

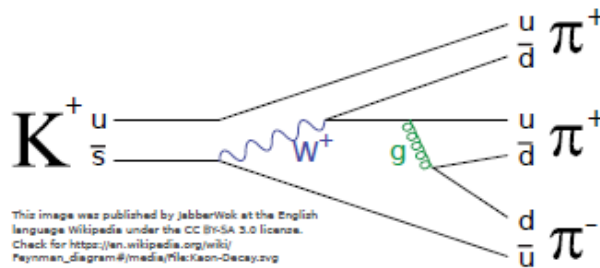
THESIS

BY

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Emission kernel of parton shower

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statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, 3. Januar 2019

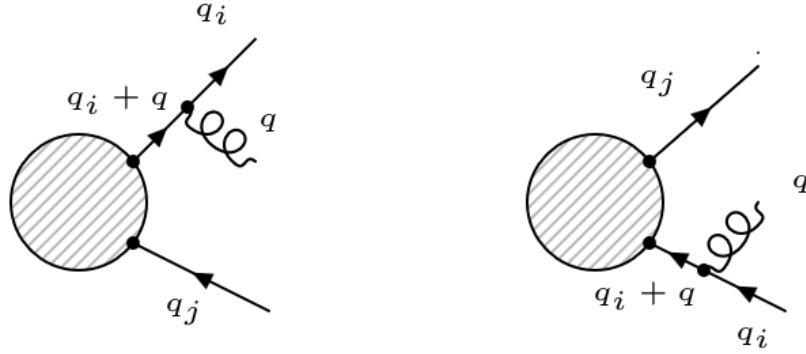
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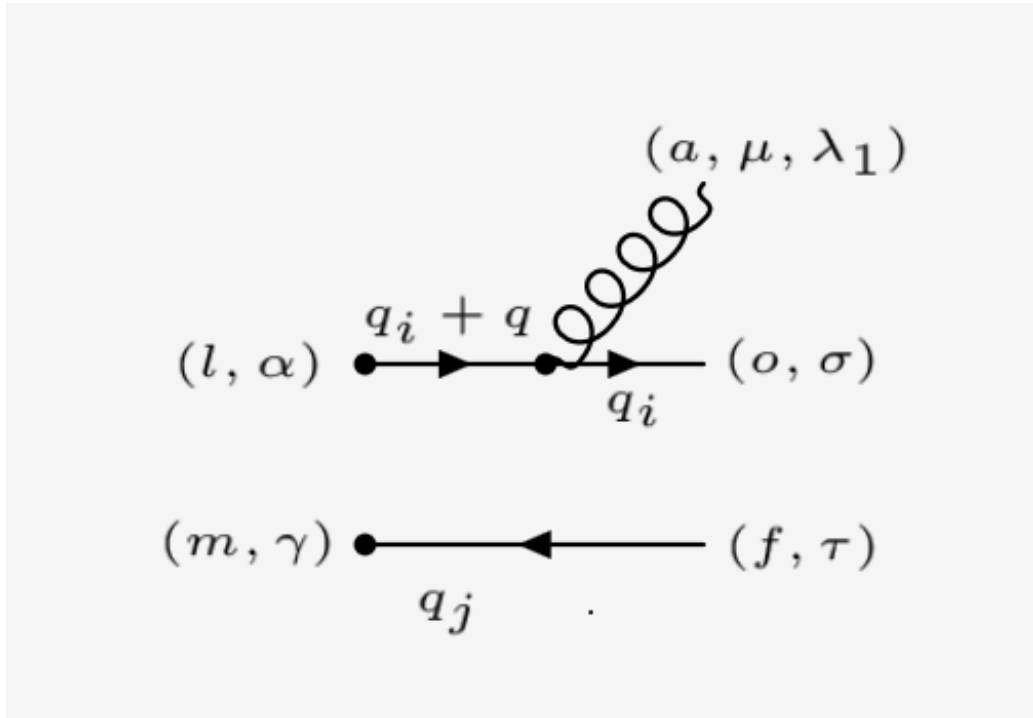
0.1 parametrisation

$$\left. \begin{aligned}
 q_i^\mu &= zp_i^\mu + y(1-z)p_j^\mu + \sqrt{zy(1-z)}m_\perp \\
 q^\mu &= (1-z)p_i^\mu + yzp_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_j^\mu &= (1-y)p_j^\mu \\
 y &= \frac{q_i q}{p_i p_j} \\
 q_i + q &= p_i + yp_j \\
 q_j + q &= (1-z)p_i^\mu + (1+yz-y)p_j^\mu - \sqrt{zy(1-z)}m_\perp
 \end{aligned} \right\} \text{parametrisation} \quad (1)$$

