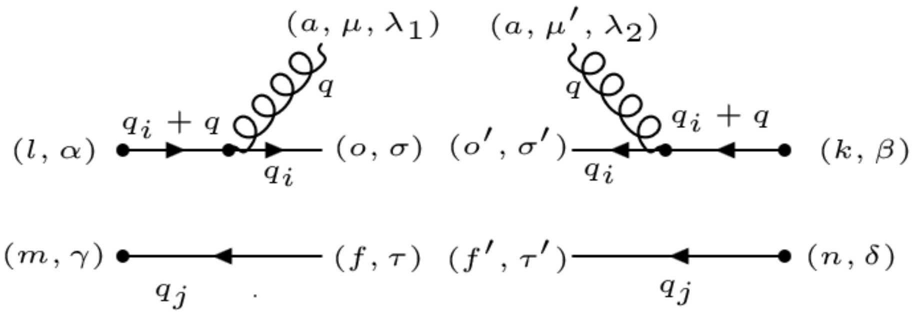
1.1 qg-$\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{1.1}$$



$$M_1{}^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{1.2}$$



$$
\begin{aligned}
|M_1|^2 = M_1\, M_1{}^{\dagger} &= [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\;\varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)] \\
&[\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)][\bar{v}_{\tau'}(q_j)]
\end{aligned}
\tag{1.3}
$$

$$
\begin{aligned}
|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\;(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q) \\
\times(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j)v_\tau(q_j)]
\end{aligned}
\tag{1.4}
$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$
\begin{aligned}
\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) = \not{q}_i, \\
\sum_{\tau,\tau'} \bar{v}_\tau(q_j)\,v_{\tau'}(q_j) = \not{q}_j
\end{aligned}
\tag{1.5}
$$

and sum over polarization index (λ_1, λ_2) :

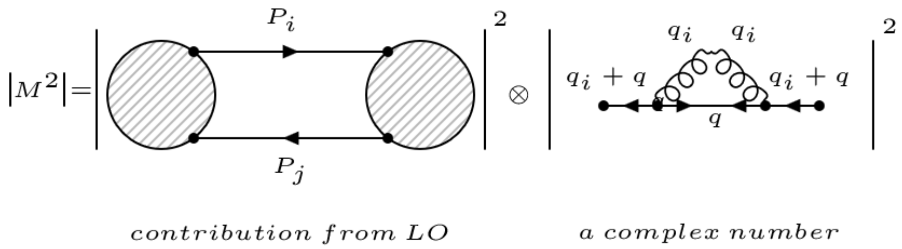
$$
\sum_{\mu,\mu'} \varepsilon^{\lambda_2}{}^*_{\mu'}(q)\varepsilon^{\lambda_1}{}_\mu(q) = -g_{\mu\mu'}
\tag{1.6}
$$

$$
|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^k\;[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,g_{\mu'\mu}\gamma^\mu\,(\not{q}_i + q)][\not{q}_j]
\tag{1.7}
$$

from here and after contraction between all indices we can actually make statements about the last result.

$$
|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^k\;[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'}\,(\not{q}_i + q)][\not{q}_j]
\tag{1.8}
$$

In other words we expect the tree level diagram from LO and a number: Which means:



$$|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[\not{P}_i][\not{P}_j] \otimes \textcolor{red}{(a\ complex\ number)} \tag{1.9}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'}\ \not{q}_i\,\gamma_{\mu'} &= q_{i\sigma}\,\gamma^{\mu'}\gamma^{\sigma}\ \gamma_{\mu'} \\ &= q_{i\sigma}\,(\{\gamma^{\mu'},\gamma^{\sigma}\}\ -\gamma^{\sigma}\gamma^{\mu'})\gamma_{\mu'} \\ &= q_{i\sigma}\,2g^{\mu'\sigma}\,\gamma_{\mu'}\ -\ d\,\gamma^{\sigma} \\ &= (2-d)\ \not{q}_i \end{aligned} \tag{1.10}$$

$$|M_1|^2 = -(2-d)\ \frac{g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\ \not{q})\ \ \not{q}_i\ \ (\not{q}_i+q)][\not{q}_j] \tag{1.11}$$

$$|M_1|^2 = -(2-d)\ \frac{g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[\not{q}_i\ \not{q}_i\ \not{q}_i +\ \not{q}_i\ \not{q}_i\ \not{q} +\ \not{q}\ \not{q}_i\ \not{q}_i +\ \not{q}\ \not{q}_i\ \not{q}][\not{q}_j] \tag{1.12}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i\ \not{q}_i &= {q_i}^2 = {m_i}^2 \\ \not{q}\ \not{q} &= q^2 = m^2 \\ \not{q}_j\ \not{q}_j &= {q_j}^2 = {m_j}^2 \end{aligned} \tag{1.13}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i\ \not{q}_i$ and $\not{q}\ \not{q}$ as well.

$$|M_1|^2 = -(2-d)\ \frac{g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(2q_iq)(2q_iq)}[\not{q}\ \not{q}_i\ \not{q}][\not{q}_j] \tag{1.14}$$

$$
\begin{aligned}
L = \not{q}\ \not{q}_i\ \not{q} &= \not{q}[q_{i\sigma} q_\mu \left(\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu\right)] \\
&\not{q}[2{q_i}^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2}]\gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}]\gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
&= \not{q}(2q_i q) - q^2\ \not{q}_i \\
&= \not{q}(2q_i q)
\end{aligned}
\tag{1.15}
$$

