The Dipole Subtraction Method

– An Introduction

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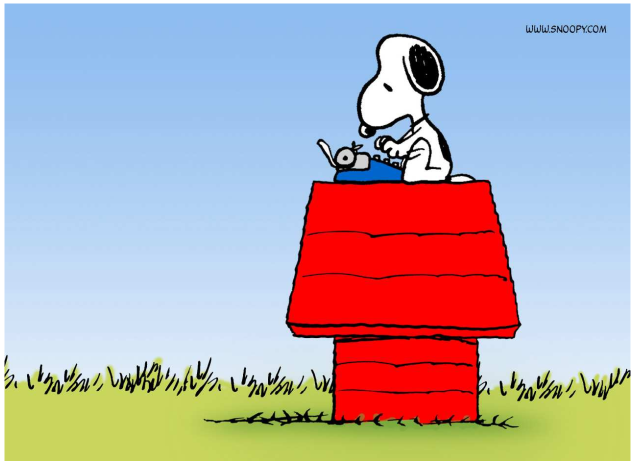


The Dipole Subtraction Method

– An Introduction

for reference (and details)

please consult

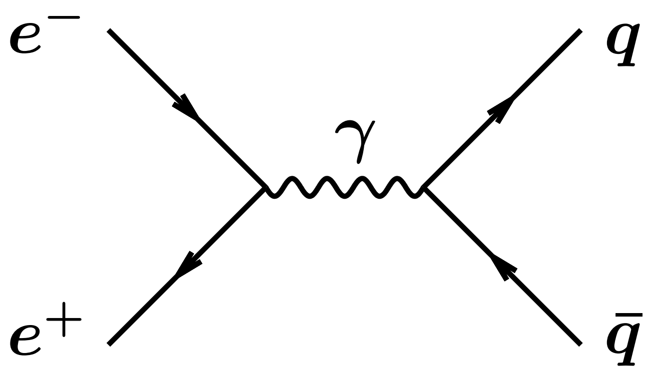
Catani, Seymour, hep-ph/9605323

Outline

the plan for the following talk is to . . .

- ✘ . . . sketch the method
  (very schematic; basic idea only)
- ✘ . . . give details
  (very technical; get you prepared for applying the method yourself)
- ✘ . . . consider basic example: $e^+e^- \longrightarrow 2$ jets
  (test your understanding)
- ✘ . . . generalize the method to processes with identified hadrons
- ✘ . . . study another example: $qq \longrightarrow qqH$ via VBF

Setting the Stage: The LO



the most transparent case:

no identified hadrons in process,
e.g. $e^+e^- \longrightarrow 2$ jets:

$m \ldots \#$ of final state partons

$$\sigma^{LO} = \int_m d\sigma^B$$

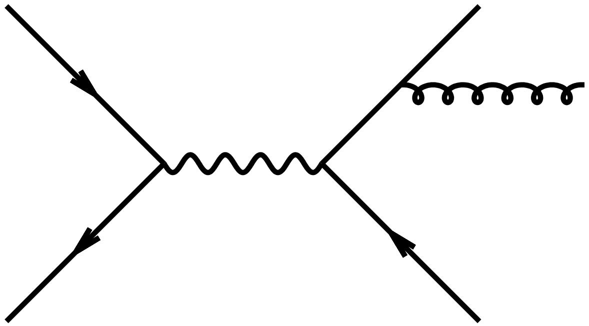
finite!
no regularization needed

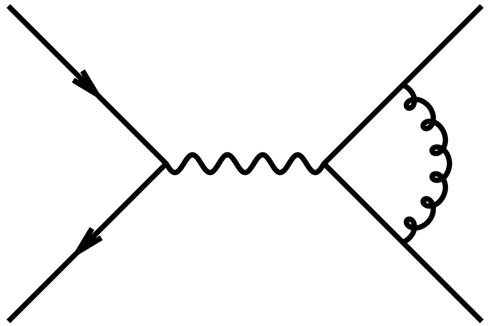
calculate in $d = 4$
dimensions

m-parton
phase space
integral

Born x-sec for
$e^+e^- \longrightarrow q\bar{q}$
$(m = 2)$

Setting the Stage: The NLO





real emission contributions
$m + 1$ parton kinematics

virtual corrections
m parton kinematics

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

IR divergent

☞regularize in $d = 4 - 2\varepsilon$ dim

The Subtraction

introduce local counterterm $d\sigma^A$ with
same singularity structure as $d\sigma^R$:

$$\sigma^{NLO} = \int_{m+1} \underbrace{\left[d\sigma^R - d\sigma^A\right]}_{\text{finite}} + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

$\downarrow$

can safely set $\varepsilon \longrightarrow 0$

perform integral numerically in
four dimension

The Subtraction

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^R - d\boldsymbol{\sigma}^A \right] \Big|_{\varepsilon=0} + \int_m d\boldsymbol{\sigma}^V + \int_{m+1} d\boldsymbol{\sigma}^A$$

$\downarrow$

integrate over one-parton PS analytically

explicitly cancel poles & then set $\varepsilon \longrightarrow 0$

$\downarrow$

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^R_{\varepsilon=0} - d\boldsymbol{\sigma}^A_{\varepsilon=0} \right] + \int_m \left[d\boldsymbol{\sigma}^V + \int_1 d\boldsymbol{\sigma}^A \right]_{\varepsilon=0}$$

The Counterterm: $d\sigma^A$

wish list:

- matches singular behavior of $d\sigma^R$ exactly in d dim
- convenient for Monte Carlo integration
- exactly integrable analytically over one-parton PS in d dim
- for given process: independent of specific observable
- extra feature: universal structure

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a solution: dipole subtraction method
`[Catani and Seymour, hep-ph/9605323]`

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

(other approaches: Ellis et al.; Kunszt and Soper; Dittmaier, . . .)

The Counterterm: $d\sigma^A$

color/spin
projection of
Born x-sec

universal dipole
factors

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

dipoles for all $(m+1)$
configurations
corresponding to given
m-parton state

PS convolution &
color/spin summation

Singularity Structure

$$\left|\mathcal{M}_{m+1}(Q\,;\,p_1,\ldots,p_i,\ldots,p_j,\ldots,p_{m+1})\right|^2$$

soft region:

$$p_j = \lambda q,\; \lambda \to 0$$

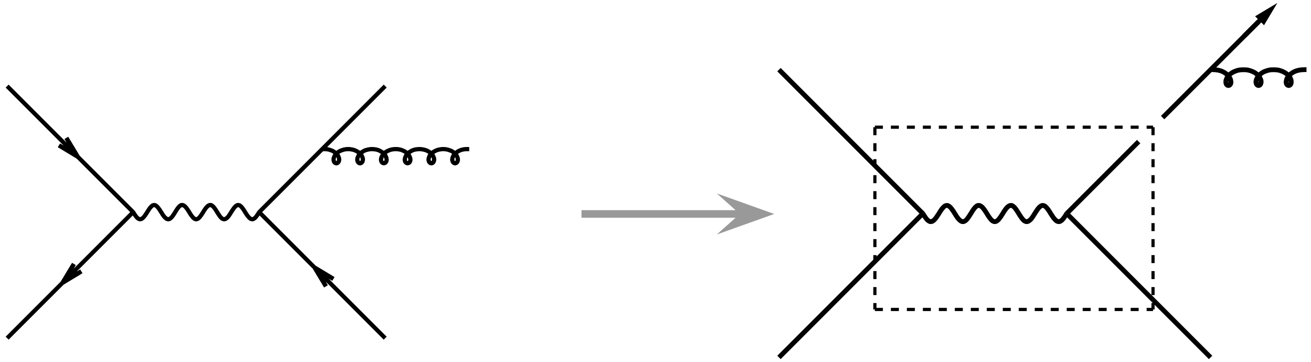
$$|\mathcal{M}_{m+1}|^2 \sim \frac{1}{\lambda^2}$$

collinear region:

$$p_j = \frac{(1-z)}{z}\,p_i$$

$$|\mathcal{M}_{m+1}|^2 \sim \frac{1}{p_i p_j}$$

e. g.:

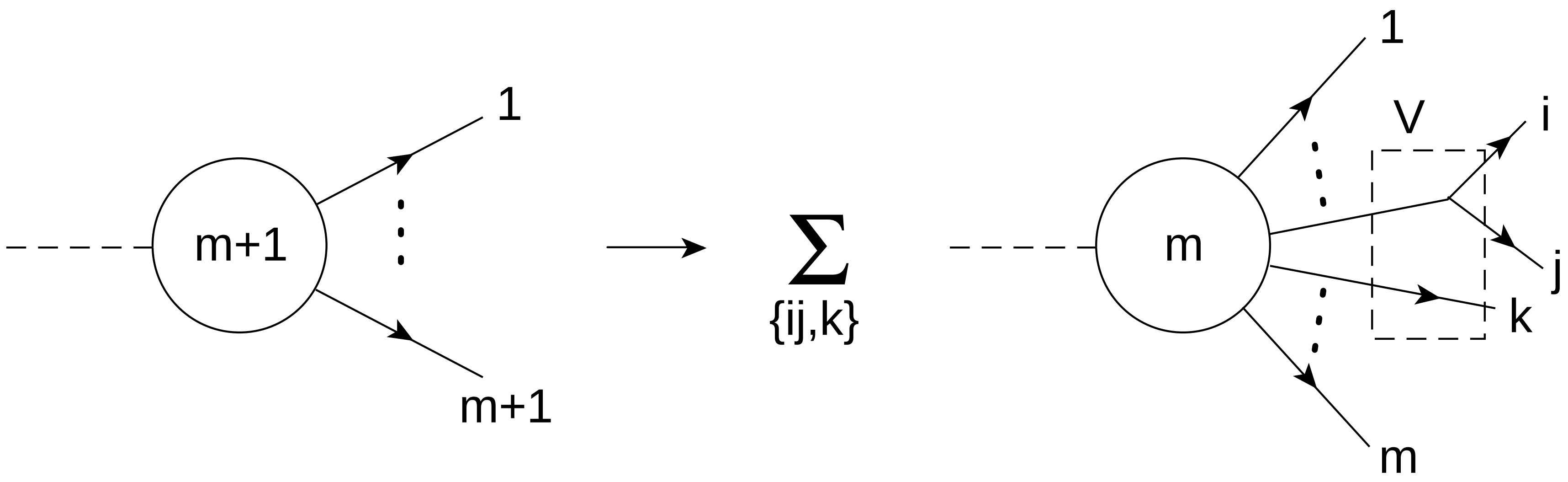


universal structure: for each singular configuration

$$|\mathcal{M}_{m+1}|^2 \to |\mathcal{M}_m|^2 \otimes \mathrm{V}_{ij,k}$$

Singularity Structure

for $|\mathcal{M}_{m+1}|^2 \rightarrow \sum |\mathcal{M}_m|^2 \otimes \mathbf{V}_{ij,k}$



$\mathbf{V}_{ij,k}$ ... contains singularities, depends on momenta & quantum numbers of partons $\boldsymbol{i, j, k}$

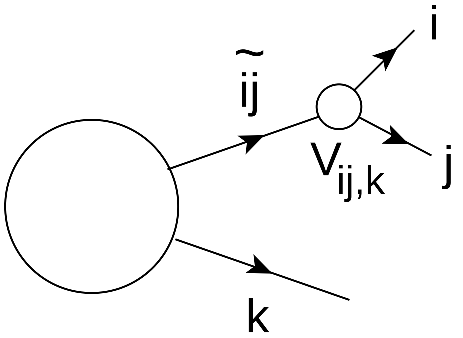
$\boldsymbol{ij}$ and $\boldsymbol{k}$ ... emitter and spectator

Dipole Formula

$$|\mathcal{M}_{m+1}|^2 = \langle 1, \ldots, m+1 || 1, \ldots, m+1 \rangle$$

$$= \underbrace{\sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1})}_{\text{divergent as } p_i \cdot p_j \to 0} + \ldots$$

finite rest

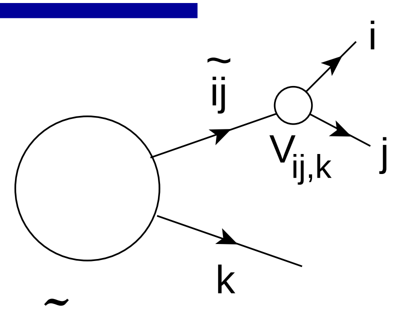


$$\mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1}) = -\frac{1}{2 p_i \cdot p_j}$$

$$\cdot_m \langle 1, \ldots, \tilde{ij}, \ldots, \tilde{k}, \ldots, m+1 | \frac{\mathrm{T_k} \cdot \mathrm{T_{ij}}}{\mathrm{T_{ij}^2}} \mathrm{V_{ij,k}} | 1, \ldots, \tilde{ij}, \ldots, \tilde{k}, \ldots, m+1 \rangle_m$$

Dipole Formula: Kinematics

$$\mathcal{D}_{ij,k}\left(p_1,\ldots,p_{m+1}\right) = -\frac{1}{2p_i\cdot p_j}$$



$$\cdot_m\langle 1,\ldots,\widetilde{ij},\ldots,\tilde{k},\ldots,m+1|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathbf{V}_{ij,k}|1,\ldots,\widetilde{ij},\ldots,\tilde{k},\ldots,m+1\rangle_m$$

kinematics:

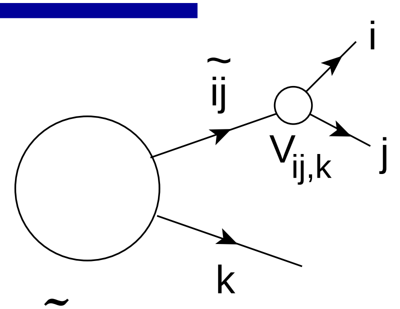
$$\tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu\ , \quad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu$$

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i}$$

$$\tilde{p}_k^2 = \tilde{p}_{ij}^2 = 0\ , \quad p_i^\mu + p_j^\mu + p_k^\mu = \tilde{p}_{ij}^\mu + \tilde{p}_k^\mu$$

Dipole Formula: Insertion Operator

$$\mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1}) = -\frac{1}{2p_i \cdot p_j}$$



$$\cdot_m \langle 1, \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots, m+1 | \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij,k} | 1, \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots, m+1 \rangle_m$$

insertion operator:

$\mathbf{T}_k, \mathbf{T}_{ij} \ldots$ color charges of spectator and emitter

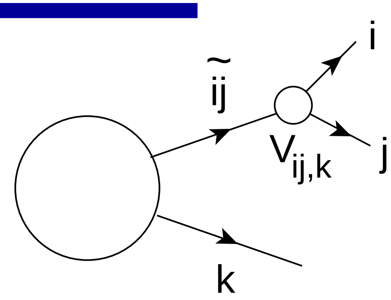
$\mathbf{V}_{ij,k} \quad \ldots$ splitting kernel in helicity space of emitter

explicit form depends on parton type

become proportional to Altarelli-Parisi splitting functions and Eikonal factors in collinear and soft limits, resp.

