

# THESIS

BY

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## Emission kernel of parton shower

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statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, February 12, 2019

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Tigran Saidnia



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## 0.1 Old parametrisation

$$\left. \begin{aligned}
 q_i^\mu &= zp_i^\mu + y(1-z)p_j^\mu + \sqrt{zy(1-z)}m_\perp \\
 q^\mu &= (1-z)p_i^\mu + yzp_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_j^\mu &= (1-y)p_j^\mu \\
 y &= \frac{q_i \cdot q}{p_i \cdot p_j} \\
 q_i + q &= p_i + yp_j \\
 q_j + q &= (1-z)p_i^\mu + (1+yz-y)p_j^\mu - \sqrt{zy(1-z)}m_\perp \\
 q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
 q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
 q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
 \end{aligned} \right\} \text{parametrisation} \quad (1)$$

## 0.2 new kinematic

$$\begin{aligned}
 k_l^\mu &= \alpha_l \Lambda^\mu{}_\nu p_i^\nu + y\beta n^\mu + \sqrt{y\alpha_l\beta_l}n_{\perp,l}^\mu \quad l = 1, \dots, m \\
 q_i^\mu &= (1 - \sum_{l=1}^m \alpha_l) \Lambda^\mu{}_\nu p_i^\nu + y(1 - \sum_{l=1}^m \beta_l) n^\mu - \sqrt{y\alpha_l\beta_l}n_{\perp,l}^\mu \\
 q_k^\mu &= \Lambda^\mu{}_\nu p_k^\nu \quad k = 1, \dots, n \quad k \neq i
 \end{aligned} \quad (2)$$

### 0.2.1 useful relations

$$\begin{aligned}
 q_i^2 &= p_i^2 = q_k^2 = k_l^2 = p_j^2 = p_k^2 = n^2 = 0 \quad \text{All hard momenta are on-shell} \\
 Q^\mu &= q_i^\mu + \sum_{l=1}^m k_l^\mu + \sum_{k=1}^m q_k^\mu = p_i^\mu + \sum_{k=1}^m p_k^\mu \quad \text{total momentum} \\
 n^\mu &= Q^\mu - \frac{Q^2}{2p_i \cdot Q} p_i^\mu \quad n^\mu \text{ is the recoil} \\
 q_i^\mu + \sum_{l=1}^m k_l^\mu &= \Lambda^\mu{}_\nu p_i^\nu + yn^\mu \\
 \Lambda^\mu{}_\nu Q^\nu &= Q^\mu - yn^\mu \\
 n_{\perp,l}^\mu \Lambda^\mu{}_\nu p_i^\nu &= n_{\perp,l} \cdot n = n_{\perp,l} \cdot Q = 0 \\
 n_{\perp,l}^\mu \cdot p_k &\neq 0 \\
 n_{\perp,l}^2 &= -2\alpha\Lambda^\mu{}_\nu p_i^\nu n_\mu \\
 n_{\perp,1}^2 &= -2p_i \cdot Q \\
 \alpha_1 &= 1 - \beta_1 \\
 \alpha &= \sqrt{1-y}
 \end{aligned} \quad (3)$$

Lorenz trafo

$$\alpha\Lambda^\mu{}_\nu = p_i^\mu p_{i\nu} \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu \quad (4)$$

$$\hat{p}_i^\mu = \alpha\Lambda^\mu{}_\nu p_i^\nu = p_i^\mu p_{i\nu} p_i^\nu \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu p_i^\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} p_i^\nu \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu p_i^\nu \quad (5)$$

$$\begin{aligned} \hat{p}_i^\mu &= p_i^\mu (Q \cdot p_i) \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} p_i^\mu \\ &= p_i^\mu \left[ \frac{y(1 + \sqrt{1-y})}{(2 + 2\sqrt{1-y} - y)} + \sqrt{1-y} \right] = p_i^\mu \end{aligned} \quad (6)$$

$$\boxed{\hat{p}_i^\mu = \alpha\Lambda^\mu{}_\nu p_i^\nu = p_i^\mu} \quad (7)$$

$$\hat{p}_k^\mu = \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu p_{i\nu} p_k^\nu \frac{-y^2 Q^2}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + p_i^\mu Q_\nu p_k^\nu \frac{y(1 + \sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + Q^\mu p_{i\nu} p_k^\nu \frac{(y^2 - y - y\sqrt{1-y})}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \eta^\mu{}_\nu p_k^\nu \quad (8)$$

$$\begin{aligned} \hat{p}_k^\mu &= \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &\quad + Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} p_k^\mu \end{aligned} \quad (9)$$

$$\begin{aligned} \hat{p}_k^\mu &= \alpha\Lambda^\mu{}_\nu p_k^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &\quad + Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} p_k^\mu \end{aligned}$$

with

$$\begin{aligned} A_1 &\equiv \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \\ A_2 &\equiv \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \end{aligned} \quad (10)$$



$$\boxed{\hat{p}_k^\mu = A_1 p_i^\mu + A_2 Q^\mu + \sqrt{1-y} p_k^\mu} \quad (11)$$

$$\begin{aligned} \hat{Q}^\mu &= \alpha \Lambda^\mu{}_\nu Q^\nu = p_i^\mu \left[ \frac{-y^2 Q^2 (p_i \cdot Q)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y}) Q^2}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \\ &+ Q^\mu \left[ \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot Q)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] + \sqrt{1-y} Q^\mu \end{aligned}$$

with

$$\begin{aligned} S_1 &\equiv \frac{Q^2}{2p_i \cdot Q} \left[ \frac{-y^2}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})}{(1 + \sqrt{1-y} - \frac{y}{2})} \right] = \frac{Q^2}{2p_i \cdot Q} y \\ S_2 &\equiv \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} = 1 - y \end{aligned} \quad (12)$$

$$\boxed{\hat{Q}^\mu = \frac{Q^2}{2p_i \cdot Q} y p_i^\mu + (1 - y) Q^\mu} \quad (13)$$

### 0.3 Single emission part

$$\begin{aligned} k_1^\mu &= (\alpha_1 - y\beta_1 (\frac{Q^2}{2p_i \cdot Q})) p_i^\mu + y\beta_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \\ q_i^\mu &= (\beta_1 - \alpha_1 y (\frac{Q^2}{2p_i \cdot Q})) p_i^\mu + y\alpha_1 n^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,l}^\mu \\ q_k^\mu &= \alpha \Lambda^\mu{}_\nu p_k^\nu \quad k = 1, \dots, n \quad k \neq i \end{aligned} \quad (14)$$

$$\begin{aligned} k_1^\mu &= \zeta_1 p_i^\mu + \lambda_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\mu \\ q_i^\mu &= \zeta_q p_i^\mu + \lambda_q Q^\mu - \sqrt{y\alpha_1\beta_1} n_{\perp,l}^\mu \\ q_k^\mu &= A_1 p_i^\mu + A_2 Q^\mu + \sqrt{1-y} p_{k\perp,l}^\mu \end{aligned}$$

$$\begin{aligned}
\zeta_1 \zeta_1 &= (\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_1 \lambda_1 &= (y\alpha_1\beta_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_1 \zeta_q &= (\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_1 \lambda_q &= (y\alpha_1^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_q \zeta_q &= (\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_q \lambda_1 &= (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) \\
\zeta_q \zeta_1 &= (\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2) \\
\zeta_q \lambda_q &= (y\beta_1\alpha_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_1 \lambda_1 &= y^2\beta_1^2 \\
\lambda_1 \zeta_q &= (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_1 \lambda_q &= y^2\beta_1\alpha_1 \\
\lambda_1 \zeta_1 &= (y\beta_1\alpha_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_q \lambda_q &= y^2\alpha_1^2 \\
\lambda_q \lambda_1 &= y^2\alpha_1\beta_1 \\
\lambda_q \zeta_q &= (y\alpha_1\beta_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})) \\
\lambda_q \zeta_1 &= (y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))
\end{aligned} \tag{15}$$

## 0.4 Common scalar products

$$\begin{aligned}
k_1 \cdot q_i &= (\zeta_1 \lambda_q + \lambda_1 \zeta_q) p_i \cdot Q + \lambda_1 \lambda_q Q^2 - y\alpha_1\beta_1 n_{\perp,1}^2 \\
&= [(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))y\alpha_1 + y\beta_1(1 - \alpha_1 - \alpha_1(\frac{Q^2}{2p_i \cdot Q}))] p_i \cdot Q \\
&\quad y^2\beta_1\alpha_1 Q^2 + 2y\alpha_1\beta_1 p_i Q
\end{aligned} \tag{16}$$

$$\begin{aligned}
\Rightarrow k_1 \cdot q_i &= [y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y\beta_1 - y\alpha_1\beta_1 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})] p_i \cdot Q \\
&\quad y^2\beta_1\alpha_1 Q^2 + 2y\alpha_1\beta_1 p_i Q
\end{aligned}$$

$$\boxed{k_1 \cdot q_i = y(\alpha_1 + \beta_1) p_i \cdot Q} \tag{17}$$

$$\begin{aligned}
k_1 \cdot q_k &= (\zeta_1 A_2 + \lambda_1 A_1) p_i \cdot Q + \zeta_1 \sqrt{1-y} p_i \cdot p_k + \lambda_1 A_2 Q^2 + \lambda_1 \sqrt{1-y} Q \cdot p_k \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1} \\
&= \left\{ \left[ (\alpha_1 - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right)) \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \right. \\
&+ y\beta_1 \left[ \frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2 (1 + \sqrt{1-y} - \frac{y}{2})} + \frac{y(1 + \sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} \right] \} p_i \cdot Q \\
&+ (\alpha_1 - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right)) \sqrt{1-y} p_i \cdot p_k + y\beta_1 \frac{(y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} Q^2 \\
&+ y\beta_1 \sqrt{1-y} Q \cdot p_k + \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{18}$$

$$\begin{aligned}
k_1 \cdot q_k &= \alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) \\
&+ y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) + y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} Q \cdot p_k \\
&+ \alpha_1 \sqrt{1-y} p_i \cdot p_k - y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \sqrt{1-y} p_i \cdot p_k \\
&+ y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} (p_i \cdot p_k) + y\beta_1 \sqrt{1-y} (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{19}$$

$$\begin{aligned}
k_1 \cdot q_k &= \left[ \alpha_1 \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1 + \sqrt{1-y} - \frac{y}{2})} + \alpha_1 \sqrt{1-y} \right. \\
&- y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) \sqrt{1-y} \left. \right] p_i \cdot p_k + \left[ y\beta_1 \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + y\beta_1 \sqrt{1-y} \right] (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{20}$$

$$\begin{aligned}
k_1 \cdot q_k &= \left\{ \alpha_1 \left[ \frac{(y^2 - y - y\sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \right] \right. \\
&+ y\beta_1 \left( \frac{Q^2}{p_i \cdot Q} \right) \left[ \frac{-y^2}{4(1 + \sqrt{1-y} - \frac{y}{2})} - \sqrt{1-y} \right] \} p_i \cdot p_k \\
&+ y\beta_1 \left[ \frac{y(1 + \sqrt{1-y})}{2(1 + \sqrt{1-y} - \frac{y}{2})} + \sqrt{1-y} \right] (Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{21}$$

$$\boxed{k_1 \cdot q_k = [\alpha_1(1-y) + y\beta_1 \left( \frac{Q^2}{2p_i \cdot Q} \right) (y-2)] p_i \cdot p_k + y\beta_1 Q \cdot p_k + \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}} \tag{22}$$



