Thesis

by

Tigran Saidnia

Emission kernel of parton shower

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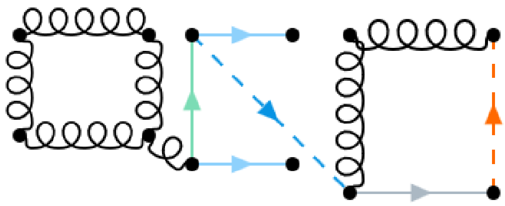

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statement of originality

I hereby confirm that I have written the accompanying thesis by myself, without contributions from any sources other than those cited in the text and acknowledgements. This applies also to all graphics, drawings, maps and images included in the thesis.

Karlsruhe, February 5, 2019

Tigran Saidnia

Contents

0.1 Old parametrisation

$$\left.\begin{aligned}
q_i{}^\mu &= zp_i{}^\mu + y(1-z)p_j{}^\mu + \sqrt{zy(1-z)}m_\perp \\
q^\mu &= (1-z)p_i{}^\mu + yzp_j{}^\mu - \sqrt{zy(1-z)}m_\perp \\
q_j{}^\mu &= (1-y)p_j{}^\mu \\
y &= \frac{q_i q}{p_i p_j} \\
q_i + q &= p_i + yp_j \\
q_j + q &= (1-z)p_i{}^\mu + (1+yz-y)p_j{}^\mu - \sqrt{zy(1-z)}m_\perp \\
q_i \cdot q &= y(1-2z+2z^2)(p_i \cdot p_j) \\
q_i \cdot q_j &= z(1-y)(p_i \cdot p_j) \\
q_j \cdot q &= (1-z)(1-y)(p_i \cdot p_j)
\end{aligned}\right\} \text{parametrisation} \tag{1}$$

0.2 new kinematic

$$\begin{aligned}
k_l{}^\mu &= \alpha_l \alpha \Lambda^\mu{}_\nu p_i{}^\nu + y\beta n^\mu + \sqrt{y\alpha_l\beta_l} n^\mu{}_{\perp,l} \qquad l = 1, ..., m \\
q_i{}^\mu &= (1 - \sum_{l=1}^{m} \alpha_l)\alpha\Lambda^\mu{}_\nu p_i{}^\nu + y(1 - \sum_{l=1}^{m} \beta_l) n^\mu - \sqrt{y\alpha_l\beta_l} n^\mu{}_{\perp,l} \\
q_k{}^\mu &= \alpha\Lambda^\mu{}_\nu p_k{}^\nu \qquad k = 1, ..., n \qquad k \neq i
\end{aligned} \tag{2}$$

0.2.1 useful relations

$$\begin{aligned}
q_i{}^2 = p_i{}^2 = q_k{}^2 = k_l^2 = p_j^2 = p_k^2 = n^2 &= 0 \quad \text{All hard momenta are on-shell} \\
Q^\mu &= q_i{}^\mu + \sum_{l=1}^{m} k_l^\mu + \sum_{k=1}^{m} q_k^\mu = p_i{}^\mu + \sum_{k=1}^{m} p_k^\mu \quad \text{total momentum} \\
n^\mu &= Q^\mu - \frac{Q^2}{2p_i \cdot Q} p_i^\mu \qquad n^\mu \text{ is the recoil} \\
q_i{}^\mu + \sum_{l=1}^{m} k_l^\mu &= \alpha\Lambda^\mu{}_\nu p_i{}^\nu + yn^\mu \\
\alpha\Lambda^\mu{}_\nu Q^\nu &= Q^\mu - yn^\mu \\
n^\mu{}_{\perp,l}\Lambda^\mu{}_\nu p_i{}^\nu &= n_{\perp,l} \cdot n = n_{\perp,l} \cdot Q = 0 \\
n^\mu{}_{\perp,l} \cdot p_k &\neq 0 \\
n_{\perp,l}^2 &= -2\alpha\Lambda^\mu{}_\nu p_i{}^\nu n_\mu \\
n_{\perp,1}^2 &= -2p_i \cdot Q \\
\alpha_1 &= 1 - \beta_1 \\
\alpha &= \sqrt{1-y}
\end{aligned} \tag{3}$$

Lorenz trafo

$$\alpha\Lambda^{\mu}{}_{\nu} = {p_i}^{\mu} p_{i\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + {p_i}^{\mu} Q_{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu} \frac{(-y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu} \tag{4}$$

$$\hat{p_i}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_i}^{\nu} = {p_i}^{\mu} p_{i\nu}{p_i}^{\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + {p_i}^{\mu} Q_{\nu}{p_i}^{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu}{p_i}^{\nu} \frac{(-y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu}{p_i}^{\nu} \tag{5}$$

$$\begin{aligned}\hat{p_i}^{\mu} &= {p_i}^{\mu}(Q\cdot p_i)\frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}{p_i}^{\mu} \\ &= {p_i}^{\mu}[\frac{y(1+\sqrt{1-y})}{(2+2\sqrt{1-y}-y)} + \sqrt{1-y}] = {p_i}^{\mu}\end{aligned} \tag{6}$$

$$\boxed{\hat{p_i}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_i}^{\nu} = {p_i}^{\mu}} \tag{7}$$

$$\hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_k}^{\nu} = {p_i}^{\mu} p_{i\nu}{p_k}^{\nu} \frac{-y^2Q^2}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + {p_i}^{\mu} Q_{\nu}{p_k}^{\nu} \frac{y(1+\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + Q^{\mu} p_{i\nu}{p_k}^{\nu} \frac{(-y^2-y-y\sqrt{1-y})}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}\eta^{\mu}{}_{\nu}{p_k}^{\nu} \tag{8}$$

$$\begin{aligned}\hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_k}^{\nu} &= {p_i}^{\mu}[\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\ &+ Q^{\mu}[\frac{(-y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}{p_k}^{\mu}\end{aligned} \tag{9}$$

$$\begin{aligned}&\hat{p_k}^{\mu} = \alpha\Lambda^{\mu}{}_{\nu}{p_k}^{\nu} = {p_i}^{\mu}[\frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\ &\quad + Q^{\mu}[\frac{(-y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}{p_k}^{\mu} \\ &\text{with} \\ &A_1 \equiv \frac{-y^2Q^2(p_i\cdot p_k)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} \\ &A_2 \equiv \frac{(-y^2-y-y\sqrt{1-y})(p_i\cdot p_k)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}\end{aligned} \tag{10}$$

$$\boxed{\hat{p_k}^\mu = A_1\, p_i{}^\mu + A_2\, Q^\mu + \sqrt{1-y} p_k{}^\mu} \tag{11}$$

$$\begin{aligned}
\hat{Q}^\mu = \alpha\Lambda^\mu{}_\nu Q^\nu = p_i{}^\mu[\frac{-y^2Q^2(p_i\cdot Q)}{4(p_i\cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})Q^2}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\
+ Q^\mu[\frac{(-y^2-y-y\sqrt{1-y})(p_i\cdot Q)}{2(p_i\cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] + \sqrt{1-y}Q^\mu \\
\text{with} \\
S_1 \equiv \frac{Q^2}{2p_i\cdot Q}[\frac{-y^2}{2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})}{(1+\sqrt{1-y}-\frac{y}{2})}] = \frac{Q^2}{2p_i\cdot Q}y \\
S_2 \equiv \frac{(-y^2-y-y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + \sqrt{1-y}
\end{aligned} \tag{12}$$

$$\boxed{\hat{Q}^\mu = \tfrac{Q^2}{2p_i\cdot Q} y\, p_i{}^\mu + S_2\, Q^\mu} \tag{13}$$

0.3 Single emission part

$$\begin{aligned}
k_1{}^\mu &= (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\mu + y\beta_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n^\mu{}_{\perp,1} \\
q_i{}^\mu &= (1-\alpha_1-\alpha_1(\frac{Q^2}{2p_i\cdot Q}))p_i{}^\mu + y\alpha_1 n^\mu - \sqrt{y\alpha_1\beta_1} n^\mu{}_{\perp,l} \\
q_k{}^\mu &= \alpha\Lambda^\mu{}_\nu p_k{}^\nu \qquad k=1,...,n \qquad k\neq i
\end{aligned} \tag{14}$$

$$\begin{aligned}
k_1{}^\mu &= \zeta_1 p_i{}^\mu + \lambda_1 Q^\mu + \sqrt{y\alpha_1\beta_1} n^\mu{}_{\perp,1} \\
q_i{}^\mu &= \zeta_q p_i{}^\mu + \lambda_q Q^\mu - \sqrt{y\alpha_1\beta_1} n^\mu{}_{\perp,l} \\
q_k{}^\mu &= A_1 p_i{}^\mu + A_2 Q^\mu + \sqrt{1-y} p^\mu_{k\perp,l}
\end{aligned}$$

