

# THESIS

BY

TIGRAN SAIDNIA

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## Emission kernel of parton shower

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Karlsruhe institute for Technology (KIT)

Institute of theoretical physics

Referents: PD Dr. Stefan Gieseke

Dr. Simon Plätzer

Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, February 20, 2019

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Tigran Saidnia



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## 0.1 Old parametrisation

$$\left. \begin{aligned}
 q_i^\mu &= zp_i^\mu + y(1-z)p_j^\mu + \sqrt{zy(1-z)}m_\perp \\
 q^\mu &= (1-z)p_i^\mu + yzp_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_j^\mu &= (1-y)p_j^\mu \\
 y &= \frac{q_i \cdot q}{p_i \cdot p_j} \\
 q_i + q &= p_i + yp_j \\
 q_j + q &= (1-z)p_i^\mu + (1+yz-y)p_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
 q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
 q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
 \end{aligned} \right\} \text{parametrisation} \quad (1)$$

## 0.2 new kinematic

$$\begin{aligned}
 k_l^\mu &= \alpha_l \Lambda^\mu{}_\nu p_i^\nu + y\beta n^\mu + \sqrt{y\alpha_l\beta_l}n_{\perp,l}^\mu \quad l = 1, \dots, m \\
 q_i^\mu &= (1 - \sum_{l=1}^m \alpha_l) \Lambda^\mu{}_\nu p_i^\nu + y(1 - \sum_{l=1}^m \beta_l) n^\mu - \sqrt{y\alpha_l\beta_l}n_{\perp,l}^\mu \\
 q_k^\mu &= \Lambda^\mu{}_\nu p_k^\nu \quad k = 1, \dots, n \quad k \neq i
 \end{aligned} \quad (2)$$

### 0.2.1 useful relations

$$\begin{aligned}
 q_i^2 &= p_i^2 = q_k^2 = k_l^2 = p_j^2 = p_k^2 = n^2 = 0 \quad \text{All hard momenta are on-shell} \\
 Q^\mu &= q_i^\mu + \sum_{l=1}^m k_l^\mu + \sum_{k=1}^m q_k^\mu = p_i^\mu + \sum_{k=1}^m p_k^\mu \quad \text{total momentum} \\
 n^\mu &= Q^\mu - \frac{Q^2}{2p_i \cdot Q} p_i^\mu \quad n^\mu \text{ is the recoil} \\
 q_i^\mu + \sum_{l=1}^m k_l^\mu &= \Lambda^\mu{}_\nu p_i^\nu + yn^\mu \\
 \Lambda^\mu{}_\nu Q^\nu &= Q^\mu - yn^\mu \\
 n_{\perp,l}^\mu \Lambda^\mu{}_\nu p_i^\nu &= n_{\perp,l} \cdot n = n_{\perp,l} \cdot Q = 0 \\
 n_{\perp,l}^\mu \cdot p_k &\neq 0 \\
 n_{\perp,l}^2 &= -2\alpha\Lambda^\mu{}_\nu p_i^\nu n_\mu \\
 n_{\perp,1}^2 &= -2p_i \cdot Q \\
 \alpha_1 &= 1 - \beta_1 \\
 \alpha &= \sqrt{1-y}
 \end{aligned} \quad (3)$$

Lorenz trafo

$$\alpha\Lambda^\mu{}_\nu = p_i^\mu p_{i\nu} \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu \quad (4)$$

$$\hat{p}_i^\mu = \alpha\Lambda^\mu{}_\nu p_i^\nu = p_i^\mu p_{i\nu} p_i^\nu \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu p_i^\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} p_i^\nu \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu p_i^\nu \quad (5)$$

$$\begin{aligned} \hat{p}_i^\mu &= p_i^\mu (Q \cdot p_i) \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} p_i^\mu \\ &= p_i^\mu \left[ \frac{y(1 + \sqrt{1-y})}{(2 + 2\sqrt{1-y} - y)} + \sqrt{1-y} \right] = p_i^\mu \end{aligned} \quad (6)$$

$$\boxed{\hat{p}_i^\mu = \alpha\Lambda^\mu{}_\nu p_i^\nu = p_i^\mu} \quad (7)$$

$$\hat{p}_k^\mu = \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu p_{i\nu} p_k^\nu \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu p_k^\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} p_k^\nu \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu p_k^\nu \quad (8)$$

$$\begin{aligned} \hat{p}_k^\mu &= \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &\quad + Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} p_k^\mu \end{aligned} \quad (9)$$

$$\begin{aligned} \hat{p}_k^\mu &= \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &\quad + Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} p_k^\mu \end{aligned}$$

with

$$\begin{aligned} A_1 &\equiv \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \\ A_2 &\equiv \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \end{aligned} \quad (10)$$



$$\boxed{\hat{p}_k^\mu = A_1 p_i^\mu + A_2 Q^\mu + \sqrt{1-y} p_k^\mu} \quad (11)$$

$$\begin{aligned} \hat{Q}^\mu &= \alpha \Lambda^\mu{}_\nu Q^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot Q)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y}) Q^2}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &\quad + Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot Q)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} Q^\mu \end{aligned}$$

with

$$\begin{aligned} S_1 &\equiv \frac{Q^2}{2p_i \cdot Q} \left[ \frac{-y^2}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})}{(1 + \sqrt{1-y} - \frac{y}{2})} \right] = \frac{Q^2}{2p_i \cdot Q} y \\ S_2 &\equiv \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} = 1 - y \end{aligned} \quad (12)$$

$$\boxed{\hat{Q}^\mu = \frac{Q^2}{2p_i \cdot Q} y p_i^\mu + (1 - y) Q^\mu} \quad (13)$$

### 0.3 Single emission part

$$\begin{aligned} k_1^\mu &= (\alpha_1 - y\beta_1 (\frac{Q^2}{2p_i \cdot Q})) p_i^\mu + y\beta_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \\ q_i^\mu &= (\beta_1 - \alpha_1 y (\frac{Q^2}{2p_i \cdot Q})) p_i^\mu + y\alpha_1 Q^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,l}^\mu \\ q_k^\mu &= \alpha \Lambda^\mu{}_\nu p_k^\nu \quad k = 1, \dots, n \quad k \neq i \end{aligned} \quad (14)$$

$$\begin{aligned} k_1^\mu &= \zeta_1 p_i^\mu + \lambda_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \\ q_i^\mu &= \zeta_q p_i^\mu + \lambda_q Q^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,l}^\mu \\ q_k^\mu &= A_1 p_i^\mu + A_2 Q^\mu + \sqrt{1-y} p_k^\mu \end{aligned}$$

$$\begin{aligned}
\zeta_1 \zeta_1 &= (\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_1 \lambda_1 &= (y\alpha_1\beta_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_1 \zeta_q &= (\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_1 \lambda_q &= (y\alpha_1^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_q \zeta_q &= (\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_q \lambda_1 &= (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_q \zeta_1 &= (\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_q \lambda_q &= (y\beta_1\alpha_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_1 \lambda_1 &= y^2\beta_1^2 \\
\lambda_1 \zeta_q &= (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_1 \lambda_q &= y^2\beta_1\alpha_1 \\
\lambda_1 \zeta_1 &= (y\beta_1\alpha_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_q \lambda_q &= y^2\alpha_1^2 \\
\lambda_q \lambda_1 &= y^2\alpha_1\beta_1 \\
\lambda_q \zeta_q &= (y\alpha_1\beta_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_q \zeta_1 &= (y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))
\end{aligned} \tag{15}$$

## 0.4 Common scalar products

$$\begin{aligned}
k_1 \cdot q_i &= (\zeta_1 \lambda_q + \lambda_1 \zeta_q) p_i \cdot Q + \lambda_1 \lambda_q Q^2 - y\alpha_1\beta_1 n_{\perp,1}^2 \\
&= [(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))y\alpha_1 + y\beta_1(1 - \alpha_1 - \alpha_1(\frac{Q^2}{2p_i \cdot Q}))] p_i \cdot Q \\
&\quad y^2\beta_1\alpha_1 Q^2 + 2y\alpha_1\beta_1 p_i Q \\
\Rightarrow k_1 \cdot q_i &= [y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y\beta_1 - y\alpha_1\beta_1 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})] p_i \cdot Q \\
&\quad y^2\beta_1\alpha_1 Q^2 + 2y\alpha_1\beta_1 p_i Q
\end{aligned} \tag{16}$$

$$\boxed{k_1 \cdot q_i = y(\alpha_1 + \beta_1) p_i \cdot Q = y p_i \cdot Q} \tag{17}$$



$$\begin{aligned}
k_1 \cdot q_k &= (\zeta_1 A_2 + \lambda_1 A_1) p_i \cdot Q + \zeta_1 \sqrt{1-y} p_i \cdot p_k + \lambda_1 A_2 Q^2 + \lambda_1 \sqrt{1-y} Q \cdot p_k \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1} \\
&= \left\{ \left[ (\alpha_1 - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right)) \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \right. \\
&+ y\beta_1 \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \} p_i \cdot Q \\
&+ (\alpha_1 - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right)) \sqrt{1-y} p_i \cdot p_k + y\beta_1 \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} Q^2 \\
&+ y\beta_1 \sqrt{1-y} Q \cdot p_k + \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{18}$$

$$\begin{aligned}
k_1 \cdot q_k &= \alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) \\
&+ y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) + y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} Q \cdot p_k \\
&+ \alpha_1 \sqrt{1-y} p_i \cdot p_k - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \sqrt{1-y} p_i \cdot p_k \\
&+ y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) + y\beta_1 \sqrt{1-y} (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{19}$$

$$\begin{aligned}
k_1 \cdot q_k &= \left[ \alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \alpha_1 \sqrt{1-y} \right. \\
&- y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \sqrt{1-y} \left. \right] p_i \cdot p_k + \left[ y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1 \sqrt{1-y} \right] (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{20}$$

$$\begin{aligned}
k_1 \cdot q_k &= \left\{ \alpha_1 \left[ \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \right] \right. \\
&+ y\beta_1 \left( \frac{Q^2}{p_i \cdot Q} \right) \left[ \frac{-y^2}{4(1 + \sqrt{1-y} - \frac{y}{2})} - \sqrt{1-y} \right] \} p_i \cdot p_k \\
&+ y\beta_1 \left[ \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \right] (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{21}$$

$$\boxed{k_1 \cdot q_k = [\alpha_1(1-y) + y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right)] p_i \cdot p_k + y\beta_1 Q \cdot p_k + \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}} \tag{22}$$

