

THESIS

BY

TIGRAN SAIDNIA

Emission kernel of parton shower

Emission kernel of parton shower



Karlsruhe institute for Technology (KIT)

Institute of theoretical physics

Referents: PD Dr. Stefan Gieseke

Dr. Simon Plätzer

Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, 13. Januar 2019

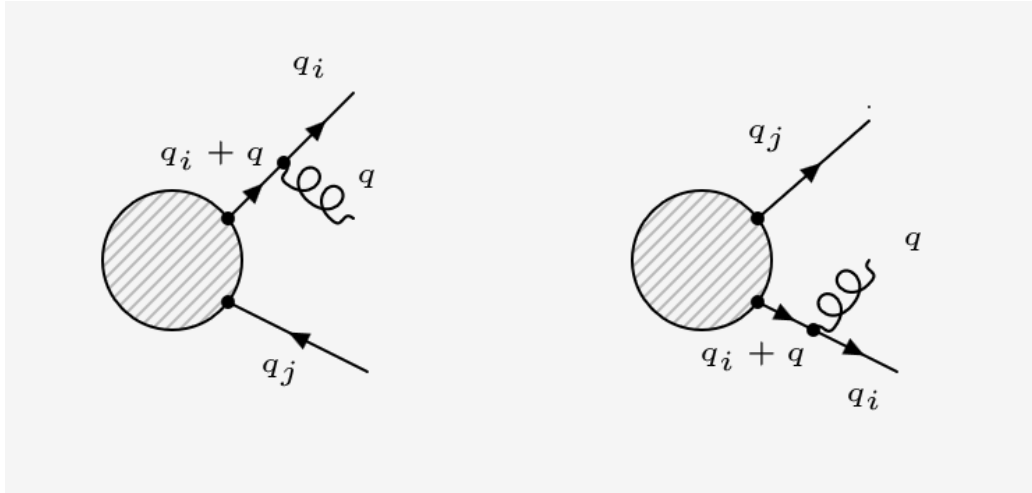
Tigran Saidnia



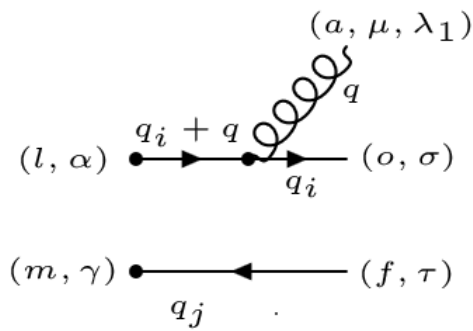
0.1 parametrisation

$$\left. \begin{aligned}
 q_i^\mu &= zp_i^\mu + y(1-z)p_j^\mu + \sqrt{zy(1-z)}m_\perp \\
 q^\mu &= (1-z)p_i^\mu + yzp_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_j^\mu &= (1-y)p_j^\mu \\
 y &= \frac{q_i q}{p_i p_j} \\
 q_i + q &= p_i + yp_j \\
 q_j + q &= (1-z)p_i^\mu + (1+yz-y)p_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_i \cdot q &= yz^2(p_i \cdot p_j) + y(1-z)^2(p_j \cdot p_i) &= y(p_i \cdot p_j) \\
 q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
 q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j) &= (1-z)(p_i \cdot p_j)
 \end{aligned} \right\} \text{parametrisation} \tag{1}$$

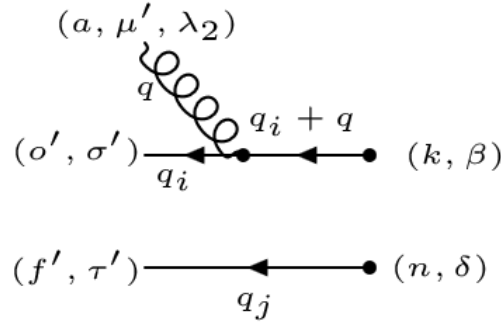
0.2 Quark/Antiquark gluon emission kernel



