# Thesis

by

Tigran Saidnia

# Emission kernel of parton shower

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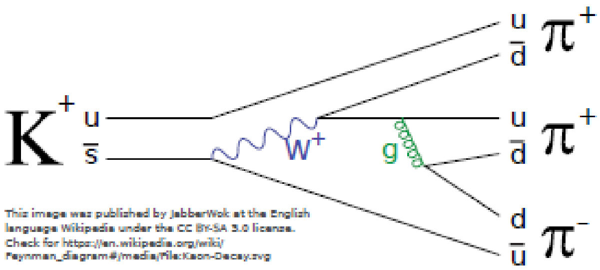



Karlsruhe institute for Technology (KIT)

Institute of theoretical physics

Referents: Dr. Stefan Gieseke

Dr. Simon Plätzle

Supervisor: Emma Simpson

statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

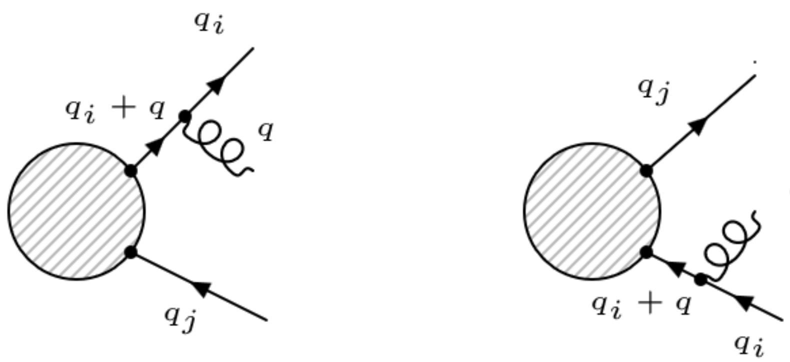
Karlsruhe, 4. Januar 2019

Tigran Saidnia

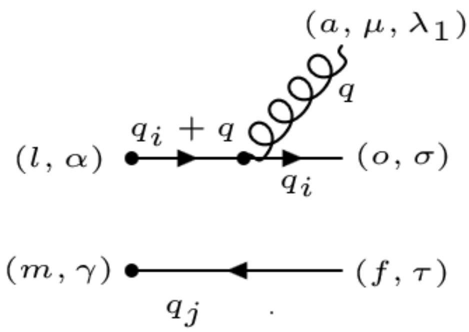
## 0.1 parametrisation

$$\left.\begin{aligned} q_i{}^{\mu} &= zp_i{}^{\mu} + y(1-z)p_j{}^{\mu} + \sqrt{zy(1-z)}m_{\perp} \\ q^{\mu} &= (1-z)p_i{}^{\mu} + yzp_j{}^{\mu} - \sqrt{zy(1-z)}m_{\perp} \\ q_j{}^{\mu} &= (1-y)p_j{}^{\mu} \\ y &= \frac{q_i q}{p_i p_j} \\ q_i + q &= p_i + yp_j \\ q_j + q &= (1-z)p_i{}^{\mu} + (1+yz-y)p_j{}^{\mu} - \sqrt{zy(1-z)}m_{\perp} \end{aligned}\right\} \text{parametrisation} \tag{1}$$

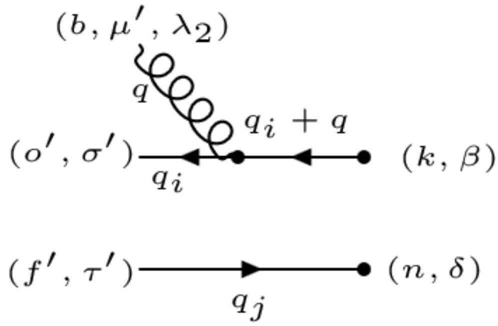
## 0.2 Quark/Antiquark gluon emission kernel