## 0.5 Parametrization in terms of $(k_1 \cdot q_i)(k_1 \cdot q_k)$

$$\boxed{(k_1 \cdot q_i)(k_1 \cdot q_k) \approx y(1 - \beta_1)(1 - y)(p_i \cdot p_k)(p_i \cdot Q)} \quad (23)$$

$$\begin{aligned} k_1^\eta k_1^{\eta'} &= [(1 - \beta_1)^2 - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})^2] p_i^\eta p_i^{\eta'} - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\ k_1^\eta q_i^{\eta'} &= [\beta_1(1 - \beta_1) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\ q_i^\eta k_1^{\eta'} &= [\beta_1(1 - \beta_1) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} \\ q_i^\eta q_i^{\eta'} &= \beta_1^2 p_i^\eta p_i^{\eta'} \\ k_1^\eta q_k^{\eta'} &= \sqrt{1 - y} p_i^\eta p_k^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_1 p_i^\eta p_i^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_2 p_i^\eta Q^{\eta'} \\ &\quad + y \beta_1 A_1 Q^\eta p_i^{\eta'} + y \beta_1 A_2 Q^\eta Q^{\eta'} \\ q_i^\eta q_k^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 p_i^\eta Q^{\eta'} + \beta_1 \sqrt{1 - y} p_i^\eta p_k^{\eta'} \\ q_k^\eta k_1^{\eta'} &= \sqrt{1 - y} p_i^\eta p_k^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_1 p_i^\eta p_i^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_2 p_i^\eta Q^{\eta'} \\ &\quad + y \beta_1 A_1 Q^\eta p_i^{\eta'} + y \beta_1 A_2 Q^\eta Q^{\eta'} \\ q_k^\eta q_i^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 Q^\eta p_i^{\eta'} + \beta_1 \sqrt{1 - y} p_k^\eta p_i^{\eta'} \end{aligned} \quad (24)$$

## 0.6 Parametrization in terms of $(k_1 \cdot q_i)(k_1 \cdot q_i)$

$$\boxed{(k_1 \cdot q_i)(k_1 \cdot q_i) = y^2(p_i \cdot Q)(p_i \cdot Q)} \quad (25)$$

$$\begin{aligned} k_1^\eta k_1^{\eta'} &= [(1 - \beta_1)^2 - 2y \beta_1 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1 (1 - \beta_1) (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} + y \beta_1 (1 - \beta_1) (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\ k_1^\eta q_i^{\eta'} &= [\beta_1(1 - \beta_1) - y(1 - \beta_1)^2 (\frac{Q^2}{2p_i \cdot Q}) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y(1 - \beta_1)^2 Q^\eta p_i^{\eta'} \\ q_i^\eta k_1^{\eta'} &= [\beta_1(1 - \beta_1) - y(1 - \beta_1)^2 (\frac{Q^2}{2p_i \cdot Q}) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y(1 - \beta_1)^2 p_i^\eta Q^{\eta'} \\ q_i^\eta q_i^{\eta'} &= [\beta_1^2 - 2y \beta_1 (1 - \beta_1) (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1 (1 - \beta_1) (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} + y \beta_1 (1 - \beta_1) (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\ k_1^\eta q_k^{\eta'} &= (1 - \beta_1) A_1 p_i^\eta p_i^{\eta'} + (1 - \beta_1) A_2 p_i^\eta Q^{\eta'} + (1 - \beta_1) \sqrt{1 - y} p_i^\eta p_k^{\eta'} \\ q_i^\eta q_k^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 p_i^\eta Q^{\eta'} + \beta_1 \sqrt{1 - y} p_i^\eta p_k^{\eta'} \\ q_k^\eta k_1^{\eta'} &= (1 - \beta_1) A_1 p_i^\eta p_i^{\eta'} + (1 - \beta_1) A_2 Q^\eta p_i^{\eta'} + (1 - \beta_1) \sqrt{1 - y} p_k^\eta p_i^{\eta'} \\ q_k^\eta q_i^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 Q^\eta p_i^{\eta'} + \beta_1 \sqrt{1 - y} p_k^\eta p_i^{\eta'} \end{aligned} \quad (26)$$

## 0.7 Altarelli-Parisi splitting functions

$$\left. \begin{aligned} \langle \hat{P}_{qq} \rangle &= C_F \left[ \frac{1+z^2}{1-z} - \varepsilon(1-z) \right] \\ \langle \hat{P}_{gq} \rangle &= T_R \left[ 1 - \frac{2z(1-z)}{1-\varepsilon} \right] \\ \langle \hat{P}_{qg} \rangle &= C_F \left[ \frac{1+(1-z)^2}{z} - \varepsilon z \right] \\ \langle \hat{P}_{gg} \rangle &= 2C_A \left[ \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \end{aligned} \right\} \text{splitting functions} \quad (27)$$

## 0.8 Colour factor calculation

fundamental representation in  $SU(2)$  and  $SU(3)$

$$\begin{aligned} T^a &= \tau^a \equiv \frac{\sigma^a}{2} \quad \text{with Pauli matrices } \sigma^a \\ T^a &= \vartheta^a \equiv \frac{\lambda^a}{2} \quad \text{with Gell - Mann matrices } \lambda^a \end{aligned} \quad (28)$$

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix} \\ \lambda^5 &= \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned} \quad (29)$$

As we can see,  $\lambda^3$  and  $\lambda^8$  are diagonal. These generators satisfy:

$$[T^a, T^b] = i\epsilon^{abc}T^c \quad (30)$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \quad (31)$$

The main relation we will use later for  $SU(N)$ :

$$\text{tr}(T^a T^b) = T_{ij}^a T_{ji}^b = T_F \delta^{ab} \quad (32)$$

$$\sum_a (T^a T^a) = C_F \delta^{ij} \quad (33)$$

$$f^{acd} f^{bcd} = C_A \delta^{ab} \quad (34)$$

With  $T_F = \frac{1}{2}$ ,  $C_A = N$  and  $C_F = \frac{N^2-1}{2N}$ .

$$f^{abc} = -2i \text{tr}(T^a [T^b, T^c]) \quad (35)$$

$$d^{abc} = 2 \text{tr}(T^a T^b, T^c) \quad (36)$$

$$T^a T^b = \frac{1}{2} \left( \frac{1}{N} \delta_{ab} + (d^{abc} + i f^{abc}) T^c \right) \quad (37)$$

$$\text{tr}(T^a T^b T^c) = \frac{1}{4} (d^{abc} + i f^{abc}) \quad (38)$$

$$\text{tr}(T^a T^b T^a T^c) = \frac{-1}{4N} \delta_{bc} \quad (39)$$

$$f^{acd} f^{bcd} = N \delta^{ab} \quad (40)$$

$$f^{acd} d^{bcd} = 0 \quad (41)$$

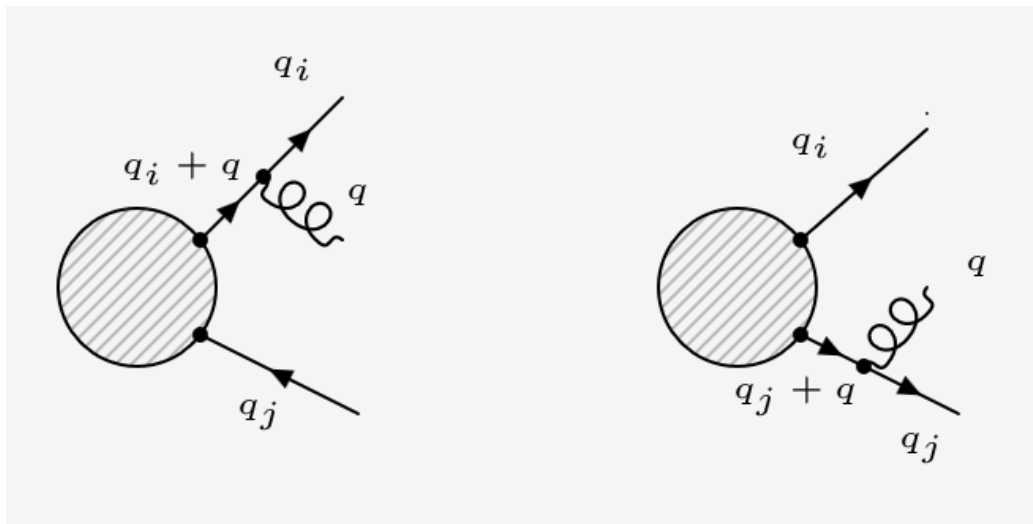
$$f^{ade} f^{bef} f^{cfd} = \frac{N}{2} f^{abc} \quad (42)$$

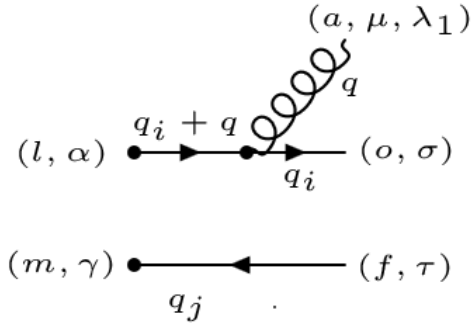
Fierz identity:

$$\sum_a T_{ij}^a T_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl}) \quad (43)$$

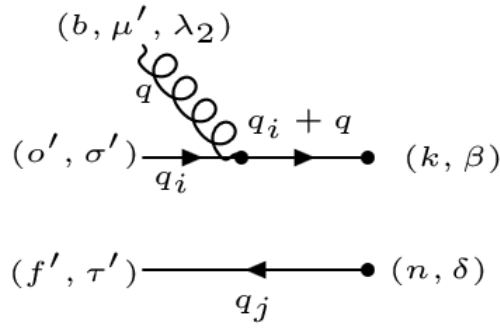
# Chapter 1

## Quark antiquark gluon emission kernel

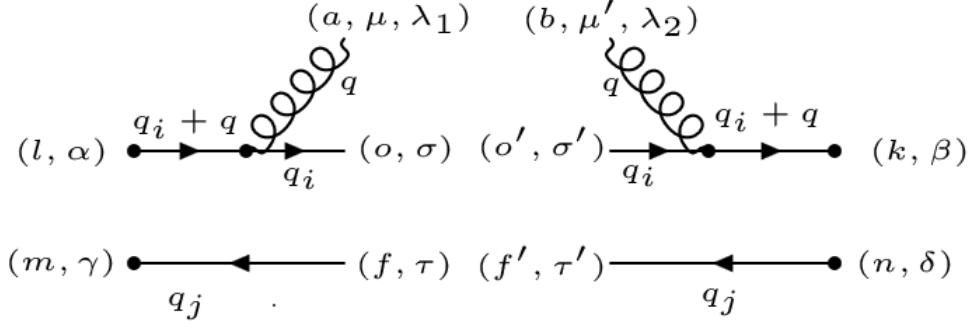


1.1  $qg\text{-}\bar{q}$ 

$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_{o^l}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)] \quad (1.1)$$



$$M_1^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s\gamma^{\mu'} \times [T^b]_{o'^k}^{o'}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q)] [\bar{v}_{\tau'}(q_j)] \quad (1.2)$$



$$|M_1|^2 = M_1 M_1^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_{o'}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q) [v_\tau(q_j)]$$

$$[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o'}^k) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}^*(q) [\bar{v}_{\tau'}(q_j)]] \quad (1.3)$$

$$|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o'}^k) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}^*(q) \varepsilon^{\lambda_1}_\mu(q)$$

$$\times (-ig_s \gamma^\mu \times [T^a]_{o'}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} [\bar{v}_{\tau'}(q_j) v_\tau(q_j)] \quad (1.4)$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i \delta^{\sigma\sigma'},$$

$$\sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \delta^{\tau\tau'} \quad (1.5)$$

and sum over polarization index  $(\lambda_1, \lambda_2)$  :

$$\sum_{\mu, \mu'} \varepsilon^{\lambda_2}_{\mu'}^*(q) \varepsilon^{\lambda_1}_\mu(q) = -g_{\mu\mu'} \delta^{ab} \quad (1.6)$$

$$|M_1|^2 = \frac{-g_s^2 [T^a]_{o'}^k [T^a]_{o'}^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i g_{\mu'\mu} \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j] \quad (1.7)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^a]_{o'}^k [T^a]_{o'}^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i \gamma_{\mu'} (\not{q}_i + \not{q})] [\not{q}_j] \quad (1.8)$$