After inserting the last result of L and simplify the term $(2q_i q)$ from the denominator and nominator because , we get:

$$
|M_1|^2 = -(2-d)\ \frac{g_s^2 [T^b]_{o'}{}^k\ [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{q}][\not{q}_j] \tag{1.16}
$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$
|M_1|^2 = (d-2)\ \frac{g_s^2 [T^b]_{o'}{}^k\ [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][(1-y)\ \not{p}_j] \tag{1.17}
$$

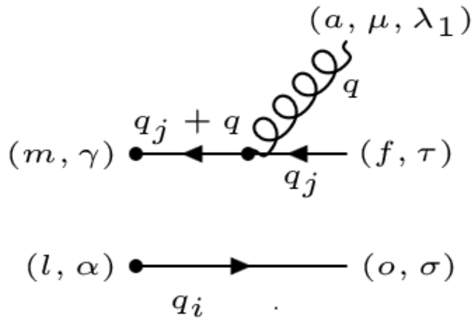
Multiplying the both sides

$$
\begin{aligned}
|M_1|^2 = (d-2)\ \frac{g_s^2 [T^b]_{o'}{}^k\ [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} &[(1-z)(1-y)\ \not{p}_i\ \not{p}_j \\
&+ zy(1-y)\ \not{p}_j\ \not{p}_j + (1-y)\sqrt{zy(1-z)}\ \not{m}_\perp\ \not{p}_j]
\end{aligned}
\tag{1.18}
$$

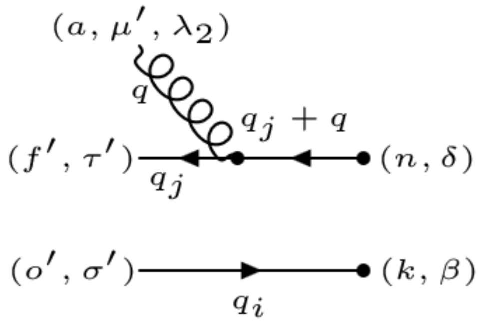
and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\ \not{p}_i$ and $\not{p}_j\ \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$
|M_1|^2 = (d-2)(1-z)(1-y)\ \frac{g_s^2 [T^b]_{o'}{}^k\ [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \tag{1.19}
$$

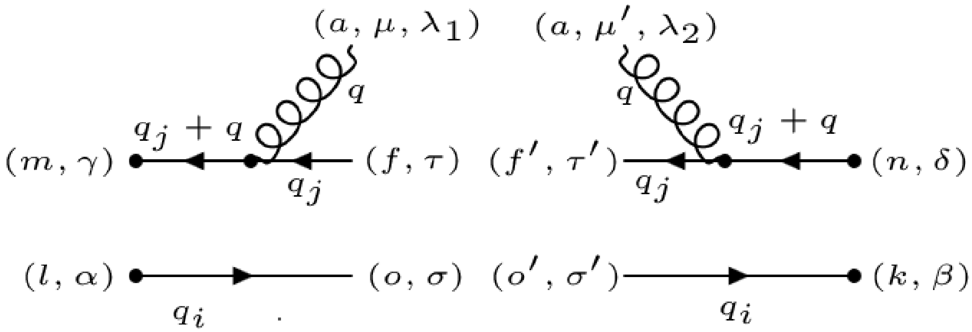
1.2 $\bar{q}$g-q



$$M_2 = [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\,[u_\sigma(q_i)] \tag{1.20}$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j)\,(ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\,\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\,\varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[\bar{u}_{\sigma'}(q_i)] \tag{1.21}$$



$$\begin{aligned}|M_2|^2 = M_2\, M_2{}^\dagger = &[\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\,[u_\sigma(q_i)] \\ &[\bar{v}_{\tau'}(q_j)\,(ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\,\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\,\varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[\bar{u}_{\sigma'}(q_i)]\end{aligned} \tag{1.22}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j+\not{q})\gamma^\nu\, v_\tau(q_j)\bar{v}_{\tau'}(q_j)\,\varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_j+\not{q})]$$
$$[u_\sigma(q_i)]\,[\bar{u}_{\sigma'}(q_i)] \tag{1.23}$$

and after sum over the lorenz index (σ,σ') and (τ,τ') and unsing the spin addition relation:

$$\begin{aligned}\sum_{\sigma,\sigma'}\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) &= \not{q}_i,\\ \sum_{\tau,\tau'}\bar{v}_\tau(q_j)\,v_{\tau'}(q_j) &= \not{q}_j\end{aligned} \tag{1.24}$$

and sum over polarization index (λ_3,λ_4) :

$$\sum_{\nu,\nu'}\varepsilon^{\lambda_4}{}^*_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'} \tag{1.25}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j+\not{q})\gamma^\nu\;\not{q}_j\,(-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_j+\not{q})]\,[\not{q}_i] \tag{1.26}$$

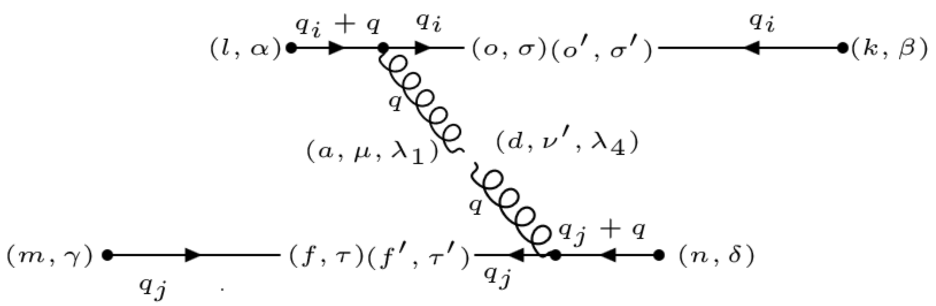
After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_j)}[\not{q}]\,[\not{q}_i] \tag{1.27}$$

finally:

$$|M_2|^2 = -(d-2)yz^2\,\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.28}$$