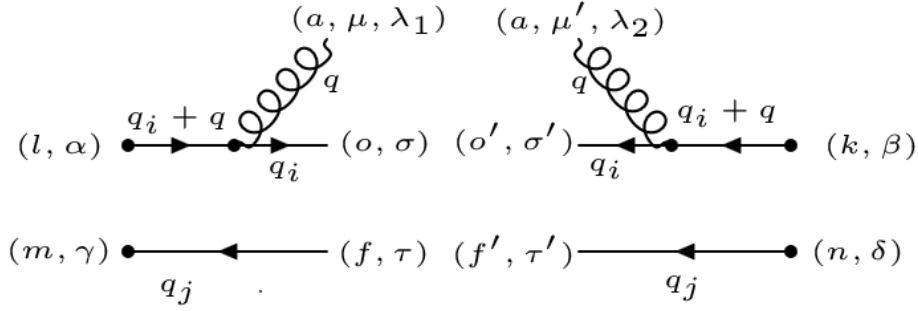
0.2.1 $qg\text{-}\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)] \quad (2)$$



$$M_1^\dagger = \left[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) \right] [\bar{v}_{\tau'}(q_j)] \quad (3)$$



$$|M_1|^2 = M_1 M_1^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q) [v_\tau(q_j)] \quad (4)$$

$$\left[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q) [\bar{v}_{\tau'}(q_j)] \right]$$

$$|M_1|^2 = \left[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o', k}) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q) \right. \quad (5)$$

$$\left. \times (-ig_s \gamma^\mu \times [T^a]_{o, l}) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \right] [\bar{v}_{\tau'}(q_j) v_\tau(q_j)]$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i, \quad (6)$$

$$\sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j$$

and sum over polarization index (λ_1, λ_2) :

$$\sum_{\mu, \mu'} \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_{\mu}(q) = -g_{\mu\mu'} \quad (7)$$

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'k} [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i g_{\mu'\mu} \gamma^{\mu} (\not{q}_i + q)] [\not{q}_j] \quad (8)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'k} [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i \gamma_{\mu'} (\not{q}_i + q)] [\not{q}_j] \quad (9)$$

In other words we expect the tree level diagram from LO and a number: Which means:

$$|M^2| = \left| \text{diagram} \right|^2 \otimes \left| \text{diagram} \right|^2$$

contribution from LO
a complex number

$$|M_1|^2 = \frac{-g_s^2 [T^b]_{o'k} [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [P_i][P_j] \otimes (\text{a complex number}) \quad (10)$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N &=: \gamma^{\mu'} \not{q}_i \gamma_{\mu'} = q_{i\sigma} \gamma^{\mu'} \gamma^{\sigma} \gamma_{\mu'} \\ &= q_{i\sigma} (\{\gamma^{\mu'}, \gamma^{\sigma}\} - \gamma^{\sigma} \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} 2g^{\mu'\sigma} \gamma_{\mu'} - d \gamma^{\sigma} \\ &= (2 - d) \not{q}_i \end{aligned} \quad (11)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^b]_{o'k} [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \not{q}_i (\not{q}_i + q)] [\not{q}_j] \quad (12)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^b]_{o'k} [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [\not{q}_i \not{q}_i \not{q}_i + \not{q}_i \not{q}_i \not{q} + \not{q} \not{q}_i \not{q}_i + \not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (13)$$

For the momenta are on-shell which means:

$$\begin{aligned}\not{q}_i \not{q}_i &= q_i^2 = m_i^2 \\ \not{q} \not{q} &= q^2 = m^2 \\ \not{q}_j \not{q}_j &= q_j^2 = m_j^2\end{aligned}\tag{14}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \not{q}_i$ and $\not{q} \not{q}$ as well.

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)(2q_i q)} [\not{q} \not{q}_i \not{q}] [\not{q}_j]\tag{15}$$

$$\begin{aligned}L &= \not{q} \not{q}_i \not{q} = \not{q} [q_{i\sigma} q_\mu (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu)] \\ &\quad \not{q} [2q_i^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu \left[\frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2} \right] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\ &= \not{q} (2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\ &= \not{q} (2q_i q) - q^2 \not{q}_i \\ &= \not{q} (2q_i q)\end{aligned}\tag{16}$$

After inserting the last result of L and simplify the term $(2q_i q)$ from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [\not{q}_i] [\not{q}_j]\tag{17}$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

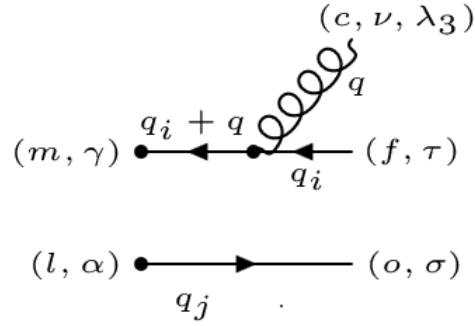
$$|M_1|^2 = (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [(1-z) \not{p}_i + zy \not{p}_j - \sqrt{zy(1-z)} \not{m}_\perp] [(1-y) \not{p}_j]\tag{18}$$