In other words we expect the tree level diagram from LO and a number: Which means:



$$|M^2| = \left| \begin{array}{c} \text{diagram with two shaded circles and momenta } P_i, P_j \\ \text{contribution from LO} \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{diagram with a loop and momenta } q_i, q, q_i+q \\ \text{a complex number} \end{array} \right|^2$$

$$|M_1|^2 = \frac{-g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [P_i][P_j] \otimes (\text{a complex number}) \quad (1.9)$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N &=: \gamma^{\mu'} \not{q}_i \gamma_{\mu'} = q_{i\sigma} \gamma^{\mu'} \gamma^\sigma \gamma_{\mu'} \\ &= q_{i\sigma} (\{\gamma^{\mu'}, \gamma^\sigma\} - \gamma^\sigma \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} 2g^{\mu'\sigma} \gamma_{\mu'} - d \gamma^\sigma \\ &= (2 - d) \not{q}_i \end{aligned} \quad (1.10)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \not{q}_i (\not{q}_i + q)] [\not{q}_j] \quad (1.11)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [\not{q}_i \not{q}_i \not{q}_i + \not{q}_i \not{q}_i \not{q} + \not{q} \not{q}_i \not{q}_i + \not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (1.12)$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \not{q}_i &= q_i^2 = m_i^2 \\ \not{q} \not{q} &= q^2 = m^2 \\ \not{q}_j \not{q}_j &= q_j^2 = m_j^2 \end{aligned} \quad (1.13)$$

we can first neglect the mass of patrons and ignore each term with  $\not{q}_i \not{q}_i$  and  $\not{q} \not{q}$  as well.

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(2q_i q)(2q_i q)} [\not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (1.14)$$

$$\begin{aligned}
L &= \not{q}_i \not{q}_\mu (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu) \\
&= \not{q}_i [2q_i^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu \left[ \frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2} \right] \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q^2 \not{q}_i \\
&= \not{q}_i (2q_i q)
\end{aligned} \tag{1.15}$$

After inserting the last result of  $L$  and simplify the term  $(2q_i q)$  from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{q}_i] [\not{q}_j] \tag{1.16}$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

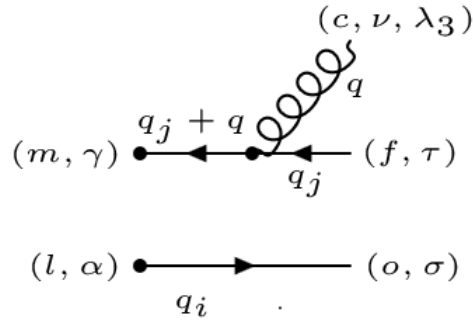
$$|M_1|^2 = (d-2) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z) \not{p}_i + zy \not{p}_j - \sqrt{zy(1-z)} \not{m}_\perp] [(1-y) \not{p}_j] \tag{1.17}$$

Multiplying the both sides

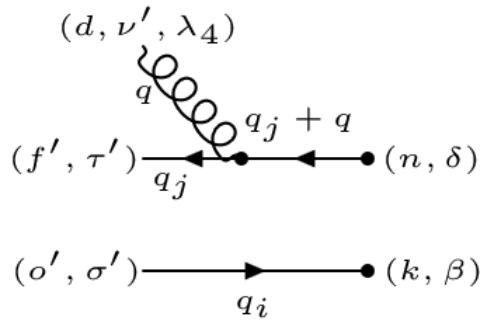
$$\begin{aligned}
|M_1|^2 &= (d-2) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)(1-y) \not{p}_i \not{p}_j \\
&\quad + zy(1-y) \not{p}_j \not{p}_j + (1-y) \sqrt{zy(1-z)} \not{m}_\perp \not{p}_j]
\end{aligned} \tag{1.18}$$

and under consideration of the fact that  $p_i$  and  $p_j$  are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with  $\not{p}_i \not{p}_i$  and  $\not{p}_j \not{p}_j$ . The  $p_i \cdot m_\perp$  and  $p_j \cdot m_\perp$  are always 0 because the  $p_i$  and  $p_j$  are lightlike, i.e. zero transverse component. So those terms can be neglected.

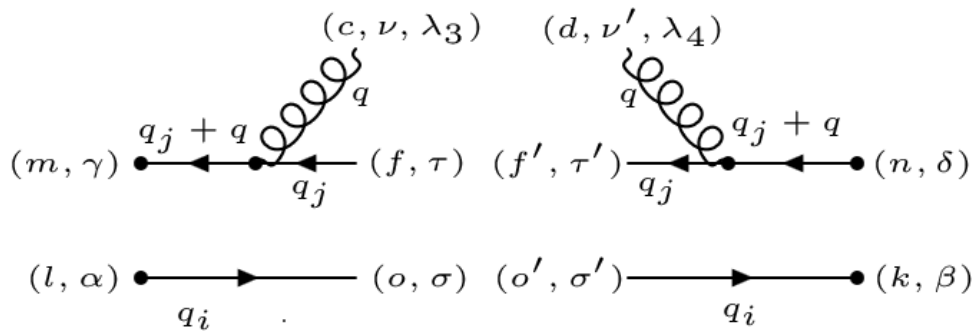
$$|M_1|^2 = (d-2)(1-z)(1-y) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \tag{1.19}$$

1.2  $\bar{q}g$ -q

$$M_2 = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_i)] \right] \quad (1.20)$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_i)]] \quad (1.21)$$



$$|M_2|^2 = M_2 M_2^\dagger = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_i)] \right] \quad (1.22)$$

$$\left[ \bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_i)] \right]$$

$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu v_\tau(q_j) \bar{v}_{\tau'}(q_j) \varepsilon^{\lambda_3}_\nu(q) \varepsilon^{\lambda_4}_{\nu'}(q) \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.23)$$

$$[u_\sigma(q_i)] [\bar{u}_{\sigma'}(q_i)]$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i \delta^{\sigma\sigma'}, \quad (1.24)$$

$$\sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \delta^{\tau\tau'}$$

and sum over polarization index  $(\lambda_3, \lambda_4)$  :

$$\sum_{\nu, \nu'} \varepsilon^{\lambda_4*}_{\nu'}(q) \varepsilon^{\lambda_3}_\nu(q) = -g_{\nu\nu'} \delta^{cd} \quad (1.25)$$

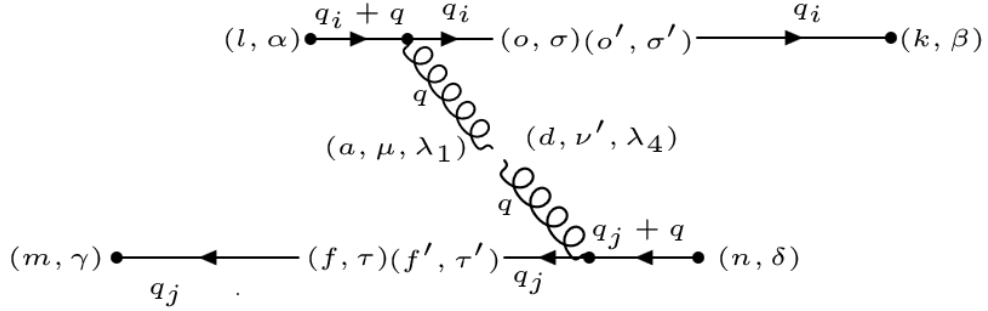
$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu \not{q}_j (-g_{\nu\nu'}) \gamma^{\nu'} (\not{q}_j + \not{q})] [\not{q}_i] \quad (1.26)$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d - 2) \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{(2qq_j)} [\not{q}] [\not{q}_i] \quad (1.27)$$

finally:

$$|M_2|^2 = -(d - 2) y z^2 \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{2(1 - z)(1 - y)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \quad (1.28)$$

1.3  $M_1 M_2^\dagger$ 

$$M_1 M_2^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)]$$

$$[\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q)] [u_{\sigma'}(q_i)] \quad (1.29)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_4}_{\nu'}(q)$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.30)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] - g_{\mu\nu'}$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.31)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j \gamma_\mu (\not{q}_j + \not{q})] \quad (1.32)$$

Expectation:

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [(\not{q}_i + \not{q}) \gamma^\mu \not{q}_i] [(\not{q}_j + \not{q}) \gamma_\mu \not{q}_j] \quad (1.33)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [-(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu + 2(\not{q}_i + \not{q}) q_i^\mu]$$

$$[-(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu + 2(\not{q}_j + \not{q}) q_{j\mu}] \quad (1.34)$$

$$|M^2| = \left| \begin{array}{c} \text{diagram with two shaded circles and arrows } P_i, P_j \\ \text{contribution from LO} \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{diagram with a shaded circle and a wavy line} \\ \text{a complex number} \end{array} \right|^2$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} \begin{aligned} & [(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu][(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu] \\ & - 2[(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu][(\not{q}_j + \not{q}) q_{j\mu}] \\ & - 2[(\not{q}_i + \not{q}) q_i^\mu][(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu] \\ & + 4[(\not{q}_i + \not{q}) q_i^\mu][(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \quad (1.35)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} \begin{aligned} & [\not{q} \not{q}_i \gamma^\mu][\not{q} \not{q}_j \gamma_\mu] \\ & - 2[\not{q} \not{q}_i \gamma^\mu][(\not{q} + \not{q}_j) q_{j\mu}] \\ & - 2[(\not{q}_i + \not{q}) q_i^\mu][\not{q} \not{q}_j \gamma_\mu] \\ & + 4[(\not{q}_i + \not{q}) q_i^\mu][(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \quad (1.36)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \begin{aligned} & [y(1-2z+2z^2) \not{p}_i \not{p}_j \gamma^\mu][(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu] \\ & + 4(q_i^\mu \cdot q_{j\mu})[(\not{q}_i + \not{q})][(\not{q}_j + \not{q})] \end{aligned} \quad (1.37)$$