1.3 $M_1 {M_2}^\dagger$



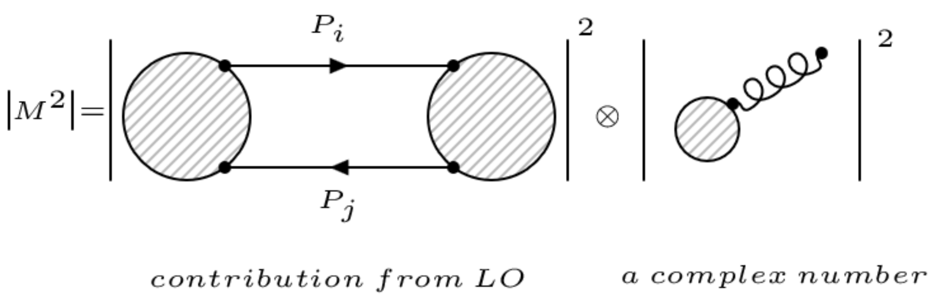
$$M_1\, {M_2}^\dagger = [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\ \varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)]$$
$$[\bar{v}_{\tau'}(q_j)\,(ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\,\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\ \varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[u_{\sigma'}(q_i)] \tag{1.29}$$

$$M_1\, {M_2}^\dagger = \frac{g_s^2 [T^a]_o{}^l\,[T^d]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,]\varepsilon^{\lambda_1}{}_\mu(q)\,\varepsilon^{\lambda_4}{}_{\nu'}(q)$$
$$[\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j + \not{q})] \tag{1.30}$$

$$M_1\, {M_2}^\dagger = \frac{g_s^2 [T^a]_o{}^l\,[T^d]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,] - g_{\mu\nu'}$$
$$[\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j + \not{q})] \tag{1.31}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2 [T^a]_o{}^l\,[T^d]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,]\,[\not{q}_j\,\gamma_\mu\,(\not{q}_j + \not{q})] \tag{1.32}$$

Expectation:



$$M_1\, {M_2}^\dagger = \frac{-g_s^2 {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(2q_iq)(2q_jq)}[(\not{q}_i+\not{q})\,\gamma^\mu\ \ \not{q}_i\,]\,[(\not{q}_j+\not{q})\,\gamma_\mu\ \ \not{q}_j] \tag{1.33}$$

$$\begin{aligned} M_1\, {M_2}^\dagger = \frac{-g_s^2 {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(2q_iq)(2q_jq)}&[-(\not{q}_i+\not{q})\ \ \not{q}_i\,\gamma^\mu\ +2(\not{q}_i+\not{q})\,{q_i}^\mu] \\ &[-(\not{q}_j+\not{q})\ \ \not{q}_j\,\gamma_\mu\ +2(\not{q}_j+\not{q})q_{j\mu}] \end{aligned} \tag{1.34}$$

$$\begin{aligned} M_1\, {M_2}^\dagger = &\frac{-g_s^2 {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(2q_iq)(2q_jq)} \\ &[(\not{q}_i+\not{q})\ \ \not{q}_i\,\gamma^\mu]\,[(\not{q}_j+\not{q})\ \ \not{q}_j\gamma_\mu] \\ &-2[(\not{q}_i+\not{q})\ \ \not{q}_i\,\gamma^\mu]\,[(\not{q}_j+\not{q})q_{j\mu}] \\ &-2[(\not{q}_i+\not{q})\,{q_i}^\mu][(\not{q}_j+\not{q})\ \ \not{q}_j\,\gamma_\mu] \\ &+4[(\not{q}_i+\not{q})\,{q_i}^\mu][(\not{q}_j+\not{q})q_{j\mu}] \end{aligned} \tag{1.35}$$

$$\begin{aligned} M_1\, {M_2}^\dagger = &\frac{-g_s^2 {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(2q_iq)(2q_jq)} \\ &[\not{q}\ \ \not{q}_i\,\gamma^\mu]\,[\not{q}\ \not{q}_j\gamma_\mu] \\ &-2[\not{q}\ \ \not{q}_i\,\gamma^\mu]\,[(\not{q}+\not{q}_j)\ \ q_{j\mu}] \\ &-2[(\not{q}_i+\not{q})\,{q_i}^\mu][\not{q}\ \ \not{q}_j\,\gamma_\mu] \\ &+4[(\not{q}_i+\not{q})\,{q_i}^\mu][(\not{q}_j+\not{q})q_{j\mu}] \end{aligned} \tag{1.36}$$

$$\begin{aligned} M_1\, {M_2}^\dagger = &\frac{-g_s^2\ {[T^a]_o}^l\, {[T^d]_{f'}}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ &[y(1-2z+z^2)\ \not{p}_i\ \ \not{p}_j\,\gamma^\mu]\,[(1-z)(1-y)\ \not{p}_i\ \not{p}_j\gamma_\mu] \\ &+4({q_i}^\mu\cdot q_{j\mu})[(\not{q}_i+\not{q})][(\not{q}_j+\not{q})] \end{aligned} \tag{1.37}$$