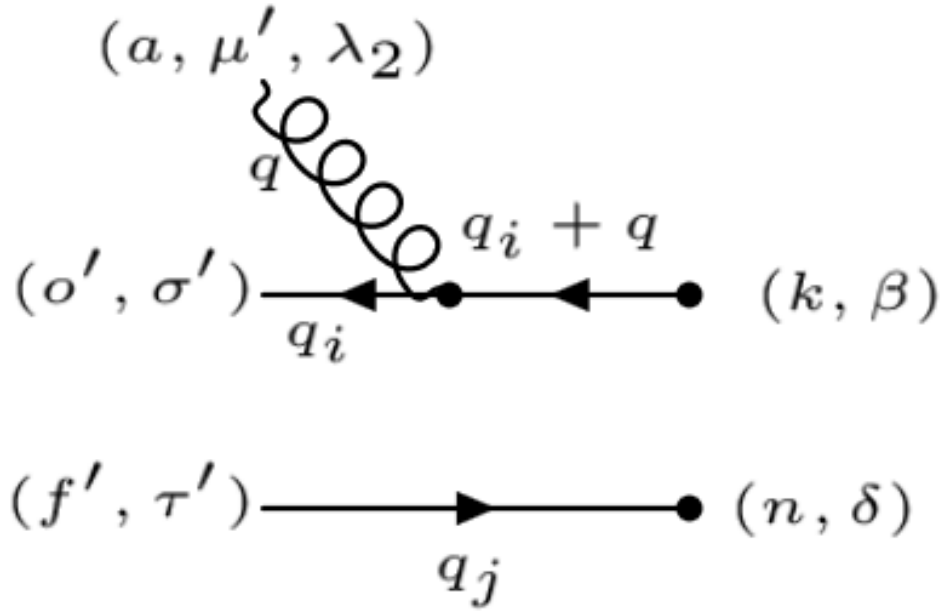
0.2 Quark/Antiquark gluon emission kernel



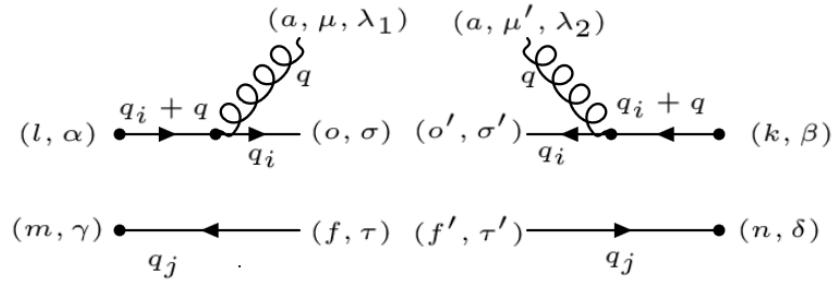
0.2.1 $qg\text{-}\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + q)}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)] \quad (2)$$



$$M_1^\dagger = \left[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) [\bar{v}_{\tau'}(q_j)] \right] \quad (3)$$



$$|M_1|^2 = M_1 \textcolor{red}{M}_1^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q) [v_\tau(q_j)] \textcolor{red}{[} \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) [\bar{v}_{\tau'}(q_j)] \textcolor{red}{]} \quad (4)$$

$$|M_1|^2 = \left[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q) \right. \\ \left. \times (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} [\bar{v}_{\tau'}(q_j) v_\tau(q_j)] \right] \quad (5)$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\begin{aligned} \sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) &= \not{q}_i, \\ \sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) &= \not{q}_j \end{aligned} \quad (6)$$

and sum over polarization index (λ_1, λ_2) :

$$\sum_{\mu, \mu'} \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q) = -g_{\mu\mu'} \quad (7)$$