Multiplying the both sides

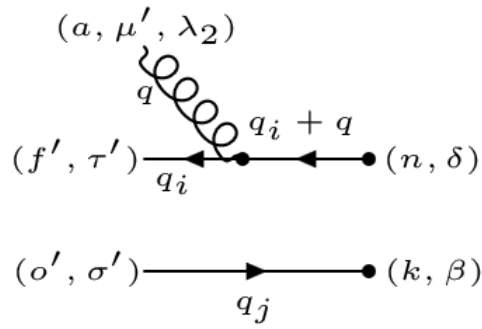
$$\begin{aligned}|M_1|^2 &= (d-2) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [(1-z)(1-y) \not{p}_i \not{p}_j \\ &\quad + zy(1-y) \not{p}_j \not{p}_j + (1-y) \sqrt{zy(1-z)} \not{m}_\perp \not{p}_j]\end{aligned}\tag{19}$$

and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i \not{p}_i$ and $\not{p}_j \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

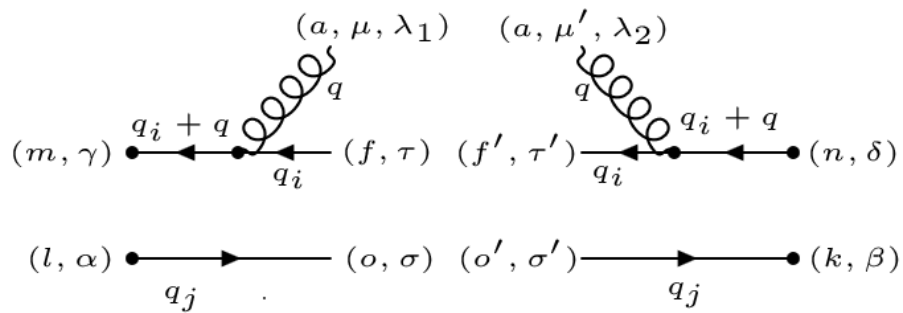
$$|M_1|^2 = (d-2)(1-z)(1-y) \frac{g_s^2 [T^b]_{o'}^k [T^a]_o^l}{(2q_i q)} [\not{p}_i] [\not{p}_j]\tag{20}$$

0.2.2 $\bar{q}g$ -q

$$M_2 = \left[\frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_i) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_j)] \right] \quad (21)$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_i) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_4}_\nu(q) [\bar{u}_{\sigma'}(q_j)] \quad (22)$$



$$|M_2|^2 = M_2 M_2^\dagger = \left[\frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_i) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_j)] \right. \\ \left. [\bar{v}_{\tau'}(q_i) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_j)] \right] \quad (23)$$

$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^\nu v_\tau(q_i) \bar{v}_{\tau'}(q_i) \varepsilon^{\lambda_3}_\nu(q) \varepsilon^{\lambda_4}_{\nu'}(q) \gamma^{\nu'} (\not{q}_i + \not{q})] \\ [u_\sigma(q_j)] [\bar{u}_{\sigma'}(q_j)] \quad (24)$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_j) u_{\sigma'}(q_j) = \not{q}_j, \\ \sum_{\tau, \tau'} \bar{v}_\tau(q_i) v_{\tau'}(q_i) = \not{q}_i \quad (25)$$

and sum over polarization index (λ_3, λ_4) :

$$\sum_{\nu, \nu'} \varepsilon^{\lambda_4*}_{\nu'}(q) \varepsilon^{\lambda_3}_\nu(q) = -g_{\nu\nu'} \quad (26)$$