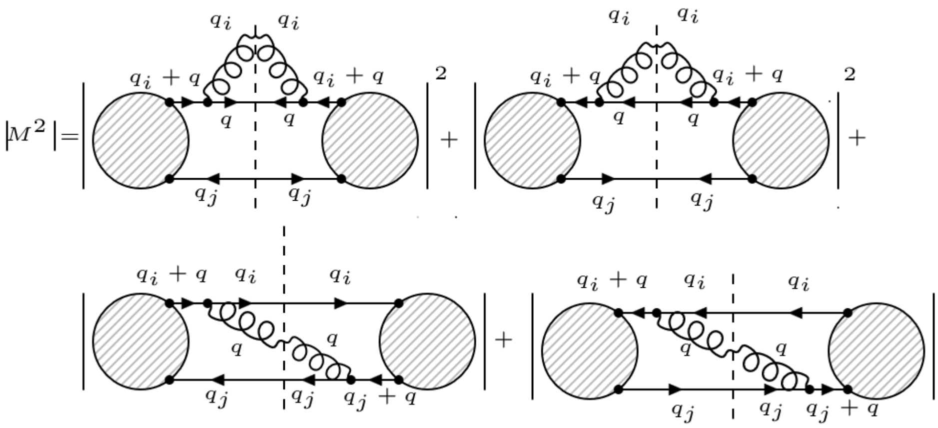
$$\begin{aligned} M_1\, {M_2}^\dagger = &\frac{-g_s^2\ {[T^a]_o}^l\, {[T^d]_{f'}}^n}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ &[y(1-2z+z^2)\ \not{p}_i\ \ \not{p}_j\,\gamma^\mu]\,[(1-z)(1-y)\ \not{p}_i\ \not{p}_j\gamma_\mu] \\ +4(p_i\cdot p_j)[(\not{p}_i+y\ \not{p}_j)][(1-z)\ \not{p}_i&+(1+yz-y)\ \not{p}_j-\sqrt{zy(1-z)}\ \not{m}] \end{aligned} \tag{1.38}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2\ {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)}z(1-y)[\not{p}_i][\not{p}_j] \tag{1.39}$$

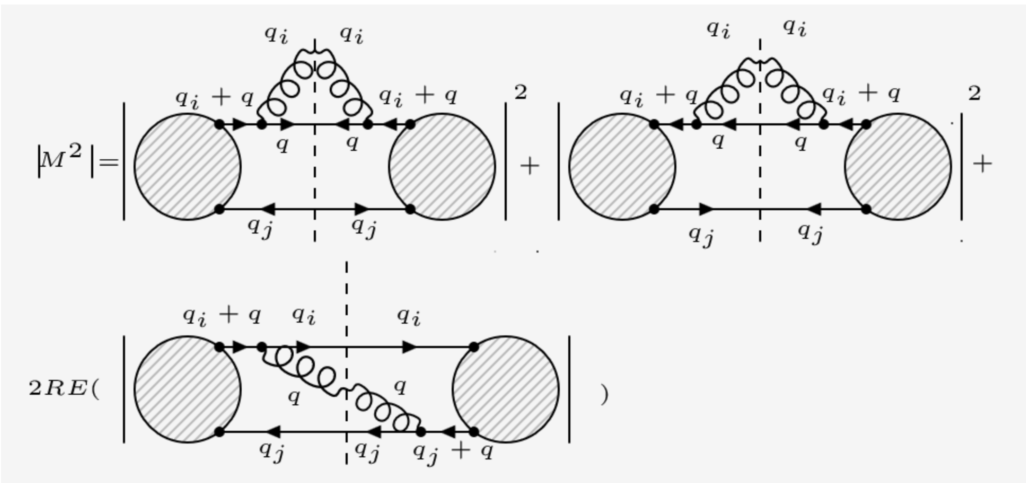
$$M_1\, {M_2}^\dagger = \frac{-g_s^2\ {[T^a]_o}^l\, {[T^d]_{f'}}^n}{(1-z)y(1-2z+z^2)(p_i\cdot p_j)}z[\not{p}_i][\not{p}_j] \tag{1.40}$$

1.4 $|M^2|$

$$| M|^2 = | M_1|^2 + | M_2|^2 + M_1\, {M_2}^\dagger + {M_1}^\dagger M_2 \tag{1.41}$$



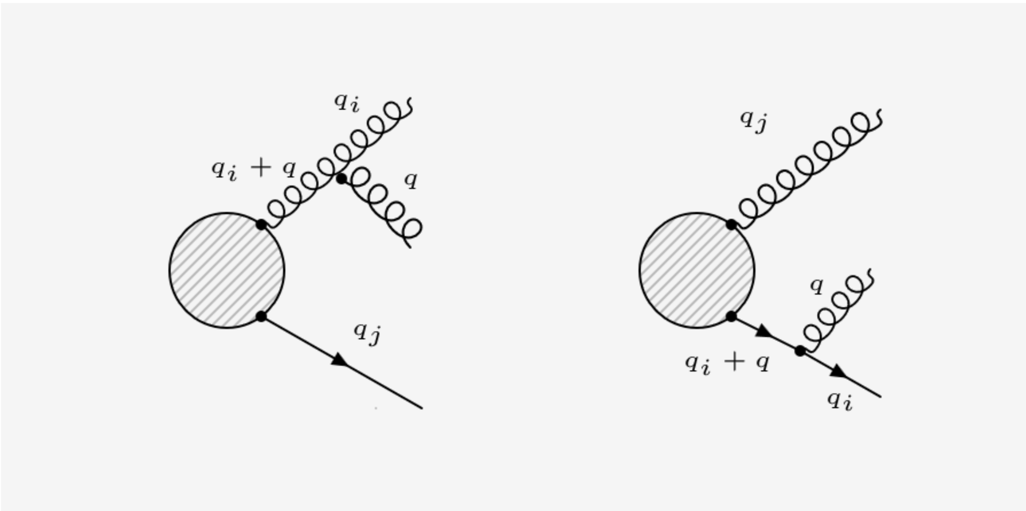
$$| M|^2 = | M_1|^2 + | M_2|^2 + 2RE(M_1\, {M_2}^\dagger) \tag{1.42}$$