Dipole Formula: Example

example: $q(ij) \to q(i)g(j)$



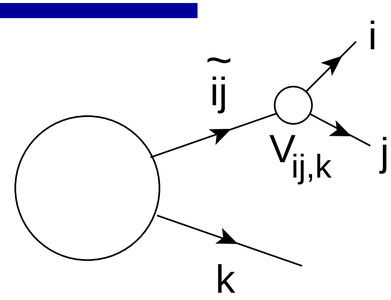
$$\langle s|\mathrm{V}_{\mathrm{q_i g_j,k}}(\tilde{z}_i; y_{ij,k})|s'\rangle = V_{q_i g_j,k}\delta_{ss'}$$

$$= 8\pi\mu^{2\varepsilon}\alpha_s C_F\left[\frac{2}{1-\tilde{z}_i(1-y_{ij,k})} - (1+\tilde{z}_i) - \varepsilon(1-\tilde{z}_i)\right]\delta_{ss'}$$

$$\tilde{z}_i = \frac{p_i p_k}{p_j p_k + p_k p_i} = \frac{p_i \tilde{p}_k}{\tilde{p}_{ij}\tilde{p}_k}$$

($s, s' \ldots$ spin index of fermion $\tilde{ij}$ in $\mathcal{M}$ and $\mathcal{M}^\star$)

Dipole Formula: Example



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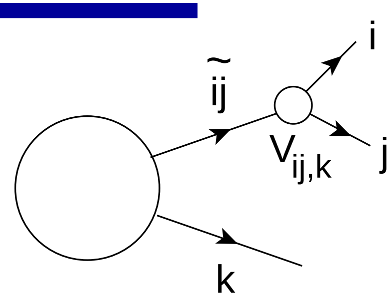
$$\tilde{z}_i = \frac{p_i p_k}{p_j p_k + p_k p_i} = \frac{p_i \tilde{p}_k}{\tilde{p}_{ij}\tilde{p}_k}$$

(looked up in *hep-ph/9605323*)

Phase Space Factorization

reminder: need $\int_m \left[d\sigma^V + \int_1 d\sigma^A\right]_{\varepsilon=0}$

factorize PS of partons i, j, k:



$$d\phi(p_i, p_j, p_k; Q) = \frac{d^d p_i}{(2\pi)^{d-1}}\delta_+(p_i^2)\frac{d^d p_j}{(2\pi)^{d-1}}\delta_+(p_j^2)\frac{d^d p_k}{(2\pi)^{d-1}}\delta_+(p_k^2) \times (2\pi)^d \delta^{(d)}(Q - p_i - p_j - p_k)$$

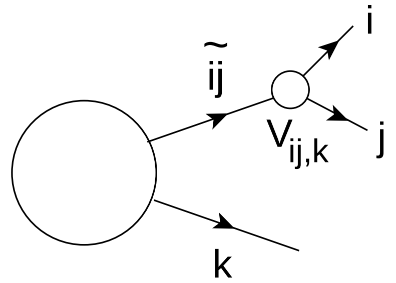
$$= d\phi(\tilde{p}_{ij}, \tilde{p}_k; Q)\, [dp_i(\tilde{p}_{ij}, \tilde{p}_k)]$$

$$\text{with } [dp_i(\tilde{p}_{ij}, \tilde{p}_k)] = \frac{d^d p_i}{(2\pi)^{d-1}}\delta_+(p_i^2)\ \mathcal{J}(p_i; \tilde{p}_{ij}, \tilde{p}_k)$$

…perform $\int [dp_i]$ explicitly once and for all $\mathrm{V}_{ij,k}$!

Integration

for $\int_1 d\sigma^A$ need



$$\int [dp_i(\tilde{p}_{ij}, \tilde{p}_k)]\, \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1})$$

$$= -\mathcal{V}_{ij,k}\ \langle \ldots, \widetilde{ij}, \tilde{k}, \ldots, m+1 | \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} | \ldots, \widetilde{ij}, \tilde{k}, \ldots, m+1 \rangle_m$$

$$\mathcal{V}_{ij,k} = \int [dp_i(\tilde{p}_{ij}, \tilde{p}_k)]\, \frac{1}{2p_i \cdot p_j} \langle \mathrm{V}_{ij,k} \rangle$$

$$\equiv \frac{\alpha_s}{2\pi} \frac{1}{\Gamma[1-\varepsilon]} \left(\frac{4\pi\mu^2}{2\tilde{p}_{ij}\tilde{p}_k} \right)^{\varepsilon} \mathcal{V}_{ij}(\varepsilon) \longrightarrow$$

compute explicitly by rewriting integral in terms of $\tilde{z}_i$ and $y_{ij,k}$

“Splitting Function”

remember: example $q \to qg$

$$\mathrm{V}_{qg,k} = 8\pi\mu^{2\varepsilon}\alpha_s C_F \left[\frac{2}{1-\tilde{z}_i(1-y_{ij,k})} - (1+\tilde{z}_i) - \varepsilon(1-\tilde{z}_i)\right]$$

$$\Downarrow$$

find

$$\mathcal{V}_{qg} = C_F \left[\frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon} + 5 - \frac{\pi^2}{2} + \mathcal{O}(\varepsilon)\right]$$

(analogous for $\mathcal{V}_{q\bar{q}}$ and $\mathcal{V}_{gg}$)

Jet-Defining Function

consider infrared-safe observables: insensitive to soft & collinear parton emission

formally: introduce jet-defining function such that

$$F_J^{(m+1)}(p_1, ..., p_i, ..., p_j, ...p_{m+1}) \longrightarrow F_J^{(m)}(p_1, ..., p, ...p_{m+1})$$

in soft / collinear regions and

$$F_J^{(m)}(p_1, ..., p_i, ..., p_j, ...p_m) \longrightarrow 0 \quad \text{for} \quad p_i \cdot p_j = 0$$

in practice: $F_J^{(n)}$ ... combination of θ-functions, δ-functions, numerical and kinematic factors

(depends on jet definition and cuts used)

crucial for feasibility of subtraction procedure

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$$\Big\downarrow$$

$$d\sigma^B \;=\; d\Phi_m(p_1, .., p_m; Q)\, |\mathcal{M}_m(p_1, .., p_m)|^2\, F_J^{(m)}(p_1, .., p_m)$$

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in soft / collinear regions and

$$F_J^{(m)}(p_1, ..., p_i, ..., p_j, ...p_m) \longrightarrow 0 \quad \text{for} \quad p_i \cdot p_j = 0$$

$\downarrow$

$$d\sigma^R = \; d\Phi_{m+1}(p_1.., p_{m+1}; Q)|\mathcal{M}_{m+1}(p_1, .., p_{m+1})|^2 F_J^{(m+1)}(p_1, .., p_{m+1})$$

crucial for feasibility of subtraction procedure

Jet-Defining Function

consider infrared-safe observables: insensitive to soft & collinear parton emission

formally: introduce jet-defining function such that

$$F_J^{(m+1)}(p_1, ..., p_i, ..., p_j, ...p_{m+1}) \longrightarrow F_J^{(m)}(p_1, ..., p, ...p_{m+1})$$

in soft / collinear regions and

$$F_J^{(m)}(p_1, ..., p_i, ..., p_j, ...p_m) \longrightarrow 0 \quad \text{for} \quad p_i \cdot p_j = 0$$

$$\downarrow$$

$$d\sigma^V = d\Phi_m(p_1, .., p_m; Q) |\mathcal{M}_m(p_1, .., p_m)|^2_{(1-loop)} F_J^{(m)}(p_1, .., p_m)$$

crucial for feasibility of subtraction procedure

Jet-Defining Function

consider infrared-safe observables: insensitive to soft & collinear parton emission

formally: introduce jet-defining function such that

$$F_J^{(m+1)}(p_1, \ldots, p_i, \ldots, p_j, \ldots p_{m+1}) \longrightarrow F_J^{(m)}(p_1, \ldots, p, \ldots p_{m+1})$$

in soft / collinear regions and

$$F_J^{(m)}(p_1, \ldots, p_i, \ldots, p_j, \ldots p_m) \longrightarrow 0 \quad \text{for} \quad p_i \cdot p_j = 0$$

$$\Downarrow$$

$$\begin{aligned} d\sigma^A &= d\Phi_{m+1}(p_1, \ldots, p_{m+1}; Q) \\ &\times \sum_{\substack{\text{pairs} \\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1}) F_J^{(m)}(\ldots, \tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1}) \end{aligned}$$

Jet Cross Sections

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A \right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

$$\begin{aligned} d\sigma^R - d\sigma^A \quad &= d\Phi_{m+1}(p_1, \ldots, p_{m+1}; Q) \\ &\cdot \Big\{ |\mathcal{M}_{m+1}(p_1, \ldots, p_{m+1})|^2 \, F_J^{(m+1)}(p_1, \ldots, p_i, p_j, p_k, p_{m+1}) \\ &- \sum_{\substack{\text{pairs} \\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1}) \, F_J^{(m)}(\ldots, \tilde{p}_{ij}, \tilde{p}_k, p_{m+1}) \Big\} \end{aligned}$$

singular regions: $d\sigma^R$ and $d\sigma^A$ separately divergent

Jet Cross Sections

$$d\sigma^R - d\sigma^A = d\Phi_{m+1}(p_1, ..., p_{m+1}; Q)$$

$$\cdot \Big\{ |\mathcal{M}_{m+1}(p_1, ..., p_{m+1})|^2 F_J^{(m+1)}(p_1, ..., p_{m+1})$$

$$- \sum_{\substack{\text{pairs}\\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, ..., p_{m+1})\, F_J^{(m)}(..., \tilde{p}_{ij}, \tilde{p}_k, p_{m+1}) \Big\}$$

singular regions:

$$|\mathcal{M}_{m+1}(p_1, ..., p_{m+1})|^2 \longrightarrow \mathcal{D}_{ij,k}(p_1, ..., p_{m+1})$$

$$\{p_1, ..., p_i, p_j, ..., p_k, ..., p_{m+1}\} \longrightarrow \{p_1, ..., \tilde{p}_{ij}, \tilde{p}_k, ..., p_{m+1}\}$$

$$F_J^{(m+1)}(p_1, .., p_i, p_j, .., p_k, ..p_{m+1}) \longrightarrow F_J^{(m)}(p_1, .., \tilde{p}_{ij}, \tilde{p}_k, .., p_{m+1})$$

☞ divergencies are cancelled!

Jet Cross Sections

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A\right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A\right]$$

$$\text{reminder}: \quad \int \left[dp_i(\tilde{p}_{ij}, \tilde{p}_k)\right] \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1})$$

$$= \; -\mathcal{V}_{ij,k} \, \langle ..., \tilde{ij}, \tilde{k}, ..., m+1 | \frac{\mathbf{T}_\mathrm{k} \cdot \mathbf{T}_\mathrm{ij}}{\mathbf{T}_\mathrm{ij}^2} | ..., \tilde{ij}, \tilde{k}, ..., m+1 \rangle_m$$

performing 1-parton PS integral yields

$$\int_{m+1} d\sigma^A \; \propto \; -\int_m d\Phi_m(p_1, ..., p_m; Q) F_J^{(m)}(p_1, ..., p_m)$$

$$\cdot \sum_i \sum_{k \neq i} {}_m\langle 1, .., m | \frac{\mathbf{T}_\mathrm{k} \cdot \mathbf{T}_\mathrm{i}}{\mathbf{T}_\mathrm{i}^2} | 1, .., m \rangle_m \mathcal{V}_{i,k}(\varepsilon)$$

remaining integral: m-parton kinematics!