0.4 Common scalar products

$$\begin{aligned}
k_1\cdot q_i &= (\zeta_1\lambda_q + \lambda_1\zeta_q)p_i\cdot Q + \lambda_1\lambda_q Q^2 - y\alpha_1\beta_1 n^2{}_{\perp,1} \\
&= [(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i\cdot Q}))y\alpha_1 + y\beta_1(1-\alpha_1-\alpha_1(\frac{Q^2}{2p_i\cdot Q}))]\, p_i\cdot Q \\
&\quad y^2\beta_1\alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_i Q \\
\Rightarrow k_1\cdot q_i &= [y\alpha_1{}^2 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q}) + y\beta_1 - y\alpha_1\beta_1 - y^2\alpha_1\beta_1(\frac{Q^2}{2p_i\cdot Q})]\, p_i\cdot Q \\
&\quad y^2\beta_1\alpha_1\, Q^2 + 2y\alpha_1\beta_1\, p_i Q
\end{aligned} \tag{15}$$

$$\boxed{k_1 \cdot q_i = 2y({\alpha_1}^2 + \beta_1)\, p_i \cdot Q} \tag{16}$$

$$\begin{aligned}
k_1 \cdot q_k &= (\zeta_1 A_2 + \lambda_1 A_1) p_i \cdot Q + \zeta_1 \sqrt{1-y}\, p_i \cdot p_k + \lambda_1 A_2\, Q^2 + \lambda_1 \sqrt{1-y}\, Q \cdot p_k \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1} \\
&= \{[(\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\frac{(-y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}] \\
&+ y\beta_1[\frac{-y^2 Q^2 (p_i \cdot p_k)}{4(p_i \cdot Q)^2(1+\sqrt{1-y}-\frac{y}{2})} + \frac{y(1+\sqrt{1-y})(Q \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}]\}\, p_i \cdot Q \\
&+ (\alpha_1 - y\beta_1(\frac{Q^2}{2p_i \cdot Q}))\sqrt{1-y}\, p_i \cdot p_k + y\beta_1 \frac{(-y^2 - y - y\sqrt{1-y})(p_i \cdot p_k)}{2(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} Q^2 \\
&+ y\beta_1 \sqrt{1-y} Q \cdot p_k + \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{17}$$

$$\begin{aligned}
k_1 \cdot q_k &= \alpha_1 \frac{(-y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i \cdot p_k) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})\frac{(-y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i \cdot p_k) \\
&+ y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})}(p_i \cdot p_k) + y\beta_1 \frac{y(1+\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}\, Q \cdot p_k \\
&+ \alpha_1 \sqrt{1-y}\, p_i \cdot p_k - y\beta_1(\frac{Q^2}{2p_i \cdot Q})\sqrt{1-y}\, p_i \cdot p_k \\
&+ y\beta_1(\frac{Q^2}{2p_i \cdot Q})\frac{(-y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i \cdot p_k) \\
&+ y\beta_1(\frac{Q^2}{2p_i \cdot Q})\frac{(-y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})}(p_i \cdot p_k) + y\beta_1 \sqrt{1-y}(Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{18}$$

$$\begin{aligned}
k_1 \cdot q_k = [&\alpha_1 \frac{(-y^2 - y - y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + y\beta_1 \frac{-y^2 Q^2}{4(p_i \cdot Q)(1+\sqrt{1-y}-\frac{y}{2})} + \alpha_1 \sqrt{1-y} \\
&- y\beta_1(\frac{Q^2}{2p_i \cdot Q})\sqrt{1-y}]\, p_i \cdot p_k + [y\beta_1 \frac{y(1+y\sqrt{1-y})}{2(1+\sqrt{1-y}-\frac{y}{2})} + y\beta_1 \sqrt{1-y}](Q \cdot p_k) \\
&+ \sqrt{\alpha_1 \beta_1 y(1-y)} p_k \cdot n_{\perp,1}
\end{aligned} \tag{19}$$

0.5 Altarelli-Parisi splitting functions

$$\left.\begin{aligned}
\langle \hat{P_{qq}} \rangle &= C_F[\frac{1+z^2}{1-z} - \varepsilon(1-z)] \\
\langle \hat{P_{gq}} \rangle &= T_R[1 - \frac{2z(1-z)}{1-\varepsilon}] \\
\langle \hat{P_{qg}} \rangle &= C_F[\frac{1+(1-z)^2}{z} - \varepsilon z] \\
\langle \hat{P_{gg}} \rangle &= 2C_A[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)]
\end{aligned}\right\} \text{splitting functions} \tag{20}$$

0.6 Colour factor calculation

fundamental representation in $SU(2)$ and $SU(3)$

$$\begin{aligned} T^a = \tau^a \equiv \frac{\sigma^2}{2} \qquad with\ Pauli\ matrices\ \sigma^a \\ T^a = \vartheta^a \equiv \frac{\lambda^2}{2} \qquad with\ Gell-Mann\ matrices\ \lambda^a \end{aligned} \tag{21}$$

$$\begin{gathered} \lambda^1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix} \\ \lambda^5 = \begin{pmatrix} & & -i \\ & 0 & \\ i & & \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{gathered} \tag{22}$$

As we can see, λ^3 and λ^8 are diagonal. These generators satisfy:

$$[T^a, T^b] = i\epsilon^{abc} T^c \tag{23}$$

The most common convention for the normalization of the generators in physics is:

$$\sum_{c,d} f^{acd} f^{bcd} = N\delta^{ab} \tag{24}$$

The main relation we will use later for SU(N):

$$tr(T^a T^b) = T_{ij}{}^a T_{ji}{}^b = T_F \delta^{ab} \tag{25}$$

$$\sum_a (T^a T^a) = C_F \delta^{ij} \tag{26}$$

$$f^{acd} f^{bcd} = C_A \delta^{ab} \tag{27}$$

With $T_F = \frac{1}{2}$, $C_A = N$ and $C_F = \frac{N^2-1}{2N}$.

$$f^{abc} = -2itr(T^a [T^b, T^c]) \tag{28}$$

$$d^{abc} = 2tr(T^a T^b, T^c) \tag{29}$$

$$T^a T^b = \frac{1}{2}(\frac{1}{N}\delta_{ab} + (d^{abc} + if^{abc})T^c) \tag{30}$$

$$tr(T^a T^b T^c) = \frac{1}{4}(d^{abc} + if^{abc}) \tag{31}$$

$$tr(T^aT^bT^aT^c) = \frac{-1}{4N}\delta_{bc} \tag{32}$$

$$f^{acd}f^{bcd} = N\delta^{ab} \tag{33}$$

$$f^{acd}d^{bcd} = 0 \tag{34}$$

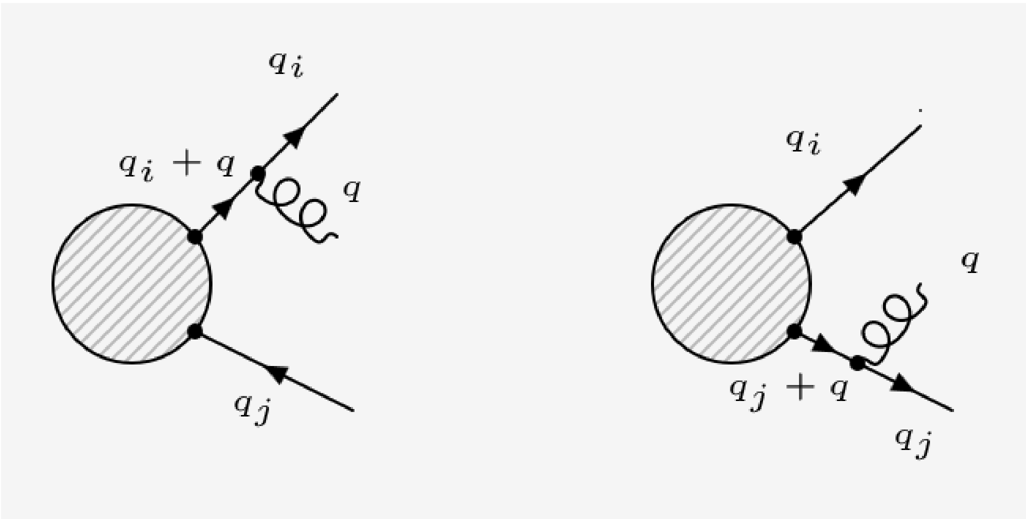
$$f^{ade}f^{bef}f^{cfd} = \frac{N}{2}f^{abc} \tag{35}$$

Fierz identity:

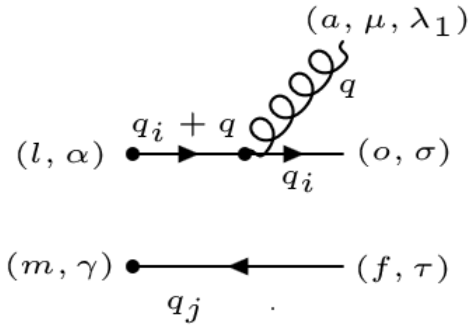
$$\sum_a T_{ij}{}^a T_{kl}{}^a = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}) \tag{36}$$