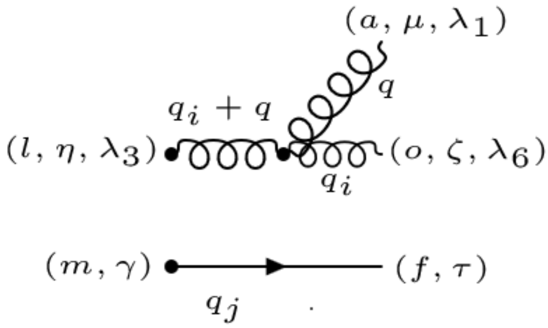
$$\begin{aligned} | M|^2 = {} & (d-2)(1-z)(1-y)\, \frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not p_i][\not p_j] \\ & -(d-2)yz^2\, \frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not p_i][\not p_j] \\ & +2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2\, [T^a]_o{}^l\, [T^d]_{f'}{}^n}{2y(1-2z+z^2)(p_i \cdot p_j)} [\not p_i][\not p_j]\right) \end{aligned} \tag{1.43}$$

Kapitel 2

Gluon quark gluon emission kernel

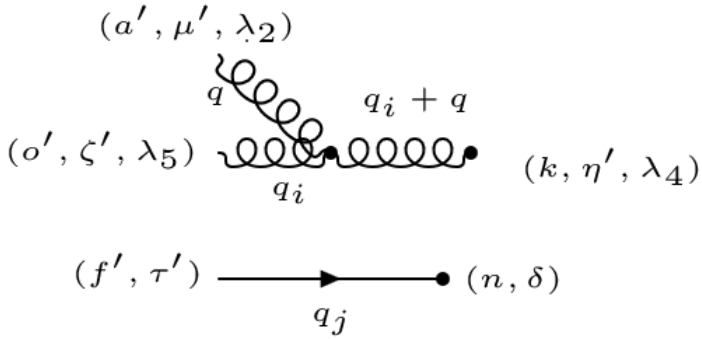


2.1 gg-q

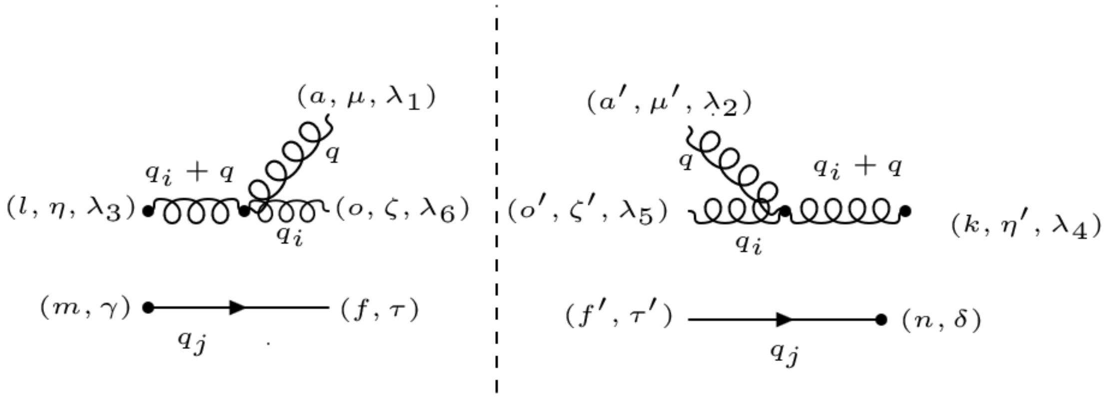


$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(-q_i+q)^{\eta} + g^{\zeta\eta}(-q-(q_i+q))^{\mu} + g^{\eta\mu}(q_i+q+q_i)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q)\varepsilon^{\lambda_6}{}_{\zeta}(q_i)][\bar{u}_{\tau}(q_j)] \tag{2.1}$$

$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^{\eta} - g^{\zeta\eta}(2q_i+q)^{\mu} + g^{\eta\mu}(2q_i+q)^{\zeta}) \\ \varepsilon^{\lambda_1}{}_{\mu}(q)\varepsilon^{\lambda_6}{}_{\zeta}(q_i)][\bar{u}_{\tau}(q_j)] \tag{2.2}$$



$$M_1{}^{\dagger} = [\frac{i}{(q_i+q)^2}(-g_s f^{\,a'\,o'\,k}(g^{\mu'\zeta'}(q-q_i)^{\eta'} - g^{\zeta'\eta'}(2q_i+q)^{\mu'} + g^{\eta'\mu'}(2q_i+q)^{\zeta'}) \\ \varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q)\varepsilon^{\lambda_5}{}_{\zeta'}{}^{*}(q_i)][u_{\tau'}(q_j)] \tag{2.3}$$



