New mapping

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1 New mapping

Defined for the general m emission case: All 'hard' momenta are taken to be massless, $p_k^2 = q_k^2 = 0$, $k = 1, ..., n$ as are the emission momenta $k_l^2 = 0$, $l = 1, ..., m$. We parametrize the splitting momenta as:

$$\begin{aligned} k_l^\mu &= \alpha_l \ \alpha \Lambda^\mu{}_\nu p_i^\nu + y \ \beta_l \ n^\mu + \sqrt{y\alpha_l\beta_l} n_{\perp,l}^\mu \qquad l = 1, ..., m \\ q_i^\mu &= \left(1 - \sum_{l=1}^m \alpha_l\right) \alpha \Lambda^\mu{}_\nu p_i^\nu + y \left(1 - \sum_{l=1}^m \beta_l\right) n^\mu - \sum_{l=1}^m \sqrt{y\alpha_l\beta_l} n_{\perp,l}^\mu \ , \\ q_k^\mu &= \alpha \Lambda^\mu{}_\nu p_k^\nu \qquad k = 1, ..., n \qquad k \neq i \ . \end{aligned} \tag{1}$$

where $n_{\perp,l}^2 = -2\alpha\Lambda^\mu{}_\nu p_i^\nu n_\mu$ and $n_{\perp,l}^\mu \Lambda^\mu{}_\nu p_{i\nu} = n_{\perp,l} \cdot n = 0$. Which are conditions required to satisfy the masslessness of the emission momenta.

The parameter y is related to the virtuality of the splitting parton,

$$q_i^\mu + \sum_{l=1}^m k_l^\mu = \alpha \Lambda^\mu{}_\nu p_i^\nu + y \ n^\mu \ . \tag{2}$$

To maintain momentum conservation the Lorentz transformation will be used to absorb the recoil.

1.1 Lorentz transformation

The transformation we need is:

$$\Lambda^\mu{}_\nu Q^\nu = \frac{Q^\mu - y n^\mu}{\alpha} \tag{3}$$

Where Q is the total momentum, ($Q = \sum_k p_k + \sum_l k_l$). In the collinear limit of $y \to 0$, $\alpha \to 1$ this transformation reduces to δ^μ_ν. The transformation times the factor α is useful to define, where $\alpha = \sqrt{1-y}$ has been used:

$$\begin{aligned} \alpha \Lambda^\mu{}_\nu =& p_i^\mu p_{i\nu} \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu \frac{y(1+\sqrt{1-y})}{2p_i \cdot Q(1 + \sqrt{1-y} - \frac{y}{2})} \\ &+ Q^\mu p_{i\mu} \frac{(y^2 - y - y\sqrt{1-y})}{2p_i \cdot Q(1 + \sqrt{1-y} - \frac{y}{2})} + \eta^\mu_\nu \sqrt{1-y} \ . \end{aligned} \tag{4}$$

1.2 Single emission case

As a first example we consider the single emission case where $l = 1$ i.e. there is only one emission k_1. Since we previously defined $n^\mu = Q^\mu - \frac{Q^2}{2p_i \cdot Q} p_i^\mu$ and for the single emission case $\alpha_1 = 1 - \beta_1$ this allows the mapping to be simplified. The action of $\Lambda^\mu{}_\nu$ on p_i^ν yields $\frac{1}{\alpha} p_i^\mu$ and hence a further simplification, $\alpha \Lambda^\mu{}_\nu p_i^\nu = p_i^\mu$.

$$
\begin{aligned}
k_1^\mu &= \left(\alpha_1 - y\beta_1\left(\frac{Q^2}{2p_i \cdot Q}\right)\right) p_i^\mu + y\beta_1\ Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \ , \\
&= \zeta_1 p_i^\mu + \lambda_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \ , \\
q_i^\mu &= \left(1 - \alpha_1 - \frac{yQ^2}{2p_i \cdot Q}(1-\beta_1)\right) p_i^\mu + y\,(1-\beta_1)\, Q^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \ , \\
&= \zeta_q p_i^\mu + \lambda_q Q^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \ , \\
q_k^\mu &= \alpha\Lambda^\mu{}_\nu p_k^\nu = A_1 p_i^\mu + A_2 Q^\mu + \sqrt{1-y}\ p_k^\mu \ .
\end{aligned} \tag{5}
$$

To investigate the mapping it is useful to determine the dot products between these vectors, where $n_{\perp,l}^2 = -2\alpha\Lambda^\mu{}_\nu p_i^\nu n_\mu$ has been used to give $n_{\perp,1}^2 = -2p_i \cdot p_k$ shown below(results need checking):

$$
\begin{aligned}
k_1 \cdot q_i &= (\alpha_1 + \beta_1) y p_i \cdot Q \ , \\
k_1 \cdot q_k &= (\zeta_1 A_2 + \lambda_1 A_1) p_i \cdot Q + (\zeta_1 \sqrt{1-y}) p_i \cdot p_k + (\lambda_1 A_2) Q^2 \\
&\quad + (\lambda_1 \sqrt{1-y}) p_k \cdot Q + (\sqrt{\alpha_1 \lambda_1 (1-y)}) n_{\perp,1} \cdot p_k \ , \\
&= ((1-y)\alpha_1 - y\beta_1 \frac{Q^2}{2p_i \cdot Q}) p_i \cdot p_k + y\beta_1 p_k \cdot Q + \sqrt{\alpha_1\beta_1 y(1-y)} p_k \cdot n_{\perp,1} \\
q_i \cdot q_k &= (\zeta_q A_2 + \lambda_q A_1) p_i \cdot Q + (\zeta_q \sqrt{1-y}) p_i \cdot p_k + (\lambda_q A_2) Q^2 \\
&\quad + (\lambda_q \sqrt{1-y}) p_k \cdot Q - (\sqrt{y\alpha_1\beta_1(1-y)}) n_{\perp,1} \cdot p_k \ .
\end{aligned} \tag{6}
$$