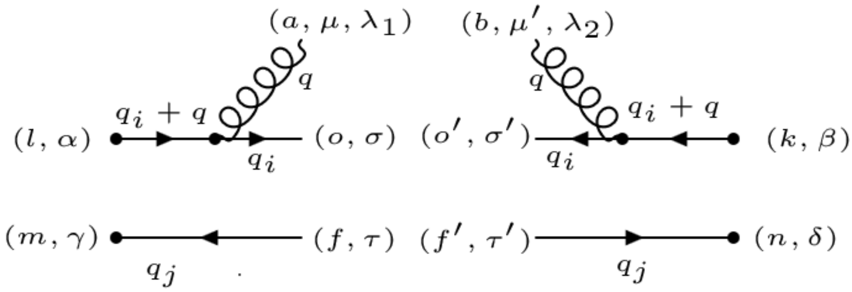

### 0.2.1 qg-$\bar{q}$




$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{2}$$




$$M_1{}^{\dagger} = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^{k})\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{3}$$




$$\begin{aligned}|M_1|^2 = M_1\, M_1{}^{\dagger} &= [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^{\mu} \times [T^a]_o{}^{l})\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\,\varepsilon^{\lambda_1}{}_{\mu}(q)][v_\tau(q_j)] \\ &[\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^{k})\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2}{}_{\mu'}{}^{*}(q)][\bar{v}_{\tau'}(q_j)]\end{aligned} \tag{4}$$

$$\begin{aligned}|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\,&(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^{k})\,\bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}^{*}_{\mu'}(q)\varepsilon^{\lambda_1}{}_{\mu}(q) \\ &\times(-ig_s\gamma^{\mu} \times [T^a]_o{}^{l})\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j)v_\tau(q_j)]\end{aligned} \tag{5}$$

and after sum over the lorenz index $(\sigma, \sigma')$ and $(\tau, \tau')$ and unsing the spin addition relation:

$$\begin{aligned}\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i) &= \not{q}_i, \\ \sum_{\tau,\tau'} \bar{v}_\tau(q_j)\, v_{\tau'}(q_j) &= \not{q}_j\end{aligned} \tag{6}$$

and sum over polarization index $(\lambda_1, \lambda_2)$ :

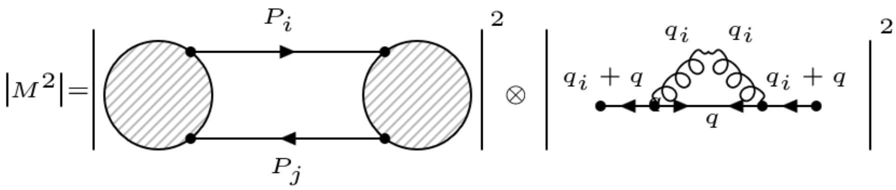
$$\sum_{\mu,\mu'} \varepsilon^{\lambda_2}{}^{*}_{\mu'}(q)\varepsilon^{\lambda_1}{}_{\mu}(q) = -g_{\mu\mu'} \tag{7}$$

$$|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,g_{\mu'\mu}\gamma^{\mu}\,(\not{q}_i+q)][\not{q}_j] \tag{8}$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'}\,(\not{q}_i+q)][\not{q}_j] \tag{9}$$

In other words we expect the tree level diagram from LO and a number: Which means:




*contribution from LO* *a complex number*

$$|M_1|^2 = \frac{-g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[\not{P}_i][\not{P}_j]\;\otimes\;(a\ complex\ number) \tag{10}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'} &= q_{i\sigma}\,\gamma^{\mu'}\gamma^{\sigma}\;\;\gamma_{\mu'} \\ &= q_{i\sigma}\,(\{\gamma^{\mu'},\gamma^{\sigma}\}\;-\gamma^{\sigma}\gamma^{\mu'})\gamma_{\mu'} \\ &= q_{i\sigma}\,2g^{\mu'\sigma}\;\gamma_{\mu'}\;-\;d\,\gamma^{\sigma} \\ &= (2-d)\;\;\not{q}_i \end{aligned} \tag{11}$$