Chapter 1

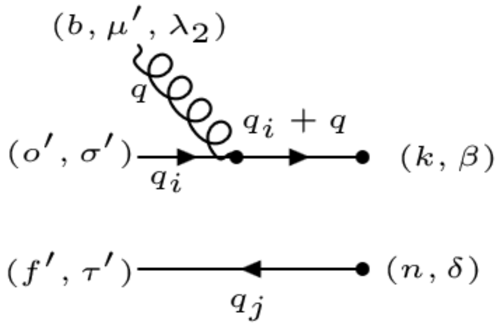
Quark antiquark gluon emission kernel



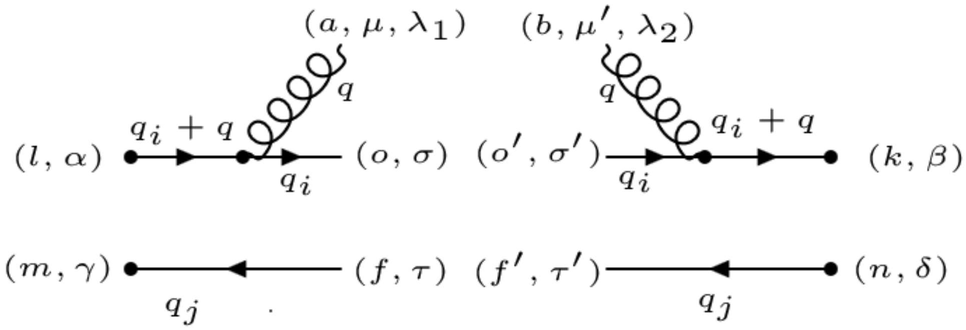
1.1 qg-$\bar{q}$



$$M_1 = [\bar{u}_\sigma(q_i)(-ig_s\gamma^\mu \times [T^a]_o{}^l)\frac{i(\not{q}_i+\not{q})}{(q_i+q)^2}\varepsilon^{\lambda_1}{}_\mu(q)]\,[v_\tau(q_j)] \tag{1.1}$$



$$M_1{}^\dagger = [\frac{-i(\not{q}_i+\not{q})}{(q_i+q)^2}\,(ig_s\gamma^{\mu'} \times [T^b]_{o'}{}^k)\,u_{\sigma'}(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}(q)][\bar{v}_{\tau'}(q_j)] \tag{1.2}$$



$$
\begin{aligned}
|M_1|^2 = M_1\, M_1{}^{\dagger} = [\bar{u}_\sigma(q_i)\, (-i g_s \gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\ \varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)] \\
[\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\, (i g_s \gamma^{\mu'} \times [T^b]_{o'}{}^k)\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2}{}_{\mu'}{}^*(q)][\bar{v}_{\tau'}(q_j)]
\end{aligned} \tag{1.3}
$$

$$
\begin{aligned}
|M_1|^2 = [\frac{-i(\not{q}_i + \not{q})}{(q_i+q)^2}\ (i g_s \gamma^{\mu'} \times [T^b]_{o'}{}^k)\, \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i)\, \varepsilon^{\lambda_2}{}^*_{\mu'}(q) \varepsilon^{\lambda_1}{}_\mu(q) \\
\times(-i g_s \gamma^\mu \times [T^a]_o{}^l)\, \frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}][\bar{v}_{\tau'}(q_j) v_\tau(q_j)]
\end{aligned} \tag{1.4}
$$

and after sum over the lorenz index (σ, σ') and (τ, τ') and unsing the spin addition relation:

$$
\begin{aligned}
\sum_{\sigma,\sigma'} \bar{u}_\sigma(q_i)\, u_{\sigma'}(q_i) = \not{q}_i \delta^{oo'}, \\
\sum_{\tau,\tau'} \bar{v}_\tau(q_j)\, v_{\tau'}(q_j) = \not{q}_j \delta^{ff'}
\end{aligned} \tag{1.5}
$$

and sum over polarization index (λ_1, λ_2) :

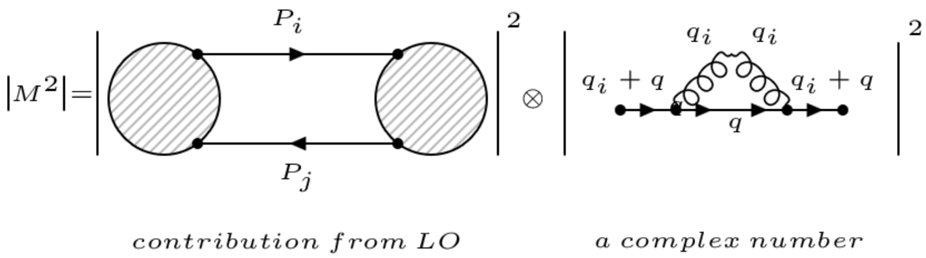
$$
\sum_{\mu,\mu'} \varepsilon^{\lambda_2}{}^*_{\mu'}(q) \varepsilon^{\lambda_1}{}_\mu(q) = -g_{\mu\mu'} \delta^{ab} \tag{1.6}
$$

$$
|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{(q_i+q)^2 (q_i+q)^2} [(\not{q}_i + \not{q})\ \gamma^{\mu'}\ \ \not{q}_i\, g_{\mu'\mu} \gamma^\mu\, (\not{q}_i + q)][\not{q}_j] \tag{1.7}
$$

from here and after contraction between all indices we can actually make statements about the last result.

$$
|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{(q_i+q)^2 (q_i+q)^2} [(\not{q}_i + \not{q})\ \gamma^{\mu'}\ \ \not{q}_i\, \gamma_{\mu'}\, (\not{q}_i + q)][\not{q}_j] \tag{1.8}
$$

In other words we expect the tree level diagram from LO and a number: Which means:



$$|M_1|^2 = \frac{-g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[\not{P}_i][\not{P}_j] \; \otimes \; (\textit{a complex number}) \tag{1.9}$$

Let's calculate the contribution and compare the final result with this expectation:

$$\begin{aligned} N =: \gamma^{\mu'} \; \not{q}_i \, \gamma_{\mu'} &= q_{i\sigma} \, \gamma^{\mu'} \gamma^{\sigma} \; \gamma_{\mu'} \\ &= q_{i\sigma} \, (\{\gamma^{\mu'}, \gamma^{\sigma}\} \; - \gamma^{\sigma}\gamma^{\mu'})\gamma_{\mu'} \\ &= q_{i\sigma} \, 2g^{\mu'\sigma} \, \gamma_{\mu'} \; - \; d \, \gamma^{\sigma} \\ &= (2-d) \; \not{q}_i \end{aligned} \tag{1.10}$$

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q}) \quad \not{q}_i \; (\not{q}_i + q)][\not{q}_j] \tag{1.11}$$

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[\not{q}_i \; \not{q}_i \; \not{q}_i + \; \not{q}_i \; \not{q}_i \; \not{q} + \; \not{q} \; \not{q}_i \; \not{q}_i + \; \not{q} \; \not{q}_i \; \not{q}][\not{q}_j] \tag{1.12}$$

For the momenta are on-shell which means:

$$\begin{aligned} \not{q}_i \; \not{q}_i &= {q_i}^2 = {m_i}^2 \\ \not{q} \; \not{q} &= q^2 = m^2 \\ \not{q}_j \; \not{q}_j &= {q_j}^2 = {m_j}^2 \end{aligned} \tag{1.13}$$

we can first neglect the mass of patrons and ignore each term with $\not{q}_i \; \not{q}_i$ and $\not{q} \; \not{q}$ as well.

$$|M_1|^2 = -(2-d) \; \frac{g_s^2 [T^a]_o{}^k \, [T^a]_o{}^l}{(2q_i q)(2q_i q)}[\not{q} \; \not{q}_i \; \not{q}][\not{q}_j] \tag{1.14}$$

$$\begin{aligned}
L = \not{q}\ \not{q}_i\ \not{q} &= \not{q}[q_{i\sigma} q_\mu \left(\{\gamma^\mu, \gamma^\sigma\} - \gamma^\sigma \gamma^\mu\right)] \\
&\not{q}[2{q_i}^\mu q_\mu - q_{i\sigma} q_\mu \gamma^\mu \gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\gamma^\mu \gamma^\mu \gamma^\sigma] \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [\frac{\gamma^\mu \gamma^\mu}{2} + \frac{\gamma^\mu \gamma^\mu}{2}]\gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q_\mu [g^{\mu\mu}]\gamma^\sigma \\
&= \not{q}(2q_i q) - q_\mu q_{i\sigma} q^\mu \gamma^\sigma \\
&= \not{q}(2q_i q) - q^2\ \not{q}_i \\
&= \not{q}(2q_i q)
\end{aligned} \tag{1.15}$$

After inserting the last result of L and simplify the term $(2q_iq)$ from the denominator and nominator because , we get:

$$|M_1|^2 = -(2-d)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)}[\not{q}][\not{q}_j] \tag{1.16}$$

Now we are going to use the parametrisation from equation (1) to reduce the 3-member matrix element to 2-member and take out the singularity term from the amplitude.

$$|M_1|^2 = (d-2)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)}[(1-z)\ \not{p}_i + zy\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}_\perp][(1-y)\ \not{p}_j] \tag{1.17}$$

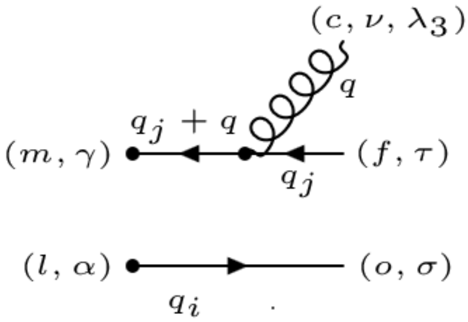
Multiplying the both sides

$$\begin{aligned}
|M_1|^2 = (d-2)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)}&[(1-z)(1-y)\ \not{p}_i\ \not{p}_j \\
&+zy(1-y)\ \not{p}_j\ \not{p}_j + (1-y)\sqrt{zy(1-z)}\ \not{m}_\perp\ \not{p}_j]
\end{aligned} \tag{1.18}$$