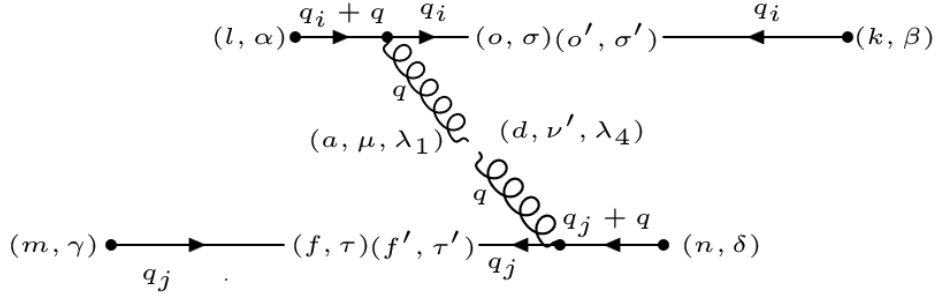
$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^\nu \not{q}_i (-g_{\nu\nu'}) \gamma^{\nu'} (\not{q}_i + \not{q})] [\not{q}_j] \quad (27)$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d - 2) \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(2qq_i)} [\not{q}] [\not{q}_j] \quad (28)$$

finally:

$$|M_1|^2 = (d - 2)(1 - z)(1 - y) \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(2qq_i)} [\not{p}_i] [\not{p}_j] \quad (29)$$

0.2.3 $M_1 M_2^\dagger$ 

$$M_1 M_2^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)]$$

$$[\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q)] [u_{\sigma'}(q_i)] \quad (30)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_4}_{\nu'}(q)$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (31)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] g_{\mu\nu'}$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (32)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j \gamma_\mu (\not{q}_j + \not{q})] \quad (33)$$

Expectation:

$$|M^2| = \left| \text{diagram with two shaded circles and two horizontal lines labeled } P_i \text{ and } P_j \right|^2 \otimes \left| \text{diagram with one shaded circle and a wavy line} \right|^2$$

contribution from LO *a complex number*

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [(\not{A}_i + \not{A}) \gamma^\mu \not{A}_i] [(\not{A}_j + \not{A}) \gamma_\mu \not{A}_j] \quad (34)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [-(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu + 2(\not{A}_i + \not{A}) q_i^\mu] \quad (35)$$

$$[-(\not{A}_j + \not{A}) \not{A}_j \gamma_\mu + 2(\not{A}_j + \not{A}) q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (36)$$

$$[(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu (\not{A}_j + \not{A}) \not{A}_j \gamma_\mu$$

$$- 2(\not{A}_i + \not{A}) \not{A}_i \gamma^\mu (\not{A}_j + \not{A}) q_{j\mu}$$

$$- 2(\not{A}_i + \not{A}) q_i^\mu (\not{A}_j + \not{A}) \not{A}_j \gamma_\mu +$$

$$4(\not{A}_i + \not{A}) q_i^\mu (\not{A}_j + \not{A}) q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (37)$$

$$[d(\not{A}_i + \not{A}) \not{A}_i (\not{A}_j + \not{A}) \not{A}_j$$

$$+ 2(\not{A}_i + \not{A}) \not{A}_i (\not{A}_j + \not{A}) \not{A}_j$$

$$- 2(\not{A}_i + \not{A}) \not{A}_i (\not{A}_j + \not{A}) \not{A}_j$$

$$+ 4(\not{A}_i + \not{A}) (\not{A}_j + \not{A}) q_i^\mu q_{j\mu}]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (38)$$

$$[d(\not{A}_i + \not{A}) \not{A}_i (\not{A}_j + \not{A}) \not{A}_j + 4(\not{A}_i + \not{A}) (\not{A}_j + \not{A}) (q_i \cdot q_j)]$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} \quad (39)$$

$$[-d(\not{A}_i + \not{A}) (\not{A}_j + \not{A}) (q_i \cdot q_j) + 4(\not{A}_i + \not{A}) (\not{A}_j + \not{A}) (q_i \cdot q_j)]$$

$$M_1 M_2^\dagger = (4 - d) \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{y(1 - z)(p_i \cdot p_j)(p_i \cdot p_j)} (q_i \cdot q_j) [\not{A}_i + \not{A}] [\not{A}_j + \not{A}] \quad (40)$$

$$M_1 M_2^\dagger = (4 - d) \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{y(1 - z)(p_i \cdot p_j)(p_i \cdot p_j)} \quad (41)$$

$$z(1 - y)(p_i \cdot p_j) [\not{A}_i + y \not{A}_j] [(1 - z) \not{A}_i + (1 + yz - y) \not{A}_j - \sqrt{zy(1 - z)} \not{A}_n]$$