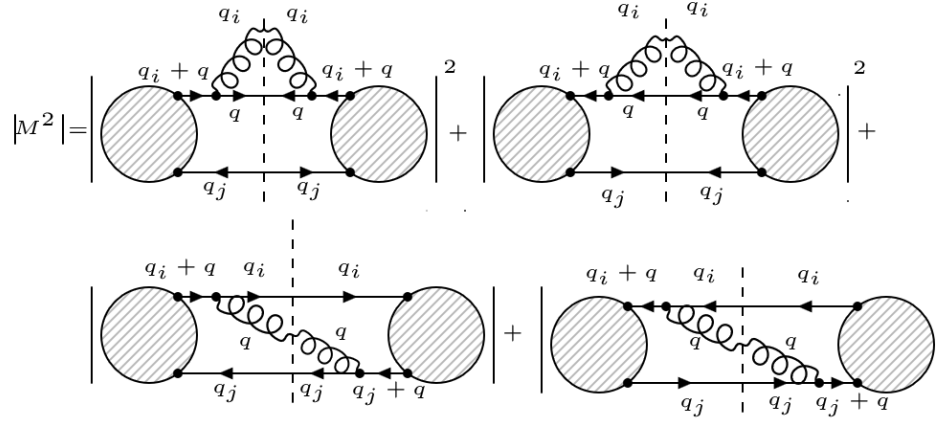
$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \begin{aligned} & [y(1-2z+2z^2) \not{p}_i \not{p}_j \gamma^\mu][(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu] \\ & + 4(p_i \cdot p_j)[(\not{p}_i + y \not{p}_j)][(1-z) \not{p}_i + (1+yz-y) \not{p}_j - \sqrt{zy(1-z)} \not{m}] \end{aligned} \quad (1.38)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)} z(1-y)[\not{p}_i][\not{p}_j] \quad (1.39)$$

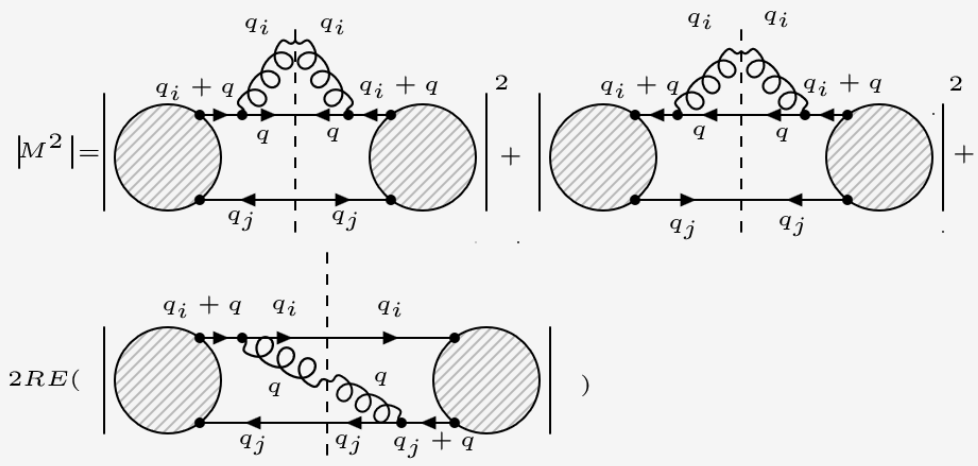
$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(1-z)y(1-2z+2z^2)(p_i \cdot p_j)} z[\not{p}_i][\not{p}_j] \quad (1.40)$$

1.4  $|M^2|$ 

$$|M|^2 = |M_1|^2 + |M_2|^2 + M_1 M_2^\dagger + M_1^\dagger M_2 \quad (1.41)$$



$$|M|^2 = |M_1|^2 + |M_2|^2 + 2RE(M_1 M_2^\dagger) \quad (1.42)$$



$$\begin{aligned}
 |M|^2 = & (d-2)(1-z)(1-y) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & - (d-2)yz^2 \frac{g_s^2 [T^c]_f^m [T^c]_f^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & + 2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2 [T^a]_o^l [T^a]_f^n}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]\right)
 \end{aligned} \quad (1.43)$$

$$T^a_{ok} T^a_{lo} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F\delta_{lk} \quad (1.44)$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a_{ok} T^a_{lo} = C_F \delta_{lk} = \frac{1}{N} \sum_{l=1}^N \delta_{lk} C_F = C_F \quad (1.45)$$

The same calculation for  $T^c_{mf} T^c_{fn}$  and  $T^a_{ol} T^a_{fn}$  turns  $C_F$  out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, which means, if:

$$y \longrightarrow 0 \quad (1.46)$$

$$\begin{aligned} |M|^2 = & (d-2)(1-z)(1-y) \frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & - (d-2)yz^2 \frac{g_s^2 C_F}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & + 2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]\right) \end{aligned} \quad (1.47)$$

$$|M|^2 = C_F \left( (d-2)(1-z) - \frac{4z}{z-1} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \quad (1.48)$$

for

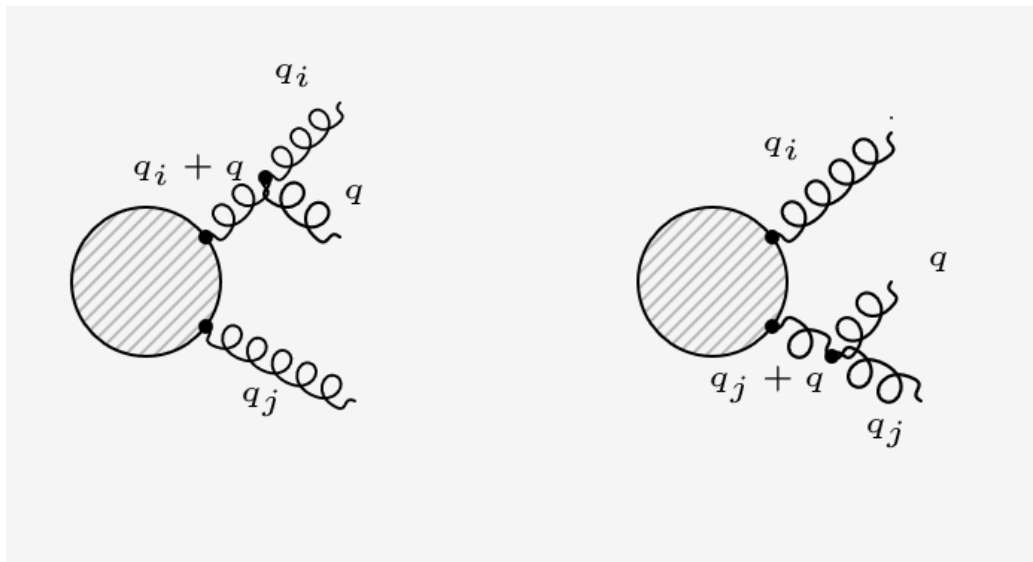
$$d = 4 - 2\epsilon \quad (1.49)$$

$$\begin{aligned} |M|^2 = & C_F \left( (4-2\epsilon-2)(1-z) + \frac{4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{2(1-\epsilon)(1-z)^2 + 4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{2-4z+2z^2-\epsilon(1-z)^2+4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{(1+z^2)}{1-z} - \epsilon(1-z) \right) \frac{g_s^2}{y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & \langle \hat{P}_{qq} \rangle \frac{g_s^2}{q_i \cdot q} [\not{p}_i][\not{p}_j] \end{aligned} \quad (1.50)$$

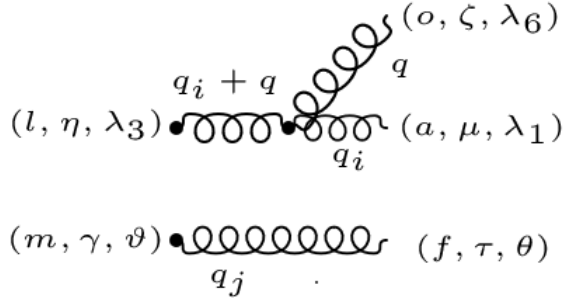


## Chapter 2

### Gluon gluon gluon emission kernel

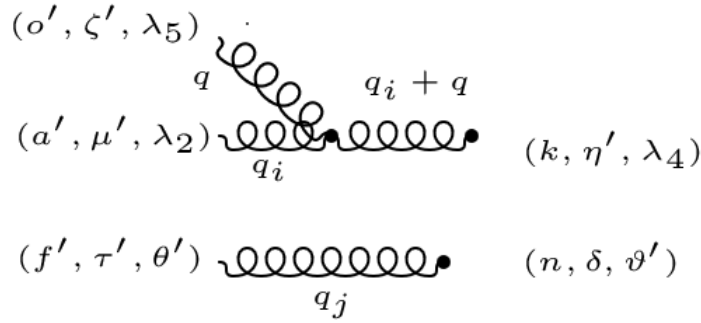


## 2.1 Gluon-Emitter Bubble



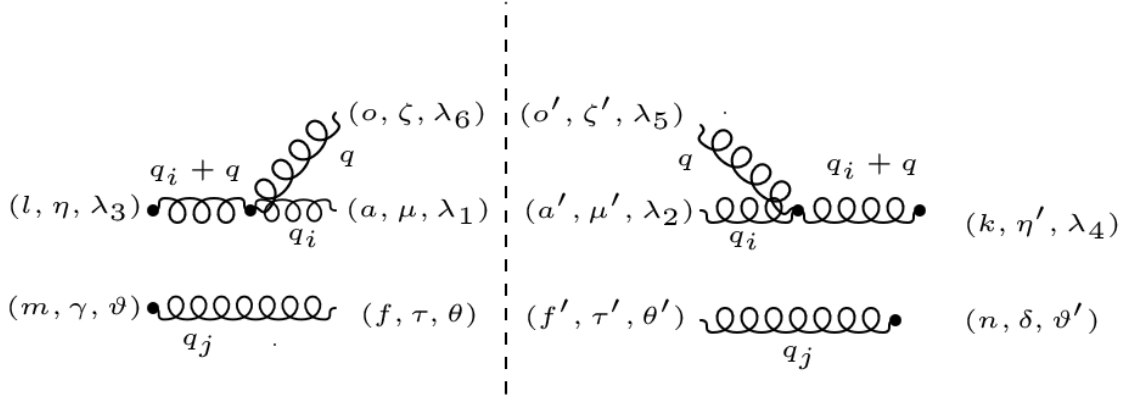
$$M_1 = \left[ \frac{-i}{(q + q_i)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta + g^{\zeta \eta} (-q - (q + q_i))^\mu + g^{\eta \mu} (q_i + q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_6}_\zeta(q) [\varepsilon^\theta_{\tau'}(q_j)] \right] \quad (2.1)$$