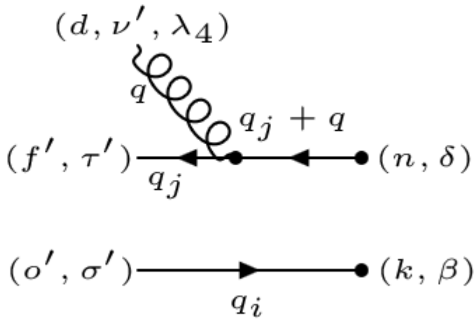
and under consideration of the fact that p_i and p_j are the on-shell momenta of the emitter and spectator partons, we can ignore the terms with $\not{p}_i\ \not{p}_i$ and $\not{p}_j\ \not{p}_j$. The $p_i \cdot m_\perp$ and $p_j \cdot m_\perp$ are always 0 because the p_i and p_j are lightlike, i.e. zero transverse component. So those terms can be neglected.

$$|M_1|^2 = (d-2)(1-z)(1-y)\ \frac{g_s^2 {[T^a]_o}^k\ {[T^a]_o}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.19}$$

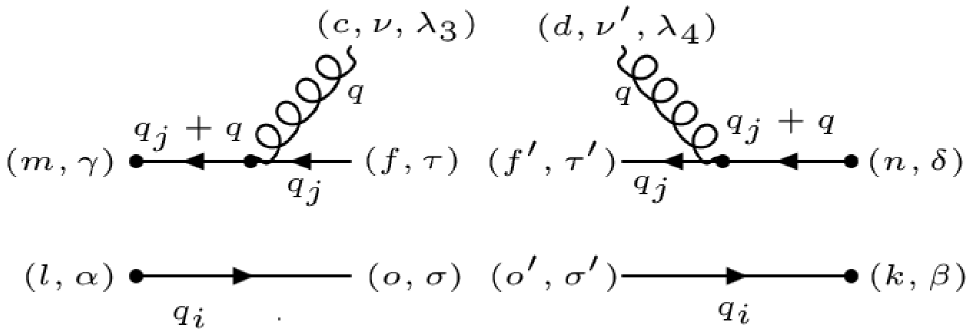
1.2 $\bar{q}$g-q



$$M_2 = [\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^\nu \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \tag{1.20}$$



$$M_2^\dagger = [\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \tag{1.21}$$



$$\begin{aligned} |M_2|^2 = M_2\, {M_2}^\dagger = &[\frac{i(\not{q}_j + \not{q})}{(q_j+q)^2}(-ig_s\gamma^\nu \times [T^c]_f{}^m)\, v_\tau(q_j)\, \varepsilon^{\lambda_3}{}_\nu(q)]\, [u_\sigma(q_i)] \\ &[\bar{v}_{\tau'}(q_j)\, (ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\, \frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\, \varepsilon^{\lambda_4}{}_{\nu'}(q)]\, [\bar{u}_{\sigma'}(q_i)] \end{aligned} \tag{1.22}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^d]_{f'}{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j+\not{q})\gamma^\nu\, v_\tau(q_j)\bar{v}_{\tau'}(q_j)\,\varepsilon^{\lambda_3}{}_\nu(q)\varepsilon^{\lambda_4}{}_{\nu'}(q)\gamma^{\nu'}(\not{q}_j+\not{q})] \\ [u_\sigma(q_i)]\,[\bar{u}_{\sigma'}(q_i)] \tag{1.23}$$

and after sum over the lorenz index (σ,σ') and (τ,τ') and unsing the spin addition relation:

$$\sum_{\sigma,\sigma'}\bar{u}_\sigma(q_i)\,u_{\sigma'}(q_i) = \not{q}_i\delta^{oo'}, \\ \sum_{\tau,\tau'}\bar{v}_\tau(q_j)\,v_{\tau'}(q_j) = \not{q}_j\delta^{ff'} \tag{1.24}$$

and sum over polarization index (λ_3,λ_4) :

$$\sum_{\nu,\nu'}\varepsilon^{\lambda_4}{}^*_{\nu'}(q)\varepsilon^{\lambda_3}{}_\nu(q) = -g_{\nu\nu'}\delta^{cd} \tag{1.25}$$

$$|M_2|^2 = \frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{(q_j+q)^2(q_j+q)^2}[(\not{q}_j+\not{q})\gamma^\nu\ \ \not{q}_j\,(-g_{\nu\nu'})\gamma^{\nu'}(\not{q}_j+\not{q})]\,[\not{q}_i] \tag{1.26}$$

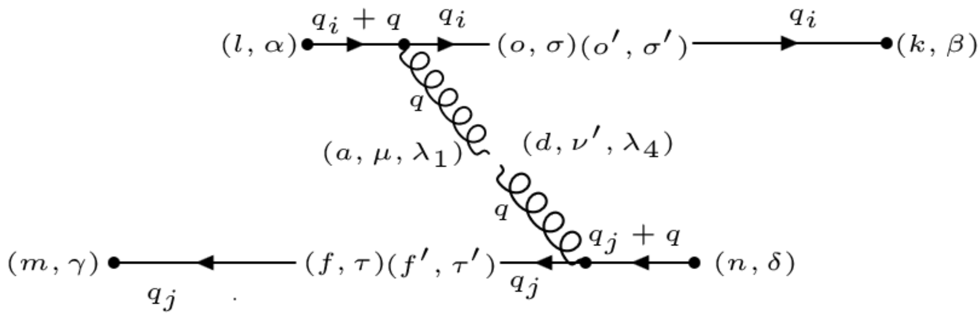
After the same calculation from the last part, we'll get:

$$|M_2|^2 = (d-2)\frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{(2qq_j)}[\not{q}]\,[\not{q}_i] \tag{1.27}$$

finally:

$$|M_2|^2 = -(d-2)yz^2\,\frac{g_s^2\,[T^c]_f{}^m\,[T^c]_f{}^n}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.28}$$

1.3 $M_1 {M_2}^\dagger$



$$M_1\, {M_2}^\dagger = [\bar{u}_\sigma(q_i)\,(-ig_s\gamma^\mu \times [T^a]_o{}^l)\,\frac{i(\not{q}_i + \not{q})}{(q_i+q)^2}\,\varepsilon^{\lambda_1}{}_\mu(q)][v_\tau(q_j)] \\ [\bar{v}_{\tau'}(q_j)\,(ig_s\gamma^{\nu'} \times [T^d]_{f'}{}^n)\,\frac{-i(\not{q}_j + \not{q})}{(q_j+q)^2}\,\varepsilon^{\lambda_4}{}_{\nu'}(q)]\,[u_{\sigma'}(q_i)] \tag{1.29}$$

$$M_1\, {M_2}^\dagger = \frac{g_s^2[T^a]_o{}^l\,[T^d]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,]\varepsilon^{\lambda_1}{}_\mu(q)\,\varepsilon^{\lambda_4}{}_{\nu'}(q) \\ [\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j + \not{q})] \tag{1.30}$$

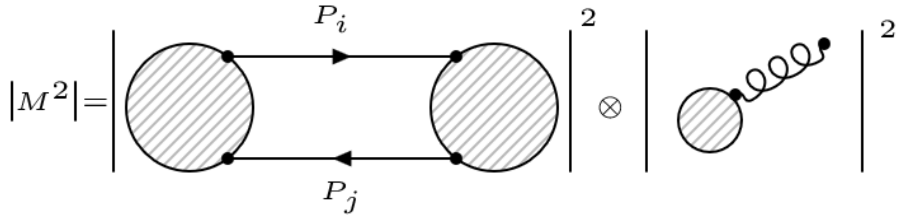
$$M_1\, {M_2}^\dagger = \frac{g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,] - g_{\mu\nu'} \\ [\not{q}_j\,\gamma^{\nu'}\,(\not{q}_j + \not{q})] \tag{1.31}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[\not{q}_i\,\gamma^\mu\,(\not{q}_i + \not{q})\,]\,[\not{q}_j\,\gamma_\mu\,(\not{q}_j + \not{q})] \tag{1.32}$$

Expectation:

$$M_1\, {M_2}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[(\not{q}_i + \not{q})\,\gamma^\mu\,\;\not{q}_i\,]\,[(\not{q}_j + \not{q})\,\gamma_\mu\,\;\not{q}_j] \tag{1.33}$$

$$M_1\, {M_2}^\dagger = \frac{-g_s^2[T^a]_o{}^l\,[T^a]_{f'}{}^n}{(2q_iq)(2q_jq)}[-(\not{q}_i + \not{q})\;\;\not{q}_i\,\gamma^\mu\;+2(\not{q}_i + \not{q})\,q_i{}^\mu] \\ [-(\not{q}_j + \not{q})\;\;\not{q}_j\,\gamma_\mu\;+2(\not{q}_j + \not{q})q_{j\mu}] \tag{1.34}$$



contribution from LO *a complex number*

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} \\ & [(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q})\ \not{q}_j \gamma_\mu] \\ & -2[(\not{q}_i + \not{q})\ \not{q}_i\, \gamma^\mu]\, [(\not{q}_j + \not{q}) q_{j\mu}] \\ & -2[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q})\ \not{q}_j\, \gamma_\mu] \\ & +4[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \tag{1.35}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{-g_s^2 [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(2q_i q)(2q_j q)} \\ & [\not{q}\ \not{q}_i\, \gamma^\mu]\, [\not{q}\ \not{q}_j \gamma_\mu] \\ & -2[\not{q}\ \not{q}_i\, \gamma^\mu]\, [(\not{q} + \not{q}_j)\ q_{j\mu}] \\ & -2[(\not{q}_i + \not{q})\, q_i{}^\mu][\not{q}\ \not{q}_j\, \gamma_\mu] \\ & +4[(\not{q}_i + \not{q})\, q_i{}^\mu][(\not{q}_j + \not{q}) q_{j\mu}] \end{aligned} \tag{1.36}$$