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i g_{\mu'\mu} \gamma^\mu (\not{q}_i + q)] [\not{q}_j] \quad (8)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i \gamma_{\mu'} (\not{q}_i + q)] [\not{q}_j] \quad (9)$$

In other words we expect the tree level diagram from LO and a number: Which means:

$$|M^2| = \left| \begin{array}{c} \text{diagram with two shaded circles and external lines } q_i + q \text{ and } q_j \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{diagram with a loop and external lines } q_i + q \text{ and } q \end{array} \right|^2$$

contribution from LO *a complex number*

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [\not{P}_i] [\not{P}_j] \otimes (\text{a complex number}) \quad (10)$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N &=: \gamma^{\mu'} \not{q}_i \gamma_{\mu'} = q_{i\sigma} \gamma^{\mu'} \gamma^\sigma \gamma_{\mu'} \\ &= q_{i\sigma} (\{\gamma^{\mu'}, \gamma^\sigma\} - \gamma^\sigma \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} 2g^{\mu'\sigma} \gamma_{\mu'} - d \gamma^\sigma \\ &= (2 - d) \not{q}_i \end{aligned} \quad (11)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \not{q}_i (\not{q}_i + q)] [\not{q}_j] \quad (12)$$

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(q_i+q)^2 (q_i+q)^2} [\not{q}_i \not{q}_i \not{q}_i + \not{q}_i \not{q}_i \not{q} + \not{q} \not{q}_i \not{q}_i + \not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (13)$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \not{q}_i &= q_i = m^2 \\ \not{q} \not{q} &= q = m^2 \\ \not{q}_j \not{q}_j &= q_j = m^2 \end{aligned} \quad (14)$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \not{q}_i$ and $\not{q} \not{q}$ as well.

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)(2q_i q)} [\not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (15)$$

$$\begin{aligned} L &= \not{q} \not{q}_i \not{q} = \not{q} [q_{i\sigma} q_\mu (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu)] \\ &\quad \not{q} [2q_i^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu \left[\frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2} \right] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\ &= \not{q} (2q_i q) - q^2 \not{q}_i \\ &= \not{q} \end{aligned} \quad (16)$$

After inserting the last result of L and simplify the term $(2q_i q)$ from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [\not{q}_i] [\not{q}_j] \quad (17)$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$|M_1|^2 = (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [z \not{p}_i + y(1-z) \not{p}_j + \sqrt{zy(1-z)} \not{m}_\perp] [(1-y) \not{p}_j^\mu] \quad (18)$$

Multiplying the both sides

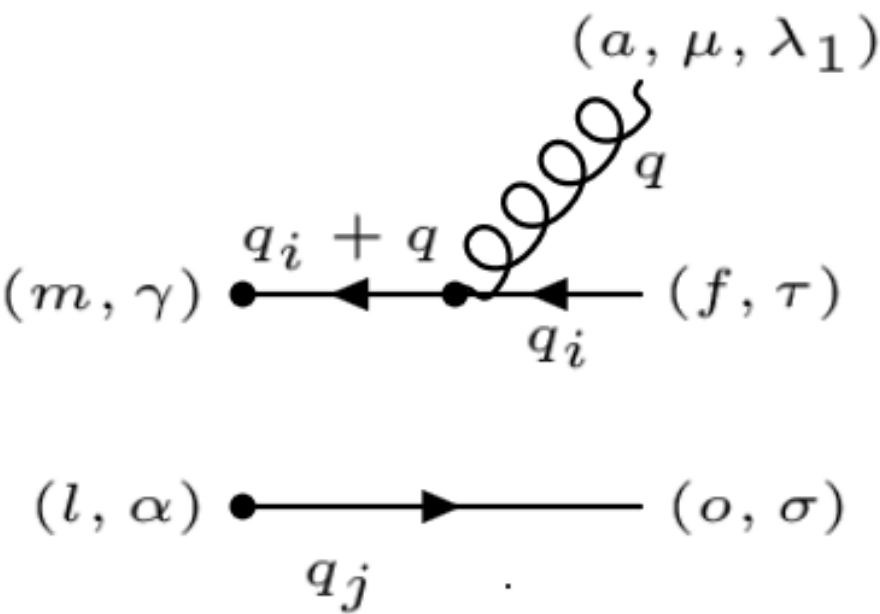
$$\begin{aligned} |M_1|^2 &= (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [(1-z)(1-y) \not{p}_i \not{p}_j \\ &\quad + zy(1-y) \not{p}_j \not{p}_j + (1-y) \sqrt{zy(1-z)} \not{m}_\perp \not{p}_j] \end{aligned} \quad (19)$$