$$M_1 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q + q_i)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q) [\varepsilon^\theta_{\tau'}(q_j)] \right] \quad (2.2)$$



$$M_1^\dagger = \left[ \frac{i}{(q_i + q)^2} (-g_s f^{a' k o'} (-g^{\mu' \eta'} (2q_i + q)^{\zeta'} + g^{\eta' \zeta'} (2q + q_i)^{\mu'} + g^{\zeta' \mu'} (q_i - q)^{\eta'}) \right. \\ \left. \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_5}_{\zeta'}(q) [\varepsilon^{\theta'}_{\tau'}(q_j)] \right] \quad (2.3)$$

$$|M_1|^2 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q + q_i)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_6}_\zeta(q) \varepsilon^{\lambda_5}_{\zeta'}(q) \right. \\ \left. (-g_s f^{a' k o'} (-g^{\mu' \eta'} (2q_i + q)^{\zeta'} + g^{\eta' \zeta'} (2q + q_i)^{\mu'} + g^{\zeta' \mu'} (q_i - q)^{\eta'}) \frac{i}{(q_i + q)^2} [g_{\gamma \delta}] \right] \quad (2.4)$$



$$\begin{aligned}
N \equiv & g_{\mu\mu'} g_{\zeta\zeta'} [-g^{\mu\zeta} g^{\mu'\eta'} (q - q_i)^\eta (2q_i + q)^{\zeta'} + g^{\mu\zeta} g^{\eta'\zeta'} (q - q_i)^\eta (2q + q_i)^{\mu'} \\
& + g^{\mu\zeta} g^{\zeta'\mu'} (q - q_i)^\eta (q_i - q)^{\eta'} + g^{\zeta\eta} g^{\mu'\zeta'} (2q + q_i)^\mu (2q_i + q)^{\zeta'} \\
& - g^{\zeta\eta} g^{\eta'\zeta'} (2q + q_i)^\mu (2q + q_i)^{\mu'} - g^{\zeta\eta} g^{\zeta'\mu'} (2q + q_i)^\mu (q_i - q)^{\eta'} \\
& - g^{\eta\mu} g^{\mu'\eta'} (2q_i + q)^\zeta (2q_i + q)^{\zeta'} + g^{\eta\mu} g^{\eta'\zeta'} (2q_i + q)^\zeta (2q + q_i)^{\mu'} \\
& + g^{\eta\mu} g^{\zeta'\mu'} (2q_i + q)^\zeta (q_i - q)^{\eta'}] [g_{\gamma\delta}]
\end{aligned} \quad (2.5)$$

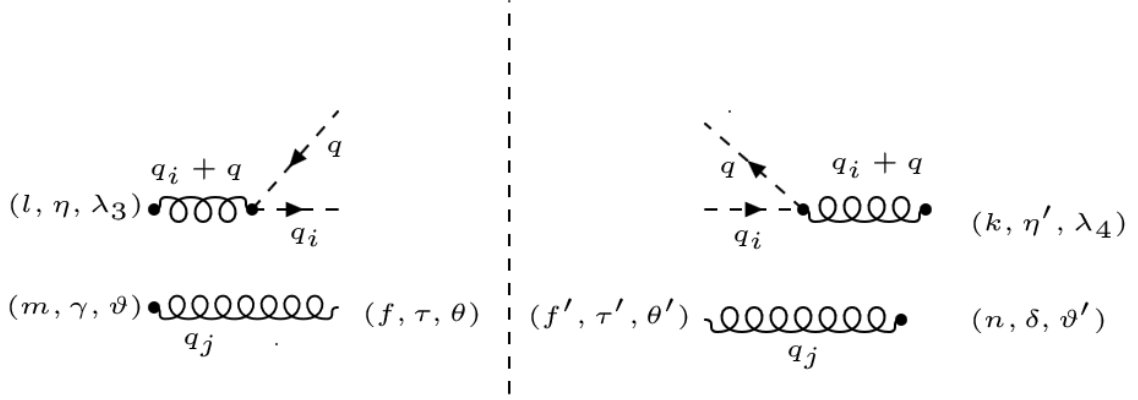
$$\begin{aligned}
N \equiv & [-(q - q_i)^\eta (2q_i + q)^{\eta'} + (q - q_i)^\eta (2q + q_i)^{\eta'} + d(q - q_i)^\eta (q_i - q)^{\eta'} \\
& + (2q + q_i)^{\eta'} (2q_i + q)^\eta - g^{\eta\eta'} (2q + q_i)^\mu (2q + q_i)_\mu - (2q + q_i)^\eta (q_i - q)^{\eta'} \\
& - g^{\eta\eta'} (2q_i + q)^\zeta (2q_i + q)_\zeta + (2q_i + q)^{\eta'} (2q + q_i)^\eta + (2q_i + q)^\eta (q_i - q)^{\eta'}] [g_{\gamma\delta}]
\end{aligned} \quad (2.6)$$

$$\begin{aligned}
N \equiv & [-(q^\eta q^{\eta'} + 2q^\eta q_i^{\eta'} - q_i^\eta q^{\eta'} - 2q_i^\eta q_i^{\eta'}) + (2q^\eta q^{\eta'} + q^\eta q_i^{\eta'} - 2q_i^\eta q^{\eta'} - q_i^\eta q_i^{\eta'}) \\
& + (dq^\eta q_i^{\eta'} - dq^\eta q^{\eta'} - dq_i^\eta q_i^{\eta'} + dq_i^\eta q^{\eta'}) + (4q^{\eta'} q_i^\eta + 2q^{\eta'} q^\eta + 2q_i^{\eta'} q_i^\eta + q_i^{\eta'} q^\eta) \\
& - (-2q^\eta q^{\eta'} + 2q^\eta q_i^{\eta'} - q_i^\eta q^{\eta'} + q_i^\eta q_i^{\eta'}) + (2q^{\eta'} q^\eta + q^{\eta'} q_i^\eta + 4q_i^{\eta'} q^\eta + 2q_i^{\eta'} q_i^\eta) \\
& + (-q^\eta q^{\eta'} + q^\eta q_i^{\eta'} - 2q_i^\eta q^{\eta'} + 2q_i^\eta q_i^{\eta'}) - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.7)$$

$$\begin{aligned}
N \equiv & [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} \\
& - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.8)$$

$$\begin{aligned}
|M_1|^2 = & \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} \\
& - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.9)$$

### 2.1.1 One-loop corrections to the gluon self-energy diagram(Gluon-Emitter Bubble)



$$|M_1|_{Ghost\ loop}^2 = \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [-q_i^\eta q^{\eta'} - q^\eta q_i^{\eta'}] [g_{\gamma\delta}] \quad (2.10)$$

$$\begin{aligned} |M'_1|^2 &= |M_1|^2 + |M_1|_{Ghost\ loop}^2 \\ &= \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} \\ &\quad + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i) - q_i^\eta q^{\eta'} - q^\eta q_i^{\eta'}] [g_{\gamma\delta}] \end{aligned} \quad (2.11)$$

$$\begin{aligned} |M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 2)q^\eta q_i^{\eta'} \\ &\quad + (d + 2)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} - g^{\eta\eta'} (8qq_i)] [g_{\gamma\delta}] \end{aligned} \quad (2.12)$$

$$\begin{aligned} |M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\ &\quad [(6 - d)(\zeta_1 p_i^\eta + \lambda_1 Q^\eta + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_1 p_i^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (d + 2)(\zeta_1 p_i^\eta + \lambda_1 Q^\eta + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_q p_i^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (d + 2)(\zeta_q p_i^\eta + \lambda_q Q^\eta - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_1 p_i^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (6 - d)(\zeta_q p_i^\eta + \lambda_q Q^\eta - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_q p_i^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad - g^{\eta\eta'} (8y(\alpha_1 + \beta_1) p_i \cdot Q)] [g_{\gamma\delta}] \end{aligned} \quad (2.13)$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)[\zeta_1 \zeta_1 p_i^\eta p_i^{\eta'} + \zeta_1 \lambda_1 p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_1 \zeta_1 Q^\eta p_i^{\eta'} + \lambda_1 \lambda_1 Q^\eta Q^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(d+2)[\zeta_1 \zeta_q p_i^\eta p_i^{\eta'} + \zeta_1 \lambda_q p_i^\eta Q^{\eta'} - \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_1 \zeta_q Q^\eta p_i^{\eta'} + \lambda_1 \lambda_q Q^\eta Q^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(d+2)[\zeta_q \zeta_1 p_i^\eta p_i^{\eta'} + \zeta_q \lambda_1 p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_q \zeta_1 Q^\eta p_i^{\eta'} + \lambda_q \lambda_1 Q^\eta Q^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(6-d)[\zeta_q \zeta_q p_i^\eta p_i^{\eta'} + \zeta_q \lambda_q p_i^\eta Q^{\eta'} - \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_q \zeta_q Q^\eta p_i^{\eta'} + \lambda_q \lambda_q Q^\eta Q^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}]
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)[(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\alpha_1\beta_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\beta_1\alpha_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\beta_1^2 Q^\eta Q^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(d+2)[(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\alpha_1^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} - \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\beta_1\alpha_1 Q^\eta Q^{\eta'} \\
&\quad - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&\quad + \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \quad (2.15) \\
&\quad [(d+2)[(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\alpha_1\beta_1 Q^\eta Q^{\eta'} \\
&\quad + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(6-d)[(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\beta_1\alpha_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} - \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\alpha_1\beta_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\alpha_1^2 Q^\eta Q^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&\quad + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)\{(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&+ y\beta_1\alpha_1 Q^\eta p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&+ y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (d+2)\{(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} \\
&- \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} + y\beta_1^2 Q^\eta p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&+ \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} \quad (2.16) \\
&+ (d+2)\{(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&+ y\alpha_1^2 Q^\eta p_i^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&- y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (6-d)\{(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1\alpha_1 p_i^\eta Q^{\eta'} \\
&- \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&- \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)\{(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\beta_1\alpha_1 Q^\eta p_i^{\eta'} \\
&+ y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} \\
&+ (d+2)\{(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} + y\beta_1^2 Q^\eta p_i^{\eta'} \quad (2.17) \\
&- y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (d+2)\{(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} \\
&+ y\alpha_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (6-d)\{(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} \\
&+ y\beta_1\alpha_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) \\
&\quad + (6-d)(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))][p_i^\eta p_i^{\eta'}] \quad (2.18) \\
&\quad + [2(6-d)y\alpha_1\beta_1 + (d+2)y(\alpha_1^2 + \beta_1^2)]p_i^\eta Q^{\eta'} \\
&\quad + [2(6-d)y\beta_1\alpha_1 + (d+2)y(\alpha_1^2 + \beta_1^2)]Q^\eta p_i^{\eta'} \\
&\quad + [2(6-d) - 2(d+2)]y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) \\
&\quad + (6-d)(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))][p_i^\eta p_i^{\eta'}] \quad (2.19) \\
&\quad + y[(4d-8)\alpha_1^2 + (8-4d)\alpha_1 + (d+2)]p_i^\eta Q^{\eta'} \\
&\quad + y[(4d-8)\alpha_1^2 + (8-4d)\alpha_1 + (d+2)]Q^\eta p_i^{\eta'} \\
&\quad + y[8-4d](\alpha_1 - \alpha_1^2)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y(1 - \beta_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \quad (2.20) \\
&[[8-4d]\beta_1(1 - \beta_1)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'}(1 - \beta_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y (p_i \cdot Q) (p_i \cdot Q)} \quad (2.21) \\
&[8[\epsilon - 1]\beta_1(1 - \beta_1)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'}p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