Jet Cross Sections

alternative notation:

$$\int_{m+1} d\boldsymbol{\sigma}^{A} \;=\; \int_m \left[d\boldsymbol{\sigma}^{B} \cdot \mathbf{I}(\varepsilon)\right]$$

replace ${}_m\langle 1, .., m|1, ..m\rangle_m$ at Born level

$$\downarrow$$

$${}_m\langle 1, .., m|\,\mathbf{I}\,(\varepsilon)\,|1, ..m\rangle_m$$

with

$$\mathbf{I}(p_1, ..., p_m; \varepsilon) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\sum_i \frac{1}{\mathbf{T}_i^2}\mathcal{V}_i(\varepsilon)\sum_{k\neq i}\mathbf{T}_\mathbf{i}\mathbf{T}_\mathbf{k}\left(\frac{4\pi\mu^2}{2p_ip_k}\right)^{\varepsilon}$$

Wake up!

. . . after this load of technical details:

need survey on main formulae . . .

Survey

need: $\boldsymbol{\sigma}^{LO}$ and $\boldsymbol{\sigma}^{NLO} = \boldsymbol{\sigma}^{NLO\,\{m+1\}} + \boldsymbol{\sigma}^{NLO\,\{m\}}$

the leading order:

$$\boldsymbol{\sigma}^{LO} = \int_m d\boldsymbol{\sigma}^B = \int d\Phi^{(m)} |\mathcal{M}_m(p_1, \ldots, p_m)|^2 F_J^{(m)}(p_1, \ldots, p_m)$$

completely finite

compute in $d = 4$ dimensions

Survey: NLO

NLO: $(m+1)$ parton kinematics:

$$\sigma^{NLO\,\{m+1\}} = \int_{m+1} \left[\left(d\sigma^R \right)_{\varepsilon=0} - \left(\sum_{dipoles} d\sigma^B \otimes dV_{dipole} \right)_{\varepsilon=0} \right]$$

$$= \int d\Phi^{(m+1)} \cdot \Big\{ |\mathcal{M}_{m+1}(p_1, \ldots, p_{m+1})|^2 F_J^{(m+1)}(p_1, \ldots, p_{m+1})$$

$$- \sum_{\substack{\text{pairs}\\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, \ldots, p_{m+1})\; F_J^{(m)}(\ldots, \tilde{p}_{ij}, \tilde{p}_k, p_{m+1}) \Big\}$$

...calculation is performed in $d = 4$ dimensions

Survey: NLO

NLO: m parton kinematics:

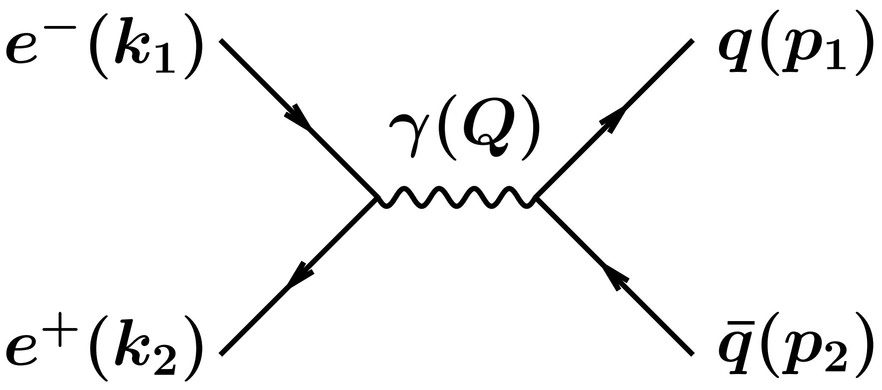
$$\sigma^{NLO\,\{m\}} = \int_m \left[d\sigma^V + d\sigma^B \otimes \mathbf{I}\right]_{\varepsilon=0}$$
$$= \int d\Phi^{(m)} \cdot \Big\{ \left|\mathcal{M}_{m+1}(p_1, ..., p_m)\right|^2_{(1-loop)}$$
$$+ {}_m\langle 1, ..., m|\, \mathbf{I}\,(\varepsilon)\, |1, ..., m\rangle_m \Big\} F_J^{(m)}(p_1, ..., p_m)$$

- first step of calculation is performed in $d = 4 - 2\varepsilon$ dimensions
- poles are cancelled analytically
- finally: $\varepsilon \longrightarrow 0$

Let's try!

. . . we are now ready to apply our knowledge . . .

$e^+e^- \longrightarrow$ 2 jets: the leading order



notation:

$$(p_1 + p_2)^2 = s = Q^2$$

$$y_{ij} = 2p_i \cdot p_j / Q^2$$

$$x_i = 2p_i \cdot Q / Q^2$$

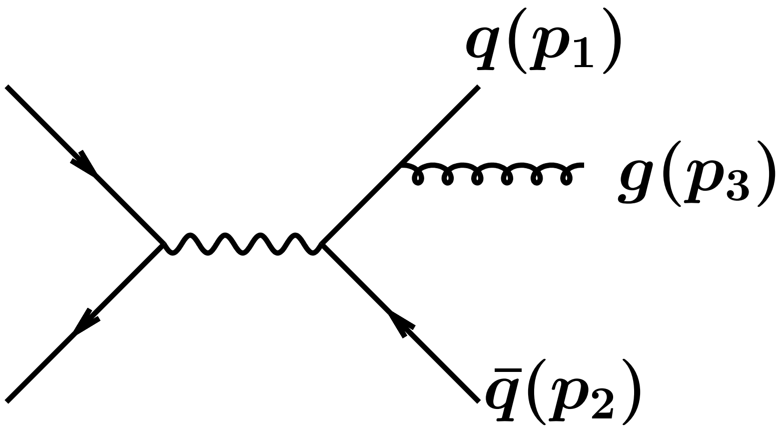
$$|\overline{\mathcal{M}_2}|^2 \; = \; \text{const.} \, \frac{(k_1 \cdot p_2)^2 + (k_1 \cdot p_1)^2}{(k_1 \cdot k_2)^2} \stackrel{c.m.s.}{=} \text{const.} \left(1 + \cos^2\theta\right)$$

average over orientation and use

$$d\Phi^{(2)} = dy_{12}\, \delta(1 - y_{12})$$

$$\sigma^{LO} = |\mathcal{M}_2|^2 \int dy_{12}\, \delta(1 - y_{12}) F_J^{(2)}(p_1, p_2)$$

$e^+e^- \to$ 2 jets: real emission



notation:

$$y_{ij} = 2p_i \cdot p_j / Q^2$$

$$x_i = 2p_i \cdot Q / Q^2$$

$$x_1 + x_2 + x_3 = 2$$

$$\sigma^R = \int d\Phi^{(3)} |\mathcal{M}_3|^2 \, F_J^{(3)}(p_1, p_2, p_3)$$

with

$$|\mathcal{M}_3(p_1, p_2, p_3)|^2 \;=\; C_F \frac{8\pi\alpha_s}{Q^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} |\mathcal{M}_2|^2$$

$$d\Phi^{(3)} \;=\; \frac{Q^2}{16\pi^2} dx_1 dx_2 \Theta(1-x_1)\Theta(1-x_2)\Theta(x_1+x_2-1)$$

reminder: for $d\sigma^{NLO\,(3)}$ need

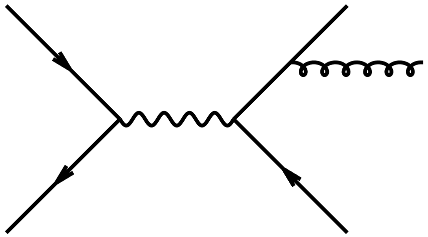
$$\sigma^A = \int d\Phi_{m+1}(p_1,\ldots,p_{m+1};Q)$$

$$\times \sum_{\substack{\text{pairs}\\ i,j}} \sum_{k\neq i,j} \mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) F_J^{(m)}(\ldots,\tilde{p}_{ij},\tilde{p}_k,\ldots,p_{m+1})$$

$$\text{with } \mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j}$$

$$\cdot_m\langle 1,\ldots,\tilde{ij},\ldots,\tilde{k},\ldots,m+1|\frac{\mathrm{T_k}\cdot\mathrm{T_{ij}}}{\mathrm{T_{ij}^2}}\mathrm{V_{ij,k}}|1,\ldots,\tilde{ij},\ldots,\tilde{k},\ldots,m+1\rangle_m$$

$e^+e^- \to$ 2 jets: dipoles $\mathcal{D}_{ij,k}$



here: need $\mathcal{D}_{13,2}$ and $\mathcal{D}_{23,1}$ $\quad (\mathcal{D}_{12,3} = 0)$

$$\mathcal{D}_{13,2} = -\frac{1}{2p_1 \cdot p_3} \langle \tilde{p}_{13}, \tilde{p}_2 | \frac{\mathbf{T}_2 \cdot \mathbf{T}_{13}}{\mathbf{T}_{13}^2} \mathbf{V}_{q_1 g_3, \bar{q}_2} | \tilde{p}_{13}, \tilde{p}_2 \rangle$$

$$\mathcal{D}_{23,1} = -\frac{1}{2p_2 \cdot p_3} \langle \tilde{p}_{23}, \tilde{p}_1 | \frac{\mathbf{T}_1 \cdot \mathbf{T}_{23}}{\mathbf{T}_{23}^2} \mathbf{V}_{\bar{q}_2 g_3, q_1} | \tilde{p}_{23}, \tilde{p}_1 \rangle$$

for color-connected amplitudes use color conservation:

$$\sum_{i=1}^{m} \mathbf{T}_i | 1, \ldots, m \rangle_m = 0 \quad \to \quad \mathbf{T}_k \cdot \mathbf{T}_{ij} | k, ij \rangle = -\mathbf{T}_{ij} \cdot \mathbf{T}_{ij} | k, ij \rangle$$

$$\big\downarrow$$

$$\langle \tilde{p}_{13}, \tilde{p}_2 | \frac{\mathbf{T}_2 \cdot \mathbf{T}_{13}}{\mathbf{T}_{13}^2} \mathbf{V}_{q_1 g_3, \bar{q}_2} | \tilde{p}_{13}, \tilde{p}_2 \rangle = -\mathbf{V}_{q_1 g_3, \bar{q}_2} \langle \tilde{p}_{13}, \tilde{p}_2 | \tilde{p}_{13}, \tilde{p}_2 \rangle$$

$$= -\mathbf{V}_{q_1 g_3, \bar{q}_2} |\mathcal{M}_2(\tilde{p}_{13}, \tilde{p}_2)|^2 = -\mathbf{V}_{q_1 g_3, \bar{q}_2} |\mathcal{M}_2|^2$$

$e^+e^- \longrightarrow$ 2 jets: dipoles

dipole kinematics:

$$\tilde{p}_2^\mu = \frac{1}{x_2} p_2^\mu \qquad \tilde{p}_{13}^\mu = Q^\mu - \frac{1}{x_2} p_2^\mu$$
$$\tilde{p}_1^\mu = \frac{1}{x_1} p_1^\mu \qquad \tilde{p}_{23}^\mu = Q^\mu - \frac{1}{x_1} p_1^\mu$$

$V_{q_1 g_3, \bar{q}_2} \longrightarrow$ look up in `hep-ph/9605323`:

$$V_{qg,q}\big|_{\varepsilon=0} = 8\pi\alpha_s C_F \left[\frac{2}{1-\tilde{z}_i(1-y_{ij,k})} - (1+\tilde{z}_i)\right]$$

and compute

$$\tilde{z}_i = \frac{x_1 + x_2 - 1}{x_2} \quad \text{and} \quad 1 - y_{13,2} = x_2$$

$e^+e^- \longrightarrow 2$ jets: dipoles

finally:

$$\mathcal{D}_{13,2} = \frac{8\pi\alpha_s C_F}{Q^2} |\mathcal{M}_2|^2 \times \left[\frac{1}{1-x_2}\left(\frac{2}{1-x_1-x_2} - (1+x_1)\right) + \frac{1-x_1}{x_2}\right]$$

for $\mathcal{D}_{23,1}$ replace $x_1 \leftrightarrow x_2$ and find:

$$\mathcal{D}_{23,1} = \frac{8\pi\alpha_s C_F}{Q^2} |\mathcal{M}_2|^2 \times \left[\frac{1}{1-x_1}\left(\frac{2}{1-x_1-x_2} - (1+x_2)\right) + \frac{1-x_2}{x_1}\right]$$

$e^+e^- \longrightarrow$ 2 jets: dipoles

collect terms:

$$\sigma^{NLO\,(3)} = \int_3 \left[d\sigma^R|_{\varepsilon=0} - d\sigma^A|_{\varepsilon=0} \right]$$

$$= |\mathcal{M}_2|^2 \frac{\alpha_s C_F}{2\pi} \int_0^1 dx_1\, dx_2\, \Theta(x_1 + x_2 - 1)$$

$$\times \left\{ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} F_J^{(3)}(p_1, p_2, p_3) \right.$$

$$- \left[\frac{1}{1-x_2} \left(\frac{2}{1-x_1-x_2} - (1+x_1) \right) + \frac{1-x_1}{x_2} \right] F_J^{(2)}(\tilde{p}_{13}, \tilde{p}_2)$$

$$\left. - \left[\frac{1}{1-x_1} \left(\frac{2}{1-x_1-x_2} - (1+x_2) \right) + \frac{1-x_2}{x_1} \right] F_J^{(2)}(\tilde{p}_{23}, \tilde{p}_1) \right\}$$

$e^+e^- \longrightarrow 2$ jets: dipoles

for real emission terms use:

$$\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} = \frac{1}{1-x_2}\left(\frac{2}{1-x_1-x_2} - (1+x_1)\right)$$

$$+ \left(x_1 \leftrightarrow x_2\right)$$

(analogous to structure of counter-terms)

---

for $x_i \longrightarrow 1$ (singular regions):

$$F_J^{(3)} \longrightarrow F_J^{(2)}$$

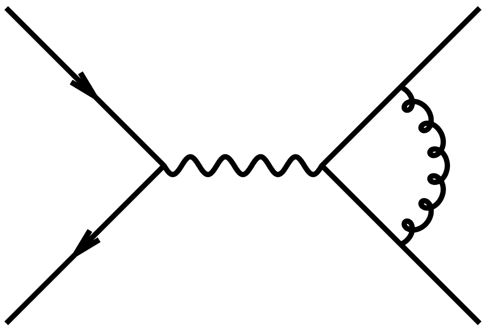
$$\Downarrow$$

“dangerous terms” cancel

integrand finite

$\sigma^{NLO\,(3)}$ well-behaved!