## 0.5 Altarelli-Parisi splitting functions

$$\left. \begin{aligned} \langle \hat{P}_{qq} \rangle &= C_F \left[ \frac{1+z^2}{1-z} - \varepsilon(1-z) \right] \\ \langle \hat{P}_{gq} \rangle &= T_R \left[ 1 - \frac{2z(1-z)}{1-\varepsilon} \right] \\ \langle \hat{P}_{qg} \rangle &= C_F \left[ \frac{1+(1-z)^2}{z} - \varepsilon z \right] \\ \langle \hat{P}_{gg} \rangle &= 2C_A \left[ \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \end{aligned} \right\} \text{splitting functions} \quad (23)$$

## 0.6 Colour factor calculation

fundamental representation in  $SU(2)$  and  $SU(3)$

$$\begin{aligned} T^a &= \tau^a \equiv \frac{\sigma^a}{2} \quad \text{with Pauli matrices } \sigma^a \\ T^a &= \vartheta^a \equiv \frac{\lambda^a}{2} \quad \text{with Gell - Mann matrices } \lambda^a \end{aligned} \quad (24)$$

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix} \\ \lambda^5 &= \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{aligned} \quad (25)$$

As we can see,  $\lambda^3$  and  $\lambda^8$  are diagonal. These generators satisfy:

$$[T^a, T^b] = i\epsilon^{abc}T^c \quad (26)$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \quad (27)$$

The main relation we will use later for  $SU(N)$ :

$$\text{tr}(T^a T^b) = T_{ij}^a T_{ji}^b = T_F \delta^{ab} \quad (28)$$

$$\sum_a (T^a T^a) = C_F \delta^{ij} \quad (29)$$

$$f^{acd} f^{bcd} = C_A \delta^{ab} \quad (30)$$

With  $T_F = \frac{1}{2}$ ,  $C_A = N$  and  $C_F = \frac{N^2-1}{2N}$ .

$$f^{abc} = -2i \text{tr}(T^a [T^b, T^c]) \quad (31)$$

$$d^{abc} = 2 \text{tr}(T^a T^b, T^c) \quad (32)$$

$$T^a T^b = \frac{1}{2} \left( \frac{1}{N} \delta_{ab} + (d^{abc} + i f^{abc}) T^c \right) \quad (33)$$

$$\text{tr}(T^a T^b T^c) = \frac{1}{4} (d^{abc} + i f^{abc}) \quad (34)$$

$$\text{tr}(T^a T^b T^a T^b) = \frac{-1}{4N} \delta_{ab} \quad (35)$$

$$f^{acd} f^{bcd} = N \delta^{ab} \quad (36)$$

$$f^{acd} d^{bcd} = 0 \quad (37)$$

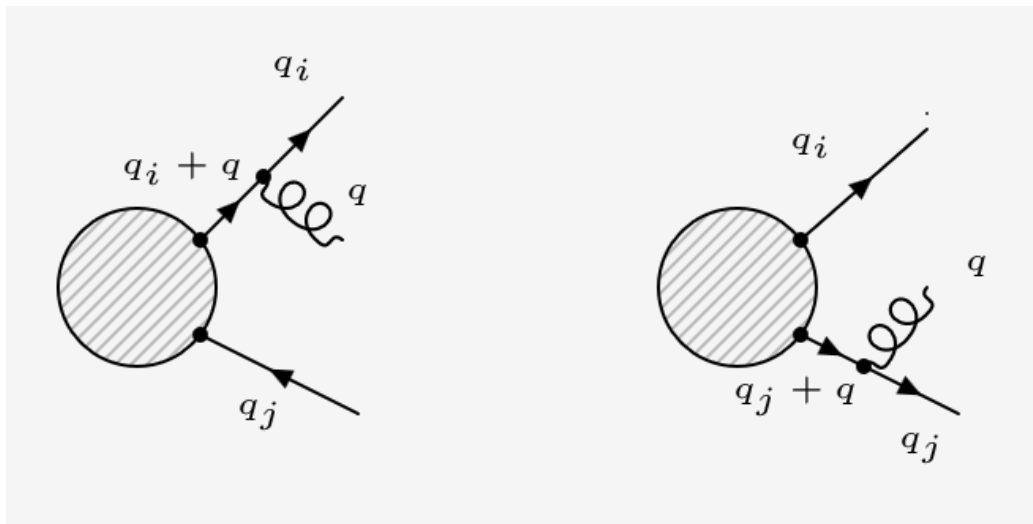
$$f^{ade} f^{bef} f^{cfd} = \frac{N}{2} f^{abc} \quad (38)$$

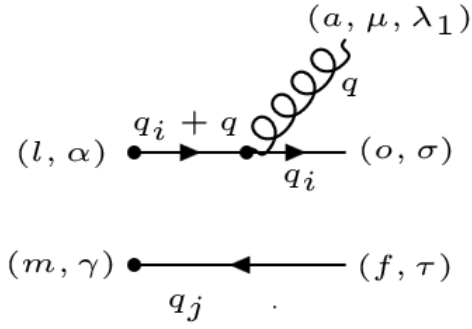
Fierz identity:

$$\sum_a T_{ij}^a T_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl}) \quad (39)$$

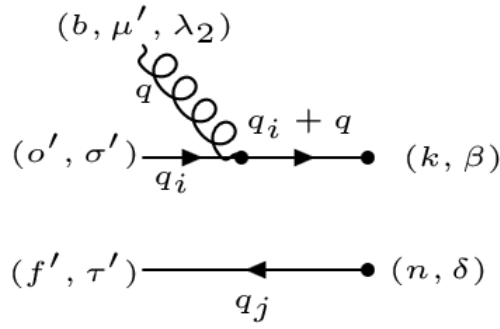
# Chapter 1

## Quark antiquark gluon emission kernel

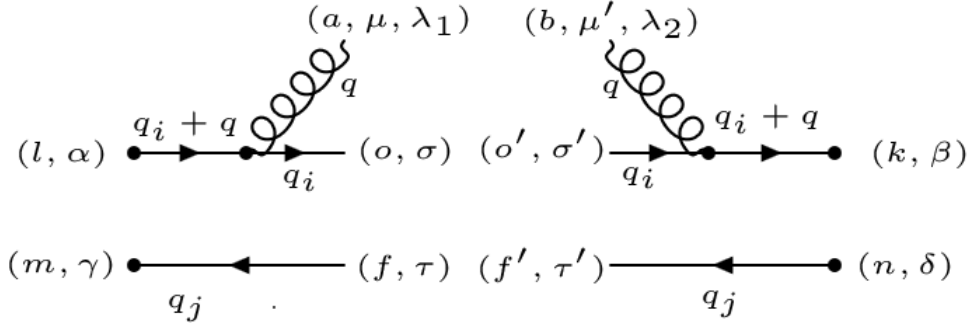


1.1  $qg\text{-}\bar{q}$ 

$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_{o^l}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)] \quad (1.1)$$



$$M_1^\dagger = [\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s\gamma^{\mu'} \times [T^b]_{o'^k}^{o'}) u_{\sigma'}(q_i) \varepsilon^{\lambda_2}_{\mu'}(q)] [\bar{v}_{\tau'}(q_j)] \quad (1.2)$$



$$|M_1|^2 = M_1 M_1^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_{o'}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q) [v_\tau(q_j)]$$

$$[\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o'}^k) u_{\sigma'}(q_i) \varepsilon^{\lambda_2*}_{\mu'}(q) [\bar{v}_{\tau'}(q_j)]] \quad (1.3)$$

$$|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i + q)^2} (ig_s \gamma^{\mu'} \times [T^b]_{o'}^k) \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q)$$

$$\times (-ig_s \gamma^\mu \times [T^a]_{o'}^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} [\bar{v}_{\tau'}(q_j) v_\tau(q_j)] \quad (1.4)$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i \delta^{\sigma\sigma'},$$

$$\sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \delta^{\tau\tau'} \quad (1.5)$$

and sum over polarization index  $(\lambda_1, \lambda_2)$  :

$$\sum_{\mu, \mu'} \varepsilon^{\lambda_2*}_{\mu'}(q) \varepsilon^{\lambda_1}_\mu(q) = -g_{\mu\mu'} \delta^{ab} \quad (1.6)$$

$$|M_1|^2 = \frac{-g_s^2 [T^a]_{o'}^k [T^a]_{o'}^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i g_{\mu'\mu} \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j] \quad (1.7)$$

from here and after contraction between all indices we can actually make statements about the last result.

$$|M_1|^2 = \frac{-g_s^2 [T^a]_{o'}^k [T^a]_{o'}^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \gamma^{\mu'} \not{q}_i \gamma_{\mu'} (\not{q}_i + \not{q})] [\not{q}_j] \quad (1.8)$$

In other words we expect the tree level diagram from LO and a number: Which means:

$$|M^2| = \left| \begin{array}{c} \text{diagram with two shaded circles and momenta } P_i, P_j \\ \text{contribution from LO} \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{diagram with a loop and momenta } q_i, q, q_i+q \\ \text{a complex number} \end{array} \right|^2$$

$$|M_1|^2 = \frac{-g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [P_i][P_j] \otimes (\text{a complex number}) \quad (1.9)$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N &=: \gamma^{\mu'} \not{q}_i \gamma_{\mu'} = q_{i\sigma} \gamma^{\mu'} \gamma^\sigma \gamma_{\mu'} \\ &= q_{i\sigma} (\{\gamma^{\mu'}, \gamma^\sigma\} - \gamma^\sigma \gamma^{\mu'}) \gamma_{\mu'} \\ &= q_{i\sigma} 2g^{\mu'\sigma} \gamma_{\mu'} - d \gamma^\sigma \\ &= (2 - d) \not{q}_i \end{aligned} \quad (1.10)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [(\not{q}_i + \not{q}) \not{q}_i (\not{q}_i + q)] [\not{q}_j] \quad (1.11)$$