Another way:

$$\begin{aligned}
k_1^\eta k_1^{\eta'} &= (\alpha_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
k_1^\eta q_i^{\eta'} &= (\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} + y\beta_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
q_i^\eta k_1^{\eta'} &= (\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} + y\alpha_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
q_i^\eta q_i^{\eta'} &= (\beta_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \quad (2.22)
\end{aligned}$$



$$\begin{aligned}
N \equiv & (6-d)(\alpha_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
& + (d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} + y\beta_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
& + (d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} + y\alpha_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
& + (6-d)(\beta_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'}
\end{aligned} \tag{2.23}$$

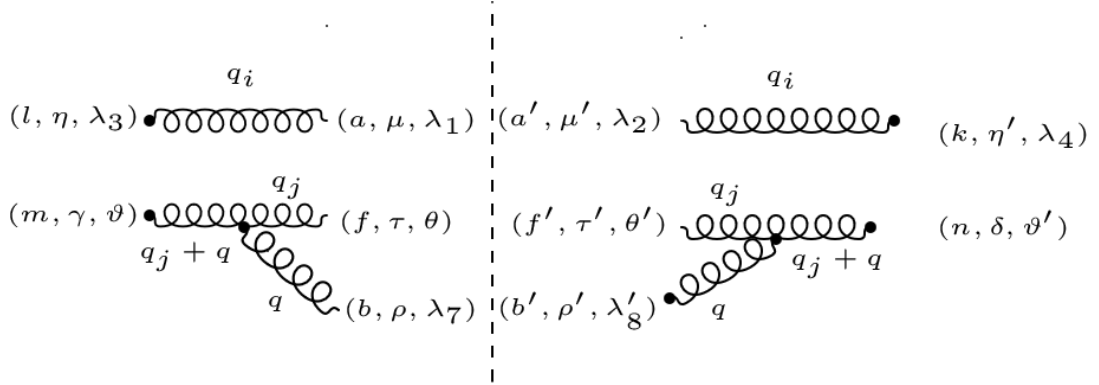
$$\begin{aligned}
N \equiv & [(6-d)(\alpha_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q})) + (d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) \\
& + (d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) + (6-d)(\beta_1^2 - 2\alpha_1\beta_1 y(\frac{Q^2}{2p_i \cdot Q}))]p_i^\eta p_i^{\eta'} \\
& + [(6-d)y\alpha_1\beta_1 + (d+2)y\alpha_1^2 + (d+2)y\beta_1^2 + (6-d)y\alpha_1\beta_1]p_i^\eta Q^{\eta'} \\
& + [(6-d)y\alpha_1\beta_1 + (d+2)y\beta_1^2 + (d+2)y\alpha_1^2 + (6-d)y\alpha_1\beta_1]Q^\eta p_i^{\eta'} \\
& + [(6-d)y\alpha_1\beta_1 - (d+2)y\alpha_1\beta_1 - (d+2)y\alpha_1\beta_1 + (6-d)y\alpha_1\beta_1]n_{\perp,1}^\eta n_{\perp,1}^{\eta'} \\
& - 8yg^{\eta\eta'} p_i \cdot Q
\end{aligned} \tag{2.24}$$

$$|M'_1|^2 = \frac{g_s^2 f^{aol} f^{ako}}{y(p_i \cdot Q)^2} [(12-2d)y\alpha_1\beta_1 - 2(d+2)y\alpha_1\beta_1]n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8yg^{\eta\eta'} p_i \cdot Q [g_{\gamma\delta}]$$

$$\Rightarrow |M'_1|^2 = \frac{g_s^2 f^{aol} f^{ako}}{(p_i \cdot Q)^2} [(12-2d)\alpha_1\beta_1 - 2(d+2)\alpha_1\beta_1]n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'} p_i \cdot Q [g_{\gamma\delta}]$$

$$\boxed{|M'_1|^2 = \frac{g_s^2 f^{aol} f^{ako}}{y(p_i \cdot Q)(p_i \cdot Q)} [8[\epsilon-1]\beta_1(1-\beta_1)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'} p_i \cdot Q] [g_{\gamma\delta}]} \tag{2.25}$$

## 2.2 Gluon-Spectator Bubble



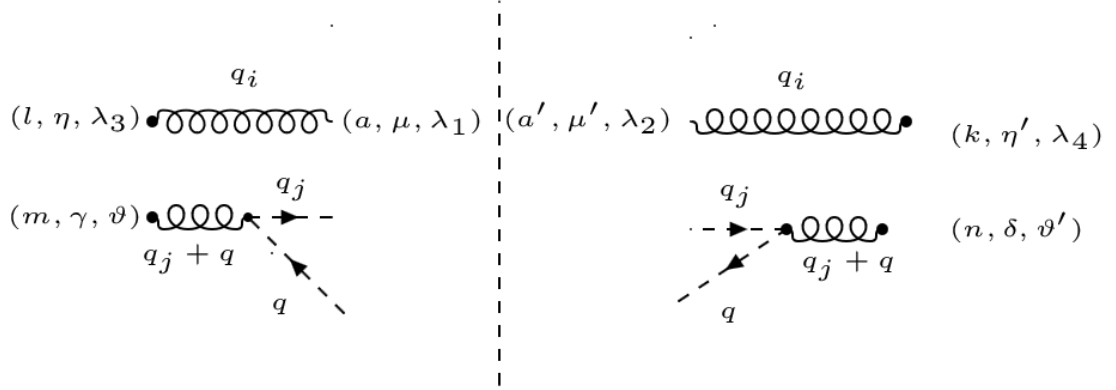
$$|M_2|^2 = \left[ \frac{-i}{(q_j + q)^2} (-g_s f^{b f m} (g^{\tau\gamma} (-2q_j - q)^\rho + g^{\gamma\rho} (2q + q_j)^\tau + g^{\rho\tau} (q_j - q)^\gamma) \right. \\ \left. g_{\tau\tau'} g_{\rho\rho'} (-g_s f^{b' n f'} (g^{\rho'\delta} (-2q - q_j)^{\tau'} + g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\tau'\rho'} (q - q_j)^\delta) \frac{i}{(q_j + q)^2} \right] [g_{\eta\eta'}] \quad (2.26)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b' n f'} \delta^{aa'} \delta^{ff'} \delta^{bb'}}{(q_j + q)^2 (q_j + q)^2} [g_{\tau\tau'} g_{\rho\rho'} (g^{\tau\gamma} (2q_j + q)^\rho g^{\rho'\delta} (2q + q_j)^{\tau'} \\ - g^{\tau\gamma} (2q_j + q)^\rho g^{\delta\tau'} (2q_j + q)^{\rho'} - g^{\tau\gamma} (2q_j + q)^\rho g^{\tau'\rho'} (q - q_j)^\delta - g^{\gamma\rho} (2q + q_j)^\tau g^{\rho'\delta} (2q + q_j)^{\tau'} \\ + g^{\gamma\rho} (2q + q_j)^\tau g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\gamma\rho} (2q + q_j)^\tau g^{\tau'\rho'} (q - q_j)^\delta - g^{\rho\tau} (q_j - q)^\gamma g^{\rho'\delta} (2q + q_j)^{\tau'} \\ + g^{\rho\tau} (q_j - q)^\gamma g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\rho\tau} (q_j - q)^\gamma g^{\tau'\rho'} (q - q_j)^\delta) [g_{\eta\eta'}] \quad (2.27)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(2q + q_j)^\gamma (2q_j + q)^\delta \\ - g^{\delta\gamma} (2q_j + q)^\rho (2q_j + q)_\rho - (2q_j + q)^\gamma (q - q_j)^\delta - g^{\delta\gamma} (2q + q_j)^\tau (2q + q_j)_\tau \\ + (2q_j + q)^\gamma (2q + q_j)^\delta + (2q + q_j)^\gamma (q - q_j)^\delta - (q_j - q)^\gamma (2q + q_j)^\delta \\ + (q_j - q)^\gamma (2q_j + q)^\delta + d(q_j - q)^\gamma (q - q_j)^\delta] [g_{\eta\eta'}] \quad (2.28)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(3 + d) q^\gamma q_j^\delta + (6 - d) q^\gamma q^\delta \\ + (6 - d) q_j^\gamma q_j^\delta + (3 + d) q_j^\gamma q^\delta - g^{\delta\gamma} (5q_j^2 + 5q^2 + 8qq_j) \\ ] [g_{\eta\eta'}] \quad (2.29)$$