$$|M_1|^2 = -(2-d)\;\frac{g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i+\not{q})\;\;\;\not{q}_i\;\;(\not{q}_i+q)][\not{q}_j] \tag{12}$$

$$|M_1|^2 = -(2-d)\;\frac{g_s^2[T^b]_{o'}{}^{k}\,[T^a]_o{}^{l}}{(q_i+q)^2(q_i+q)^2}[\not{q}_i\;\not{q}_i\;\not{q}_i+\;\not{q}_i\;\not{q}_i\;\not{q}+\;\not{q}\;\not{q}_i\;\not{q}_i+\;\not{q}\;\not{q}_i\;\not{q}][\not{q}_j] \tag{13}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i\;\;\not{q}_i &= q_i = m^2 \\ \not{q}\;\;\not{q} &= q = m^2 \\ \not{q}_j\;\;\not{q}_j &= q_j = m^2 \end{aligned} \tag{14}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \not{q}_i$ and $\not{q} \not{q}$ as well.

$$|M_1|^2 = -(2-d)\,\frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{(2q_iq)(2q_iq)}[\not{q}\, \not{q}_i\, \not{q}][\not{q}_j] \tag{15}$$

$$\begin{aligned} L = \not{q}\, \not{q}_i\, \not{q} = &\not{q}[q_{i\sigma}q_\mu\left(\{\gamma^\mu,\gamma^\sigma\} - \gamma^\sigma\gamma^\mu\right)] \\ &\not{q}[2q_i{}^\mu q_\mu - q_{i\sigma}q_\mu\gamma^\mu\gamma^\sigma \\ =&\not{q}(2q_iq) - q_\mu q_{i\sigma}q_\mu[\gamma^\mu\gamma^\mu\gamma^\sigma] \\ =&\not{q}(2q_iq) - q_\mu q_{i\sigma}q_\mu[\frac{\gamma^\mu\gamma^\mu}{2} + \frac{\gamma^\mu\gamma^\mu}{2}]\gamma^\sigma \\ =&\not{q}(2q_iq) - q_\mu q_{i\sigma}q_\mu[g^{\mu\mu}]\gamma^\sigma \\ =&\not{q}(2q_iq) - q_\mu q_{i\sigma}q^\mu\gamma^\sigma \\ =&\not{q}(2q_iq) - q^2\, \not{q}_i \\ =&\not{q} \end{aligned} \tag{16}$$

After inserting the last result of $L$ and simplify the term $(2q_iq)$ from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d)\,\frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{(2q_iq)}[\not{q}_i][\not{q}_j] \tag{17}$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$|M_1|^2 = (d-2)\,\frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{(2q_iq)}[(1-z)\, \not{p}_i + zy\, \not{p}_j - \sqrt{zy(1-z)}\, \not{m}_\perp]\, \not{m}_\perp][(1-y)\, \not{p}_j{}^\mu] \tag{18}$$

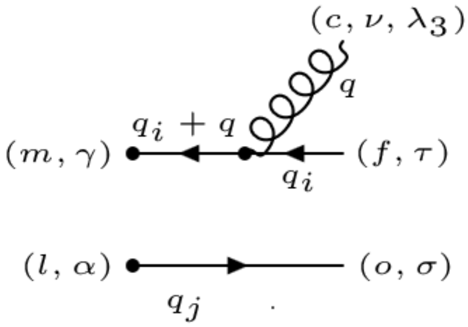
Multiplying the both sides

$$\begin{aligned} |M_1|^2 = (d-2)\,&\frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{(2q_iq)}[(1-z)(1-y)\, \not{p}_i\, \not{p}_j \\ &+ zy(1-y)\, \not{p}_j\, \not{p}_j + (1-y)\sqrt{zy(1-z)}\, \not{m}_\perp\, \not{p}_j] \end{aligned} \tag{19}$$