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(q_i + q)^2 (q_i + q)^2} [\not{q}_i \not{q}_i \not{q}_i + \not{q}_i \not{q}_i \not{q} + \not{q} \not{q}_i \not{q}_i + \not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (1.12)$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \not{q}_i &= q_i^2 = m_i^2 \\ \not{q} \not{q} &= q^2 = m^2 \\ \not{q}_j \not{q}_j &= q_j^2 = m_j^2 \end{aligned} \quad (1.13)$$

we can first neglect the mass of patrons and ignore each term with  $\not{q}_i \not{q}_i$  and  $\not{q} \not{q}$  as well.

$$|M_1|^2 = -(2 - d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{(2q_i q)(2q_i q)} [\not{q} \not{q}_i \not{q}] [\not{q}_j] \quad (1.14)$$

$$\begin{aligned}
L &= \not{q}_i \not{q}_\mu (\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu) \\
&= \not{q}_i [2q_i^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu \left[ \frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2} \right] \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}] \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
&= \not{q}_i (2q_i q) - q^2 \not{q}_i \\
&= \not{q}_i (2q_i q)
\end{aligned} \tag{1.15}$$

After inserting the last result of  $L$  and simplify the term  $(2q_i q)$  from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{q}_i] [\not{q}_j] \tag{1.16}$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$|M_1|^2 = (d-2) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z) \not{p}_i + zy \not{p}_j - \sqrt{zy(1-z)} \not{m}_\perp] [(1-y) \not{p}_j] \tag{1.17}$$

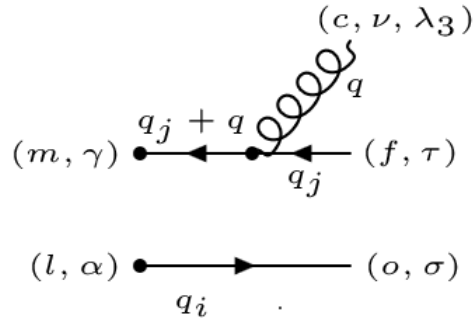
Multiplying the both sides

$$\begin{aligned}
|M_1|^2 &= (d-2) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [(1-z)(1-y) \not{p}_i \not{p}_j \\
&\quad + zy(1-y) \not{p}_j \not{p}_j + (1-y) \sqrt{zy(1-z)} \not{m}_\perp \not{p}_j]
\end{aligned} \tag{1.18}$$

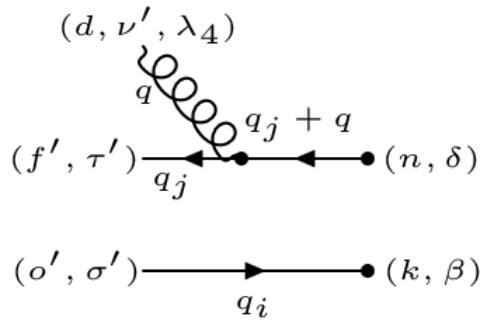
and under consideration of the fact that  $p_i$  and  $p_j$  are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with  $\not{p}_i \not{p}_i$  and  $\not{p}_j \not{p}_j$ . The  $p_i \cdot m_\perp$  and  $p_j \cdot m_\perp$  are always 0 because the  $p_i$  and  $p_j$  are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$|M_1|^2 = (d-2)(1-z)(1-y) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \tag{1.19}$$

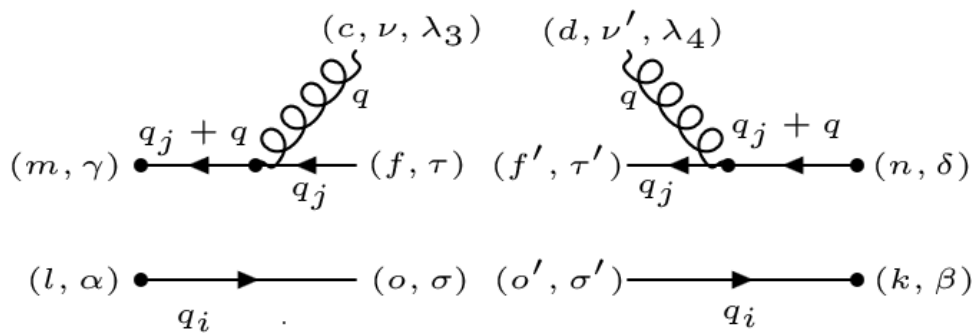


1.2  $\bar{q}g$ -q

$$M_2 = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_i)] \right] \quad (1.20)$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_i)]] \quad (1.21)$$



$$|M_2|^2 = M_2 M_2^\dagger = \left[ \frac{i(\not{q}_j + \not{q})}{(q_j + q)^2} (-ig_s \gamma^\nu \times [T^c]_f^m) v_\tau(q_j) \varepsilon^{\lambda_3}_\nu(q) [u_\sigma(q_i)] \right] \quad (1.22)$$

$$\left[ \bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q) [\bar{u}_{\sigma'}(q_i)] \right]$$

$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^d]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu v_\tau(q_j) \bar{v}_{\tau'}(q_j) \varepsilon^{\lambda_3}_\nu(q) \varepsilon^{\lambda_4}_{\nu'}(q) \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.23)$$

$$[u_\sigma(q_i)] [\bar{u}_{\sigma'}(q_i)]$$

and after sum over the lorenz index  $(\sigma, \sigma')$  and  $(\tau, \tau')$  and unsing the spin addition relation:

$$\sum_{\sigma, \sigma'} \bar{u}_\sigma(q_i) u_{\sigma'}(q_i) = \not{q}_i \delta^{\sigma\sigma'}, \quad (1.24)$$

$$\sum_{\tau, \tau'} \bar{v}_\tau(q_j) v_{\tau'}(q_j) = \not{q}_j \delta^{\tau\tau'}$$

and sum over polarization index  $(\lambda_3, \lambda_4)$  :

$$\sum_{\nu, \nu'} \varepsilon^{\lambda_4*}_{\nu'}(q) \varepsilon^{\lambda_3}_\nu(q) = -g_{\nu\nu'} \delta^{cd} \quad (1.25)$$

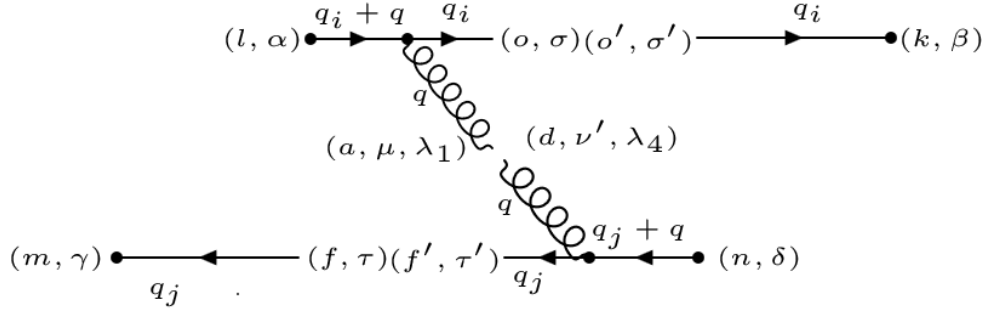
$$|M_2|^2 = \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{(q_j + q)^2 (q_j + q)^2} [(\not{q}_j + \not{q}) \gamma^\nu \not{q}_j (-g_{\nu\nu'}) \gamma^{\nu'} (\not{q}_j + \not{q})] [\not{q}_i] \quad (1.26)$$

After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d - 2) \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{(2qq_j)} [\not{q}] [\not{q}_i] \quad (1.27)$$

finally:

$$|M_2|^2 = -(d - 2) y z^2 \frac{g_s^2 [T^c]_f^m [T^c]_{f'}^n}{2(1 - z)(1 - y)(p_i \cdot p_j)} [\not{p}_i] [\not{p}_j] \quad (1.28)$$

1.3  $M_1 M_2^\dagger$ 

$$M_1 M_2^\dagger = [\bar{u}_\sigma(q_i) (-ig_s \gamma^\mu \times [T^a]_o^l) \frac{i(\not{q}_i + \not{q})}{(q_i + q)^2} \varepsilon^{\lambda_1}_\mu(q)] [v_\tau(q_j)]$$

$$[\bar{v}_{\tau'}(q_j) (ig_s \gamma^{\nu'} \times [T^d]_{f'}^n) \frac{-i(\not{q}_j + \not{q})}{(q_j + q)^2} \varepsilon^{\lambda_4}_{\nu'}(q)] [u_{\sigma'}(q_i)] \quad (1.29)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^d]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_4}_{\nu'}(q)$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.30)$$

$$M_1 M_2^\dagger = \frac{g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] - g_{\mu\nu'}$$

$$[\not{q}_j \gamma^{\nu'} (\not{q}_j + \not{q})] \quad (1.31)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [\not{q}_i \gamma^\mu (\not{q}_i + \not{q})] [\not{q}_j \gamma_\mu (\not{q}_j + \not{q})] \quad (1.32)$$

Expectation:

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [(\not{q}_i + \not{q}) \gamma^\mu \not{q}_i] [(\not{q}_j + \not{q}) \gamma_\mu \not{q}_j] \quad (1.33)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} [-(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu + 2(\not{q}_i + \not{q}) q_i^\mu]$$

$$[-(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu + 2(\not{q}_j + \not{q}) q_{j\mu}] \quad (1.34)$$

$$|M^2| = \left| \begin{array}{c} \text{diagram with two shaded circles and arrows } P_i, P_j \\ \text{contribution from LO} \end{array} \right|^2 \otimes \left| \begin{array}{c} \text{diagram with a shaded circle and a wavy line} \\ \text{a complex number} \end{array} \right|^2$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} \begin{aligned} & [(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu] [(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu] \\ & - 2[(\not{q}_i + \not{q}) \not{q}_i \gamma^\mu] [(\not{q}_j + \not{q}) q_{j\mu}] \\ & - 2[(\not{q}_i + \not{q}) q_i^\mu] [(\not{q}_j + \not{q}) \not{q}_j \gamma_\mu] \\ & + 4[(\not{q}_i + \not{q}) q_i^\mu] [(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \quad (1.35)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(2q_i q)(2q_j q)} \begin{aligned} & [\not{q} \not{q}_i \gamma^\mu] [\not{q} \not{q}_j \gamma_\mu] \\ & - 2[\not{q} \not{q}_i \gamma^\mu] [(\not{q} + \not{q}_j) q_{j\mu}] \\ & - 2[(\not{q}_i + \not{q}) q_i^\mu] [\not{q} \not{q}_j \gamma_\mu] \\ & + 4[(\not{q}_i + \not{q}) q_i^\mu] [(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \quad (1.36)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \begin{aligned} & [y(1-2z+2z^2) \not{p}_i \not{p}_j \gamma^\mu] [(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu] \\ & + 4(q_i^\mu \cdot q_{j\mu}) [(\not{q}_i + \not{q})] [(\not{q}_j + \not{q})] \end{aligned} \quad (1.37)$$