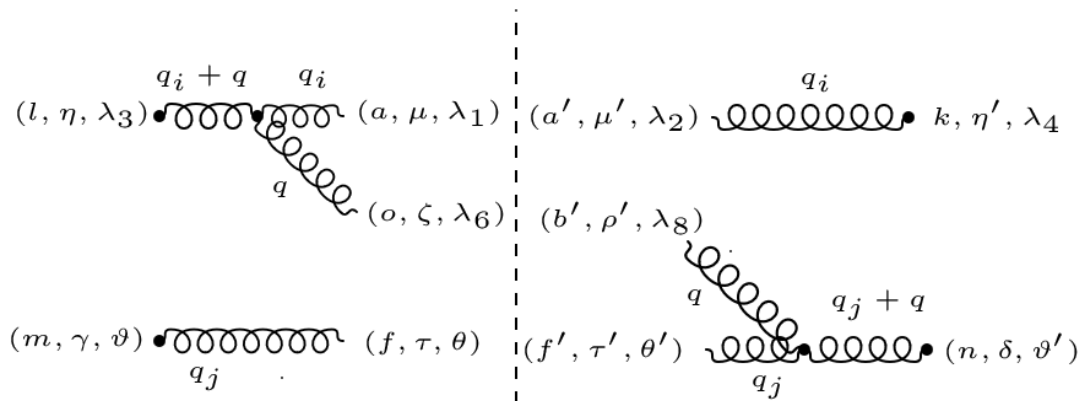
### 2.2.1 One-loop corrections to the gluon self-energy diagram (Gluon-Spectator Bubble)



$$|M_1|_{Ghost\ loop}^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [-q_j^\gamma q^\delta - q^\delta q_j^\gamma] [g_{\gamma\delta}] \quad (2.30)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(2 + d)q^\gamma q_j^\delta + (6 - d)q^\gamma q^\delta + (6 - d)q_j^\gamma q_j^\delta + (2 + d)q_j^\gamma q^\delta - g^{\delta\gamma} (8q q_j)] [g_{\eta\eta'}] \quad (2.31)$$

### 2.3 Interference term $M_1 M_2^\dagger$



$$\begin{aligned}
M_1 M_2^\dagger = & \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{l a o} (g^{\eta\mu} (2q_i + q)^\zeta + g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q + q_i)^\mu) \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q)) \right. \\
& [\varepsilon^{\theta^*}_{\tau'}(q_j)] \\
& \left[ \frac{i}{(q + q_j)^2} (-g_s f^{f' b' n} (g^{\tau'\rho'} (q_j - q)^\delta + g^{\rho'\delta} (2q + q_j)^{\tau'} - g^{\delta\tau'} (2q_j + q)^{\rho'}) \varepsilon^{\theta^*}_{\tau'}(q_j) \varepsilon^{\lambda_8}_{\rho'}(q)) \right. \\
& \left. [\varepsilon^{\lambda_2}_{\mu'}(q_i)] \right]
\end{aligned} \tag{2.32}$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{l a o} f^{f' b' n} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i + q)^2 (q_j + q)^2} [g_\mu^{\eta'} g_{\tau\tau'} (g^{\eta\mu} (2q_i + q)^\zeta + g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q + q_i)^\mu) \\
& g_{\zeta\rho'} (g^{\tau'\rho'} (q_j - q)^\delta + g^{\rho'\delta} (2q + q_j)^{\tau'} - g^{\delta\tau'} (2q_j + q)^{\rho'})]
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{l a o} f^{f' b' n} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i + q)^2 (q_j + q)^2} \\
& [g^{\eta\eta'} (2q_i + q)^\gamma (q_j - q)^\delta + g^{\eta\eta'} (2q + q_j)^\gamma (2q_i + q)^\delta - g^{\eta\eta'} g^{\gamma\delta} (2q_i + q) \cdot (2q_j + q) \\
& + g^{\gamma\eta'} (q - q_i)^\eta (q_j - q)^\delta + g^{\eta'\delta} (q - q_i)^\eta (2q + q_j)^\gamma - g^{\gamma\delta} (q - q_i)^\eta (2q_j + q)^{\eta'} \\
& - g^{\gamma\eta} (2q + q_i)^{\eta'} (q_j - q)^\delta - g^{\eta\delta} (2q + q_i)^{\eta'} (2q + q_j)^\gamma + g^{\gamma\delta} (2q_j + q)^\eta (2q + q_i)^{\eta'}]
\end{aligned} \tag{2.34}$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{l a o} f^{f o n}}{4(q \cdot q_i)(q \cdot q_j)} \\
& \{g^{\eta\eta'} [2q_i^\gamma q_j^\delta + 2q_i^\gamma q^\delta + q^\gamma q_j^\delta + q^\gamma q^\delta + 4q^\gamma q_i^\delta + 2q^\gamma q^\delta + 2q_j^\gamma q_i^\delta + q_j^\gamma q^\delta] \\
& - g^{\eta\eta'} g^{\gamma\delta} (2q \cdot q_j + q \cdot q + 4q_i \cdot q_j + 2q_i \cdot q) + g^{\gamma\eta'} [q^\eta q_j^\delta - q^\eta q^\delta - q_i^\eta q_j^\delta + q_i^\eta q^\delta] \\
& + g^{\eta'\delta} [2q^\eta q^\gamma + q^\eta q_j^\gamma + q_i^\eta q^\gamma + q_i^\eta q_j^\gamma] - g^{\gamma\delta} [2q^\eta q_j^{\eta'} + q^\eta q^{\eta'} - 2q_i^\eta q_j^{\eta'} - q_i^\eta q^{\eta'}] \\
& - g^{\gamma\eta} [2q^{\eta'} q_j^\delta - 2q^{\eta'} q^\delta + q_i^{\eta'} q_j^\delta - q_i^{\eta'} q^\delta] - g^{\eta\delta} [4q^{\eta'} q^\gamma + 2q^{\eta'} q_j^\gamma + 2q_i^{\eta'} q^\gamma + q_i^{\eta'} q_j^\gamma] \\
& + g^{\gamma\delta} [4q_j^\eta q^{\eta'} + 2q_j^\eta q_i^{\eta'} + q^\eta q^{\eta'} + q^\eta q_i^{\eta'}]\}
\end{aligned} \tag{2.35}$$

$$\begin{aligned}
k_1^\eta k_1^{\eta'} &= [(1 - \beta_1)^2 - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})^2] p_i^\eta p_i^{\eta'} - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} - y^2 \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\
k_1^\eta q_i^{\eta'} &= [\beta_1(1 - \beta_1) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) Q^\eta p_i^{\eta'} \\
q_i^\eta k_1^{\eta'} &= [\beta_1(1 - \beta_1) - y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q})] p_i^\eta p_i^{\eta'} + y \beta_1^2 (\frac{Q^2}{2p_i \cdot Q}) p_i^\eta Q^{\eta'} \\
q_i^\eta q_i^{\eta'} &= \beta_1^2 p_i^\eta p_i^{\eta'} \\
k_1^\eta q_k^{\eta'} &= \sqrt{1 - y} p_i^\eta p_k^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_1 p_i^\eta p_i^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_2 p_i^\eta Q^{\eta'} \\
&\quad + y \beta_1 A_1 Q^\eta p_i^{\eta'} + y \beta_1 A_2 Q^\eta Q^{\eta'} \\
q_i^\eta q_k^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 p_i^\eta Q^{\eta'} + \beta_1 \sqrt{1 - y} p_i^\eta p_k^{\eta'} \\
q_k^\eta k_1^{\eta'} &= \sqrt{1 - y} p_i^\eta p_k^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_1 p_i^\eta p_i^{\eta'} - y \beta_1 (\frac{Q^2}{2p_i \cdot Q}) A_2 p_i^\eta Q^{\eta'} \\
&\quad + y \beta_1 A_1 Q^\eta p_i^{\eta'} + y \beta_1 A_2 Q^\eta Q^{\eta'} \\
q_k^\eta q_i^{\eta'} &= A_1 \beta_1 p_i^\eta p_i^{\eta'} + A_2 \beta_1 Q^\eta p_i^{\eta'} + \beta_1 \sqrt{1 - y} p_k^\eta p_i^{\eta'}
\end{aligned} \tag{2.36}$$

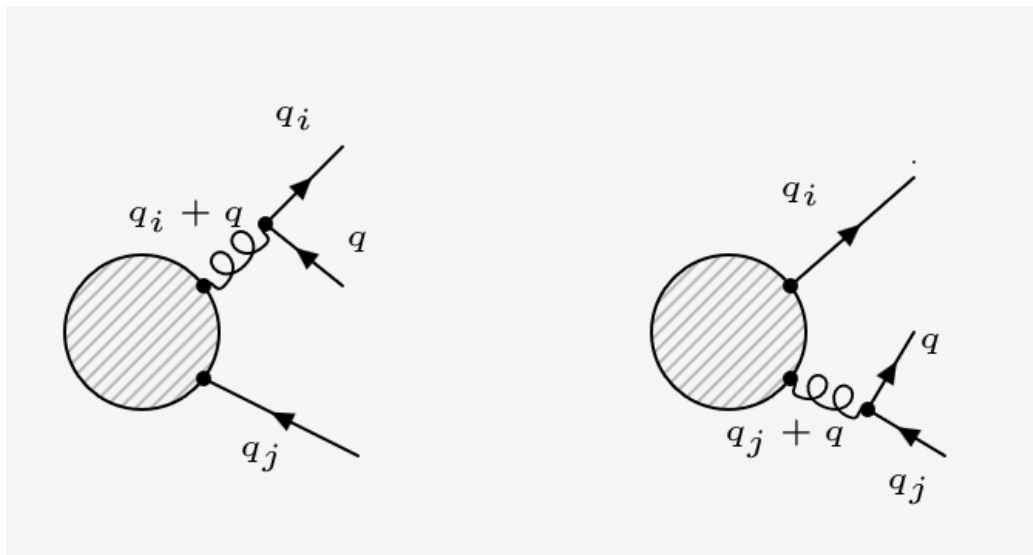
$$\begin{aligned}
M_1 M_2^\dagger &= \frac{g_s^2 f^{l a o f f o n}}{4(q_i \cdot q)(q_j \cdot q)} \\
&\{ g^{\eta\eta'} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\gamma p_i^\delta - g^{\eta\eta'} g^{\gamma\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i \cdot p_i + g^{\gamma\eta'} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^\delta \\
&\quad + g^{\eta'\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^\gamma - g^{\gamma\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^{\eta'} - g^{\gamma\eta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^{\eta'} p_i^\delta \\
&\quad - g^{\eta\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^{\eta'} p_i^\gamma + g^{\gamma\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^{\eta'} \}
\end{aligned} \tag{2.37}$$

$$\begin{aligned}
M_1 M_2^\dagger &= \frac{g_s^2 f^{l a o f f o n}}{4(q_i \cdot q)(q_j \cdot q)} \{ g^{\eta\eta'} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\gamma p_i^\delta + g^{\eta\eta'} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^\delta \\
&\quad + g^{\eta'\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^\eta p_i^\gamma - g^{\gamma\eta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^{\eta'} p_i^\delta - g^{\eta\delta} (\alpha_1^2 + \alpha_1 \beta_1) p_i^{\eta'} p_i^\gamma \}
\end{aligned} \tag{2.38}$$