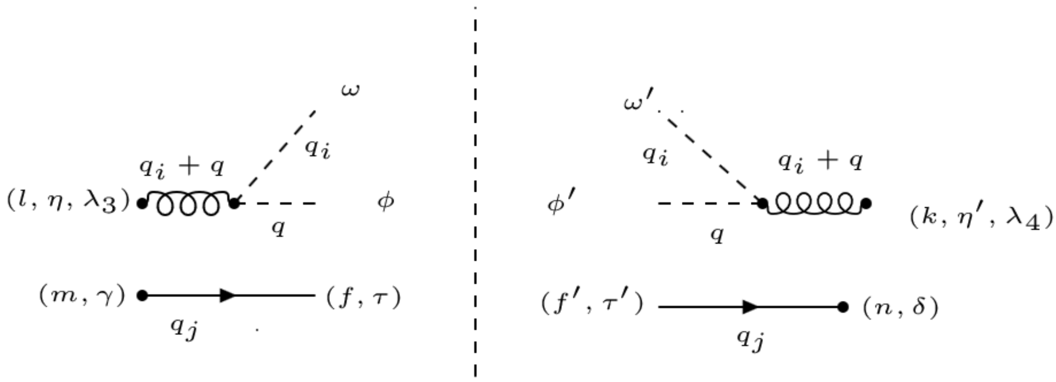
$$
\begin{aligned}
|M_1|^2 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\
\varepsilon^{\lambda_1}{}_\mu(q)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)\varepsilon^{\lambda_6}{}_\zeta(q_i)\,\varepsilon^{\lambda_5}{}_{\zeta'}{}^*(q_i) \\
(-g_s f^{\,a'\,o'\,k}(g^{\mu'\zeta'}(q-q_i)^{\eta'} - g^{\zeta'\eta'}(2q_i+q)^{\mu'} + g^{\eta'\mu'}(2q_i+q)^{\zeta'})\frac{i}{(q_i+q)^2}][\bar{u}_\tau(q_j)u_{\tau'}(q_j)]
\end{aligned}
\tag{2.4}
$$

$$
\begin{aligned}
|M_1|^2 = \frac{g_s{}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2} \\
[(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\
g_{\mu\mu'}g^{\zeta\zeta'}(g_{\mu'\zeta'}(q-q_i)_{\eta'} - g_{\zeta'\eta'}(2q_i+q)_{\mu'} + g_{\eta'\mu'}(2q_i+q)_{\zeta'})][\not{q}_j]
\end{aligned}
\tag{2.5}
$$

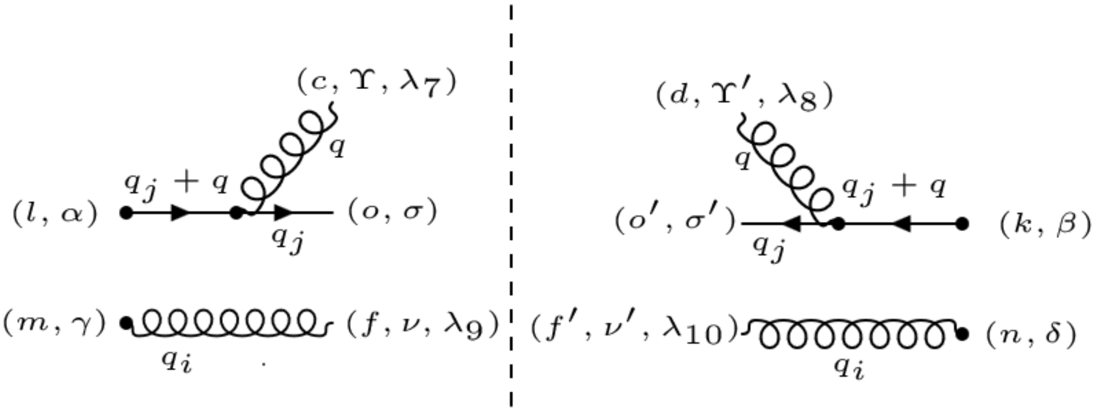
$$
\begin{aligned}
|M_1|^2 = \frac{g_s{}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2} \\
[(g^{\mu'\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)_{\mu'} + g^{\eta}{}_{\mu'}(2q_i+q)^\zeta) \\
(g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} + g_{\eta'\mu'}(2q_i+q)^\zeta)][\not{q}_j]
\end{aligned}
\tag{2.6}
$$

$$
\begin{aligned}
|M_1|^2 = \frac{g_s{}^2 f^{\,a\,o\,l} f^{\,a'\,o'\,k}}{(q_i+q)^2(q_i+q)^2} \\
[g^{\mu'\zeta}(q-q_i)^\eta\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\mu'\zeta}(q-q_i)^\eta\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} \\
+g^{\mu'\zeta}(q-q_i)^\eta\, g_{\eta'\mu'}(2q_i+q)^\zeta) - g^{\zeta\eta}(2q_i+q)_{\mu'}\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} \\
+g^{\zeta\eta}(2q_i+q)_{\mu'}\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} - g^{\zeta\eta}(2q_i+q)_{\mu'}\, g_{\eta'\mu'}(2q_i+q)^\zeta) \\
+g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g^{\eta}{}_{\mu'}(q-q_i)_{\eta'} - g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g^{\zeta}{}_{\eta'}(2q_i+q)_{\mu'} \\
+g^{\eta}{}_{\mu'}(2q_i+q)^\zeta\, g_{\eta'\mu'}(2q_i+q)^\zeta)][\not{q}_j]
\end{aligned}
\tag{2.7}
$$