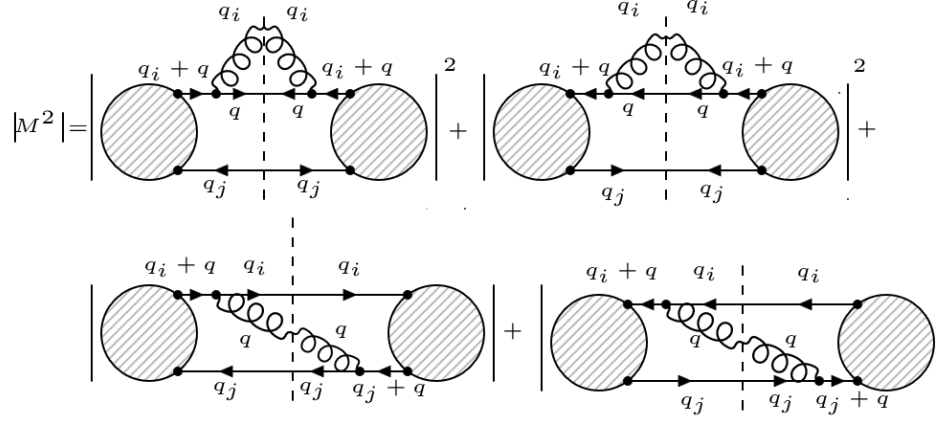
$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \begin{aligned} & [y(1-2z+2z^2) \not{p}_i \not{p}_j \gamma^\mu] [(1-z)(1-y) \not{p}_i \not{p}_j \gamma_\mu] \\ & + 4(p_i \cdot p_j) [(\not{p}_i + y \not{p}_j)] [(1-z) \not{p}_i + (1+yz-y) \not{p}_j - \sqrt{zy(1-z)} \not{m}] \end{aligned} \quad (1.38)$$

$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)} z(1-y) [\not{p}_i] [\not{p}_j] \quad (1.39)$$

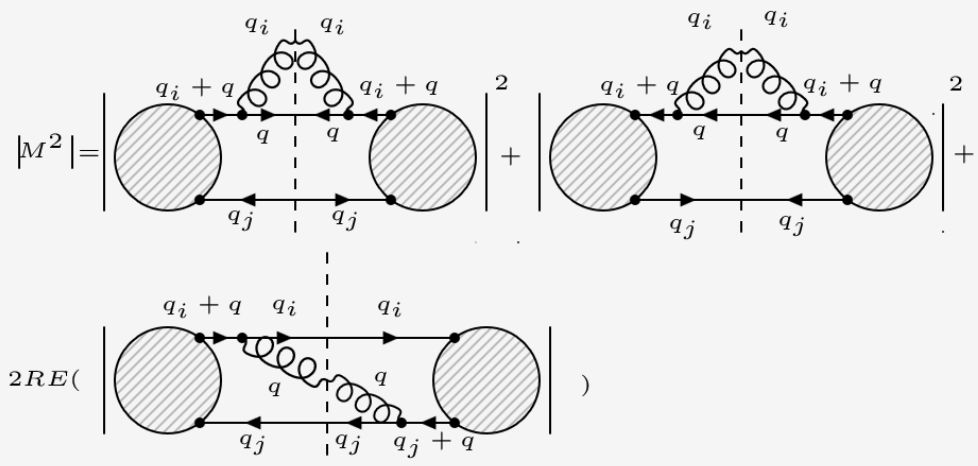
$$M_1 M_2^\dagger = \frac{-g_s^2 [T^a]_o^l [T^a]_{f'}^n}{(1-z)y(1-2z+2z^2)(p_i \cdot p_j)} z [\not{p}_i] [\not{p}_j] \quad (1.40)$$

1.4  $|M^2|$ 

$$|M|^2 = |M_1|^2 + |M_2|^2 + M_1 M_2^\dagger + M_1^\dagger M_2 \quad (1.41)$$



$$|M|^2 = |M_1|^2 + |M_2|^2 + 2RE(M_1 M_2^\dagger) \quad (1.42)$$



$$\begin{aligned}
 |M|^2 = & (d-2)(1-z)(1-y) \frac{g_s^2 [T^a]_o^k [T^a]_o^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & - (d-2)yz^2 \frac{g_s^2 [T^c]_f^m [T^c]_f^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
 & + 2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2 [T^a]_o^l [T^a]_f^n}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]\right)
 \end{aligned} \quad (1.43)$$

$$T^a_{ok} T^a_{lo} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F\delta_{lk} \quad (1.44)$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a_{ok} T^a_{lo} = C_F \delta_{lk} = \frac{1}{N} \sum_{l=1}^N \delta_{lk} C_F = C_F \quad (1.45)$$

The same calculation for  $T^c_{mf} T^c_{fn}$  and  $T^a_{ol} T^a_{fn}$  turns  $C_F$  out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, which means, if:

$$y \longrightarrow 0 \quad (1.46)$$

$$\begin{aligned} |M|^2 = & (d-2)(1-z)(1-y) \frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & - (d-2)yz^2 \frac{g_s^2 C_F}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ & + 2RE\left(\left(\frac{-2z}{z-1}\right) \frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j]\right) \end{aligned} \quad (1.47)$$

$$|M|^2 = C_F \left( (d-2)(1-z) - \frac{4z}{z-1} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \quad (1.48)$$

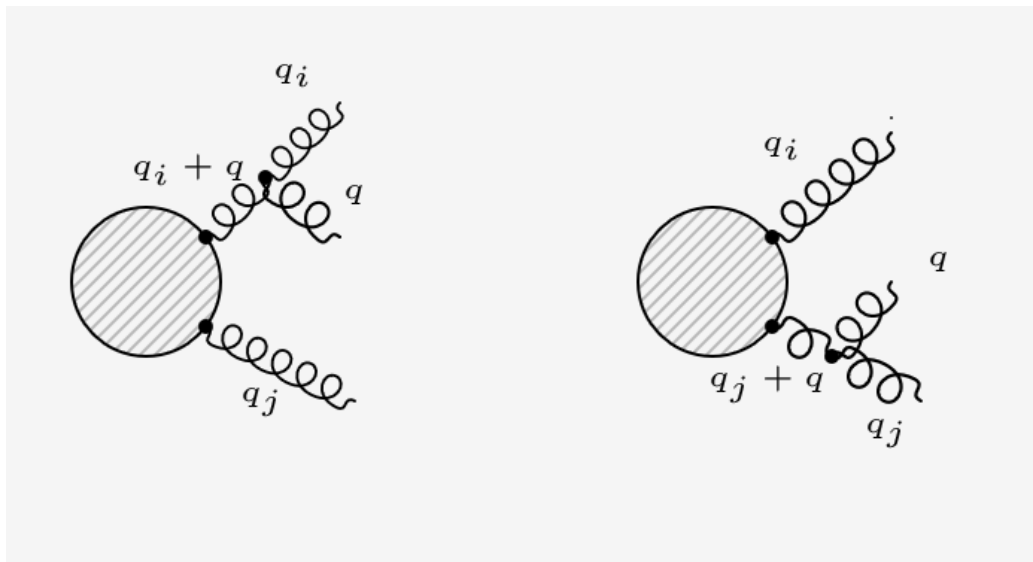
for

$$d = 4 - 2\epsilon \quad (1.49)$$

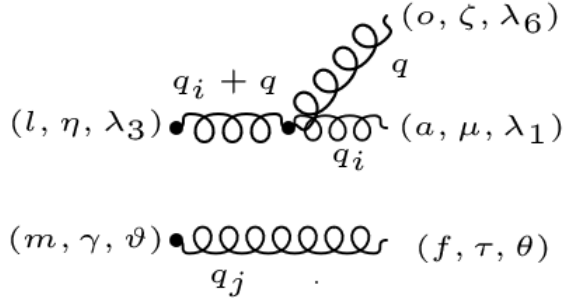
$$\begin{aligned} |M|^2 = & C_F \left( (4-2\epsilon-2)(1-z) + \frac{4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{2(1-\epsilon)(1-z)^2 + 4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{2-4z+2z^2-\epsilon(1-z)^2+4z}{1-z} \right) \frac{g_s^2}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & C_F \left( \frac{(1+z^2)}{1-z} - \epsilon(1-z) \right) \frac{g_s^2}{y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\ = & \langle \hat{P}_{qq} \rangle \frac{g_s^2}{q_i \cdot q} [\not{p}_i][\not{p}_j] \end{aligned} \quad (1.50)$$

## Chapter 2

### Gluon gluon gluon emission kernel

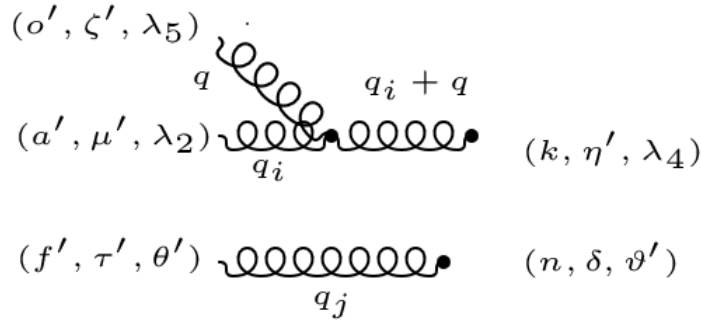


## 2.1 Gluon-Emitter Bubble



$$M_1 = \left[ \frac{-i}{(q + q_i)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta + g^{\zeta \eta} (-q - (q + q_i))^\mu + g^{\eta \mu} (q_i + q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q) \varepsilon^{\lambda_6}_\zeta(q) [\varepsilon^\theta_{\tau'}(q_j)] \right] \quad (2.1)$$