$e^+e^- \longrightarrow$ 2 jets: virtuals



$$\sigma^V = \int d\Phi^{(V)} |\mathcal{M}_2|^2_{(1-loop)} F_J^{(2)}(p_1, p_2)$$

combining self-energy and vertex corrections in $\overline{\mathrm{MS}}$ renormalization scheme yields

$$|\mathcal{M}_2(p_1, p_2)|^2_{(1-loop)} = |\mathcal{M}_2|^2 \frac{C_F \alpha_s}{2\pi} \frac{1}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \times \left\{ \frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 10 - \pi^2 + \mathcal{O}(\varepsilon) \right\}$$

$e^+e^- \longrightarrow$ 2 jets: integral of counterterm

reminder: counterterm integrated over one-parton PS

$$\int_1 d\sigma^A = {}_m\langle 1, \ldots, m|\, I(\varepsilon)\, |1, \ldots, m\rangle_m F_J^{(m)}(p_1, \ldots, p_m)$$

with

$$I(p_1, \ldots, p_m; \varepsilon) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\sum_i \frac{1}{\mathrm{T}_i^2}\mathcal{V}_i(\varepsilon)\sum_{k\neq i}\mathrm{T_i T_k}\left(\frac{4\pi\mu^2}{2p_i p_k}\right)^{\varepsilon}$$

tabulated:

$$\mathcal{V}_i(\varepsilon) = \mathrm{T}_i^2\left(\frac{1}{\varepsilon^2}-\frac{\pi^2}{3}\right)+\gamma_i\frac{1}{\varepsilon}+\gamma_i+K_i$$

$$\gamma_q = \frac{3}{2}C_F$$

$$K_q = \left(\frac{7}{2}-\frac{\pi^2}{6}\right)C_F$$

$e^+e^- \to 2$ jets: integral of counterterm

$$
\begin{aligned}
\langle 1,2|\,\mathbf{I}(p_1,p_2;\varepsilon)\,|1,2\rangle &= 2\,\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)} \\
&\quad\times\langle 1,2|\frac{1}{T_{q_1}^2}\mathcal{V}_{q_1}(\varepsilon)\mathbf{T}_{q_1}\mathbf{T}_{q_2}\left(\frac{4\pi\mu^2}{2p_1p_2}\right)^{\varepsilon}|1,2\rangle \\
= 2\,\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{2p_1p_2}\right)^{\varepsilon} & \\
\times\langle 1,2|\frac{1}{T_{q_1}^2}&\left[\mathbf{T}_{q_1}^2\left(\frac{1}{\varepsilon^2}-\frac{\pi^2}{3}\right)+\gamma_q\frac{1}{\varepsilon}+\gamma_q+K_q\right]\mathbf{T}_{q_1}\mathbf{T}_{q_2}|1,2\rangle \\
= -2C_F\,\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}&\left(\frac{4\pi\mu^2}{2p_1p_2}\right)^{\varepsilon}\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}-\frac{\pi^2}{2}+\frac{10}{2}\right]\langle 1,2|1,2\rangle
\end{aligned}
$$

$e^+e^- \longrightarrow$ 2 jets: 2-parton contribution

$$\begin{aligned}\boldsymbol{\sigma}^{NLO\,(2)} &= \int_2 \left[d\boldsymbol{\sigma}^V + \int_1 d\boldsymbol{\sigma}^A \right]_{\varepsilon=0} \\ &= |\mathcal{M}_2|^2 \, \frac{C_F \alpha_s}{\pi} \int dy_{12}\, \delta(1 - y_{12})\, F_J^{(2)}(p_1, p_2)\end{aligned}$$

$\downarrow$

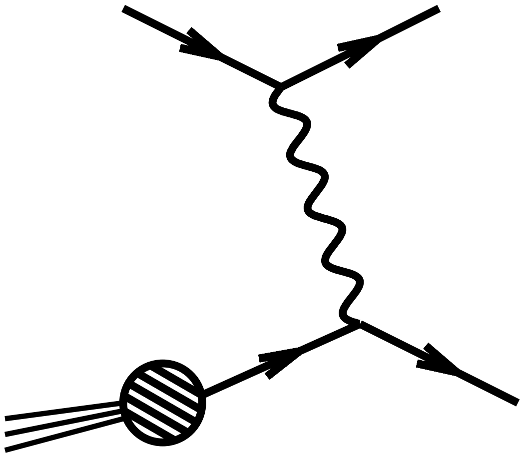
singularities in $d\boldsymbol{\sigma}^V$ and $\int_1 d\boldsymbol{\sigma}^A$ cancel analytically

final result completely finite!

. . . but . . .

. . . e^+e^- collisions are not the full story . . .

Identified Hadrons

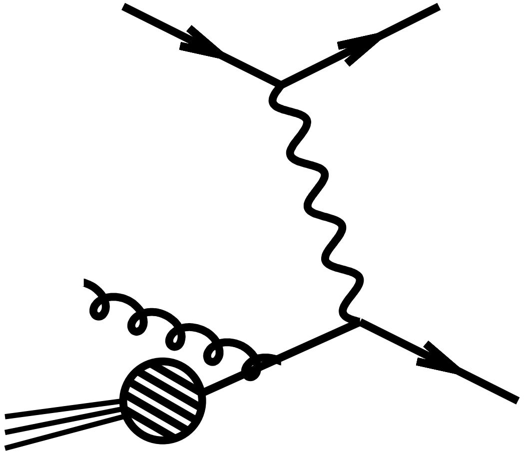


real world:
need to deal with identified
hadrons in initial and / or final state
(DIS, pp / $p\bar{p}$ collider, etc.)

additional type of singularities:
emission collinear to identified particle

standard procedure: absorbed by PDFs or fragmentation
functions *(factorization)*

Identified Hadrons



real world:
need to deal with identified
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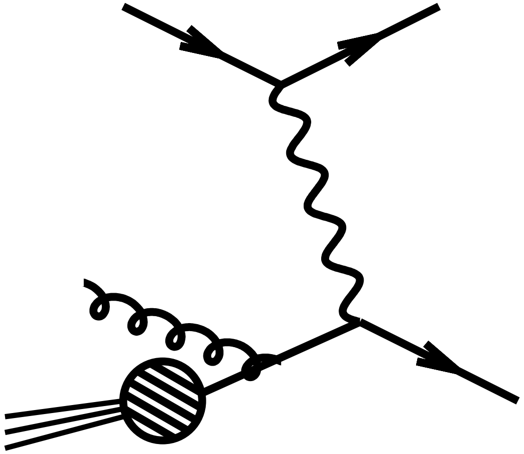
(DIS, pp / $p\bar{p}$ collider, etc.)

additional type of singularities:
emission collinear to identified particle

standard procedure: absorbed by PDFs or fragmentation functions *(factorization)*

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int_m d\sigma^C$$

Identified Hadrons



need modified subtraction term:

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes \left(dV_{dipole} + dV'_{dipole} \right)$$

dV'_{dipole} . . . matches new singularities from regions collinear to identified particles

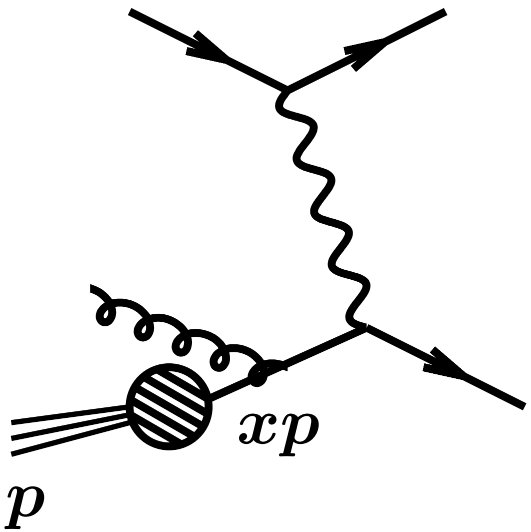
Identified Hadrons

$$\boldsymbol{\sigma}^{NLO}(p) = \boldsymbol{\sigma}^{NLO\,\{m+1\}}(p) + \boldsymbol{\sigma}^{NLO\,\{m\}}(p) + \int_0^1 dx\, \hat{\boldsymbol{\sigma}}^{NLO\,\{m\}}(x; xp)$$

$$= \int_{m+1} \left[d\boldsymbol{\sigma}^R(p)|_{\varepsilon=0} - \sum_{dipoles} d\boldsymbol{\sigma}^B \otimes \left(dV_{dipole} + dV'_{dipole} \right)_{\varepsilon=0} \right]$$

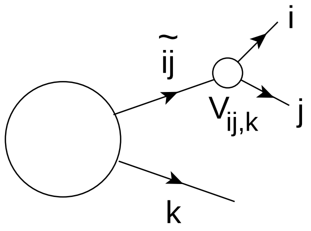
$$+ \int_m \left[d\boldsymbol{\sigma}^V(p) + d\boldsymbol{\sigma}^B \otimes \mathbf{I} \right]_{\varepsilon=0}$$

$$+ \int_0^1 dx \int_m \left[d\boldsymbol{\sigma}^B(xp) \otimes (\mathbf{P} + \mathbf{K} + \mathbf{H})(x) \right]_{\varepsilon=0}$$



Dipoles

$$|\mathcal{M}_{m+1}|^2 \;=\; \langle 1,\ldots,m+1;a,\ldots||1,\ldots,m+1;a,\ldots\rangle$$



$$= \underbrace{\sum_{k\neq i,j} \mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1};p_a)}$$

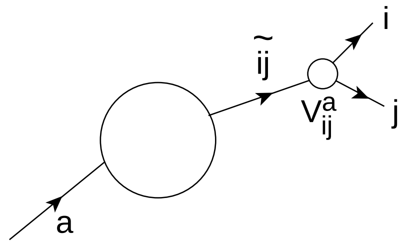
divergent as $p_i \cdot p_j \longrightarrow 0$

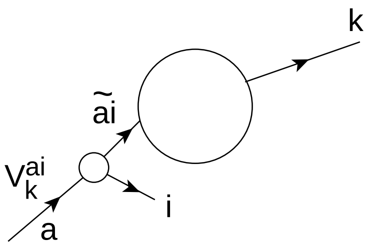
spectator k

$$+ \quad \underbrace{\mathcal{D}_{ij}^{a}(p_1,\ldots,p_{m+1};p_a)}$$

divergent as $p_i \cdot p_j \longrightarrow 0$

spectator a





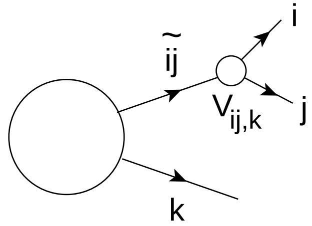
$$+ \underbrace{\sum_{k\neq i} \mathcal{D}_{k}^{ia}(p_1,\ldots,p_{m+1};p_a)} \;+\; \ldots$$

divergent as $p_i \cdot p_a \longrightarrow 0$

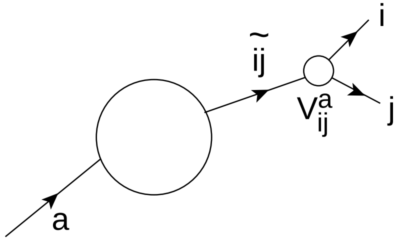
spectator k

Dipoles

$$\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1};p_a) = -\frac{1}{2p_i\cdot p_j}$$

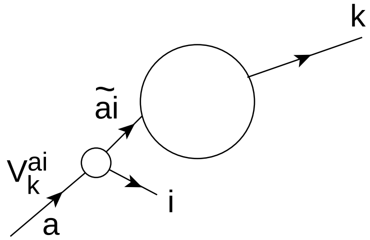


$$\cdot_{m,a}\langle ..,\tilde{ij},..,\tilde{k},..,m+1;a|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathbf{V}_{ij,k}|..,\tilde{ij},..,\tilde{k}..,m+1;a\rangle_{m,a}$$



$$\mathcal{D}_{ij}^{a}(p_1,\ldots,p_{m+1};p_a) = -\frac{1}{2p_i\cdot p_j}\frac{1}{x_{ij,a}}$$

$$\cdot_{m,a}\langle ..,\tilde{ij},..,m+1;\tilde{a}|\frac{\mathbf{T}_a\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathbf{V}_{ij}^{a}|..,\tilde{ij},..m+1;\tilde{a},..\rangle_{m,a}$$



$$\mathcal{D}_{k}^{ia}(p_1,\ldots,p_{m+1};p_a) = -\frac{1}{2p_i\cdot p_a}\frac{1}{x_{ik,a}}$$