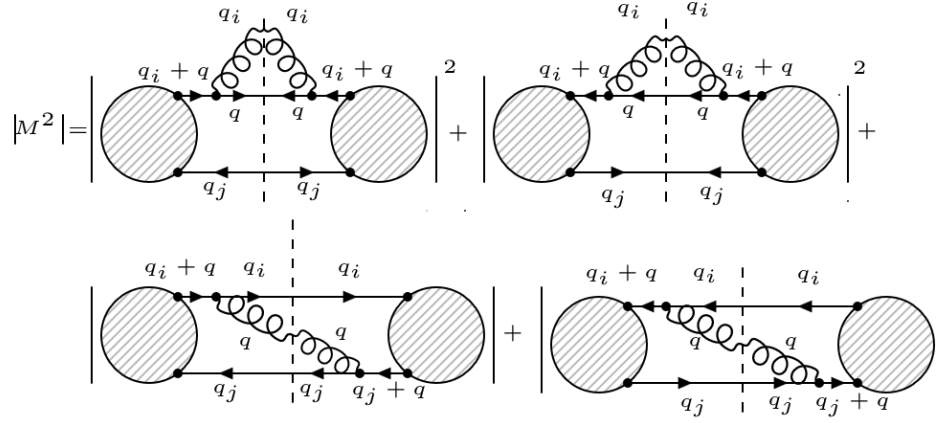
$$M_1 M_2^\dagger = (4-d) \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{y(1-z)(p_i \cdot p_j)} z(1-y)[(1+yz-y) \not{p}_i \not{p}_j + y(1-z) \not{p}_j \not{p}_i] \quad (42)$$

$$M_1 M_2^\dagger = (4-d) \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{y(1-z)(p_i \cdot p_j)} z(1-y)[(1+yz-y) \not{p}_i \not{p}_j - y(1-z) \not{p}_i \not{p}_j] \quad (43)$$

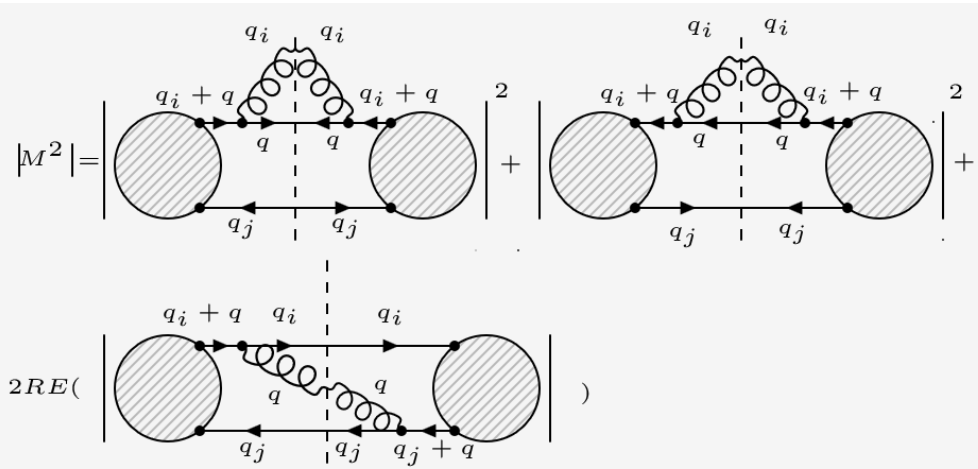
$$M_1 M_2^\dagger = (4-d) \frac{-g_s^2 [T^a]_o^l [T^d]_{f'}^n}{y(1-z)(p_i \cdot p_j)} z(1-y)(1+yz-y)[\not{p}_i][\not{p}_j] \quad (44)$$