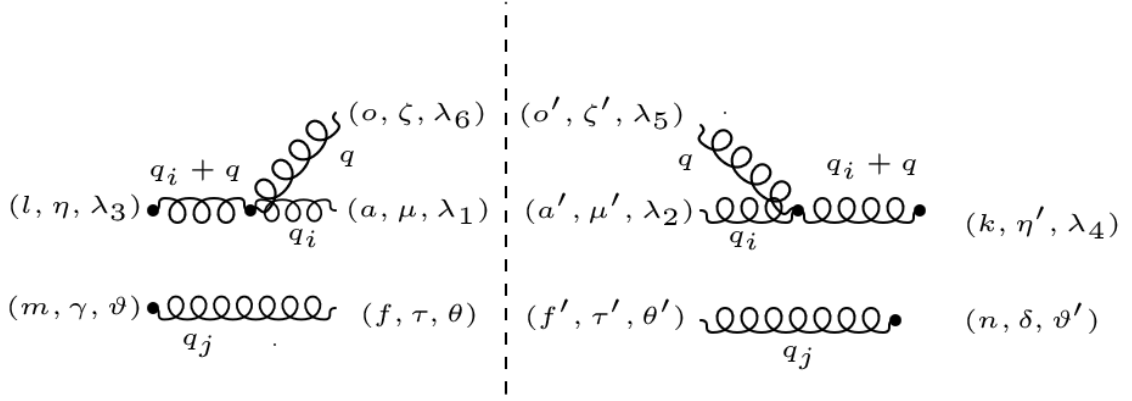
$$M_1 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q + q_i)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_6}_\zeta(q) [\varepsilon^\theta_{\tau'}(q_j)] \right] \quad (2.2)$$



$$M_1^\dagger = \left[ \frac{i}{(q_i + q)^2} (-g_s f^{a' k o'} (-g^{\mu' \eta'} (2q_i + q)^{\zeta'} + g^{\eta' \zeta'} (2q + q_i)^{\mu'} + g^{\zeta' \mu'} (q_i - q)^{\eta'}) \right. \\ \left. \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_5}_{\zeta'}(q) [\varepsilon^{\theta'}_{\tau'}(q_j)] \right] \quad (2.3)$$

$$|M_1|^2 = \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a o l} (g^{\mu \zeta} (q - q_i)^\eta - g^{\zeta \eta} (2q + q_i)^\mu + g^{\eta \mu} (2q_i + q)^\zeta) \right. \\ \left. \varepsilon^{\lambda_1}_\mu(q_i) \varepsilon^{\lambda_2}_{\mu'}(q_i) \varepsilon^{\lambda_6}_\zeta(q) \varepsilon^{\lambda_5}_{\zeta'}(q) \right. \\ \left. (-g_s f^{a' k o'} (-g^{\mu' \eta'} (2q_i + q)^{\zeta'} + g^{\eta' \zeta'} (2q + q_i)^{\mu'} + g^{\zeta' \mu'} (q_i - q)^{\eta'}) \frac{i}{(q_i + q)^2} [g_{\gamma \delta}] \right] \quad (2.4)$$





$$\begin{aligned}
N \equiv & g_{\mu\mu'} g_{\zeta\zeta'} [-g^{\mu\zeta} g^{\mu'\eta'} (q - q_i)^\eta (2q_i + q)^{\zeta'} + g^{\mu\zeta} g^{\eta'\zeta'} (q - q_i)^\eta (2q + q_i)^{\mu'} \\
& + g^{\mu\zeta} g^{\zeta'\mu'} (q - q_i)^\eta (q_i - q)^{\eta'} + g^{\zeta\eta} g^{\mu'\zeta'} (2q + q_i)^\mu (2q_i + q)^{\zeta'} \\
& - g^{\zeta\eta} g^{\eta'\zeta'} (2q + q_i)^\mu (2q + q_i)^{\mu'} - g^{\zeta\eta} g^{\zeta'\mu'} (2q + q_i)^\mu (q_i - q)^{\eta'} \\
& - g^{\eta\mu} g^{\mu'\eta'} (2q_i + q)^\zeta (2q_i + q)^{\zeta'} + g^{\eta\mu} g^{\eta'\zeta'} (2q_i + q)^\zeta (2q + q_i)^{\mu'} \\
& + g^{\eta\mu} g^{\zeta'\mu'} (2q_i + q)^\zeta (q_i - q)^{\eta'}] [g_{\gamma\delta}]
\end{aligned} \quad (2.5)$$

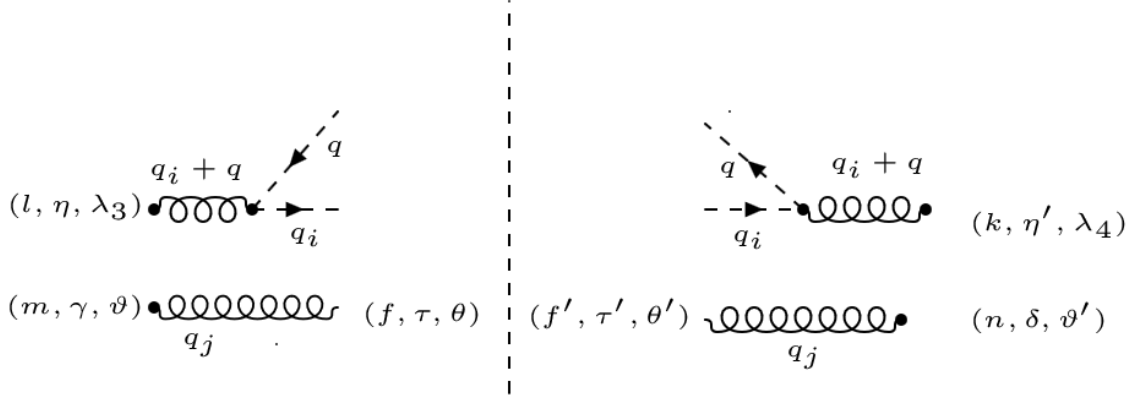
$$\begin{aligned}
N \equiv & [-(q - q_i)^\eta (2q_i + q)^{\eta'} + (q - q_i)^\eta (2q + q_i)^{\eta'} + d(q - q_i)^\eta (q_i - q)^{\eta'} \\
& + (2q + q_i)^{\eta'} (2q_i + q)^\eta - g^{\eta\eta'} (2q + q_i)^\mu (2q + q_i)_\mu - (2q + q_i)^\eta (q_i - q)^{\eta'} \\
& - g^{\eta\eta'} (2q_i + q)^\zeta (2q_i + q)_\zeta + (2q_i + q)^{\eta'} (2q + q_i)^\eta + (2q_i + q)^\eta (q_i - q)^{\eta'}] [g_{\gamma\delta}]
\end{aligned} \quad (2.6)$$

$$\begin{aligned}
N \equiv & [-(q^\eta q^{\eta'} + 2q^\eta q_i^{\eta'} - q_i^\eta q^{\eta'} - 2q_i^\eta q_i^{\eta'}) + (2q^\eta q^{\eta'} + q^\eta q_i^{\eta'} - 2q_i^\eta q^{\eta'} - q_i^\eta q_i^{\eta'}) \\
& + (dq^\eta q_i^{\eta'} - dq^\eta q^{\eta'} - dq_i^\eta q_i^{\eta'} + dq_i^\eta q^{\eta'}) + (4q^{\eta'} q_i^\eta + 2q^{\eta'} q^\eta + 2q_i^{\eta'} q_i^\eta + q_i^{\eta'} q^\eta) \\
& - (-2q^\eta q^{\eta'} + 2q^\eta q_i^{\eta'} - q_i^\eta q^{\eta'} + q_i^\eta q_i^{\eta'}) + (2q^{\eta'} q^\eta + q^{\eta'} q_i^\eta + 4q_i^{\eta'} q^\eta + 2q_i^{\eta'} q_i^\eta) \\
& + (-q^\eta q^{\eta'} + q^\eta q_i^{\eta'} - 2q_i^\eta q^{\eta'} + 2q_i^\eta q_i^{\eta'}) - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.7)$$

$$\begin{aligned}
N \equiv & [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} \\
& - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.8)$$

$$\begin{aligned}
|M_1|^2 = & \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} \\
& - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i)] [g_{\gamma\delta}]
\end{aligned} \quad (2.9)$$

### 2.1.1 One-loop corrections to the gluon self-energy diagram(Gluon-Emitter Bubble)



$$|M_1|_{Ghost\ loop}^2 = \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [-q_i^\eta q^{\eta'} - q^\eta q_i^{\eta'}] [g_{\gamma\delta}] \quad (2.10)$$

$$\begin{aligned} |M'_1|^2 &= |M_1|^2 + |M_1|_{Ghost\ loop}^2 \\ &= \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 3)q^\eta q_i^{\eta'} \\ &\quad + (d + 3)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} - g^{\eta\eta'} (5q^2 + 5q_i^2 + 8qq_i) - q_i^\eta q^{\eta'} - q^\eta q_i^{\eta'}] [g_{\gamma\delta}] \end{aligned} \quad (2.11)$$

$$\begin{aligned} |M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{(q_i + q)^2 (q_i + q)^2} [(6 - d)q^\eta q^{\eta'} + (d + 2)q^\eta q_i^{\eta'} \\ &\quad + (d + 2)q_i^\eta q^{\eta'} + (6 - d)q_i^\eta q_i^{\eta'} - g^{\eta\eta'} (8qq_i)] [g_{\gamma\delta}] \end{aligned} \quad (2.12)$$