$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \\ & [y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\, \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu] \\ & +4(q_i{}^\mu \cdot q_{j\mu})[(\not{q}_i + \not{q})][(\not{q}_j + \not{q})] \end{aligned} \tag{1.37}$$

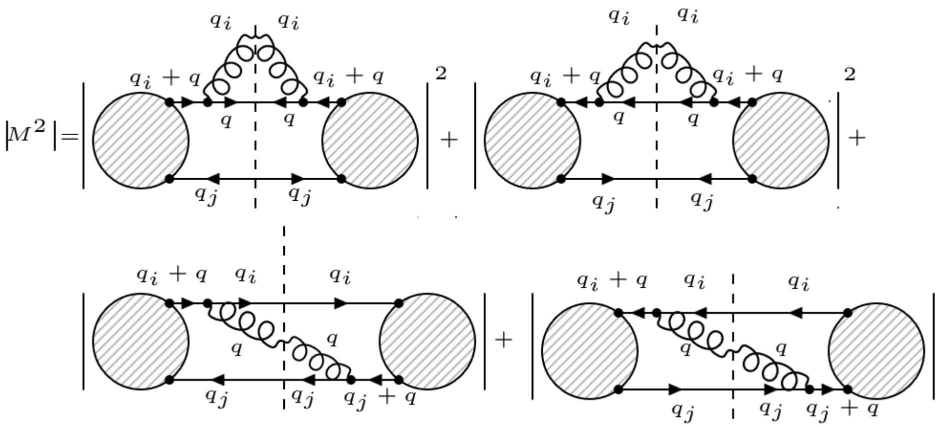
$$\begin{aligned} M_1\, M_2{}^\dagger = & \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{4(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)(p_i \cdot p_j)} \\ & [y(1-2z+2z^2)\ \not{p}_i\ \ \not{p}_j\, \gamma^\mu]\, [(1-z)(1-y)\ \not{p}_i\ \not{p}_j \gamma_\mu] \\ & +4(p_i \cdot p_j)[(\not{p}_i + y\ \not{p}_j)][(1-z)\ \not{p}_i + (1+yz-y)\ \not{p}_j - \sqrt{zy(1-z)}\ \not{m}] \end{aligned} \tag{1.38}$$

$$M_1\, M_2{}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)(1-y)y(1-2z+2z^2)(p_i \cdot p_j)} z(1-y)[\not{p}_i][\not{p}_j] \tag{1.39}$$

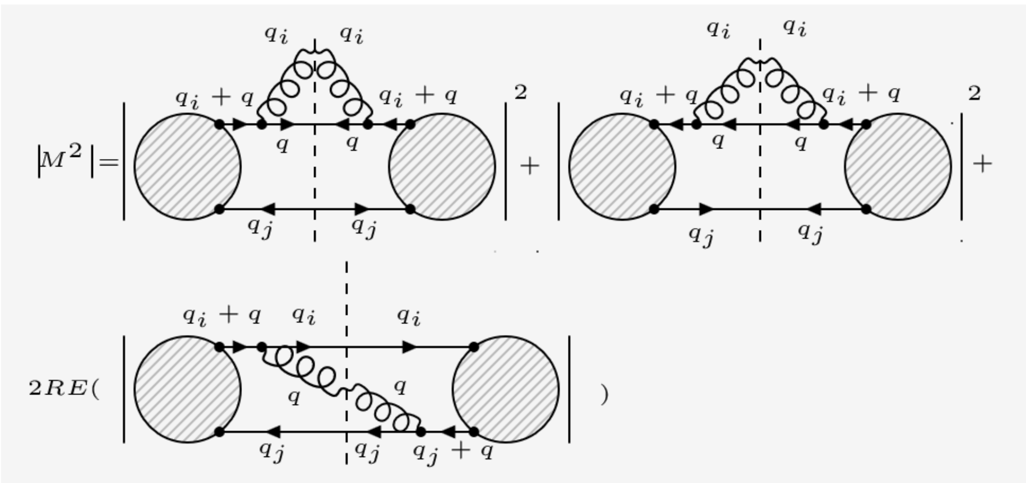
$$M_1\, M_2{}^\dagger = \frac{-g_s^2\ [T^a]_o{}^l\, [T^a]_{f'}{}^n}{(1-z)y(1-2z+2z^2)(p_i \cdot p_j)} z[\not{p}_i][\not{p}_j] \tag{1.40}$$

1.4 $|M^2|$

$$| M|^2 = | M_1|^2 + | M_2|^2 + M_1\, M_2{}^{\dagger} + M_1{}^{\dagger} M_2 \tag{1.41}$$



$$| M|^2 = | M_1|^2 + | M_2|^2 + 2RE(M_1\, M_2{}^{\dagger}) \tag{1.42}$$



$$\begin{aligned}
| M|^2 = (d-2)(1-z)(1-y)\, & \frac{g_s^2 [T^a]_o{}^k\, [T^a]_o{}^l}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
-(d-2)yz^2\, & \frac{g_s^2\, [T^c]_f{}^m\, [T^c]_f{}^n}{2(1-z)(1-y)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j] \\
+2RE((\frac{-2z}{z-1}) & \frac{g_s^2\, [T^a]_o{}^l\, [T^a]_f{}^n}{2y(1-2z+2z^2)(p_i \cdot p_j)} [\not{p}_i][\not{p}_j])
\end{aligned} \tag{1.43}$$

$$T^a{}_{o\,k}\, T^a{}_{l\,o} = \frac{1}{2}(\delta_{oo}\delta_{lk} - \frac{1}{N}\delta_{ok}\delta_{lo}) = \frac{1}{2}(N\delta_{lk} - \frac{1}{N}\delta_{lk}) = C_F \delta_{lk} \tag{1.44}$$

After summation over the final colour states and averaging over initial colour states we get:

$$T^a{}_{o\,k}\,T^a{}_{l\,o} = C_F\delta_{lk} = \frac{1}{N}\sum_{l=1}^{N}\delta_{lk}C_F = C_F \tag{1.45}$$

The same calculation for $T^c{}_{m\,f}\,T^c{}_{f\,n}$ and $T^a{}_{o\,l}\,T^a{}_{f\,n}$ turns C_F out as the colour factor. Now we are going to compute the splitting function in the case of the colinearity, wich means, if:

$$y \longrightarrow 0 \tag{1.46}$$

$$\begin{aligned}
|\,M|^2 = (d-2)(1-z)(1-y)\,&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
-(d-2)yz^2\,&\frac{g_s^2\,C_F}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
+2RE((\frac{-2z}{z-1})&\frac{g_s^2 C_F}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j]
\end{aligned} \tag{1.47}$$

$$|\,M|^2 = C_F((d-2)(1-z) - \frac{4z}{z-1})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \tag{1.48}$$

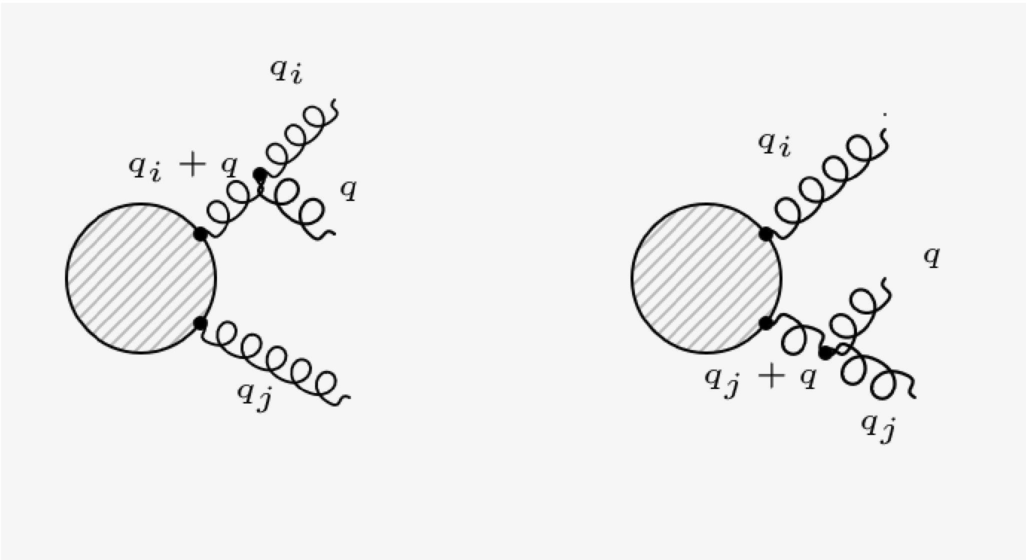
for

$$d = 4-2\epsilon \tag{1.49}$$

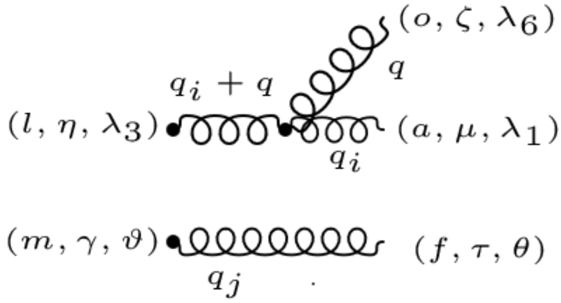
$$\begin{aligned}
|\,M|^2 &= C_F((4-2\epsilon-2)(1-z)+\frac{4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
&= C_F(\frac{2(1-\epsilon)(1-z)^2+4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
&C_F(\frac{2-4z+2z^2-\epsilon(1-z)^2+4z}{1-z})\,\frac{g_s^2}{2y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
&= C_F(\frac{(1+z^2)}{1-z}-\epsilon(1-z))\,\frac{g_s^2}{y(1-2z+2z^2)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \\
&= \langle\,\hat{P}_{qq}\rangle\,\frac{g_s^2}{q_i\cdot q}[\not{p}_i][\not{p}_j]
\end{aligned} \tag{1.50}$$