0.2.4 $|M^2|$

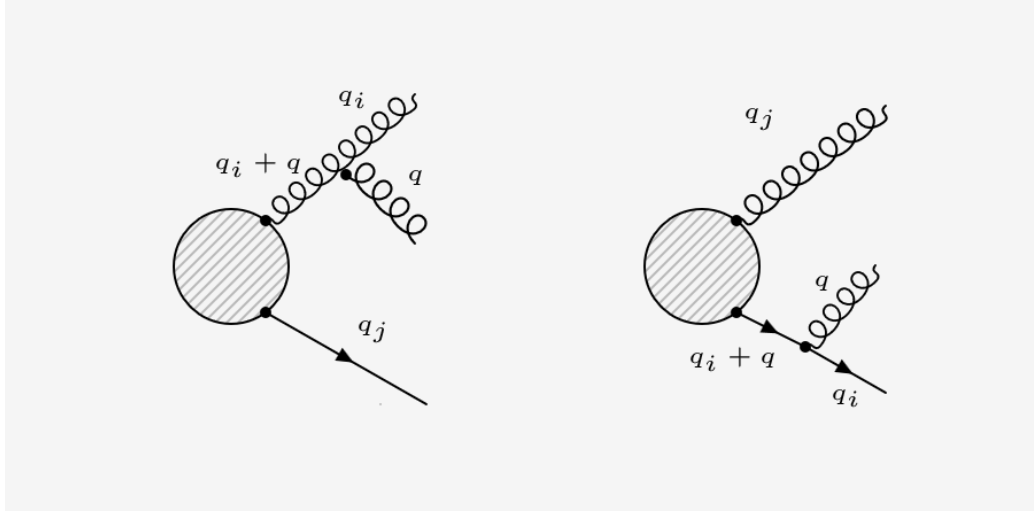
$$|M|^2 = |M_1|^2 + |M_2|^2 + M_1 M_2^\dagger + M_1^\dagger M_2 \quad (45)$$



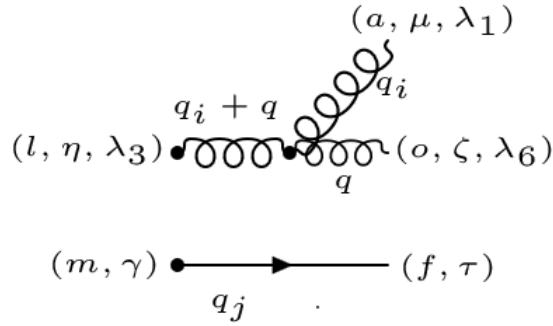
$$|M|^2 = |M_1|^2 + |M_2|^2 + 2RE(M_1 M_2^\dagger) \quad (46)$$



0.3 Quark/Gluon gluon emission kernel

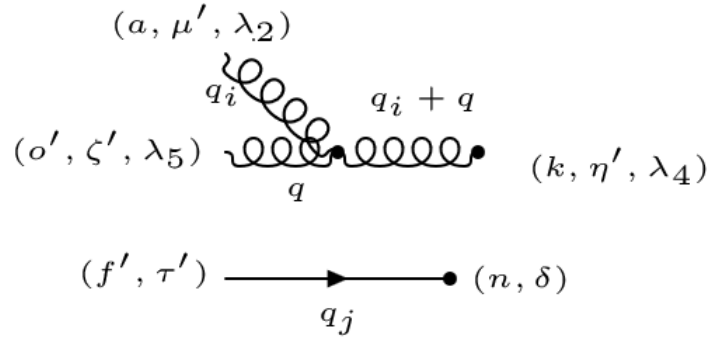


0.3.1 gg-q

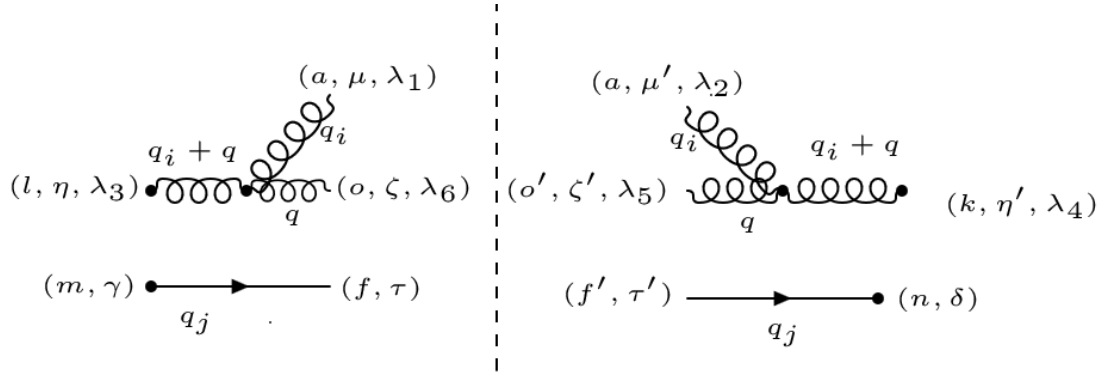


$$M_1 = \left[\frac{-ig_{\eta\mu}}{(q_i + q)^2} (-g_s f^{aol} (g^{\mu\zeta} (-q_i + q)^\eta + g^{\zeta\eta} (-q - (q_i + q))^\mu + g^{\eta\mu} (q_i + q + q_i)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q) \right] [\bar{u}_\tau(q_j)] \quad (47)$$

$$M_1 = \left[\frac{-ig_{\eta\mu}}{(q_i + q)^2} (-g_s f^{aol} (g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q) \right] [\bar{u}_\tau(q_j)] \quad (48)$$