$$\begin{aligned} |M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\ &\quad [(6 - d)(\zeta_1 p_i^\eta + \lambda_1 Q^\eta + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_1 p_i^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (d + 2)(\zeta_1 p_i^\eta + \lambda_1 Q^\eta + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_q p_i^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (d + 2)(\zeta_q p_i^\eta + \lambda_q Q^\eta - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_1 p_i^{\eta'} + \lambda_1 Q^{\eta'} + \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad + (6 - d)(\zeta_q p_i^\eta + \lambda_q Q^\eta - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^\eta)(\zeta_q p_i^{\eta'} + \lambda_q Q^{\eta'} - \sqrt{y\alpha_1\beta_1} n_{\perp,1}^{\eta'}) \\ &\quad - g^{\eta\eta'} (8y(\alpha_1 + \beta_1) p_i \cdot Q)] [g_{\gamma\delta}] \end{aligned} \quad (2.13)$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)[\zeta_1 \zeta_1 p_i^\eta p_i^{\eta'} + \zeta_1 \lambda_1 p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_1 \zeta_1 Q^\eta p_i^{\eta'} + \lambda_1 \lambda_1 Q^\eta Q^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&[(d+2)[\zeta_1 \zeta_q p_i^\eta p_i^{\eta'} + \zeta_1 \lambda_q p_i^\eta Q^{\eta'} - \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_1 \zeta_q Q^\eta p_i^{\eta'} + \lambda_1 \lambda_q Q^\eta Q^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&[(d+2)[\zeta_q \zeta_1 p_i^\eta p_i^{\eta'} + \zeta_q \lambda_1 p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_q \zeta_1 Q^\eta p_i^{\eta'} + \lambda_q \lambda_1 Q^\eta Q^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&[(6-d)[\zeta_q \zeta_q p_i^\eta p_i^{\eta'} + \zeta_q \lambda_q p_i^\eta Q^{\eta'} - \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + \lambda_q \zeta_q Q^\eta p_i^{\eta'} + \lambda_q \lambda_q Q^\eta Q^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}]
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)[(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\alpha_1\beta_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\beta_1\alpha_1 - y^2\beta_1^2(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\beta_1^2 Q^\eta Q^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(d+2)[(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\alpha_1^2 - y^2\beta_1\alpha_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} - \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\beta_1\alpha_1 Q^\eta Q^{\eta'} \\
&\quad - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&\quad + \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \quad (2.15) \\
&\quad [(d+2)[(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\beta_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\alpha_1^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\alpha_1\beta_1 Q^\eta Q^{\eta'} \\
&\quad + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}] \\
&\quad [(6-d)[(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}) + y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q})^2)p_i^\eta p_i^{\eta'} \\
&\quad + (y\beta_1\alpha_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta Q^{\eta'} - \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&\quad + (y\alpha_1\beta_1 - y^2\alpha_1^2(\frac{Q^2}{2p_i \cdot Q}))Q^\eta p_i^{\eta'} + y^2\alpha_1^2 Q^\eta Q^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} \\
&\quad - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&\quad + \sqrt{y\alpha_1\beta_1} \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} n^{\eta'}_{\perp,1}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)\{(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&+ y\beta_1\alpha_1 Q^\eta p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} + \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&+ y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (d+2)\{(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} \\
&- \zeta_1 \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} + y\beta_1^2 Q^\eta p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} + \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&+ \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} - y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} \quad (2.16) \\
&+ (d+2)\{(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} + \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} \\
&+ y\alpha_1^2 Q^\eta p_i^{\eta'} + \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} - \zeta_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} - \lambda_1 \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} \\
&- y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (6-d)\{(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1\alpha_1 p_i^\eta Q^{\eta'} \\
&- \zeta_q \sqrt{y\alpha_1\beta_1} p_i^\eta n^{\eta'}_{\perp,1} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} - \lambda_q \sqrt{y\alpha_1\beta_1} Q^\eta n^{\eta'}_{\perp,1} - \zeta_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} p_i^{\eta'} \\
&- \lambda_q \sqrt{y\alpha_1\beta_1} n^\eta_{\perp,1} Q^{\eta'} + y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{a o l} f^{a k o}}{y^2 (\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)\{(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1\beta_1 p_i^\eta Q^{\eta'} + y\beta_1\alpha_1 Q^\eta p_i^{\eta'} \\
&+ y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} \\
&+ (d+2)\{(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\alpha_1^2 p_i^\eta Q^{\eta'} + y\beta_1^2 Q^\eta p_i^{\eta'} \quad (2.17) \\
&- y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (d+2)\{(\beta_1\alpha_1 - y(\beta_1^2 + \alpha_1^2)(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} + y\beta_1^2 p_i^\eta Q^{\eta'} \\
&+ y\alpha_1^2 Q^\eta p_i^{\eta'} - y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} + (6-d)\{(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))p_i^\eta p_i^{\eta'} \\
&+ y\beta_1\alpha_1 p_i^\eta Q^{\eta'} + y\alpha_1\beta_1 Q^\eta p_i^{\eta'} + y\alpha_1\beta_1 n^\eta_{\perp,1} n^{\eta'}_{\perp,1}\} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

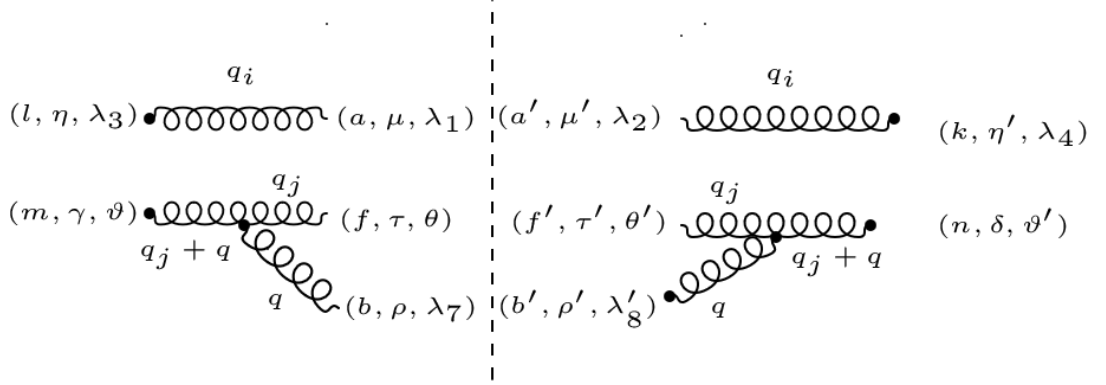
$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) \\
&\quad + (6-d)(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))][p_i^\eta p_i^{\eta'}] \quad (2.18) \\
&\quad + [2(6-d)y\alpha_1\beta_1 + (d+2)y(\alpha_1^2 + \beta_1^2)]p_i^\eta Q^{\eta'} \\
&\quad + [2(6-d)y\beta_1\alpha_1 + (d+2)y(\alpha_1^2 + \beta_1^2)]Q^\eta p_i^{\eta'} \\
&\quad + [2(6-d) - 2(d+2)]y\alpha_1\beta_1 n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y^2(\alpha_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \\
&[(6-d)(\alpha_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q})) + 2(d+2)(\alpha_1\beta_1 - y(\alpha_1^2 + \beta_1^2)(\frac{Q^2}{2p_i \cdot Q})) \\
&\quad + (6-d)(\beta_1^2 - 2y\alpha_1\beta_1(\frac{Q^2}{2p_i \cdot Q}))][p_i^\eta p_i^{\eta'}] \quad (2.19) \\
&\quad + y[(4d-8)\alpha_1^2 + (8-4d)\alpha_1 + (d+2)]p_i^\eta Q^{\eta'} \\
&\quad + y[(4d-8)\alpha_1^2 + (8-4d)\alpha_1 + (d+2)]Q^\eta p_i^{\eta'} \\
&\quad + y[8-4d](\alpha_1 - \alpha_1^2)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8yg^{\eta\eta'}(\alpha_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y(1 - \beta_1 + \beta_1)^2 (p_i \cdot Q) (p_i \cdot Q)} \quad (2.20) \\
&[[8-4d]\beta_1(1 - \beta_1)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'}(1 - \beta_1 + \beta_1)p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

$$\begin{aligned}
|M'_1|^2 &= \frac{g_s^2 f^{aol} f^{ako}}{y(p_i \cdot Q) (p_i \cdot Q)} \quad (2.21) \\
&[8[\epsilon - 1]\beta_1(1 - \beta_1)n_{\perp,1}^\eta n_{\perp,1}^{\eta'} - 8g^{\eta\eta'}p_i \cdot Q][g_{\gamma\delta}]
\end{aligned}$$

## 2.2 Gluon-Spectator Bubble



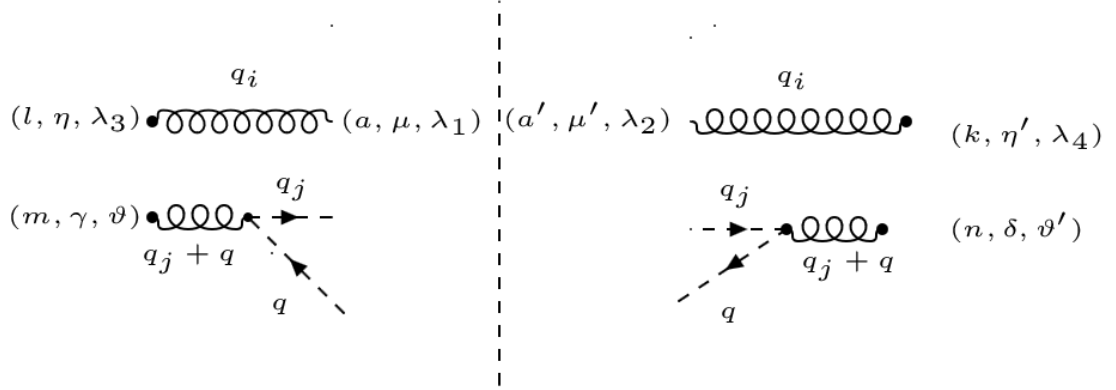
$$|M_2|^2 = \left[ \frac{-i}{(q_j + q)^2} (-g_s f^{b f m} (g^{\tau\gamma} (-2q_j - q)^\rho + g^{\gamma\rho} (2q + q_j)^\tau + g^{\rho\tau} (q_j - q)^\gamma) \right. \\ \left. g_{\tau\tau'} g_{\rho\rho'} (-g_s f^{b' n f'} (g^{\rho'\delta} (-2q - q_j)^{\tau'} + g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\tau'\rho'} (q - q_j)^\delta) \frac{i}{(q_j + q)^2} \right] [g_{\eta\eta'}] \quad (2.22)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b' n f'} \delta^{aa'} \delta^{ff'} \delta^{bb'}}{(q_j + q)^2 (q_j + q)^2} [g_{\tau\tau'} g_{\rho\rho'} (g^{\tau\gamma} (2q_j + q)^\rho g^{\rho'\delta} (2q + q_j)^{\tau'} \\ - g^{\tau\gamma} (2q_j + q)^\rho g^{\delta\tau'} (2q_j + q)^{\rho'} - g^{\tau\gamma} (2q_j + q)^\rho g^{\tau'\rho'} (q - q_j)^\delta - g^{\gamma\rho} (2q + q_j)^\tau g^{\rho'\delta} (2q + q_j)^{\tau'} \\ + g^{\gamma\rho} (2q + q_j)^\tau g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\gamma\rho} (2q + q_j)^\tau g^{\tau'\rho'} (q - q_j)^\delta - g^{\rho\tau} (q_j - q)^\gamma g^{\rho'\delta} (2q + q_j)^{\tau'} \\ + g^{\rho\tau} (q_j - q)^\gamma g^{\delta\tau'} (2q_j + q)^{\rho'} + g^{\rho\tau} (q_j - q)^\gamma g^{\tau'\rho'} (q - q_j)^\delta) [g_{\eta\eta'}] \quad (2.23)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(2q + q_j)^\gamma (2q_j + q)^\delta \\ - g^{\delta\gamma} (2q_j + q)^\rho (2q_j + q)_\rho - (2q_j + q)^\gamma (q - q_j)^\delta - g^{\delta\gamma} (2q + q_j)^\tau (2q + q_j)_\tau \\ + (2q_j + q)^\gamma (2q + q_j)^\delta + (2q + q_j)^\gamma (q - q_j)^\delta - (q_j - q)^\gamma (2q + q_j)^\delta \\ + (q_j - q)^\gamma (2q_j + q)^\delta + d(q_j - q)^\gamma (q - q_j)^\delta] [g_{\eta\eta'}] \quad (2.24)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(3 + d) q^\gamma q_j^\delta + (6 - d) q^\gamma q^\delta \\ + (6 - d) q_j^\gamma q_j^\delta + (3 + d) q_j^\gamma q^\delta - g^{\delta\gamma} (5q_j^2 + 5q^2 + 8qq_j) \\ ] [g_{\eta\eta'}] \quad (2.25)$$