Chapter 2

Gluon gluon gluon emission kernel

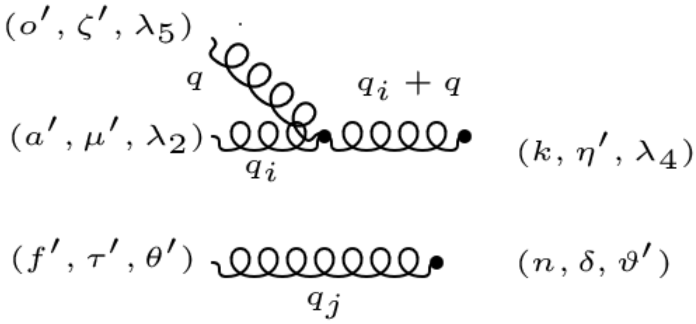


2.1 Gluon-Emitter Bubble



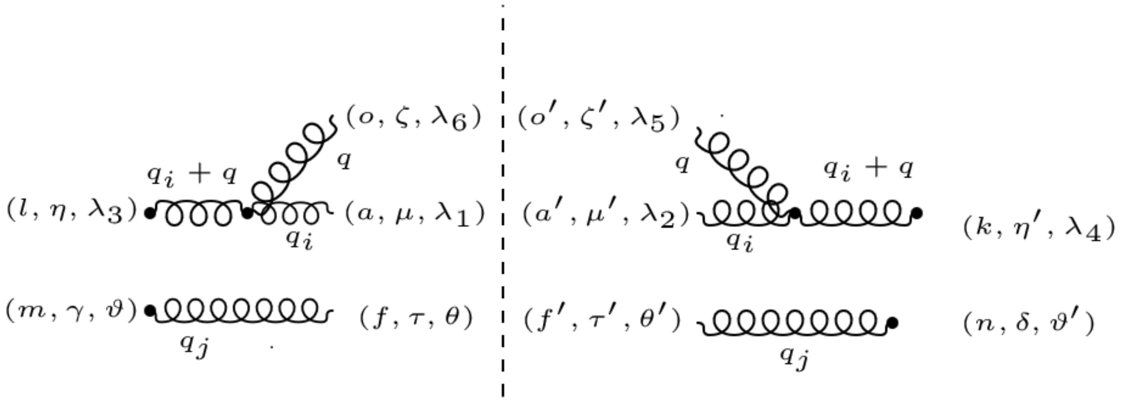
$$M_1 = [\frac{-i}{(q+q_i)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta + g^{\zeta\eta}(-q-(q+q_i))^\mu + g^{\eta\mu}(q_i+q_i+q)^\zeta) \\ \varepsilon^{\lambda_1}{}_\mu(q)\varepsilon^{\lambda_6}{}_\zeta(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \quad (2.1)$$

$$M_1 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q+q_i)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\ \varepsilon^{\lambda_1}{}_\mu(q_i)\varepsilon^{\lambda_6}{}_\zeta(q)][\varepsilon^{\theta}{}_{\tau'}(q_j)] \quad (2.2)$$



$$M_1{}^\dagger = [\frac{i}{(q_i+q)^2}(-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'}) \\ \varepsilon^{\lambda_2}{}_{\mu'}{}^*(q_i)\varepsilon^{\lambda_5}{}_{\zeta'}{}^*(q)][\varepsilon^{\theta'}{}_{\tau'}{}^*(q_j)] \quad (2.3)$$

$$|M_1|^2 = [\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,o\,l}(g^{\mu\zeta}(q-q_i)^\eta - g^{\zeta\eta}(2q+q_i)^\mu + g^{\eta\mu}(2q_i+q)^\zeta) \\ \varepsilon^{\lambda_1}{}_\mu(q_i)\,\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q_i)\varepsilon^{\lambda_6}{}_\zeta(q)\,\varepsilon^{\lambda_5}{}_{\zeta'}{}^*(q) \quad (2.4) \\ (-g_s f^{\,a'\,k\,o'}(-g^{\mu'\eta'}(2q_i+q)^{\zeta'} + g^{\eta'\zeta'}(2q+q_i)^{\mu'} + g^{\zeta'\mu'}(q_i-q)^{\eta'})\frac{i}{(q_i+q)^2}][g_{\gamma\delta}]$$



$$\begin{aligned}N \equiv g_{\mu\mu'}g_{\zeta\zeta'}[&-g^{\mu\zeta}g^{\mu'\eta'}(q-q_i)^\eta(2q_i+q)^{\zeta'}+g^{\mu\zeta}g^{\eta'\zeta'}(q-q_i)^\eta(2q+q_i)^{\mu'}\\&+g^{\mu\zeta}g^{\zeta'\mu'}(q-q_i)^\eta(q_i-q)^{\eta'}+g^{\zeta\eta}g^{\mu'\zeta'}(2q+q_i)^\mu(2q_i+q)^{\zeta'}\\&-g^{\zeta\eta}g^{\eta'\zeta'}(2q+q_i)^\mu(2q+q_i)^{\mu'}-g^{\zeta\eta}g^{\zeta'\mu'}(2q+q_i)^\mu(q_i-q)^{\eta'}\\&-g^{\eta\mu}g^{\mu'\eta'}(2q_i+q)^\zeta(2q_i+q)^{\zeta'}+g^{\eta\mu}g^{\eta'\zeta'}(2q_i+q)^\zeta(2q+q_i)^{\mu'}\\&+g^{\eta\mu}g^{\zeta'\mu'}(2q_i+q)^\zeta(q_i-q)^{\eta'}][g_{\gamma\delta}]\end{aligned} \tag{2.5}$$

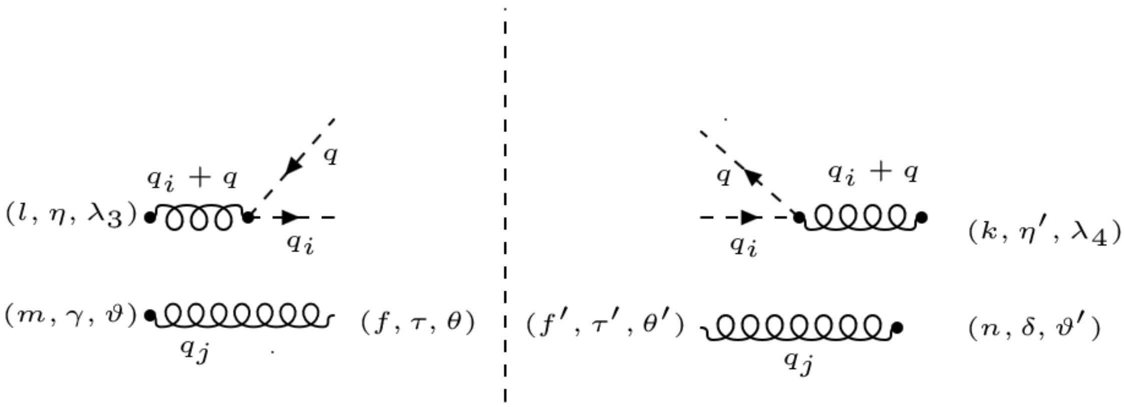
$$\begin{aligned}N \equiv [&-(q-q_i)^\eta(2q_i+q)^{\eta'}+(q-q_i)^\eta(2q+q_i)^{\eta'}+d(q-q_i)^\eta(q_i-q)^{\eta'}\\&+(2q+q_i)^{\eta'}(2q_i+q)^\eta-g^{\eta\eta'}(2q+q_i)^\mu(2q+q_i)_\mu-(2q+q_i)^\eta(q_i-q)^{\eta'}\\&-g^{\eta\eta'}(2q_i+q)^\zeta(2q_i+q)_\zeta+(2q_i+q)^{\eta'}(2q+q_i)^\eta+(2q_i+q)^\eta(q_i-q)^{\eta'}][g_{\gamma\delta}]\end{aligned} \tag{2.6}$$