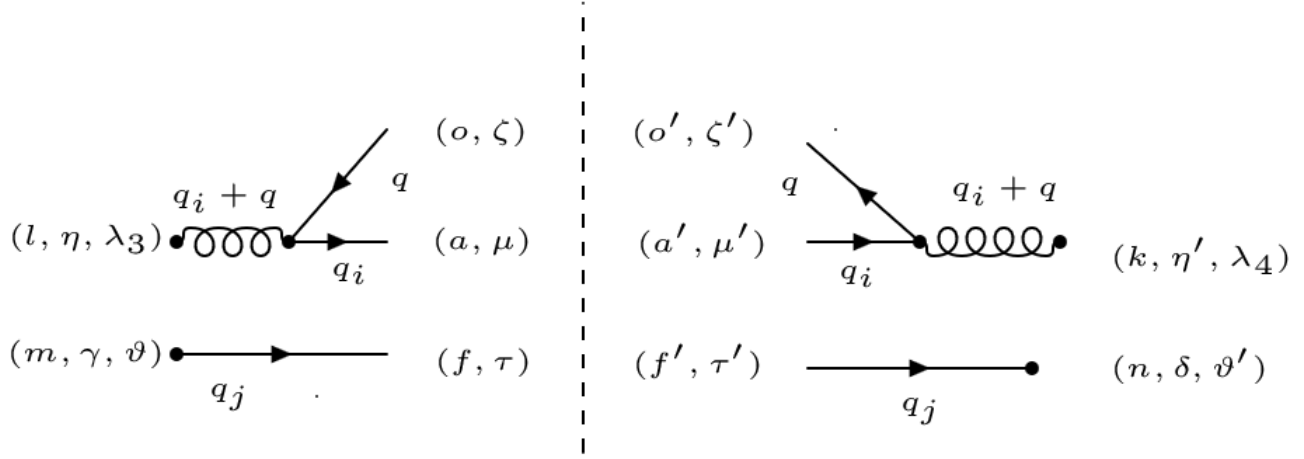
## 2.4 $|M^2|$

## Chapter 3

### Quark gluon quark emission kernel



### 3.1 Quark loop

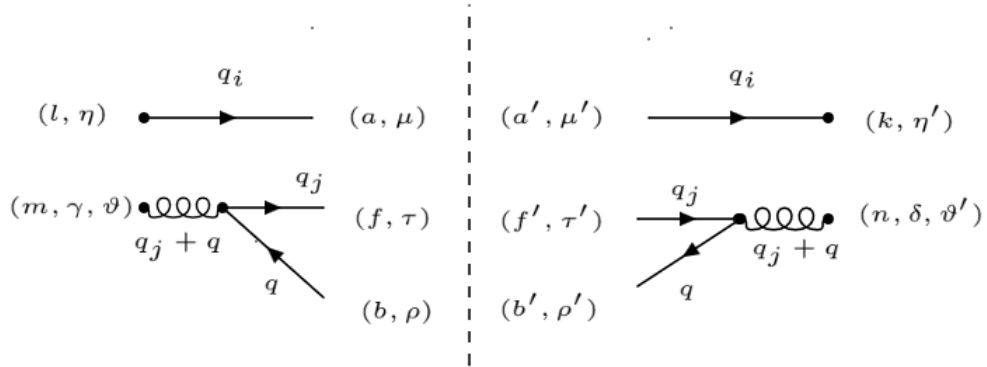


$$|M_1|^2 = \left[ \frac{-i}{(q_i + q)^2} \not{q}_i (-ig_s \gamma^\eta \times [T^l]_a^o) \not{q} (ig_s \gamma^{\eta'} \times [T^k]_{o'}^{a'}) \frac{i}{(q_i + q)^2} \right] [\not{q}_j] \quad (3.1)$$

$$|M_1|^2 = \frac{g_s^2 [T^l]_a^o [T^k]_{o'}^{a'}}{4(qq_i)(qq_i)} [\not{q}_i \gamma^\eta \not{q} \gamma^{\eta'}] [\not{q}_j] \quad (3.2)$$

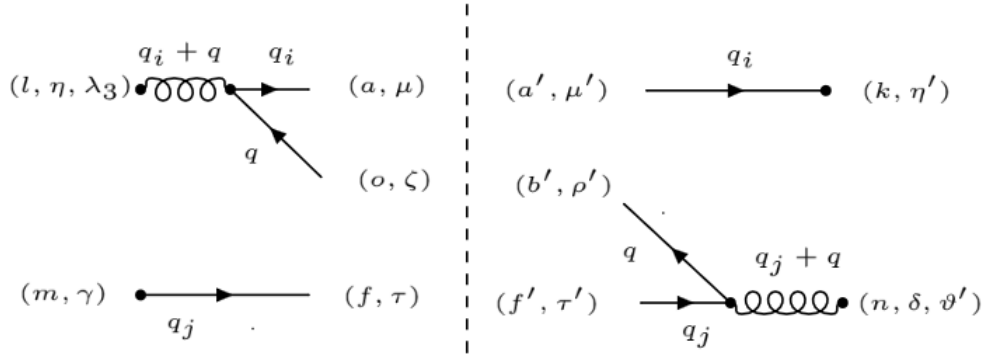


## 3.2 Spectator Quark loop



$$|M_2|^2 = \frac{g_s^2 [T^m]_f^b [T^m]_f^b}{4(qq_j)(qq_j)} [\not{A}_j \gamma^\gamma \not{A} \gamma^\delta] [\not{A}_i] \quad (3.3)$$

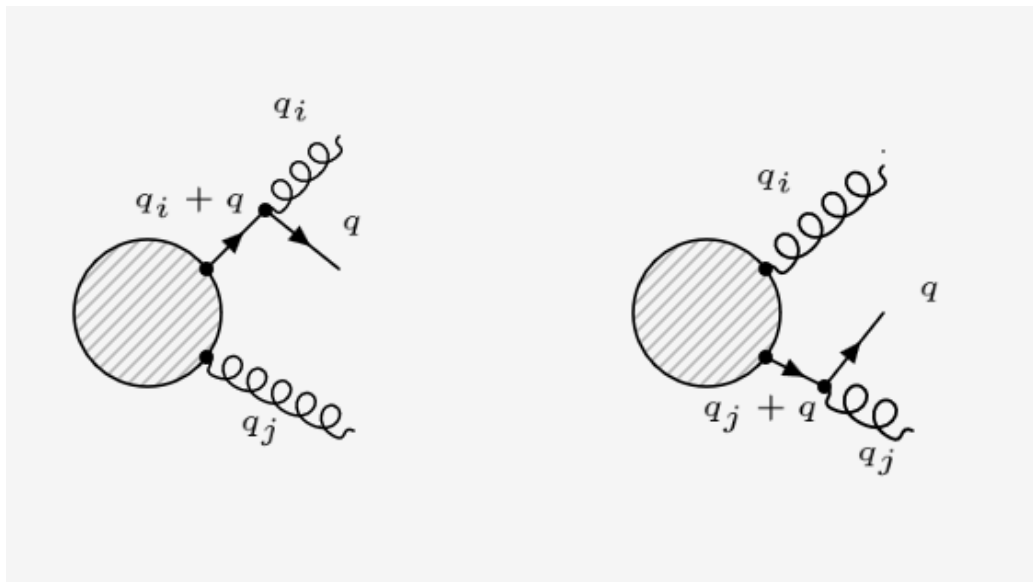
### 3.3 Interference term

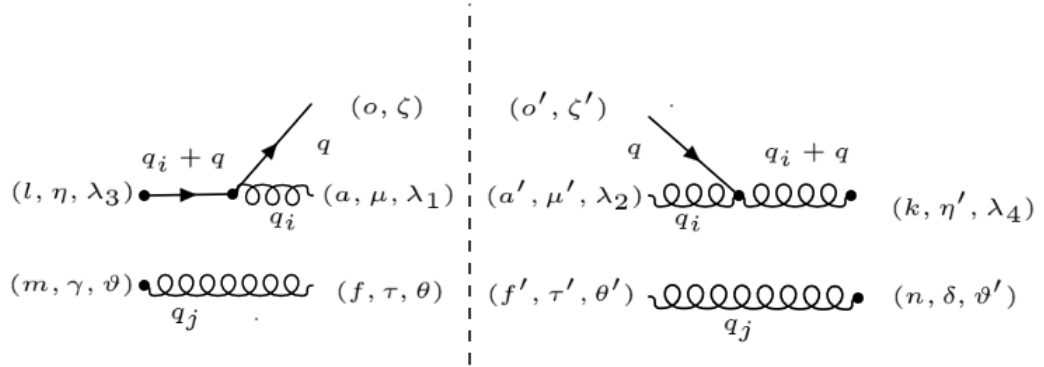


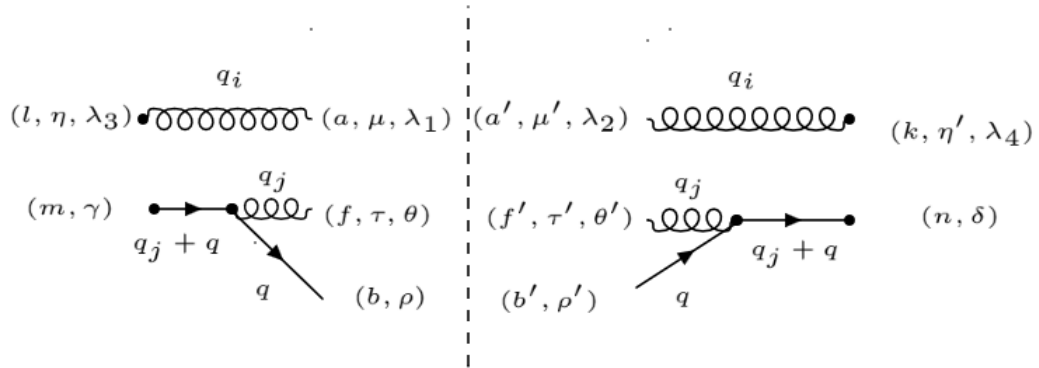
$$M_1 M_2^\dagger = \frac{g_s^2 [T^l]_a^o [T^n]_{f'}^o}{4(qq_i)(qq_j)} [\not{q}_i \gamma^\eta \not{q} \gamma^\delta \not{q}_j] \quad (3.4)$$

## Chapter 4

### Gluon quark quark emission kernel



4.1  $M_1$ 

4.2  $M_2$ 

4.3  $M1M_2^\dagger$ 