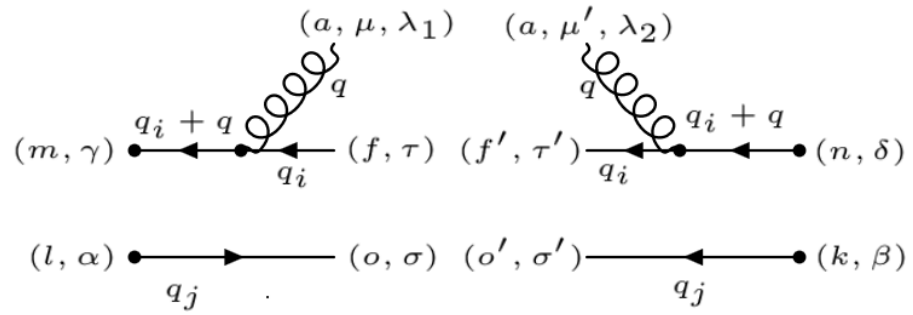
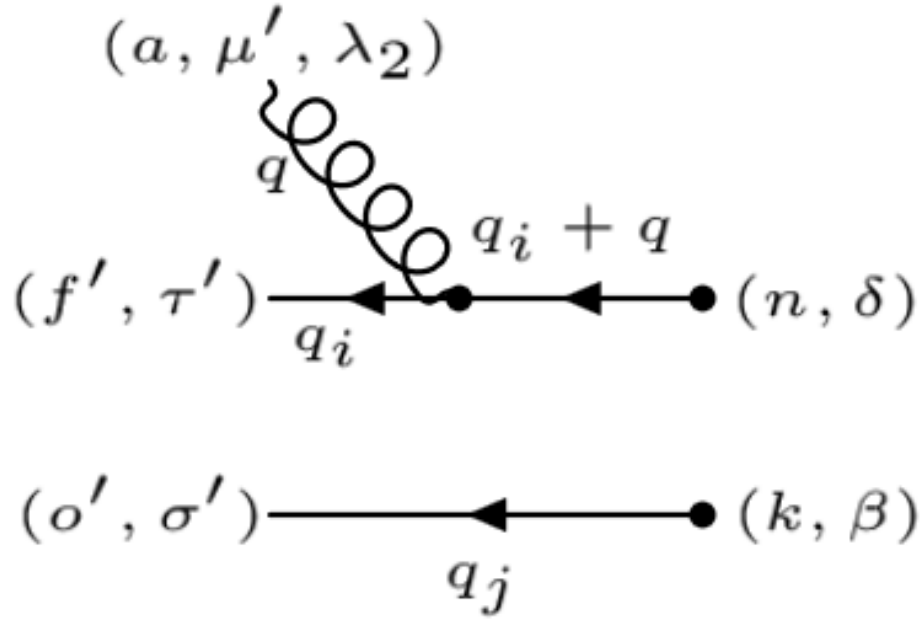
and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i \not{p}_i$ and $\not{p}_j \not{p}_j$. The $p_i \cdot m_\perp$ and