$$\begin{aligned}N \equiv [&-(q^\eta q^{\eta'}+2q^\eta {q_i}^{\eta'}-{q_i}^\eta q^{\eta'}-2{q_i}^\eta {q_i}^{\eta'})+(2q^\eta q^{\eta'}+q^\eta {q_i}^{\eta'}-2{q_i}^\eta q^{\eta'}-{q_i}^\eta {q_i}^{\eta'})\\&+(dq^\eta {q_i}^{\eta'}-dq^\eta q^{\eta'}-d{q_i}^\eta {q_i}^{\eta'}+d{q_i}^\eta q^{\eta'})+(4q^{\eta'}{q_i}^\eta+2q^{\eta'}q^\eta+2{q_i}^{\eta'}{q_i}^\eta+{q_i}^{\eta'}q^\eta)\\&-(-2q^\eta q^{\eta'}+2q^\eta {q_i}^{\eta'}-{q_i}^\eta q^{\eta'}+{q_i}^\eta {q_i}^{\eta'})+(2q^{\eta'}q^\eta+q^{\eta'}{q_i}^\eta+4{q_i}^{\eta'}q^\eta+2{q_i}^{\eta'}{q_i}^\eta)\\&+(-q^\eta q^{\eta'}+q^\eta {q_i}^{\eta'}-2{q_i}^\eta q^{\eta'}+2{q_i}^\eta {q_i}^{\eta'})-g^{\eta\eta'}(5q^2+5{q_i}^2+8qq_i)][g_{\gamma\delta}]\end{aligned} \tag{2.7}$$

$$\begin{aligned}N \equiv [&(6-d)q^\eta q^{\eta'}+(d+3)q^\eta {q_i}^{\eta'}+(d+3){q_i}^\eta q^{\eta'}+(6-d){q_i}^\eta {q_i}^{\eta'}\\&-g^{\eta\eta'}(5q^2+5{q_i}^2+8qq_i)][g_{\gamma\delta}]\end{aligned} \tag{2.8}$$

$$\begin{aligned}|M_1|^2 = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[&(6-d)q^\eta q^{\eta'}+(d+3)q^\eta {q_i}^{\eta'}+(d+3){q_i}^\eta q^{\eta'}+(6-d){q_i}^\eta {q_i}^{\eta'}\\&-g^{\eta\eta'}(5q^2+5{q_i}^2+8qq_i)][g_{\gamma\delta}]\end{aligned} \tag{2.9}$$

2.1.1 One-loop corrections to the gluon self-energy diagram(Gluon-Emitter Bubble)

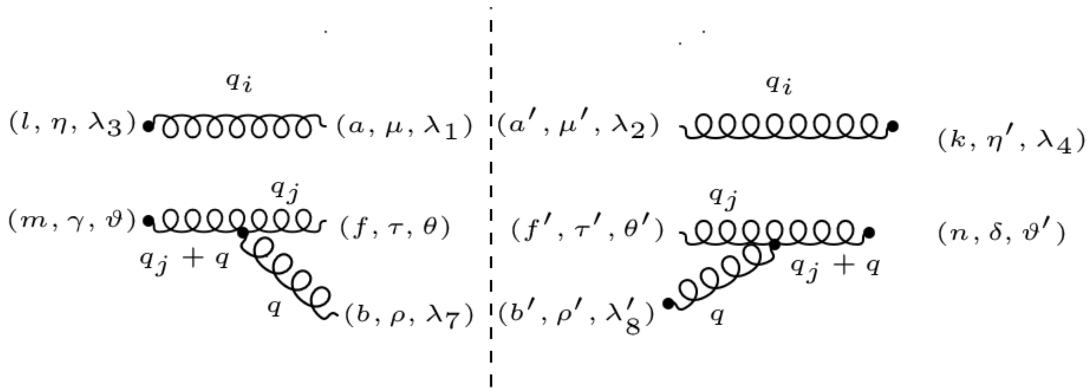


$$|M_1|^2_{Ghost\, loop} = \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[-{q_i}^{\eta} q^{\eta'} - q^{\eta} {q_i}^{\eta'}][g_{\gamma\delta}] \tag{2.10}$$

$$\begin{aligned} |M_1'|^2 &= |M_1|^2 + |M_1|^2_{Ghost\, loop} \\ &= \frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[(6-d)q^{\eta} q^{\eta'} + (d+3) q^{\eta} {q_i}^{\eta'} \\ &+(d+3){q_i}^{\eta} q^{\eta'} + (6-d){q_i}^{\eta}{q_i}^{\eta'} - g^{\eta\eta'}(5q^2 + 5{q_i}^2 + 8qq_i) - {q_i}^{\eta} q^{\eta'} - q^{\eta}{q_i}^{\eta'}][g_{\gamma\delta}] \end{aligned} \tag{2.11}$$

$$\begin{aligned} |M_1'|^2 = &\frac{g_s^2\, f^{\,a\,o\,l}\, f^{\,a\,k\,o}}{(q_i+q)^2(q_i+q)^2}[(6-d)q^{\eta} q^{\eta'} + (d+2) q^{\eta} {q_i}^{\eta'} \\ &+(d+2){q_i}^{\eta} q^{\eta'} + (6-d){q_i}^{\eta}{q_i}^{\eta'} - g^{\eta\eta'}(5q^2 + 5{q_i}^2 + 8qq_i)][g_{\gamma\delta}] \end{aligned} \tag{2.12}$$

2.2 Gluon-Spectator Bubble



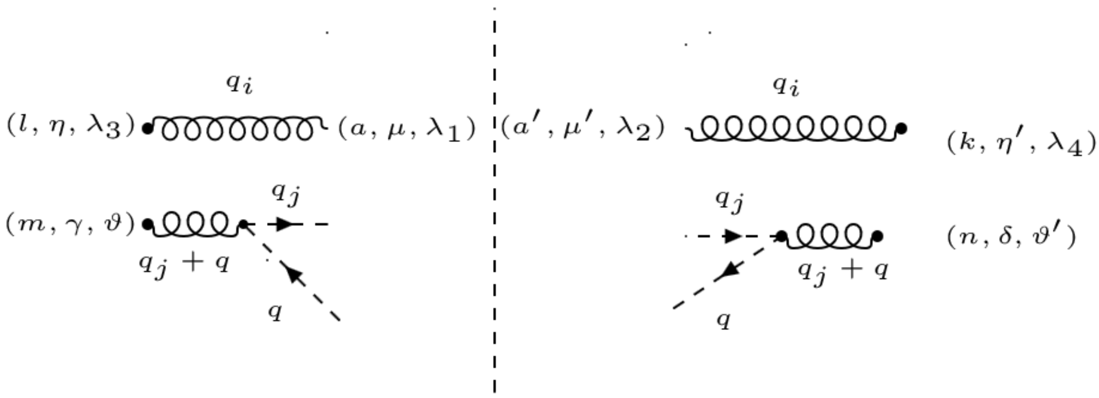
$$|M_2|^2 = [\frac{-i}{(q_j+q)^2}(-g_s f^{\,b\,f\,m}(g^{\tau\gamma}(-2q_j - q)^\rho + g^{\gamma\rho}(2q+q_j)^\tau + g^{\rho\tau}(q_j-q)^\gamma)$$
$$g_{\tau\tau'}g_{\rho\rho'}(-g_s f^{\,b'\,n\,f'}(g^{\rho'\delta}(-2q-q_j)^{\tau'} + g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\tau'\rho'}(q-q_j)^\delta)\frac{i}{(q_j+q)^2}][g_{\eta\eta'}] \tag{2.13}$$

$$|M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b'\,n\,f'}\delta^{aa'}\delta^{ff'}\delta^{bb'}}{(q_j+q)^2(q_j+q)^2}[g_{\tau\tau'}g_{\rho\rho'}(g^{\tau\gamma}(2q_j+q)^\rho g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$-g^{\tau\gamma}(2q_j+q)^\rho g^{\delta\tau'}(2q_j+q)^{\rho'} - g^{\tau\gamma}(2q_j+q)^\rho g^{\tau'\rho'}(q-q_j)^\delta - g^{\gamma\rho}(2q+q_j)^\tau g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$+g^{\gamma\rho}(2q+q_j)^\tau g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\gamma\rho}(2q+q_j)^\tau g^{\tau'\rho'}(q-q_j)^\delta - g^{\rho\tau}(q_j-q)^\gamma g^{\rho'\delta}(2q+q_j)^{\tau'}$$
$$+g^{\rho\tau}(q_j-q)^\gamma g^{\delta\tau'}(2q_j+q)^{\rho'} + g^{\rho\tau}(q_j-q)^\gamma g^{\tau'\rho'}(q-q_j)^\delta][g_{\eta\eta'}] \tag{2.14}$$

$$\begin{aligned} |M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[(2q+q_j)^\gamma(2q_j+q)^\delta \\ -g^{\delta\gamma}(2q_j+q)^\rho(2q_j+q)_\rho - (2q_j+q)^\gamma(q-q_j)^\delta - g^{\delta\gamma}(2q+q_j)^\tau(2q+q_j)_\tau \\ +(2q_j+q)^\gamma(2q+q_j)^\delta + (2q+q_j)^\gamma(q-q_j)^\delta - (q_j-q)^\gamma(2q+q_j)^\delta \\ +(q_j-q)^\gamma(2q_j+q)^\delta + d(q_j-q)^\gamma(q-q_j)^\delta][g_{\eta\eta'}] \end{aligned} \tag{2.15}$$

$$\begin{aligned} |M_2|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[(3+d)q^\gamma {q_j}^\delta + (6-d)q^\gamma q^\delta \\ +(6-d){q_j}^\gamma {q_j}^\delta + (3+d){q_j}^\gamma q^\delta - g^{\delta\gamma}(5{q_j}^2 + 5q^2 + 8qq_j) \\ ][g_{\eta\eta'}] \end{aligned} \tag{2.16}$$