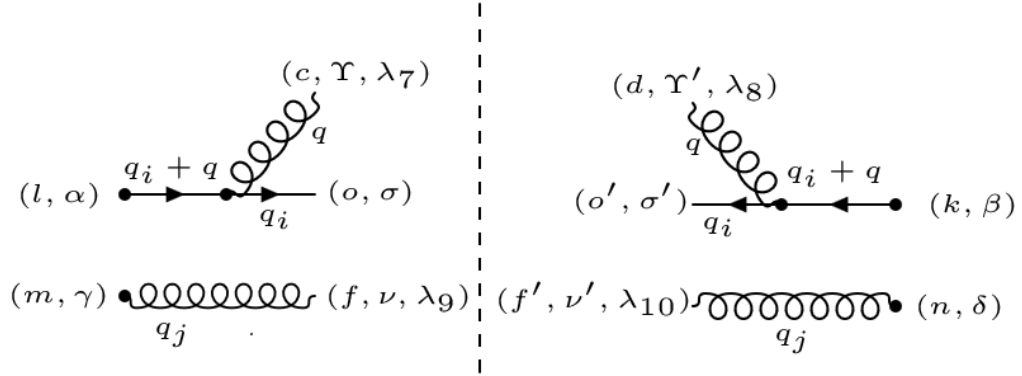
$$M_1^\dagger = \left[\frac{i g_{\eta' \mu'}}{(q_i + q)^2} (-g_s f^{a' o' k} (g^{\mu' \zeta'} (q - q_i)^{\eta'} - g^{\zeta' \eta'} (2q_i + q)^{\mu'} + g^{\eta' \mu'} (2q_i + q)^{\zeta'}) \right. \\ \left. \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_5}_{\zeta'}(q) \right] [u_{\tau'}(q_j)] \quad (49)$$



$$|M_1|^2 = \left[\frac{-i g_{\eta \mu}}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (-q_i + q)^\eta + g^{\zeta \eta} (-q - (q_i + q))^\mu + g^{\eta \mu} (q_i + q + q_i)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_6}_\zeta(q) \varepsilon^{\lambda_5}_{\zeta'}(q) \right. \\ \left. (-g_s f^{a' o' k} (g^{\mu' \zeta'} (q - q_i)^{\eta'} - g^{\zeta' \eta'} (2q_i + q)^{\mu'} + g^{\eta' \mu'} (2q_i + q)^{\zeta'}) \frac{i g_{\eta' \mu'}}{(q_i + q)^2} [\bar{u}_\tau(q_j) u_{\tau'}(q_j)] \right] \quad (50)$$

$$|M_1|^2 = \frac{g_s^2 f^{a o l} f^{a' o' k}}{(q_i + q)^2 (q_i + q)^2} \\ [g_{\eta \mu} (g^{\mu \zeta} (-q_i + q)^\eta + g^{\zeta \eta} (-q - (q_i + q))^\mu + g^{\eta \mu} (q_i + q + q_i)^\zeta) \\ g_{\mu \mu'} g_{\zeta \zeta'} (g^{\mu' \zeta'} (q - q_i)^{\eta'} - g^{\zeta' \eta'} (2q_i + q)^{\mu'} + g^{\eta' \mu'} (2q_i + q)^{\zeta'}) g_{\eta' \mu'}] [\not{A}_j] \quad (51)$$

0.3.2 qg-g



$$|M_2|^2 = M_2 \mathbf{M}_2^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^c]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_7} \Upsilon(q)] [\varepsilon^{\lambda_9}{}_\nu(q_j)]$$

$$[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^d]_{o'}^k) u_{\sigma'}(q_i) \varepsilon^{\lambda_8} \Upsilon'^*(q)] [\varepsilon^{\lambda_{10}}{}_{\nu'}(q_j)] \quad (52)$$

$$|M_2|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (-ig_s \gamma^\mu \times [T^c]_o^l) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_7} \Upsilon(q) \varepsilon^{\lambda_8} \Upsilon'^*(q)]$$

$$\times (ig_s \gamma^{\mu'} \times [T^d]_{o'}^k) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} [\varepsilon^{\lambda_{10}}{}_{\nu'}(q_j) \varepsilon^{\lambda_9}{}_\nu(q_j)] \quad (53)$$