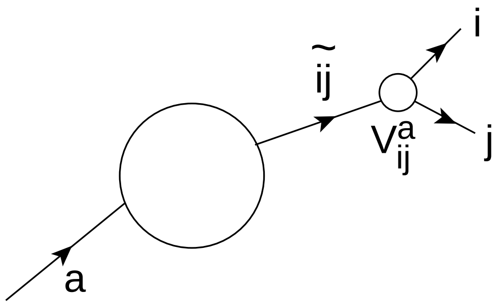
$$\cdot_{m,a}\langle ..,\tilde{k},..,m+1;\widetilde{ai}|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ai}}{\mathbf{T}_{ai}^2}\mathbf{V}_{k}^{ai}|..,\tilde{k},..m+1;\widetilde{ai},..\rangle_{m,a}$$

Phase Space Convolution ($\mathcal{D}_{ij}^{a}$)

rewrite phase space integral:

$$d\phi(p_i, p_j; Q + p_a) = \frac{d^d p_i}{(2\pi)^{d-1}} \delta_+(p_i^2) \frac{d^d p_j}{(2\pi)^{d-1}} \delta_+(p_j^2)$$

$$\times (2\pi)^d \delta^{(d)}(Q + p_a - p_j - p_k)$$



$$= \int_0^1 dx \, d\phi(\tilde{p}_{ij}; Q + x p_a) \, [dp_i \, (\tilde{p}_{ij}; p_a, x)]$$

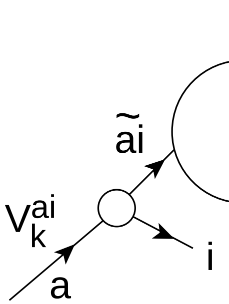
perform $\displaystyle \int [dp_i(\tilde{p}_{ij}; p_a, x)] \frac{1}{2 p_i p_j} \langle \mathrm{V}^a_{ij} \rangle \propto \left(\frac{4\pi\mu^2}{2 \tilde{p}_{ij} p_a} \right)^{\varepsilon} \mathcal{V}_{ij} \, (x; \varepsilon)$

cf. $\mathcal{D}_{ij,k}$: no x-dependence in $\mathcal{V}_{ij}(\varepsilon)$

Phase Space Convolution ($\mathcal{D}_k^{ai}$)

rewrite phase space integral:

$$d\phi(p_i, p_k; Q + p_a) \;=\; \int_0^1 dx\, d\phi(\tilde{p}_k; Q + xp_a)\,[dp_i(\tilde{p}_k; p_a, x)]$$



$$\int [dp_i(\tilde{p}_k; p_a, x)]\, \frac{1}{2p_ip_a}\, \langle \mathrm{V}_k^{ai} \rangle \propto \left(\frac{4\pi\mu^2}{2\tilde{p}_kp_a}\right)^{\varepsilon} \mathcal{V}^{a,ai}(x;\varepsilon)$$

$\mathcal{V}^{a,ai}(x;\varepsilon)$ . . . closely related to Altarelli-Parisi splitting functions $P_{a,ai}(x)$

Jet Cross Sections

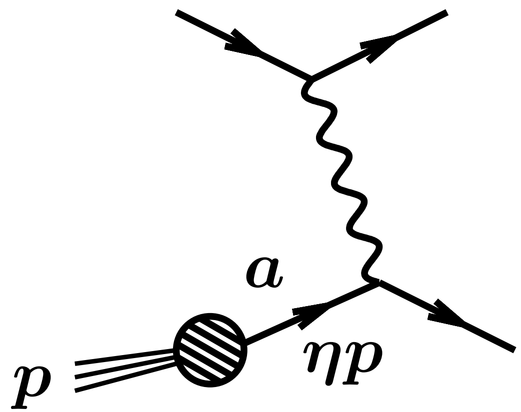
convolution

partonic cross sections

$$\sigma(p) \;=\; \sum_a \int_0^1 d\eta\, f_a(\eta, \mu_F^2)\, \left[\sigma_a^{LO}(\eta p) + \sigma_a^{NLO}(\eta p; \mu_F^2)\right]$$

hadronic cross section

PDF for parton type a



Born cross section:

$$\sigma_a^{LO}(p_a) \;=\; \int_m d\Phi^{(m)} |\mathcal{M}_{m,a}(p_1, \ldots, p_m; p_a)|^2 F_J^{(m)}(p_1, \ldots, p_m; p_a)$$

Jet Cross Sections

$$\sigma_a^{NLO}(p_a;\mu_F^2) \;=\; \int_{m+1}\left[d\sigma_a^R(p_a) - d\sigma_a^A(p_a)\right]$$
$$+\left[\int_{m+1} d\sigma_a^A(p_a) + \int_m d\sigma_a^V(p_a) + \int_m d\sigma_a^C(p_a;\mu_F^2)\right]$$

$$d\sigma_a^R(p_a) - d\sigma_a^A(p_a) \quad = d\Phi_{m+1}(p_1,\ldots,p_{m+1};p_a+Q)$$
$$\cdot\Big\{|\mathcal{M}_{m+1}(p_1,\ldots,p_{m+1};p_a)|^2 F_J^{(m+1)}(p_1,\ldots,p_{m+1};p_a)$$
$$-\sum_{\text{pairs } i,j}\;\sum_{k\neq i,j}\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1};p_a)\;F_J^{(m)}(\ldots,\tilde{p}_{ij},\tilde{p}_k,p_{m+1};p_a)$$
$$-\sum_{\text{pairs } i,j}\mathcal{D}_{ij}^a(p_1,\ldots,p_{m+1};p_a)\;F_J^{(m)}(\ldots,\tilde{p}_{ij},..,p_{m+1};\tilde{p}_a)$$
$$-\sum_i\sum_{k\neq i}\mathcal{D}_k^{ai}(p_1,\ldots,p_{m+1};p_a)\;F_J^{(m)}(\ldots,,\tilde{p}_k,p_{m+1};\tilde{p}_{ai})\Big\}$$

Jet Cross Sections

$$\sigma_a^{NLO}(p_a;\mu_F^2) = \int_{m+1}\left[d\sigma_a^R(p_a) - d\sigma_a^A(p_a)\right]$$

$$+\left[\int_{m+1} d\sigma_a^A(p_a) + \int_m d\sigma_a^V(p_a) + \int_m d\sigma_a^C(p_a;\mu_F^2)\right]$$

$$\int_{m+1} d\sigma_a^A(p) + \int_m d\sigma_a^C(p;\mu_F^2) = \int_m d\sigma_a^B(p)\cdot \mathbf{I}(\varepsilon)$$

$$+\sum_{ai}\int_0^1 dx \int_m \left[\mathbf{K}^{a,ai}(x)\cdot d\sigma_{ai}^B(xp)\right]$$

$$+\sum_{ai}\int_0^1 dx \int_m \left[\mathbf{P}^{a,ai}(xp,x;\mu_F^2)\cdot d\sigma_{ai}^B(xp)\right]$$

$d\sigma_a^B \cdot \mathbf{I}\ldots$ cancels ε-poles in $d\sigma_a^V$

$d\sigma_a^B \cdot (\mathbf{K}+\mathbf{P})$: finite remainders from factorization of initial-state radiation into PDFs

Survey

✘ have identified all pieces that are needed for applying the subtraction method to processes with one identified hadron:

- $d\sigma_a^R - d\sigma_a^A$
- $d\sigma_a^A + d\sigma_a^C$
- $d\sigma_a^V$

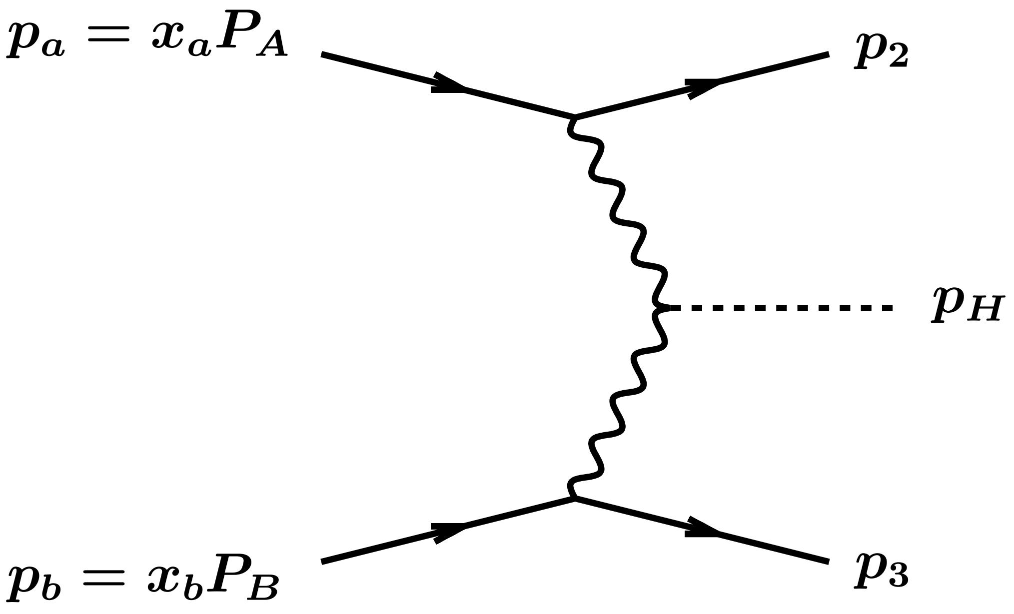
✘ generalization to processes with two initial state hadrons or identified hadrons in the final state: straightforward

another example

we are now ready to apply our knowledge

to Higgs production via vector boson fusion (VBF)

$qq' \longrightarrow qq'H$ via VBF: Born kinematics

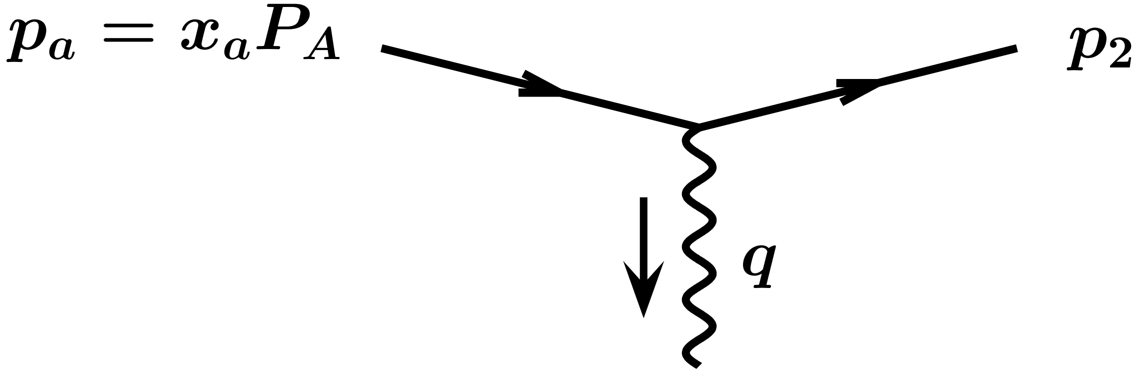


no color exchange between upper and lower quark lines

↓

consider independently
(two DIS-like processes)

$qq' \to qq'H$ via VBF: Born kinematics



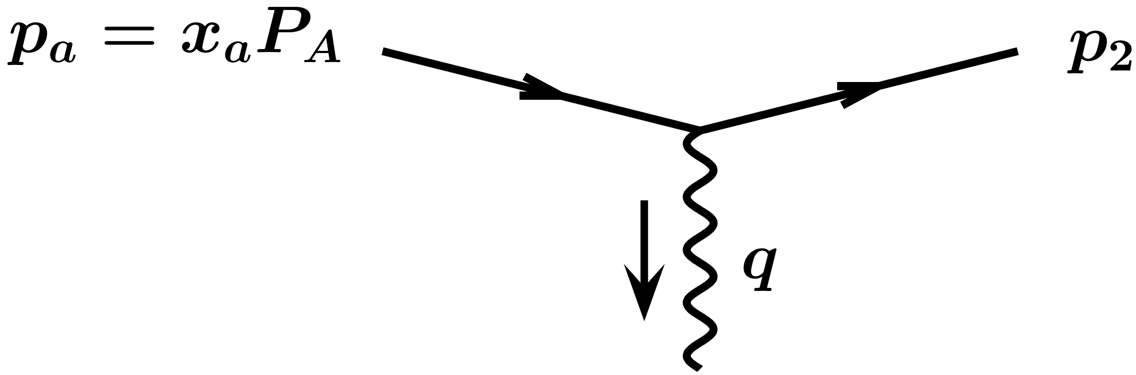
$$p_a^2 = p_2^2 = 0$$
$$q^2 = (p_2 - p_a)^2 = -Q^2$$

no color exchange between upper and lower quark lines

↓

consider independently
(two DIS-like processes)