2.1.1 One-loop corrections to the gluon self-energy diagram



2.2 qg-g

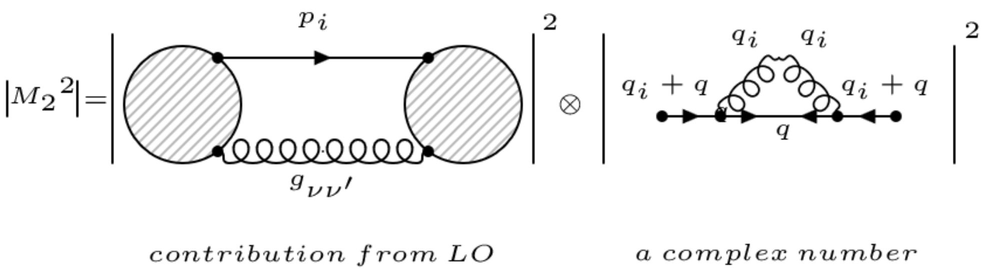


$$|M_2|^2 = M_2\, M_2{}^\dagger = [\bar{u}_\sigma(q_j)\,(-ig_s\gamma^\Upsilon \times [T^c]_o{}^l)\,\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}\;\varepsilon^{\lambda_7}{}_\Upsilon(q)][\varepsilon^{\lambda_9}{}_\nu(q_i)]$$
$$[\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\,(ig_s\gamma^{\Upsilon'} \times [T^d]_{o'}{}^k)\,u_{\sigma'}(q_j)\,\varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q)][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)] \tag{2.8}$$

$$|M_2|^2 = [\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\;(-ig_s\gamma^\Upsilon \times [T^c]_o{}^l)\,\bar{u}_\sigma(q_j)\,u_{\sigma'}(q_j)\,\varepsilon^{\lambda_7}{}_\Upsilon(q)\varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q)$$
$$\times(ig_s\gamma^{\Upsilon'} \times [T^d]_{o'}{}^k)\,\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)\varepsilon^{\lambda_9}{}_\nu(q_i)] \tag{2.9}$$

$$|M_2|^2 = \frac{g_s{}^2[T^c]_o{}^l[T^d]_{o'}{}^k}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\;\gamma^\Upsilon\;\;\not{q}_j\,(-g_{\Upsilon\Upsilon'})$$
$$\gamma^{\Upsilon'}\,(\not{q}_j + \not{q})][-g_{\nu'\nu}] \tag{2.10}$$

Expect:



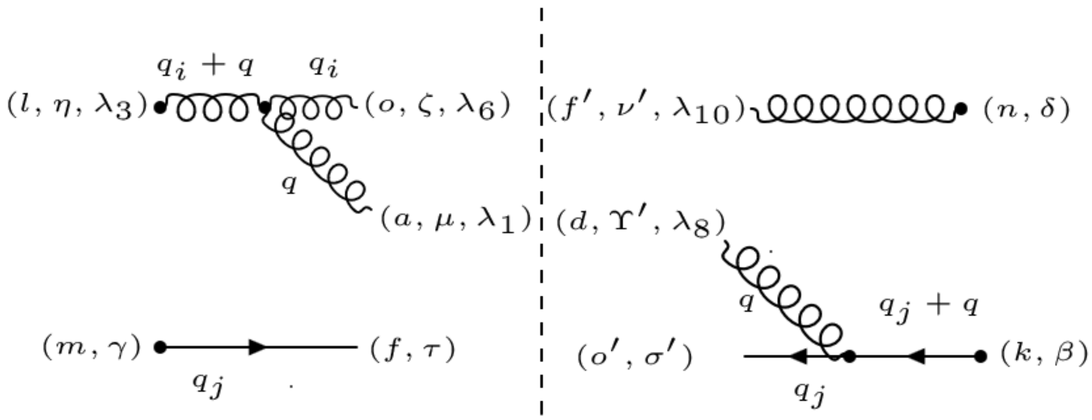
$$|M_2|^2 = -\frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j + \not{q})\ \gamma^{\Upsilon}\ \ \not{q}_j\ \gamma_{\Upsilon}\ (\not{q}_j + \not{q})][-g_{\nu'\nu}] \tag{2.11}$$

$$|M_2|^2 = (d-2)\frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{(2qq_j)}[\not{q}][-g_{\nu'\nu}] \tag{2.12}$$

$$|M_2|^2 = (d-2)\frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{2(1-z)(1-y)(p_i \cdot p_j)}[(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][-g_{\nu'\nu}] \tag{2.13}$$

$$|M_2|^2 = (d-2)zy\frac{{g_s}^2 [T^c]_o{}^l [T^d]_{o'}{}^k}{2(1-z)(1-y)(p_i \cdot p_j)}[\not{p}_j][-g_{\nu'\nu}] \tag{2.14}$$

2.3 $M_1 {M_2}^\dagger$

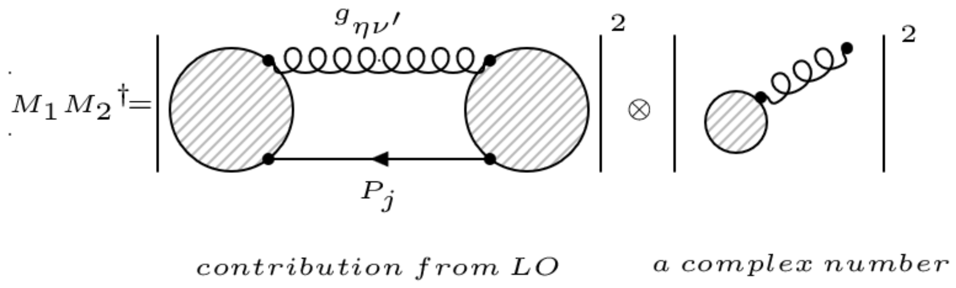


$$\begin{aligned} M_1\ {M_2}^\dagger = {} & [\frac{-ig_{\eta\zeta}}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\ & \varepsilon^{\lambda_1}{}_\mu(q_i)\varepsilon^{\lambda_6}{}_\zeta(q)][\bar{u}_\tau(q_j)][\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\ (ig_s\gamma^{\mu'} \times [T^d]_{o'}{}^k)\ u_{\sigma'}(q_j)\ \varepsilon^{\lambda_8}{}_{\Upsilon'}{}^*(q)][\varepsilon^{\lambda_{10}}{}_{\nu'}{}^*(q_i)] \end{aligned} \tag{2.15}$$