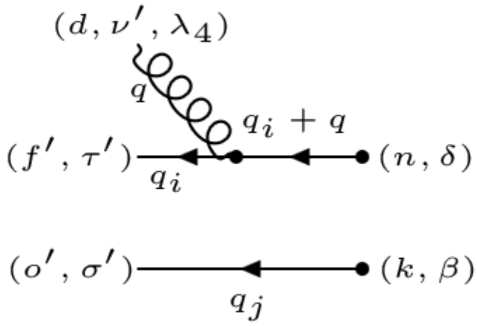
and under consideration of the fact that $p_i$ and $p_j$ are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\, \not{p}_i$ and $\not{p}_j\, \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the $p_i$ and $p_j$ are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$|M_1|^2 = (d-2)(1-z)(1-y)\,\frac{g_s^2 [T^b]_{o'}{}^k\, [T^a]_o{}^l}{(2q_iq)}[\not{p}_i][\not{p}_j] \tag{20}$$

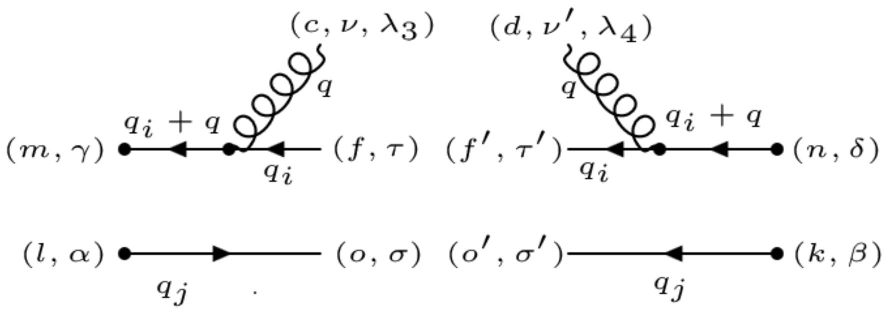
### 0.2.2 $\bar{q}$g-q




$$M_2 = [\frac{i(\not{q}_i + \not{q})}{(q_i + q)^2}(-ig_s\gamma^{\nu} \times [T^c]_f{}^m)\, v_\tau(q_i)\, \varepsilon^{\lambda_3}{}_{\nu}(q)]\, [u_\sigma(q_j)] \tag{21}$$




$$M_2^{\dagger} = [\bar{v}_{\tau'}(q_i)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_j)] \tag{22}$$



$$|M_2|^2 = M_2\, {M_2}^\dagger = [\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}(-ig_s\gamma^\nu \times [T^c]_f{}^m)\, v_\tau(q_i)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_j)]$$
$$[\bar{v}_{\tau'}(q_i)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_j)] \tag{23}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\gamma^\nu\, v_\tau(q_i)\bar{v}_{\tau'}(q_i)\, \varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_i + \not{q})]$$
$$[u_\sigma(q_j)]\, [\bar{u}_{\sigma'}(q_j)] \tag{24}$$

and after sum over the lorenz index $(\sigma, \sigma')$ and $(\tau, \tau')$ and unsing the spin addition relation:

$$\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_j)\, u_{\sigma'}(q_j) = \not{q}_j,$$
$$\sum_{\tau,\tau'} \bar{v}_\tau(q_i)\, v_{\tau'}(q_i) = \not{q}_i \tag{25}$$

and sum over polarization index $(\lambda_3, \lambda_4)$ :

$$\sum_{\nu,\nu'} \varepsilon^{\lambda_4}{}^*_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'} \tag{26}$$

$$|M_2|^2 = \frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\gamma^\nu\ \ \not{q}_i\, (-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_i + \not{q})]\, [\not{q}_j] \tag{27}$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(2qq_i)}[\not{q}]\, [\not{q}_j] \tag{28}$$