$qq' \longrightarrow qq'H$ via VBF: Born kinematics

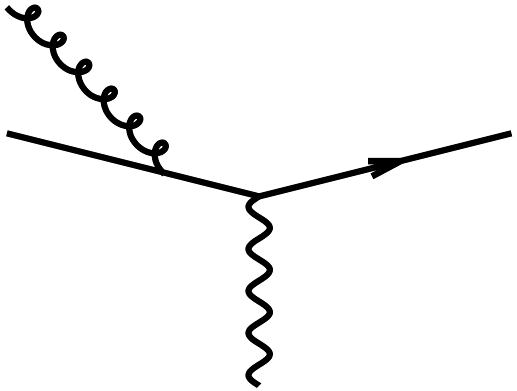


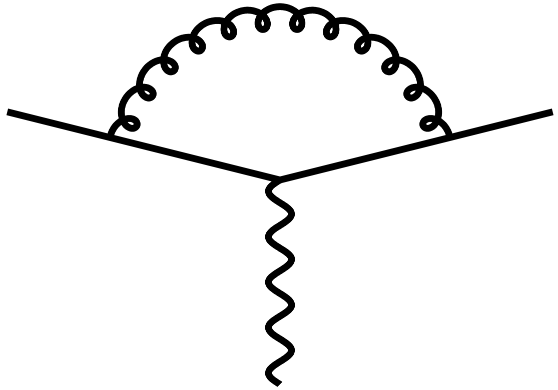
hadronic cross section contribution $d\sigma_a^{LO}(p_a)$ from parton a:

$$d\sigma_a^{LO} = \int_0^1 dx_a f_a(x_a, \mu_F) \int_{m=1} d\Phi^{(1)}(p_2; p_a)\, F_J^{(1)}(p_2; p_a)\, |\mathcal{M}_{1,a}(p_2; p_a)|^2$$

(dependence on q implicitly understood)

$qq' \longrightarrow qq'H$ via VBF @ NLO



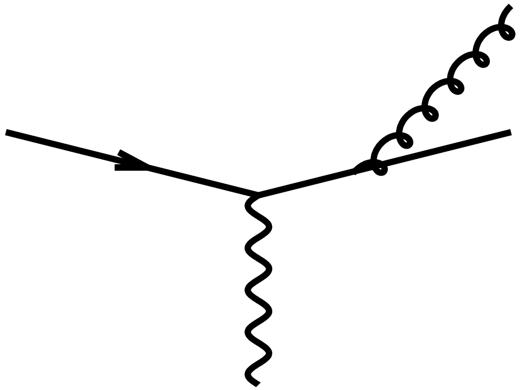


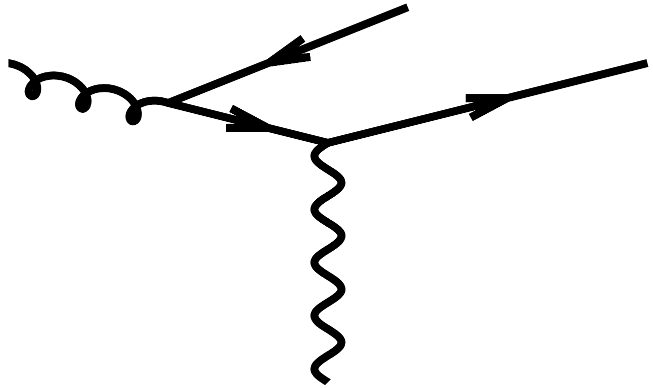
hadronic cross section contribution $\boldsymbol{\sigma_a^{NLO}(p_a;\mu_F^2)}$
from parton $\boldsymbol{a}$:

$$\boldsymbol{\sigma_a^{NLO}(p_a;\mu_F^2) \;=\; \int_{m+1}\left[d\sigma_a^R(p_a) - d\sigma_a^A(p_a)\right]}$$

$$\boldsymbol{+\left[\int_{m+1} d\sigma_a^A(p_a) + \int_m d\sigma_a^V(p_a) + \int_m d\sigma_a^C(p_a;\mu_F^2)\right]}$$

Counterterms





reminder: for processes with one initial state hadron generally need

$$d\sigma_a^A(p_a) = d\Phi_{m+1}(p_1,\ldots,p_{m+1};p_a,p_\ell)$$

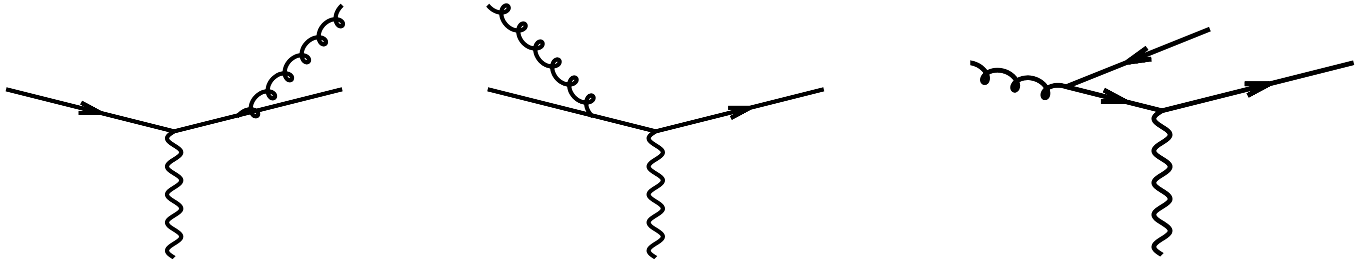
$$\cdot\Big\{\sum_{\text{pairs } i,j}\ \sum_{k\neq i,j}\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1};p_a)\ F_J^{(m)}(\ldots,\tilde{p}_{ij},\tilde{p}_k,p_{m+1};p_a)$$

$$+\sum_{\text{pairs } i,j}\mathcal{D}_{ij}^a(p_1,\ldots,p_{m+1};p_a)\ F_J^{(m)}(\ldots,\tilde{p}_{ij},..,p_{m+1};\tilde{p}_a)$$

$$+\sum_i\sum_{k\neq i}\mathcal{D}_k^{ai}(p_1,\ldots,p_{m+1};p_a)\ F_J^{(m)}(\ldots,,\tilde{p}_k,p_{m+1};\tilde{p}_{ai})\Big\}$$

Counterterms

in this case: have only one initial and two final state partons

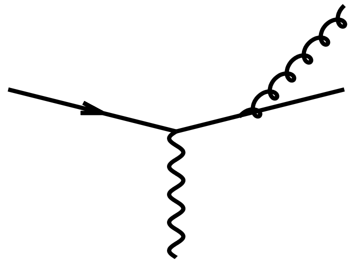


$$m = 1$$
$$m + 1 = 2$$
extra parton $\longrightarrow p_1$

dipole	for VBF
$\sum\sum \mathcal{D}_{ij,k}(p_1, p_2; p_a)$	not needed (only two final state partons)
$\sum \mathcal{D}_{ij}^{a}(p_1, p_2; p_a)$	need $\mathcal{D}_{12}^{a=q}(p_1, p_2; p_a)$
$\sum \mathcal{D}_{k}^{ai}(p_1, p_2; p_a)$	need $\mathcal{D}_{2}^{a=q,1}$, $\mathcal{D}_{2}^{a=g,1}$, $\mathcal{D}_{1}^{a=g,2}$

Counterterms: $\mathcal{D}^q_{12}$

reminder:



$$\mathcal{D}^a_{ij}(p_1,\ldots,p_{m+1};p_a) \;=\; -\frac{1}{2p_i\cdot p_j}\frac{1}{x_{ij,a}}$$

$$\cdot_{m,a}\langle .., \tilde{ij}, .., m+1; \tilde{a}|\frac{\mathbf{T_a}\cdot\mathbf{T_{ij}}}{\mathbf{T_{ij}^2}}\mathbf{V^a_{ij}}|.., \tilde{ij}, ..m+1; \tilde{a}\rangle_{m,a}$$

$$\downarrow$$

$$\mathcal{D}^q_{12}(p_1,p_2;p_a) = -\frac{1}{2p_1\cdot p_2}\frac{1}{x_{12,a}}\langle\tilde{p}_{12};\tilde{p}_a|\frac{\mathbf{T_a}\cdot\mathbf{T_{12}}}{\mathbf{T_{12}^2}}\mathbf{V^{q_a}_{g_1q_2}}|\tilde{p}_{12};\tilde{p}_a\rangle$$

using color conservation we find:

$$\langle\tilde{p}_{12};\tilde{p}_a|\frac{\mathbf{T_a}\cdot\mathbf{T_{12}}}{\mathbf{T_{12}^2}}\mathbf{V^{q_a}_{g_1q_2}}|\tilde{p}_{12};\tilde{p}_a\rangle$$

$$= -V^{q_a}_{g_1q_2}\langle\tilde{p}_{12};\tilde{p}_a|\tilde{p}_{12};\tilde{p}_a\rangle = -V^{q_a}_{g_1q_2}|\mathcal{M}^q_B(\tilde{p}_{12};\tilde{p}_a)|^2$$

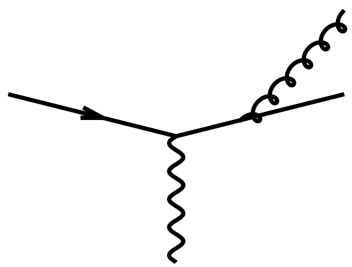
Counterterms: $\mathcal{D}^q_{12}$

$V^a_{q_2 g_1}$ $\longrightarrow$ look up in `hep-ph/9605323`:

$$V^a_{q_i g_j} = 8\pi\alpha_s C_F \left[\frac{2}{1-\tilde{z}_i(1-x_{ij,k})} - (1+\tilde{z}_i) - \varepsilon(1-\tilde{z}_i)\right]$$

dipole kinematics:

$$x_{12,a} \equiv x = \frac{p_1 p_a + p_2 p_a - p_1 p_2}{(p_1+p_2)\cdot p_a}$$

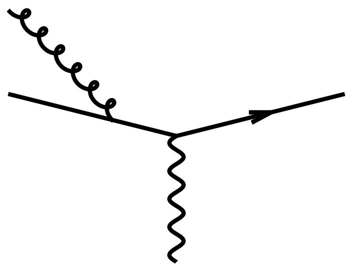


$$\tilde{z}_2 \equiv z = \frac{p_2 \cdot p_a}{(p_1+p_2)\cdot p_a}$$

$$\tilde{p}_a = x p_a \,, \quad \tilde{p}_{12} \equiv \tilde{p}_f = p_1 + p_2 - (1-x) p_a$$

Counterterms: $\mathcal{D}_2^{q1}$

reminder:



$$\mathcal{D}_k^{ia}(p_1,\ldots,p_{m+1};p_a) = -\frac{1}{2p_i\cdot p_a}\frac{1}{x_{ik,a}}$$

$$\cdot_{m,a}\langle..,\tilde{k},..,m+1;\widetilde{ai}|\frac{\mathrm{T_k}\cdot\mathrm{T_{ai}}}{\mathrm{T_{ai}^2}}\mathrm{V_k^{ai}}|..,\tilde{k},..m+1;\widetilde{ai}\rangle_{m,a}$$

$$\downarrow$$

$$\mathcal{D}_2^{q1}(p_1,p_2;p_a) = -\frac{1}{2p_1\cdot p_a}\frac{1}{x_{12,a}}\langle\tilde{p}_2;\tilde{p}_{a1}|\frac{\mathrm{T_2}\cdot\mathrm{T_{a1}}}{\mathrm{T_{a1}^2}}\mathrm{V_{q_2}^{q_a g_1}}|\tilde{p}_2;\tilde{p}_{a1}\rangle$$

using color conservation we find:

$$\langle\tilde{p}_2;\tilde{p}_{a1}|\frac{\mathrm{T_2}\cdot\mathrm{T_{a1}}}{\mathrm{T_{a1}^2}}\mathrm{V_{q_2}^{q_a g_1}}|\tilde{p}_2;\tilde{p}_{a1}\rangle$$
$$= -V_{q_2}^{q_a g_1}\langle\tilde{p}_2;\tilde{p}_{a1}|\tilde{p}_2;\tilde{p}_{a1}\rangle = -V_{q_2}^{q_a g_1}|\mathcal{M}_B^q(\tilde{p}_2;\tilde{p}_{a1})|^2$$

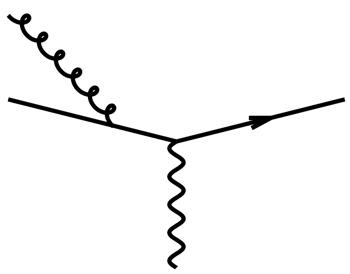
Counterterms: $\mathcal{D}_2^{q1}$

$V_{q_2}^{q_a g_1} \longrightarrow$ look up in `hep-ph/9605323`:

$$V_{q_2}^{q_a g_1} = 8\pi\alpha_s C_F \left[\frac{2}{1 - x_{ik,a} + u_i} - (1 + x_{ik,a}) - \varepsilon(1 - x_{ik,a})\right]$$

dipole kinematics:

$$x_{12,a} = \frac{p_1 p_a + p_2 p_a - p_1 p_2}{(p_1 + p_2) \cdot p_a} = x$$

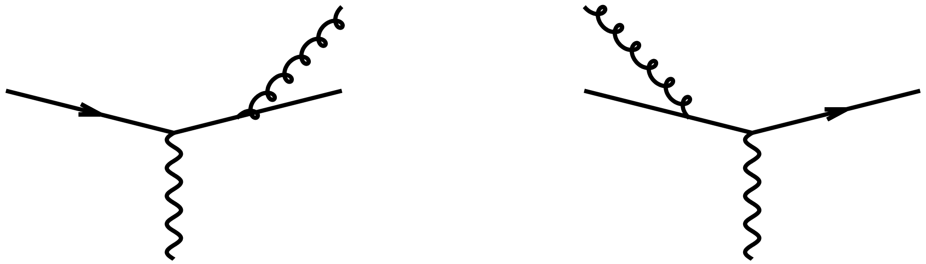


$$u_1 = \frac{p_1 \cdot p_a}{(p_1 + p_2) \cdot p_a}$$

$$\tilde{p}_{a1} = x p_a \; = \tilde{p}_a, \quad \tilde{p}_2 = p_1 + p_2 - (1 - x) p_a = \tilde{p}_f$$