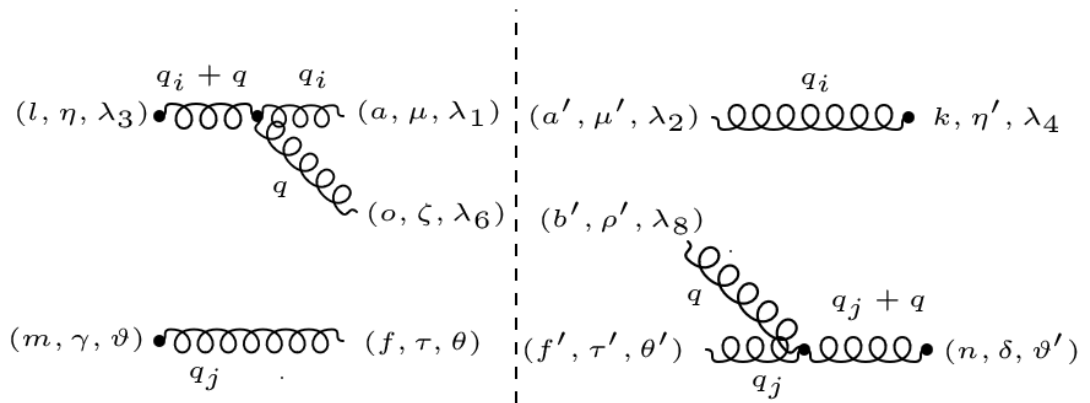
### 2.2.1 One-loop corrections to the gluon self-energy diagram (Gluon-Spectator Bubble)



$$|M_1|_{Ghost\ loop}^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [-q_j^\gamma q^\delta - q^\delta q_j^\gamma] [g_{\gamma\delta}] \quad (2.26)$$

$$|M_2|^2 = \frac{g_s^2 f^{b f m} f^{b n f}}{(q_j + q)^2 (q_j + q)^2} [(2 + d) q^\gamma q_j^\delta + (6 - d) q^\gamma q^\delta + (6 - d) q_j^\gamma q_j^\delta + (2 + d) q_j^\gamma q^\delta - g^{\delta\gamma} (8 q q_j)] [g_{\eta\eta'}] \quad (2.27)$$

### 2.3 Interference term $M_1 M_2^\dagger$





$$\begin{aligned}
M_1 M_2^\dagger = & \left[ \frac{-i}{(q_i + q)^2} (-g_s f^{a l o} (-g^{\mu\eta} (2q_i + q)^\zeta + g^{\eta\zeta} (2q + q_i)^\mu + g^{\zeta\mu} (q_i - q)^\eta) \right. \\
& \left. \varepsilon^{\lambda_1}{}_\mu(q_i) \varepsilon^{\lambda_6}{}_\zeta(q) [\varepsilon^{\theta}{}_\tau{}^*(q_j)] \right. \\
& \left[ \frac{i}{(q + q_j)^2} (-g_s f^{f' n b'} (g^{\tau'\delta} (2q_j + q)^{\rho'} - g^{\delta\rho'} (2q + q_j)^{\tau'} + g^{\rho'\tau'} (q - q_j)^\delta) \right. \\
& \left. \varepsilon^{\theta'}{}_{\tau'}{}^*(q_j) \varepsilon^{\lambda_8}{}_{\rho'}{}^*(q) [\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q_i)] \right]
\end{aligned} \quad (2.28)$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{a l o} f^{f' n b'} \delta^{aa'} \delta^{ob'} \delta^{ff'}}{(q_i + q)^2 (q_j + q)^2} [g_{\mu\mu'} g_{\tau\tau'} \\
& (-g^{\mu\eta} (2q_i + q)^\zeta + g^{\eta\zeta} (2q + q_i)^\mu + g^{\zeta\mu} (q_i - q)^\eta) \\
& g_{\zeta\rho'} (g^{\tau'\delta} (2q_j + q)^{\rho'} - g^{\delta\rho'} (2q + q_j)^{\tau'} + g^{\rho'\tau'} (q - q_j)^\delta)]
\end{aligned} \quad (2.29)$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{a l o} f^{f n o}}{(q_i + q)^2 (q_j + q)^2} [g_{\mu\eta'} g_{\gamma\tau'} \\
& (-g^{\mu\eta} g_{\zeta\rho'} g^{\tau'\delta} (2q_i + q)^\zeta (2q_j + q)^{\rho'} + g^{\mu\eta} g_{\zeta\rho'} g^{\delta\rho'} (2q_i + q)^\zeta (2q + q_j)^{\tau'} \\
& - g^{\mu\eta} g_{\zeta\rho'} g^{\rho'\tau'} (2q_i + q)^\zeta (q - q_j)^\delta + g^{\eta\zeta} g_{\zeta\rho'} g^{\tau'\delta} (2q + q_i)^\mu (2q_j + q)^{\rho'} \\
& - g^{\eta\zeta} g_{\zeta\rho'} g^{\delta\rho'} (2q + q_i)^\mu (2q + q_j)^{\tau'} + g^{\eta\zeta} g_{\zeta\rho'} g^{\rho'\tau'} (2q + q_i)^\mu (q - q_j)^\delta \\
& + g^{\zeta\mu} g_{\zeta\rho'} g^{\tau'\delta} (q_i - q)^\eta (2q_j + q)^{\rho'} - g^{\zeta\mu} g_{\zeta\rho'} g^{\delta\rho'} (q_i - q)^\eta (2q + q_j)^{\tau'} \\
& + g^{\zeta\mu} g_{\zeta\rho'} g^{\rho'\tau'} (q_i - q)^\eta (q - q_j)^\delta)]
\end{aligned} \quad (2.30)$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{a l o} f^{f n o}}{(q_i + q)^2 (q_j + q)^2} [g_{\mu\eta'} g_{\gamma\tau'} \\
& (-g^{\mu\eta} g^{\tau'\delta} (2q_i + q)^\zeta (2q_j + q)_\zeta + g^{\mu\eta} (2q + q_j)^{\tau'} (2q_i + q)^\delta - g^{\mu\eta} (2q_i + q)^{\tau'} (q - q_j)^\delta \\
& + g^{\tau'\delta} (2q + q_i)^\mu (2q_j + q)^\eta - g^{\eta\delta} (2q + q_i)^\mu (2q + q_j)^{\tau'} + g^{\eta\tau'} (2q + q_i)^\mu (q - q_j)^\delta \\
& + g^{\tau'\delta} (2q_j + q)^\mu (q_i - q)^\eta - g^{\mu\delta} (q_i - q)^\eta (2q + q_j)^{\tau'} + g^{\mu\tau'} (q_i - q)^\eta (q - q_j)^\delta)]
\end{aligned} \quad (2.31)$$

$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{a l o} f^{f n o}}{(q_i + q)^2 (q_j + q)^2} \\
& [-g_{\eta'}{}^\eta g_\gamma{}^\delta (2q_i + q) \cdot (2q_j + q) + g_{\eta'}{}^\eta (2q + q_j)_\gamma (2q_i + q)^\delta - g_{\eta'}{}^\eta (2q_i + q)_\gamma (q - q_j)^\delta \\
& + g_\gamma{}^\delta (2q + q_i)_{\eta'} (2q_j + q)^\eta - g^{\eta\delta} (2q + q_i)_{\eta'} (2q + q_j)_\gamma + g_\gamma{}^\eta (2q + q_i)_{\eta'} (q - q_j)^\delta \\
& + g_\gamma{}^\delta (2q_j + q)_{\eta'} (q_i - q)^\eta - g_{\eta'}{}^\delta (q_i - q)^\eta (2q + q_j)_\gamma + g_{\gamma\eta'} (q_i - q)^\eta (q - q_j)^\delta]
\end{aligned} \quad (2.32)$$

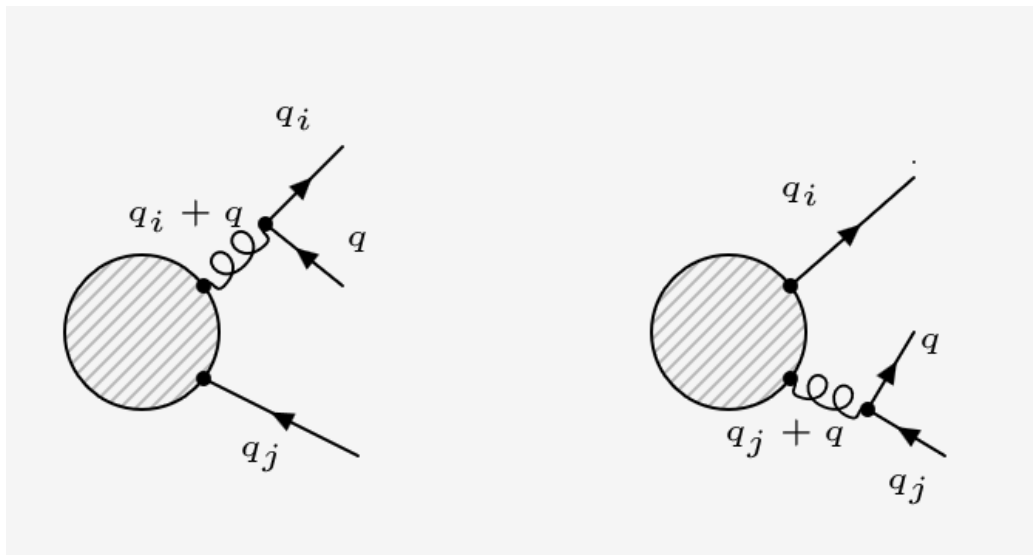
$$\begin{aligned}
M_1 M_2^\dagger = & \frac{g_s^2 f^{a l o} f^{f n o}}{4(q_i \cdot q)(q_j \cdot q)} \\
& [-g_{\eta'}{}^\eta g_\gamma{}^\delta (2q_i + q) \cdot (2q_j + q) + g_{\eta'}{}^\eta [(2q + q_j)_\gamma (2q_i + q)^\delta - (2q_i + q)_\gamma (q - q_j)^\delta] \\
& + g_\gamma{}^\delta [(2q + q_i)_{\eta'} (2q_j + q)^\eta + (2q_j + q)_{\eta'} (q_i - q)^\eta] - g^{\eta\delta} (2q + q_i)_{\eta'} (2q + q_j)_\gamma \\
& + g_\gamma{}^\eta (2q + q_i)_{\eta'} (q - q_j)^\delta - g_{\eta'}{}^\delta (q_i - q)^\eta (2q + q_j)_\gamma + g_{\gamma\eta'} (q_i - q)^\eta (q - q_j)^\delta]
\end{aligned} \quad (2.33)$$

$$\begin{aligned}
M_1 M_2^\dagger &= \frac{g_s^2 f^{a l o f f n o}}{4(q_i \cdot q)(q_j \cdot q)} \\
&[-g_{\eta'}^\eta g_\gamma^\delta (2q \cdot q_i + q \cdot q + 4q_j \cdot q_i + 2q_j \cdot q) + g_{\eta'}^\eta [2q_\gamma q_i^\delta + q_\gamma q^\delta + 4q_{j\gamma} q_i^\delta + 2q_{j\gamma} q^\delta] \\
&\quad + g_\gamma^\delta [2q_{\eta'} q_i^\eta + q_{\eta'} q^\eta + 4q_{j\eta'} q_i^\eta + 2q_{j\eta'} q^\eta] - g^{\eta\delta} (2q + q_i)_{\eta'} (2q + q_j)_\gamma \\
&\quad + g_\gamma^\eta (2q + q_i)_{\eta'} (q - q_j)^\delta - g_{\eta'}^\delta (q_i - q)^\eta (2q + q_j)_\gamma + g_{\gamma\eta'} (q_i - q)^\eta (q - q_j)^\delta] \quad (2.34)
\end{aligned}$$