In this case we have to be careful because the quark is the emitter and we have to insert the right parametrisation for this, namely:

$$q_j{}^\mu = zp_i{}^\mu + y(1-z)p_j{}^\mu + \sqrt{zy(1-z)}m_\perp \tag{29}$$

To avoid such irritating problems we ought to compute this matrix element with the exact same initialized $i, j$ from $|M_1|^2$. But it's also possible to do that in reverse order as far as we know what we do. After parametrisation we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\, [T^c]_f{}^m\, [T^d]_{f'}{}^n}{(2qq_i)}[z\ \not{p}_i + y(1-z)\ \not{p}_j + \sqrt{zy(1-z)}\ \not{m}_\perp]$$
$$[z\ \not{p}_i + y(1-z)\ \not{p}_j + \sqrt{zy(1-z)}\ \not{m}_\perp] \tag{30}$$

Multiplying the both side gives:

$$|M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_i)}[(1-z)\,\not{p}_i + zy\,\not{p}_j - \sqrt{zy(1-z)}\,\not{m}_\perp]$$
$$[z\,\not{p}_i + y(1-z)\,\not{p}_j + \sqrt{zy(1-z)}\,\not{m}_\perp] \tag{31}$$

$$\implies |M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_i)}[(1-z)z\,\not{p}_i\,\not{p}_i + y(1-z)^2\,\not{p}_i\,\not{p}_j$$
$$+(1-z)\sqrt{zy(1-z)}\,\not{p}_i\,\not{m}_\perp + z^2y\,\not{p}_j\,\not{p}_i + zy^2(1-y)\,\not{p}_j\,\not{p}_j + zy\sqrt{zy(1-z)}\,\not{p}_j\,\not{m}_\perp$$
$$-z\sqrt{zy(1-z)}\,\not{m}_\perp\,\not{p}_i - y(1-z)\sqrt{zy(1-z)}\,\not{m}_\perp\,\not{p}_j - zy(1-z)\,\not{m}_\perp\,\not{m}_\perp] \tag{32}$$

$$\implies |M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_i)}[y(1-z)^2\,\not{p}_i\,\not{p}_j + z^2y\,\not{p}_j\,\not{p}_i] \tag{33}$$

we're changing the position of two matrices to be able to sum the coefficients

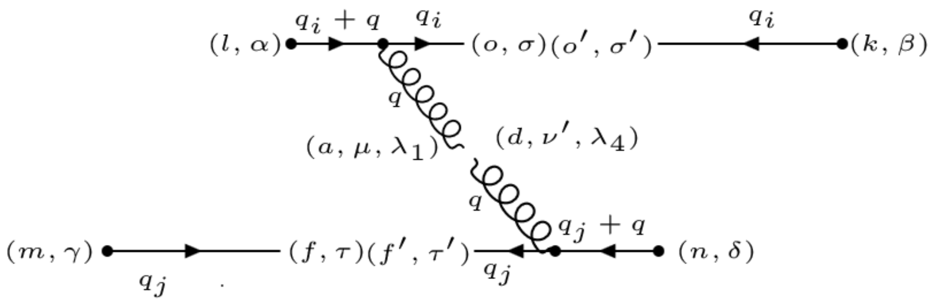
$$\not{p}_j\,\not{p}_i = -\,\not{p}_i\,\not{p}_j \tag{34}$$

$$\implies |M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_i)}[y(1-z)^2\,\not{p}_i\,\not{p}_j - z^2y\,\not{p}_i\,\not{p}_j] \tag{35}$$