Counterterms: $\mathcal{D}_{12}^{q} + \mathcal{D}_{2}^{q1}$

now combine two dipole terms contributing to $d\sigma_q^A$



$$\mathcal{D}_{12}^{q} + \mathcal{D}_{2}^{q1} = -V_{g_1 q_2}^{q_a} |\mathcal{M}_q(\tilde{p}_{12};\tilde{p}_a)|^2 - V_{q_2}^{q_a g_1} |\mathcal{M}_q(\tilde{p}_2;\tilde{p}_{a1})|^2$$

insert expressions for $V_{g_1 q_2}^{q_a}$ and $V_{q_2}^{q_a g_1}$ and find that after rewriting all $\{\tilde{p}\}$ in terms of $\{p_1, p_2, p_a\}$ both Born amplitudes have same arguments $\{\tilde{p}_f;\tilde{p}_a\}$

$\Downarrow$

after some algebra find

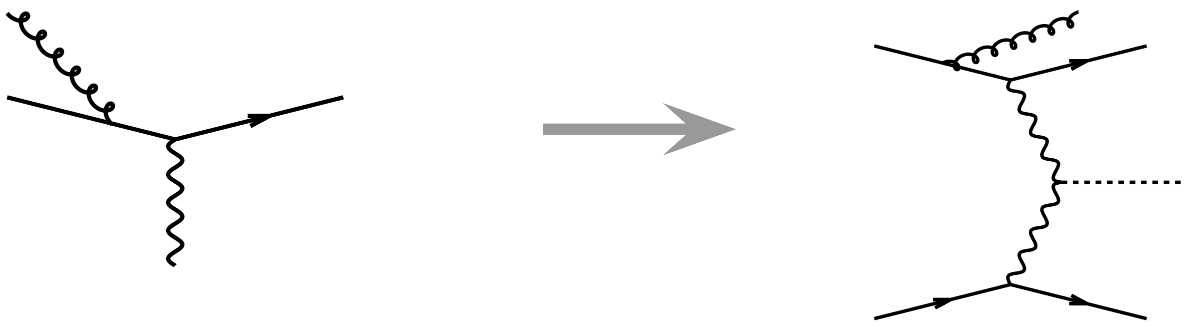
$$\mathcal{D}_{12}^{q} + \mathcal{D}_{2}^{q1} = 8\pi\alpha_s C_F \frac{1}{Q^2} \frac{x^2+z^2}{(1-x)(1-z)} |\mathcal{M}_B^q(\tilde{p}_f;\tilde{p}_a)|^2$$

Counterterms: $\mathcal{D}_{12}^{q} + \mathcal{D}_{2}^{q1}$

$$\mathcal{D}_{12}^{q} + \mathcal{D}_{2}^{q1} = 8\pi\alpha_s C_F \frac{1}{Q^2} \frac{x^2+z^2}{(1-x)(1-z)} |\mathcal{M}_B^q(\tilde{p}_f;\tilde{p}_a)|^2$$

can use this result for DIS-like process, but also for VBF
(singularity structure along quark line doesn't change)

replace Born amplitude $\mathcal{M}_B^q$ for $q \to qV$ with
$\mathcal{M}_B(qq' \to qq'H)$
("effective polarization vector" for V)



$qq' \to qq'gH$: $d\sigma_q^R - d\sigma_q^A$

$$\int_{m+1} \left[d\sigma_q^R(p_a) - d\sigma_q^A(p_a) \right]$$

$$= \int_0^1 dx_a dx_b \, f_q(x_a, \mu_F) f_{q'}(x_b, \mu_F) d\Phi^{(3+1)}(p_1, p_2, p_3, p_H; p_a, p_b)$$

$$\times \left\{ |\mathcal{M}_R^q|^2 F_J^{(3)}(p_1, p_2, p_3) - \left[\mathcal{D}_{12}^q + \mathcal{D}_2^{q1} \right] F_J^{(2)}(\tilde{p}_2, p_3) \right\}$$

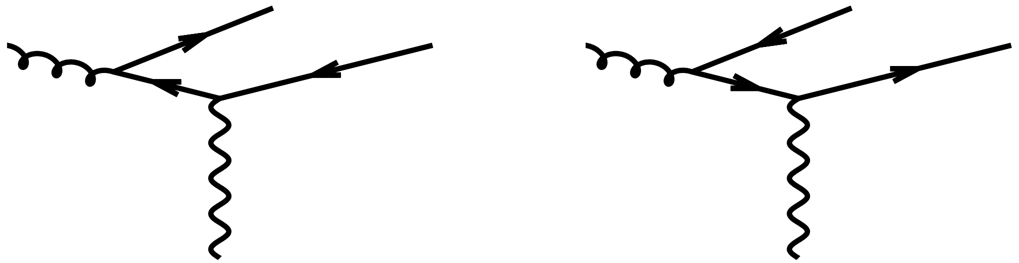
with $F_J^{(3)} \to F_J^{(2)}$ in the singular regions

integrand $\{\ldots\}$ finite

$\downarrow$

perform all integrals numerically in $d = 4$ dimensions

Counterterms: $\mathcal{D}_2^{g1} + \mathcal{D}_1^{g2}$



for dipoles contributing to $d\sigma_g^A$ proceed analogously and find

$$\mathcal{D}_2^{g1} + \mathcal{D}_1^{g2} = 8\pi\alpha_s T_F \frac{1}{Q^2}\left[\frac{x^2+(1-x)^2}{1-z}\,|\mathcal{M}_B^{\bar{q}}(\tilde{p}_f;\tilde{p}_a)|^2 + \frac{x^2+(1-x)^2}{z}\,|\mathcal{M}_B^{q}(\tilde{p}_f;\tilde{p}_a)|^2\right]$$

note: need Born amplitudes for

$$\bar{q}(\tilde{p}_a) \longrightarrow \bar{q}(\tilde{p}_f)V(q) \text{ and } q(\tilde{p}_a) \longrightarrow q(\tilde{p}_f)V(q)$$

$gq' \longrightarrow q\bar{q}q'H$: $d\sigma_g^R - d\sigma_g^A$

$$\int_{m+1} \left[d\sigma_g^R(p_a) - d\sigma_g^A(p_a) \right]$$

$$= \int dx_a dx_b\, f_g(x_a, \mu_F) f_{q'}(x_b, \mu_F) d\Phi^{(3+1)}(p_1, p_2, p_3, p_H; p_a, p_b)$$

$$\times \left\{ |\mathcal{M}_R^g|^2 F_J^{(3)}(p_1, p_2, p_3) - \left[\mathcal{D}_2^{g1} + \mathcal{D}_1^{g2} \right] F_J^{(2)}(\tilde{p}_2, p_3) \right\}$$

with $F_J^{(3)} \longrightarrow F_J^{(2)}$ in the singular regions

integrand $\{\ldots\}$ finite

$\downarrow$

perform all integrals numerically in $d = 4$ dimensions

$d\sigma_q^A$: integral of counterterm

reminder: counterterm integrated over one-parton PS

$$\int_1 d\sigma^A = {}_m\langle 1,\dots,m;a|\, \boldsymbol{I}(\varepsilon)\, |1,\dots,m;a\rangle_m F_J^{(m)}(p_1,\dots,p_m)$$

with

$$\begin{aligned} \boldsymbol{I}(p_1,\dots,p_m,p_a;\varepsilon) &= -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)} \\ &\times \left\{ \sum_i \frac{1}{\mathbf{T}_i^2}\mathcal{V}_i(\varepsilon) \left[\sum_{k\neq i} \mathbf{T_i T_k} \left(\frac{4\pi\mu^2}{2p_ip_k}\right)^{\varepsilon} + \mathbf{T_i T_a}\left(\frac{4\pi\mu^2}{2p_ip_a}\right)^{\varepsilon} \right] \right. \\ &\left. + \frac{1}{\mathbf{T_a}^2}\mathcal{V}_a(\varepsilon) \sum_i \mathbf{T_i T_a}\left(\frac{4\pi\mu^2}{2p_ip_a}\right)^{\varepsilon} \right\} \end{aligned}$$

$d\sigma_q^A$: integral of counterterm

here: need only contributions stemming from integration of $\mathcal{D}_{12}^{q}$ and $\mathcal{D}_{2}^{q1}$

$$I(p_1,\ldots,p_m,p_a;\varepsilon) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\sum_i\left(\frac{4\pi\mu^2}{2p_ip_a}\right)^{\varepsilon} \times\left\{\frac{1}{\mathrm{T}_i^2}\mathcal{V}_i(\varepsilon)\mathrm{T_i T_a}+\frac{1}{\mathrm{T_a}^2}\mathcal{V}_a(\varepsilon)\mathrm{T_i T_a}\right\}$$

$$= -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{2p_2p_a}\right)^{\varepsilon}\left\{\frac{\mathrm{T_{q_2}T_a}}{\mathrm{T}_{q_2}^2}\mathcal{V}_{q_2}(\varepsilon)+\frac{\mathrm{T_{q_2}T_a}}{\mathrm{T_a}^2}\mathcal{V}_a(\varepsilon)\right\}$$

need to look up

$$\mathcal{V}_q^{DR}(\varepsilon) = \mathrm{T}_q^2\left(\frac{1}{\varepsilon^2}-\frac{\pi^2}{3}\right)+\gamma_i\frac{1}{\varepsilon}+\gamma_i+K_i-\frac{1}{2}C_F$$

$d\sigma_q^A$: integral of counterterm

collect terms and find

$$\langle p_2; p_a | I(p_2, p_a; \varepsilon) | p_2; p_a \rangle$$

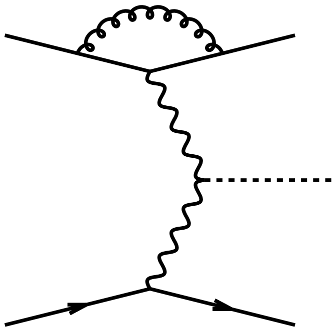
$$= |\mathcal{M}_B^q(p_2; p_a)|^2 \, \frac{\alpha_s(\mu)}{2\pi} \frac{C_F}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^{\varepsilon} \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 9 - \frac{4\pi^2}{3} \right]$$

note:

$\mathcal{M}_B \ldots$ Born kinematics $\{p_2; p_a\}$ rather than $\{\tilde{p}_2; \tilde{p}_a\}$

$\mu \ldots$ renormalization scale

Virtuals

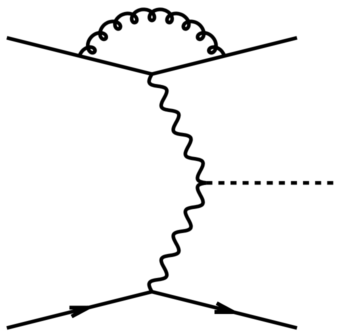


$$\int_m d\sigma_q^V(p_a) \;=\; \int dx_a dx_b\, f_q(x_a,\mu_F) f_{q'}(x_b,\mu_F)$$
$$d\Phi^{(2+1)}(p_2,p_3,p_H;p_a,p_b)\,|\mathcal{M}^q|^2_{(1-loop)}\,F_J^{(2)}(p_2,p_3)$$

combining self-energy and vertex corrections in $\overline{\mathrm{MS}}$ renormalization scheme yields for $|\mathcal{M}^q|^2_{(1-loop)}$:

$$2\,\mathrm{Re}[\mathcal{M}_B^q \mathcal{M}_V^{q\star}(p_2,p_3)|^2 \;=\; |\mathcal{M}_B^q(p_2,p_3)|^2\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{1}{\Gamma(1-\varepsilon)}$$
$$\times\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}+c_{virt}\right]$$

Virtuals & counterterms



combine virtuals with suitable pieces of counterterms

$$\hat{\sigma}_q^{NLO\,\{2\}}(p_a) = \int_m \left[d\hat{\sigma}_q^V(p_a) + d\hat{\sigma}_q^B(p_a) \otimes \mathbf{I} \right]_{\varepsilon=0}$$

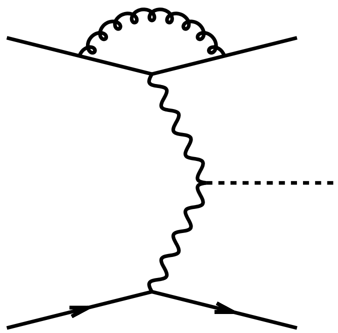
$$= \int d\Phi^{(2+1)}(p_2, p_3, p_H; p_a, p_b)$$

$$\times \left\{ |\mathcal{M}^q|^2_{(1-loop)} + \langle p_2; p_a | I(p_2, p_a; \varepsilon) | p_2; p_a \rangle \right\} F_J^{(2)}(p_2, p_3)$$

$$= \int d\Phi^{(2+1)} \frac{\alpha_s}{2\pi} \frac{C_F}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^{\varepsilon} |\mathcal{M}_B^q|^2$$

$$\times \left\{ \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{virt} \right] + \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 9 - \frac{4\pi^2}{3} \right] \right\} F_J^{(2)}(p_2, p_3)$$