$p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

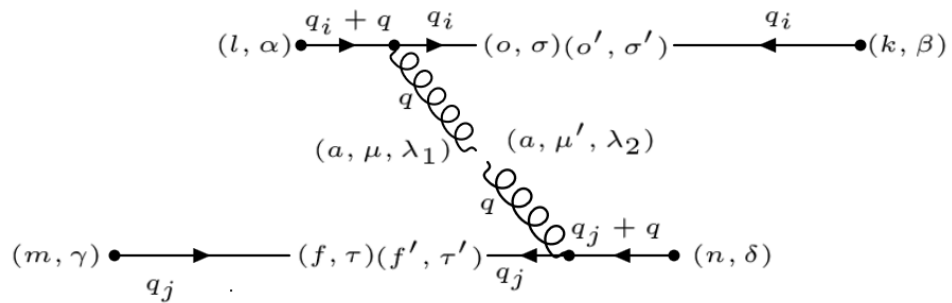
$$|M_1|^2 = (d-2)(1-z)(1-y) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [\not{p}_i] [\not{p}_j] \quad (20)$$

0.2.2 $\bar{q}g$ -q





0.2.3 $M_1 M_2^\dagger$



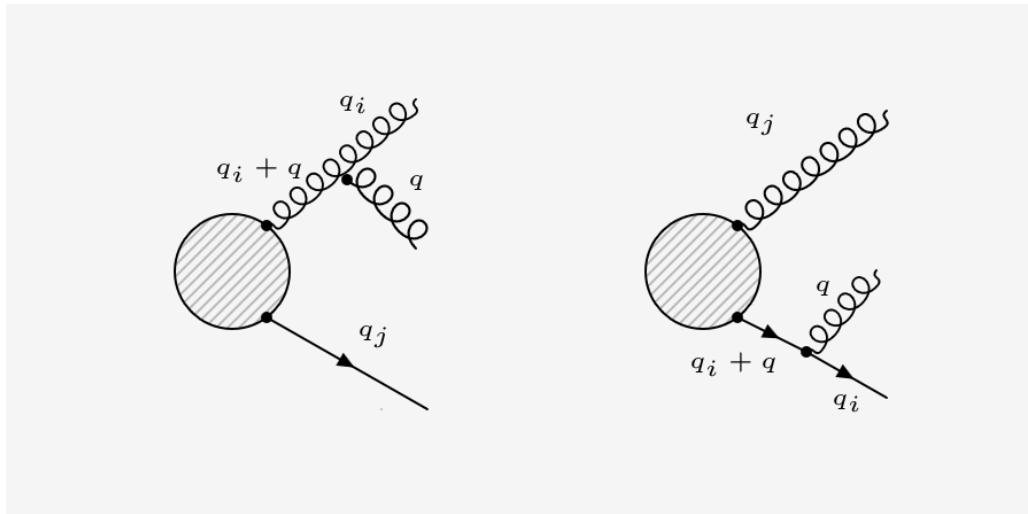
0.2.4 $|M^2|$

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1} \\ \vdots \\ \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \\ \vdots \\ \text{Diagram 4} \end{array} \right|^2 +$$

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1} \\ \vdots \\ \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \\ \vdots \\ \text{Diagram 4} \end{array} \right|^2 +$$

$$2RE \left(\left| \begin{array}{c} \text{Diagram 5} \\ \vdots \\ \text{Diagram 6} \end{array} \right| \right)$$

0.3 Quark/Gluon gluon emission kernel



$$|M^2| = \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right|^2 + \dots$$

The diagrams in the equation represent different topologies for gluon emission from a quark line. Diagram 1 and 2 show a quark line with a gluon loop and a gluon emission. Diagram 3 and 4 show a quark line with a gluon loop and a gluon emission, with different momentum assignments. The diagrams are separated by a vertical dashed line, indicating a cut in the process.

Abbildung 1: Die Landkarte.

The figure displays three Feynman diagrams representing the two-loop self-energy of a gluon. Each diagram consists of two shaded circular vertices connected by a horizontal gluon line with momentum q_j .
 - The first diagram on the left shows a gluon loop (curly line) with momentum q and a ghost loop (dashed line) with momentum q_i .
 - The second diagram in the middle shows a gluon loop with momentum q and a gluon loop with momentum q_i .
 - The third diagram on the right shows a gluon loop with momentum q and a ghost loop with momentum q_i , with an additional gluon line connecting the vertices with momentum $q_j + q$.
 The diagrams are enclosed in large vertical bars, and the first two are summed and multiplied by $|M^2|$, while the third is multiplied by $+2RE(\dots)$.

Abbildung 2: Die Landkarte.

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