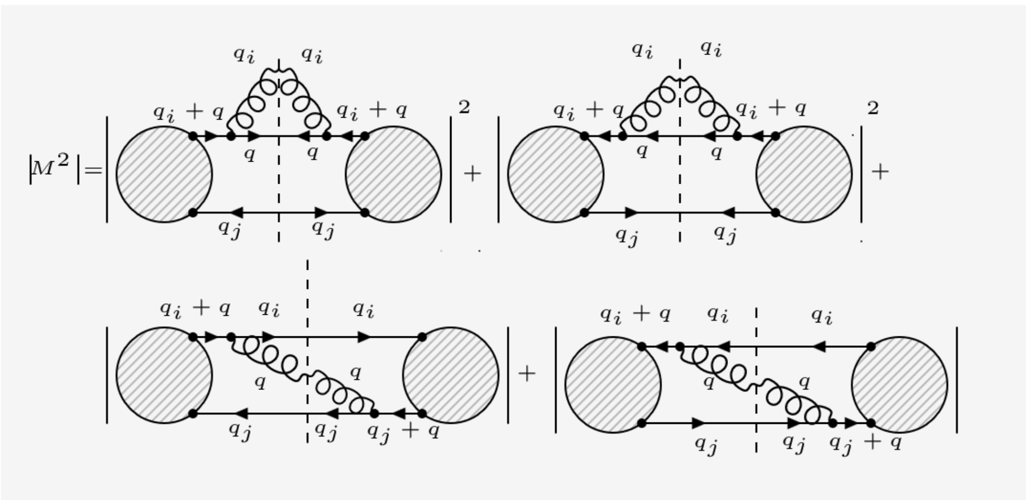
finally:

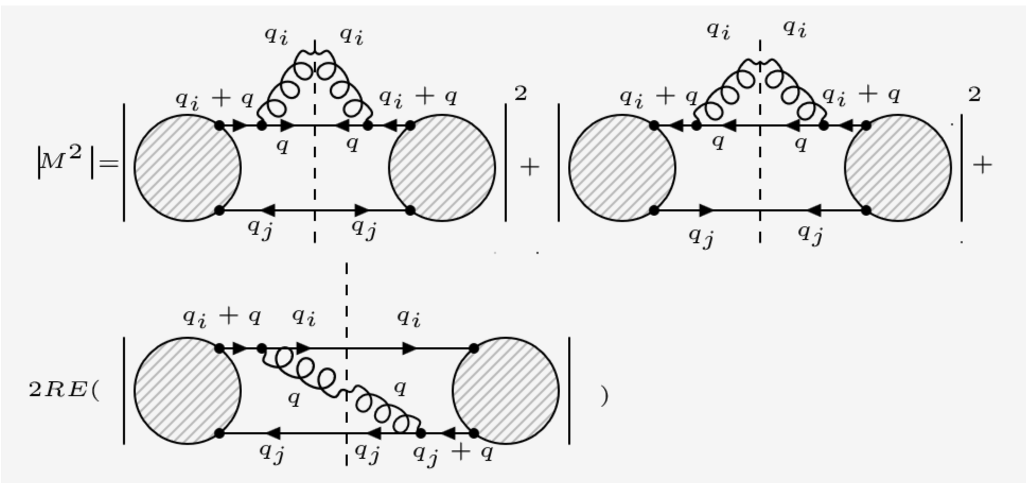
$$\implies |M_2|^2 = (d-2)y(1-2z)\frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(2qq_i)}[\,\not{p}_i\,\not{p}_j] \tag{36}$$