Virtuals & counterterms



combine virtuals with suitable pieces of counterterms

$$\hat{\sigma}_q^{NLO\,\{2\}}(p_a) = \int_m \left[d\hat{\sigma}_q^V(p_a) + d\hat{\sigma}_q^B(p_a) \otimes \mathbf{I} \right]_{\varepsilon=0}$$

$$\hat{\sigma}_q^{NLO\,\{2\}}(p_a) = \int d\Phi^{(2+1)} \frac{\alpha_s}{2\pi} \frac{C_F}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^{\varepsilon} |\mathcal{M}_B^q|^2 \times \left\{ c_{virt} + 9 - \frac{4\pi^2}{3} \right\} F_J^{(2)}(p_2, p_3)$$

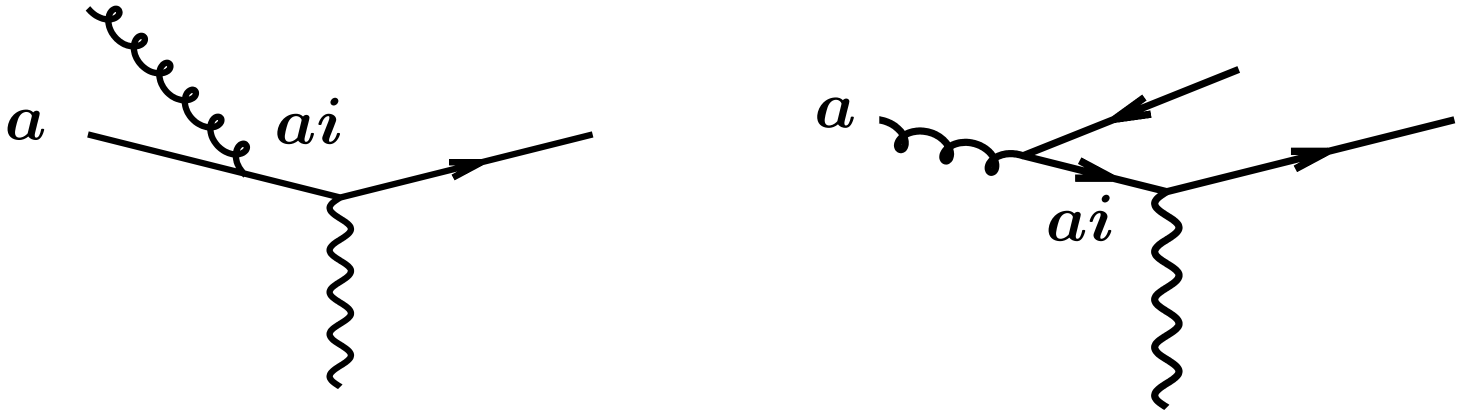
- completely finite: can set $\varepsilon \longrightarrow 0$
- obtain σ_q out of $\hat{\sigma}_q$ by convolution with quark PDFs

Collinear terms

needed:

$$\int_m d\hat{\sigma}_a^C(p_a;\mu_F^2) = \int_0^1 dz\,\hat{\sigma}_a^{NLO\,\{m\}}(z;zp_a;\mu_F^2)$$

$$= \sum_{ai}\int_0^1 dz \int_m \left[d\hat{\sigma}_{ai}^B(zp_a)\otimes(\mathbf{K}+\mathbf{P})^{a,ai}\right]$$



ai ... parton emerging from parton a

Collinear terms

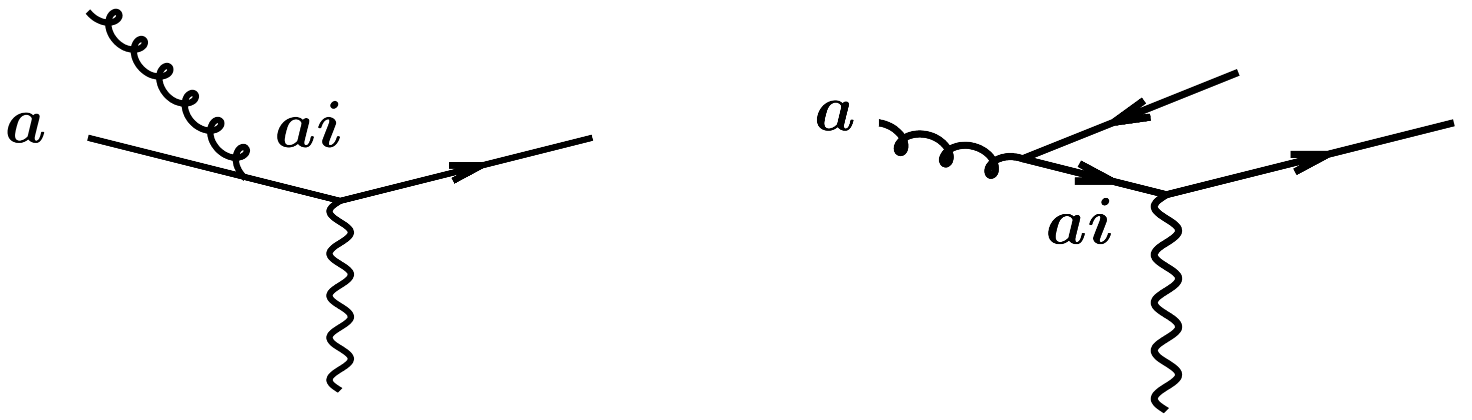
$$\sum_{ai}\int_0^1 dz \int_m \left[d\hat{\sigma}^B_{ai}(zp_a)\otimes(\mathbf{K}+\mathbf{P})^{a,ai}\right]$$

literature: rename $ai \longrightarrow b$

(here: misleading $\longrightarrow$ keep ai)

$$= \sum_{ai}\int_0^1 dz \int d\Phi^{(2+1)}(p_2,p_3,p_H;zp_a,p_b)F_J^{(2)}(p_2,p_3)$$

$$\times\langle p_2;zp_a|\mathbf{K}^{a,ai}(z)+\mathbf{P}^{a,ai}(zp_a,z;\mu_F^2)|p_2;zp_a\rangle$$

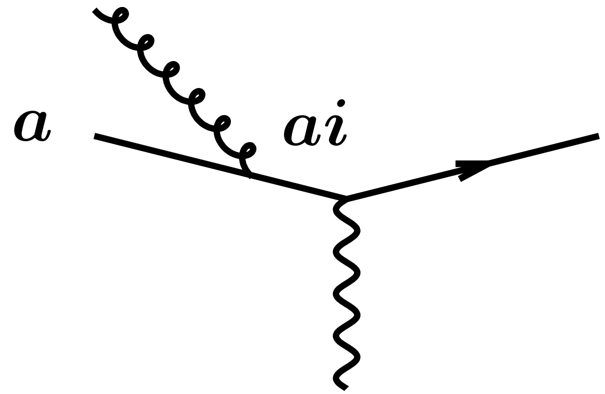


Collinear terms: $d\sigma_q^C$

$$\sum_{ai} \int_0^1 dz \int d\Phi^{(2+1)}(p_2, p_3, p_H; zp_a, p_b) F_J^{(2)}(p_2, p_3)$$
$$\times \langle p_2; zp_a | \mathbf{K}^{a,ai}(z) + \mathbf{P}^{a,ai}(zp_a, z; \mu_F^2) | p_2; zp_a \rangle$$

for $d\sigma_q^C$ ... have $a = q$

$\sum_{ai}$ ... one contribution only ($ai = q$)



$$\int_0^1 dz \int d\Phi^{(2+1)}(p_2, p_3, p_H; zp_a, p_b) F_J^{(2)}(p_2, p_3)$$
$$\times \langle p_2; zp_a | \mathbf{K}^{q,q}(z) + \mathbf{P}^{q,q}(zp_a, z; \mu_F^2) | p_2; zp_a \rangle$$

... look up $\mathbf{K}^{q,q}$ and $\mathbf{P}^{q,q}$

Collinear terms: $d\sigma_q^C$

insert tabulated results for $\mathbf{K}^{q,q}$ and $\mathbf{P}^{q,q}$ and find

$$\sigma_q^C(p_a;\mu_F^2) \;=\; \frac{\alpha_s}{2\pi}C_F\int_0^1 dx_a\,dx_b\,f_q(x_a,\mu_F^2)f_{q'}(x_b,\mu_F^2)\int_0^1 dz$$

$$\int d\Phi^{(2+1)}(p_2,p_3,p_H;zp_a,p_b)F_J^{(2)}(p_2,p_3;zp_a)\langle p_2;zp_a|p_2;zp_a\rangle$$

$$\times\left\{\left(\frac{2}{1-z}\ln\frac{1-z}{z}\right)_+ -(1+z)\ln\frac{1-z}{z}+(1-z)\right.$$

$$-\delta(1-z)(5-\pi^2)-\frac{3}{2}\left[\left(\frac{1}{1-z}\right)_+ +\delta(1-z)\right]$$

$$\left.-\left(\frac{1+z^2}{1-z}\right)_+\ln\frac{\mu_F^2}{2zp_a\cdot p_2}\right\}$$

Collinear terms: $d\sigma_q^C$

let's abbreviate $\{\ldots\}$ by $E(z,\mu_F^2)$ so that

$$\sigma_q^C(p_a;\mu_F^2) \;=\; \int_0^1 dx_a\, dx_b\, f_q(x_a,\mu_F^2) f_{q'}(x_b,\mu_F^2)$$

$$\int_0^1 dz \int d\Phi^{(2+1)}(p_2,p_3,p_H;zp_a,p_b) F_J^{(2)}(p_2,p_3;zp_a)$$

$$\times \frac{\alpha_s}{2\pi} C_F \left|\mathcal{M}_q^B(p_2;zp_a)\right|^2 E(z,\mu_F^2)$$

and keep in mind the meaning of plus distributions

$$\int_0^1 dz\, g(z) f(z)_+ = \int_0^1 dz\, [g(z)-g(1)]\, f(z)$$

Collinear terms: $d\sigma^C$

$$\sigma_q^C(p_a;\mu_F^2) \;=\; \int_0^1 dx_a\, dx_b\, f_q(x_a,\mu_F^2) f_{q'}(x_b,\mu_F^2)$$

$$\int_0^1 dz \int d\Phi^{(2+1)}(p_2,p_3,p_H;zp_a,p_b) F_J^{(2)}(p_2,p_3;zp_a)$$

$$\times\, \frac{\alpha_s}{2\pi} C_F \left|\mathcal{M}_q^B(p_2;zp_a)\right|^2 E(z,\mu_F^2)$$

for $\int dz$ apply two tricks: substitute $x_a = x/z$ and use

$$zp_a = z(x_a P_A) \longrightarrow z\left(\frac{x}{z} P_A\right) = xP_A$$

then:

repeat the same procedure for gluon contribution $d\sigma_g^C$

Collinear terms: $d\sigma_q^C + d\sigma_g^C$

after some algebra find

$$\sigma_{tot}^C(p_a;\mu_F^2) \;=\; \int_0^1 dx \int_0^1 dx_b\, f_q^c(x,\mu_F,\mu)\, f_{q'}(x_b,\mu_F^2)$$
$$\times\, d\Phi^{(2+1)}(p_2,p_3,p_H;xP_A,p_b)F_J^{(2)}(p_2,p_3;xP_A)\,|\mathcal{M}_q^B(p_2;xP_A)|^2$$

where now $xP_A = p_a$ and

$$\begin{aligned} f_q^c(x,\mu_F,\mu) \;=\; & \frac{\alpha_s(\mu)}{2\pi}\int_x^1 \frac{dz}{z}\left\{ f_g\left(\frac{x}{z},\mu_F^2\right) A(z) \right. \\ & + \left[f_q\left(\frac{x}{z},\mu_F^2\right) - z f_q\left(x,\mu_F^2\right)\right] B(z) \\ & \left. + f_q\left(\frac{x}{z},\mu_F^2\right) C(z) \right\} + \frac{\alpha_s(\mu)}{2\pi} f_q\left(x,\mu_F^2\right) D(x) \end{aligned}$$

$qq' \longrightarrow qq'H$ via VBF @ NLO

reminder: building blocks needed for hadronic cross section contribution $\sigma_a^{NLO}(p_a;\mu_F^2)$ from parton a:

$$\sigma_a^{NLO}(p_a;\mu_F^2) = \int_{m+1}\left[d\sigma_a^R(p_a) - d\sigma_a^A(p_a)\right]$$

$$+\left[\int_{m+1} d\sigma_a^A(p_a) + \int_m d\sigma_a^V(p_a) + \int_m d\sigma_a^C(p_a;\mu_F^2)\right]$$

have computed all pieces

↓

can determine σ_a^{NLO} numerically!

Summary

✘ sketched the basic ideas of the dipole subtraction method suggested by *Catani & Seymour* for the case of reactions with

· no identified hadrons

· one identified hadron

✘ illustrated the approach by two examples:

· $e^+e^- \longrightarrow 2$ jets

· $qq \longrightarrow qqH$ in VBF

$\Downarrow$

general, powerful approach

numerically stable

implementation requires care