$$\begin{aligned} M_1\ {M_2}^\dagger = {} & \frac{ig_s^2 f^{\,a\,o\,l}\ [T^d]_{o'}{}^k}{4(qq_i)(qq_j)}[(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)^\zeta)] \\ & (-g_{\zeta\nu'})(-g_{\mu\Upsilon'})[(\not{q}_j + \not{q})\gamma^{\Upsilon'}\ \not{q}_j] \end{aligned} \tag{2.16}$$

$$\begin{aligned} M_1\ {M_2}^\dagger = {} & \frac{ig_s^2 f^{\,a\,o\,l}\ [T^d]_{o'}{}^k}{4(qq_i)(qq_j)}[(g^\mu{}_{\nu'}(q-q_i)^\eta - g^\eta{}_{\nu'}(2q_i+q)^\mu + g^{\eta\mu}(2q_i+q)_{\nu'})] \\ & [(\not{q}_j + \not{q})\gamma_\mu\ \not{q}_j] \end{aligned} \tag{2.17}$$

Expect:



$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[-g^{\eta}{}_{\nu'}(2q_i+q)^{\mu}][-(\not{q}_j+\not{q})\,\not{q}_j\gamma_\mu + 2(\not{q}_j+\not{q})q_{j\mu}] \tag{2.18}$$

$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[-g^{\eta}{}_{\nu'}(2q_i+q)^{\mu}][-(qq_j)\gamma_\mu + 2(\not{q}_j+\not{q})q_{j\mu}] \tag{2.19}$$

$$M_1\, M_2{}^\dagger = \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(qq_i)(qq_j)}[(qq_j)g^{\eta}{}_{\nu'}(2\,\not{q}_i+\not{q}) - 2((2q_i+q)\cdot q_j)\,g^{\eta}{}_{\nu'}(\not{q}_j+\not{q})] \tag{2.20}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(qq_j)(2\,\not{q}_i+\not{q}) - 2((2q_i+q)\cdot q_j)\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.21}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(p_i\cdot p_j)(2\,\not{q}_i+\not{q}) - (4q_i\cdot q_j + 2q\cdot q_j)\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.22}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(p_i\cdot p_j)(2\,\not{q}_i+\not{q}) \\ & -(4(z(1-y)(p_i\cdot p_j)) + 2((1-z)(1-y)(p_i\cdot p_j)))\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.23}$$

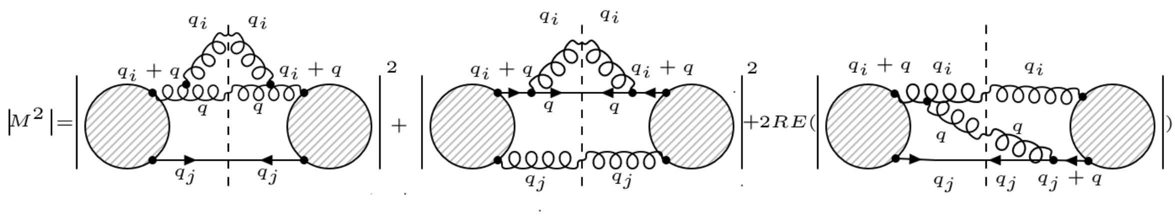
$$\begin{aligned} M_1\, M_2{}^\dagger = {} & \frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)} \\ & [g^{\eta}{}_{\nu'}][(1-z)(1-y)(2\,\not{q}_i+\not{q}) \\ & -(4(z(1-y)) + 2((1-z)(1-y)))\,(\not{q}_j+\not{q})] \end{aligned} \tag{2.24}$$

$$
\begin{aligned}
M_1\, M_2{}^{\dagger} = &\frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)} \\
&[g^{\eta}{}_{\nu'}][(1-z)(1-y)(2y(1-z)\ \not{p}_j + yz\ \not{p}_j) \\
&-2(1+z)(1-y)\ ((1+yz-z)\ \not{p}_j)]
\end{aligned}
\tag{2.25}
$$

$$
\begin{aligned}
M_1\, M_2{}^{\dagger} = &\frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)(1-y)y(1-2z+z^2)(p_i\cdot p_j)} \\
&[g^{\eta}{}_{\nu'}][-2(1+z)(1-y)\ (1+yz-z)\ \not{p}_j]
\end{aligned}
\tag{2.26}
$$

$$
M_1\, M_2{}^{\dagger} = -2(1+z)\,(1+yz-z)\frac{i g_s^2 f^{\,a\,o\,l}\,[T^d]_{o'}{}^{k}}{4(1-z)y(1-2z+z^2)(p_i\cdot p_j)}[g^{\eta}{}_{\nu'}][\,\not{p}_j] \tag{2.27}
$$

2.4 $|M^2|$


$|M^2| =$
q_i q_i
$q_i + q$ $q_i + q$
q q
q_j q_j
2
+
q_i q_i
$q_i + q$ $q_i + q$
q q
q_j q_j
2
$+2RE($
$q_i + q$ q_i q_i
q q
q_j q_j $q_j + q$
)