### 0.2.3 $M_1M_2^\dagger$

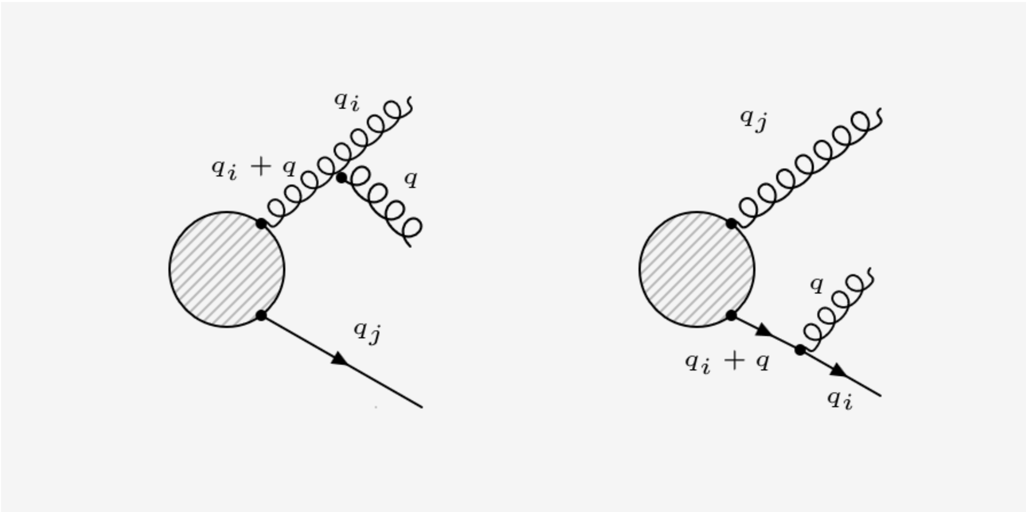



### 0.2.4 $|M^2|$



## 0.3 Quark/Gluon gluon emission kernel



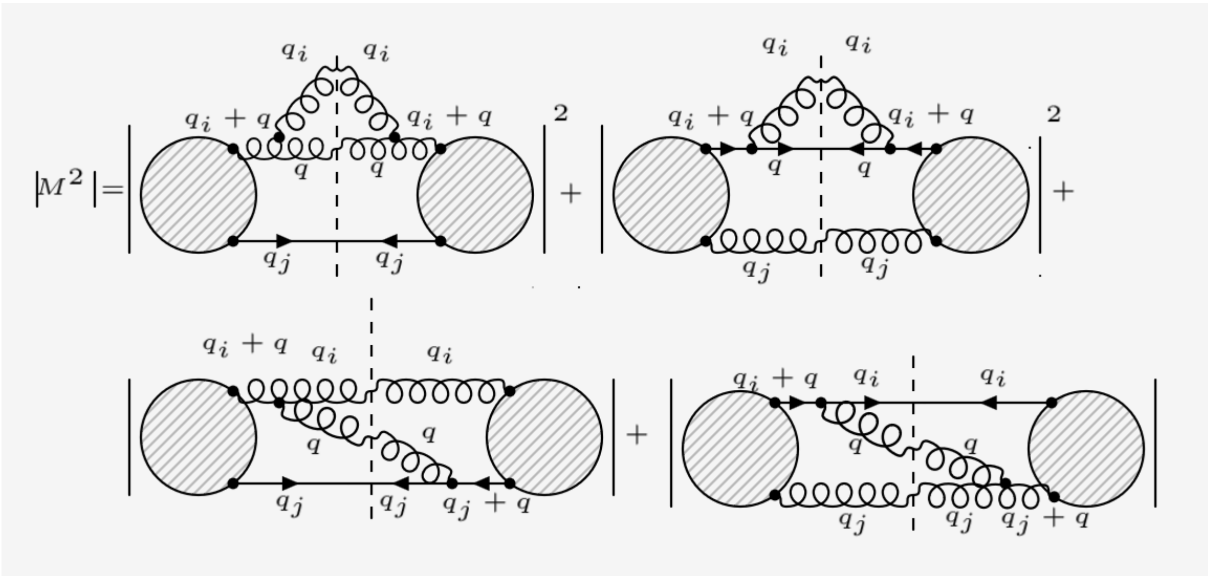




**Abbildung 1:** Die Landkarte.

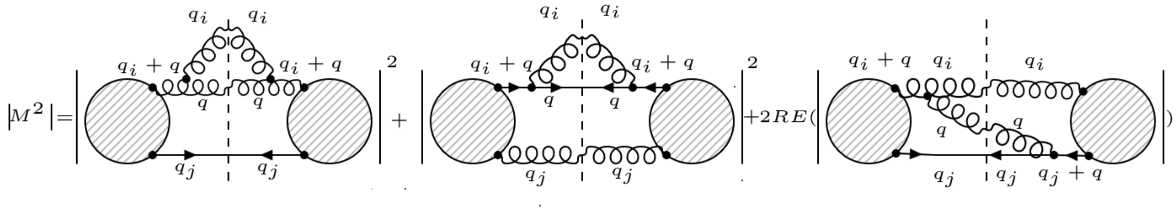



**Abbildung 2:** Die Landkarte.

# Inhaltsverzeichnis