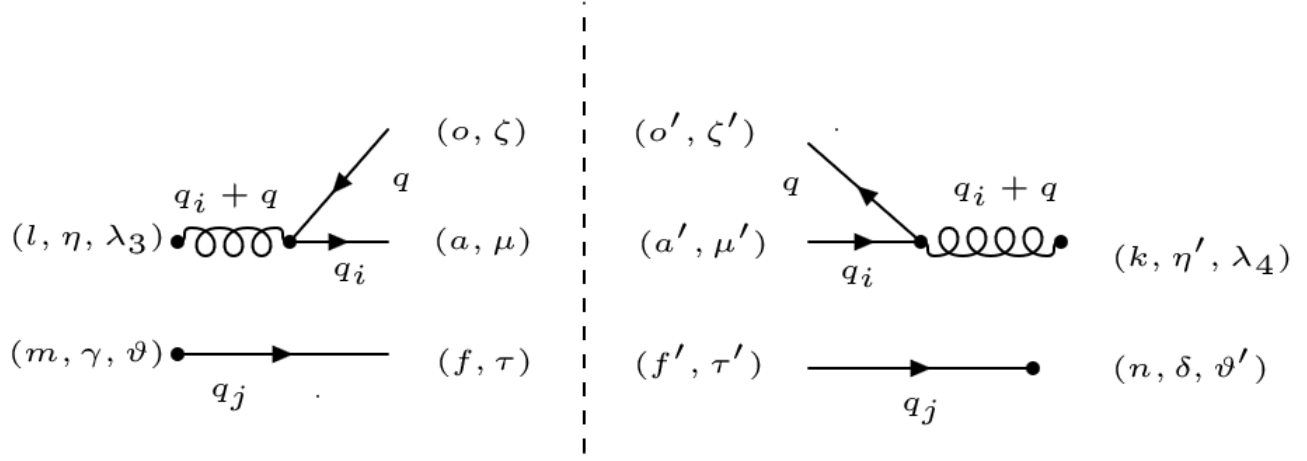
## 2.4 $|M^2|$

## Chapter 3

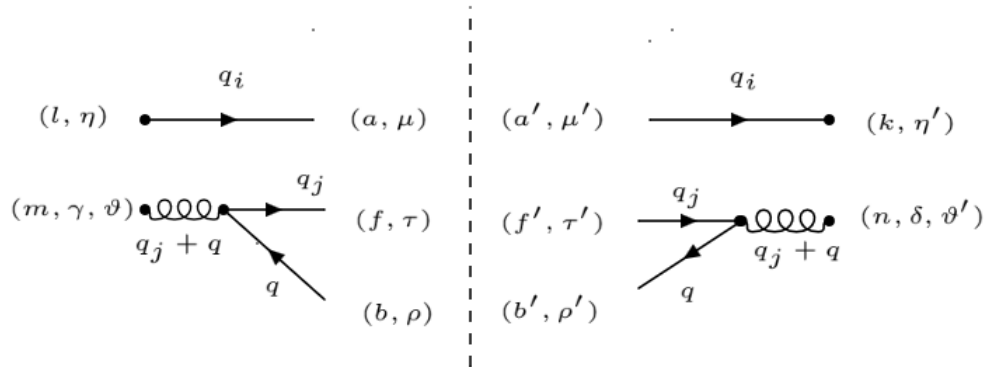
### Quark gluon quark emission kernel



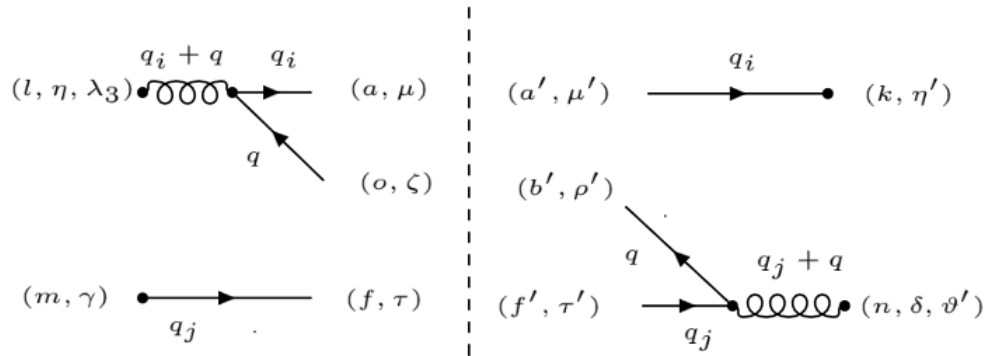
### 3.1 Gluon-Emitter Quark loop



## 3.2 Gluon-Spectator Quark loop

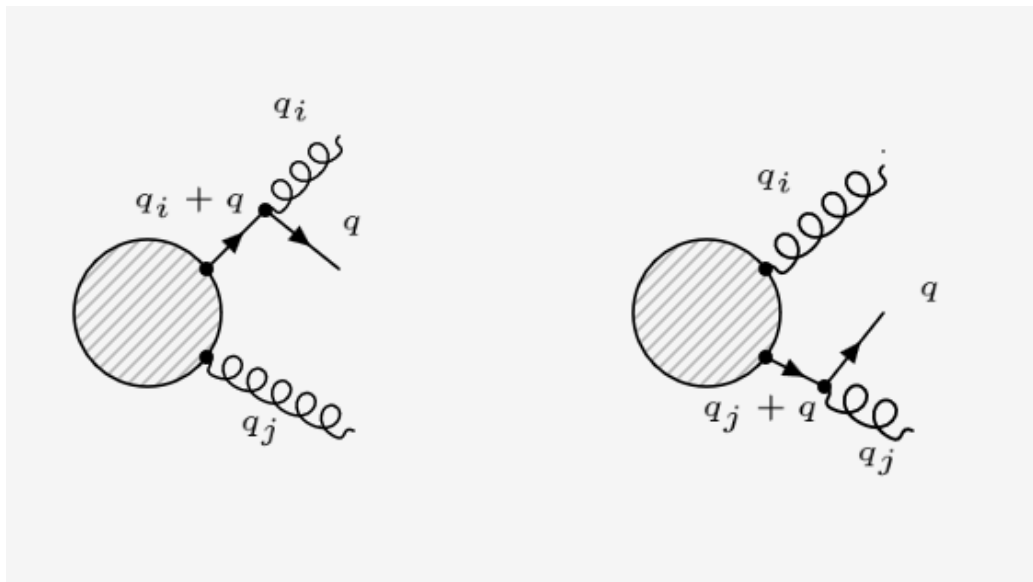


### 3.3 Gluon-Emitter Quark loop

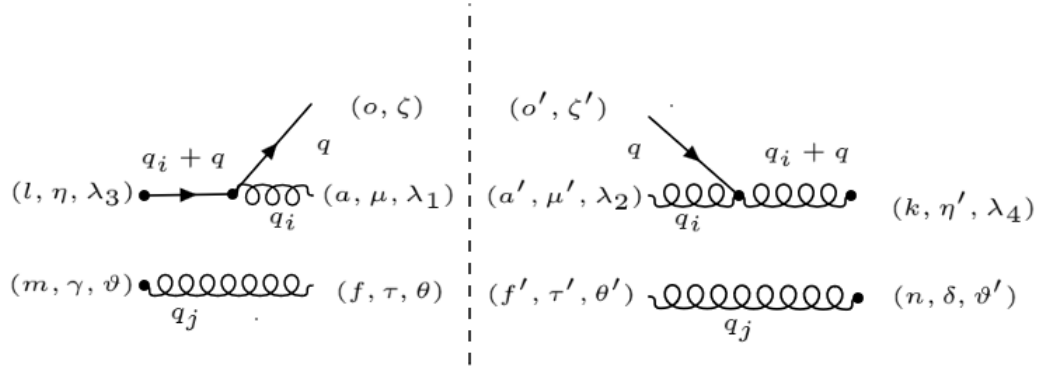


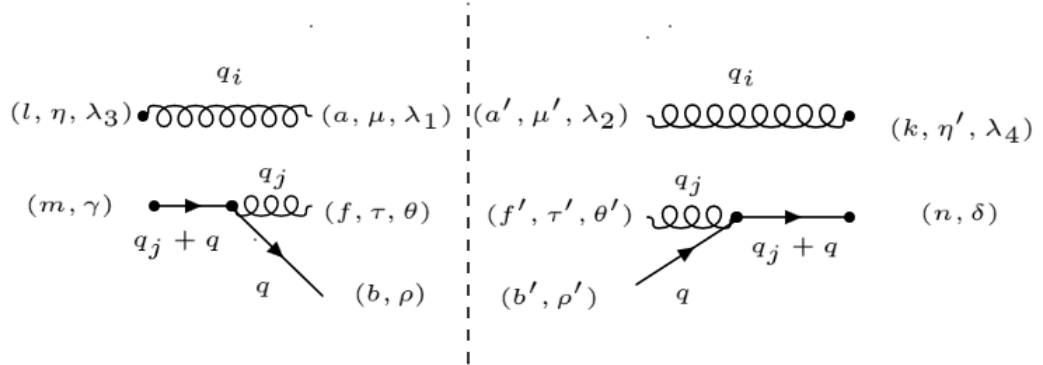
## Chapter 4

### Gluon quark quark emission kernel





4.1  $M_1$ 

4.2  $M_2$ 

4.3  $M_1 M_2^\dagger$ 