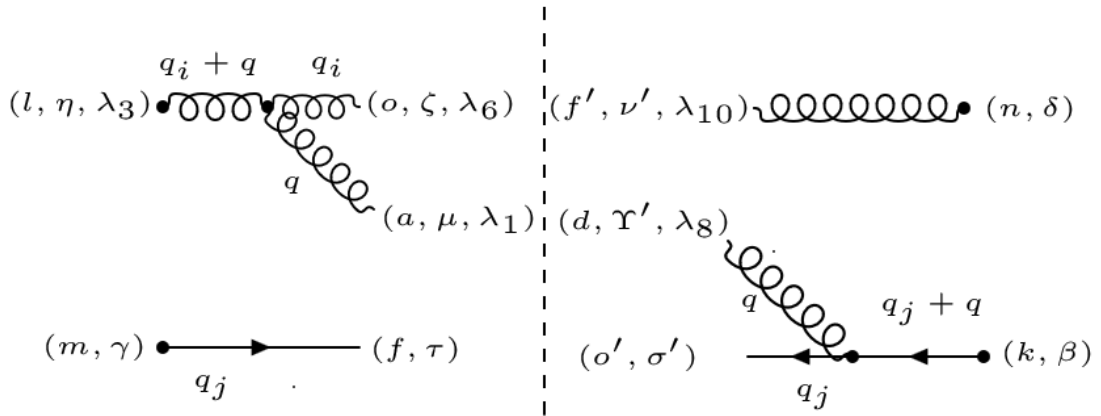
$$|M_2|^2 = \frac{g_s^2 [T^c]_o^l [T^d]_{o'}^k}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^\mu \not{q}_i (-g_{\Upsilon \Upsilon'})$$

$$\gamma^{\mu'} (\not{q}_i + \not{q})] [-g_{\nu' \nu}] \quad (54)$$

0.3.3 $M_1 M_2^\dagger$

$$M_1 \mathbf{M}_2^\dagger = [\frac{-ig_{\eta\mu}}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu\zeta} (q - q_i)^\eta - g^{\zeta\eta} (2q_i + q)^\mu + g^{\eta\mu} (2q_i + q)^\zeta)$$

$$\varepsilon^{\lambda_1}{}_\mu(q_i) \varepsilon^{\lambda_6}{}_\zeta(q)] [\bar{u}_\tau(q_j)] [\frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} (ig_s \gamma^{\mu'} \times [T^d]_{o'}^k) u_{\sigma'}(q_j) \varepsilon^{\lambda_8} \Upsilon'^*(q)] [\varepsilon^{\lambda_{10}}{}_{\nu'}(q_i)] \quad (55)$$



0.3.4 $|M^2|$

$$|M^2| = \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 4} \\ \text{Diagram 5} \end{array} \right|^2 + 2RE \left(\begin{array}{c} \text{Diagram 6} \\ \text{Diagram 7} \end{array} \right)$$

The diagram shows the calculation of the squared magnitude of the amplitude $|M^2|$. It consists of three terms: the first term is the squared magnitude of a sum of two diagrams (Diagram 1 and Diagram 2), the second term is the squared magnitude of a sum of two diagrams (Diagram 4 and Diagram 5), and the third term is the real part of the product of two diagrams (Diagram 6 and Diagram 7) multiplied by 2. The diagrams involve various particle lines (solid and wavy) and shaded circular regions representing interaction vertices. Momenta q_i , q_j , and q are labeled on the lines.

Inhaltsverzeichnis

0.1	parametrisation	1
0.2	Quark/Antiquark gluon emission kernel	2
0.2.1	$qg-\bar{q}$	2
0.2.2	$\bar{q}g-q$	6
0.2.3	$M_1M_2^\dagger$	8
0.2.4	$ M^2 $	11
0.3	Quark/Gluon gluon emission kernel	12
0.3.1	$gg-q$	12
0.3.2	$qg-g$	14
0.3.3	$M_1M_2^\dagger$	14
0.3.4	$ M^2 $	15
Literaturverzeichnis		2