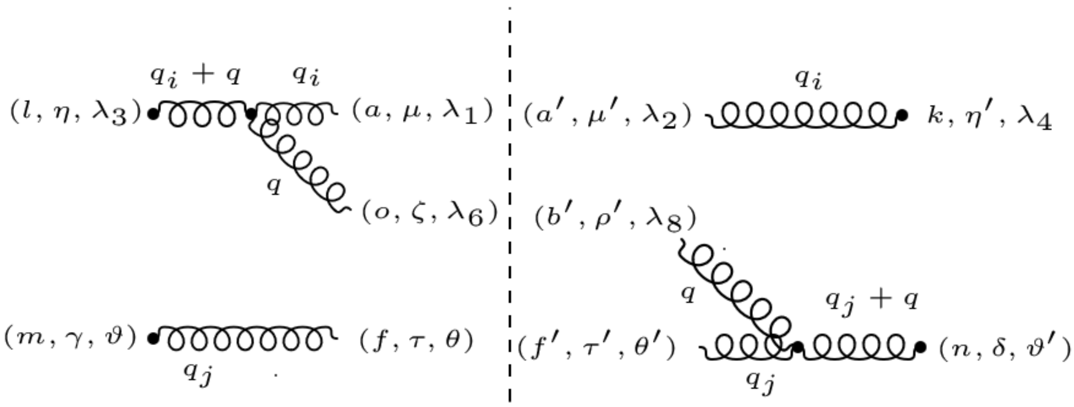
2.2.1 One-loop corrections to the gluon self-energy diagram (Gluon-Spectator Bubble)



$$|M_1|^2_{Ghost\ loop} = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[-{q_j}^{\gamma}q^{\delta} - q^{\delta}{q_j}^{\gamma}][g_{\gamma\delta}] \tag{2.17}$$

$$\begin{aligned}|M_2'|^2 = \frac{g_s^2\, f^{\,b\,f\,m} f^{\,b\,n\,f}}{(q_j+q)^2(q_j+q)^2}[&(2+d)q^{\gamma}{q_j}^{\delta} + (6-d)q^{\gamma}q^{\delta} \\ +(6-d){q_j}^{\gamma}{q_j}^{\delta} + (2+d){q_j}^{\gamma}q^{\delta} &- g^{\delta\gamma}(5{q_j}^2 + 5q^2 + 8qq_j) \\ &][g_{\eta\eta'}]\end{aligned} \tag{2.18}$$

2.3 Interference term $M_1{M_2}^{\dagger}$



$$
\begin{aligned}
M_1 M_2{}^\dagger = [&\frac{-i}{(q_i+q)^2}(-g_s f^{\,a\,l\,o}(-g^{\mu\eta}(2q_i+q)^\zeta + g^{\eta\zeta}(2q+q_i)^\mu + g^{\zeta\mu}(q_i-q)^\eta) \\
&\varepsilon^{\lambda_1}{}_\mu(q_i)\varepsilon^{\lambda_6}{}_\zeta(q)][\varepsilon^\theta{}_\tau{}^*(q_j)] \\
[&\frac{i}{(q+q_j)^2}(-g_s f^{\,f'\,n\,b'}(-g^{\tau'\delta}(2q_j+q)^{\rho'} + g^{\delta\rho'}(2q+q_j)^{\tau'} + g^{\rho'\tau'}(q_j-q)^\delta) \\
&\varepsilon^{\theta'}{}_{\tau'}{}^*(q_j)\varepsilon^{\lambda_8}{}_{\rho'}{}^*(q)][\varepsilon^{\lambda_2}{}_{\mu'}{}^*(q_i)]
\end{aligned}
\tag{2.19}
$$

$$
\begin{aligned}
M_1 M_2{}^\dagger = &\frac{g_s^2 f^{\,a\,l\,o} f^{\,f'\,n\,b'}\delta^{aa'}\delta^{ob'}\delta^{ff'}}{(q_i+q)^2(q_j+q)^2}[g_{\mu\mu'}g_{\zeta\rho'}g_{\tau\tau'} \\
&(-g^{\mu\eta}(2q_i+q)^\zeta + g^{\eta\zeta}(2q+q_i)^\mu + g^{\zeta\mu}(q_i-q)^\eta) \\
&(-g^{\tau'\delta}(2q_j+q)^{\rho'} + g^{\delta\rho'}(2q+q_j)^{\tau'} + g^{\rho'\tau'}(q_j-q)^\delta)]
\end{aligned}
\tag{2.20}
$$

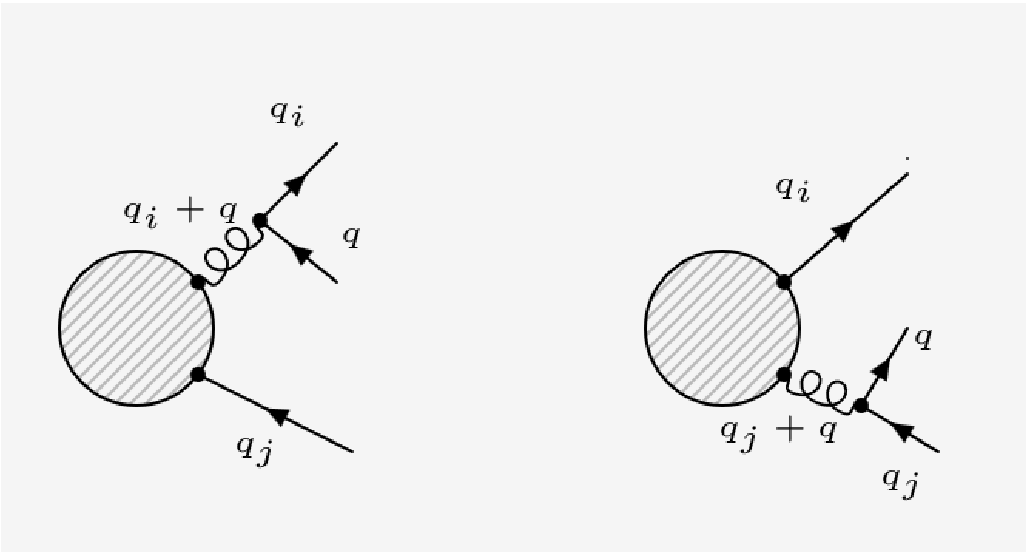
$$
\begin{aligned}
M_1 M_2{}^\dagger = &\frac{g_s^2 f^{\,a\,l\,o} f^{\,f\,n\,o}}{(q_i+q)^2(q_j+q)^2}[g_{\mu\mu'}g_{\zeta\rho'}g_{\tau\tau'} \\
&(g^{\mu\eta}(2q_i+q)^\zeta g^{\tau'\delta}(2q_j+q)^{\rho'} - g^{\mu\eta}(2q_i+q)^\zeta g^{\delta\rho'}(2q+q_j)^{\tau'} - g^{\mu\eta}(2q_i+q)^\zeta g^{\rho'\tau'}(q_j-q)^\delta \\
&-g^{\eta\zeta}(2q+q_i)^\mu g^{\tau'\delta}(2q_j+q)^{\rho'} + g^{\eta\zeta}(2q+q_i)^\mu g^{\delta\rho'}(2q+q_j)^{\tau'} + g^{\eta\zeta}(2q+q_i)^\mu g^{\rho'\tau'}(q_j-q)^\delta \\
&-g^{\zeta\mu}(q_i-q)^\eta g^{\tau'\delta}(2q_j+q)^{\rho'} + g^{\zeta\mu}(q_i-q)^\eta g^{\delta\rho'}(2q+q_j)^{\tau'} + g^{\zeta\mu}(q_i-q)^\eta g^{\rho'\tau'}(q_j-q)^\delta)]
\end{aligned}
\tag{2.21}
$$

$$
\begin{aligned}
M_1 M_2{}^\dagger = &\frac{g_s^2 f^{\,a\,l\,o} f^{\,f\,n\,o}}{(q_i+q)^2(q_j+q)^2}[g_{\eta'}{}^\eta g_\gamma{}^\delta(2q_i+q)_{\rho'}(2q_j+q)^{\rho'} \\
&-g_{\eta'}{}^\eta(2q_i+q)^\eta(2q+q_j)_\gamma - g_{\eta'}{}^\eta(2q_i+q)^\delta(q_j-q)^\delta \\
&-g_\gamma{}^\delta(2q+q_i)_{\eta'}(2q_j+q)^\eta + g^{\eta\delta}(2q+q_i)_{\eta'}(2q+q_j)_\gamma \\
&+g_\gamma{}^\eta(2q+q_i)_{\eta'}(q_j-q)^\eta - g_\gamma{}^\delta(q_i-q)^\eta(2q_j+q)_{\eta'} \\
&+g_{\eta'}{}^\delta(q_i-q)^\eta(2q+q_j)_\gamma + g_{\eta'}{}^{\tau'}g_{\gamma\tau'}(q_i-q)^\eta(q_j-q)^\delta]
\end{aligned}
\tag{2.22}
$$

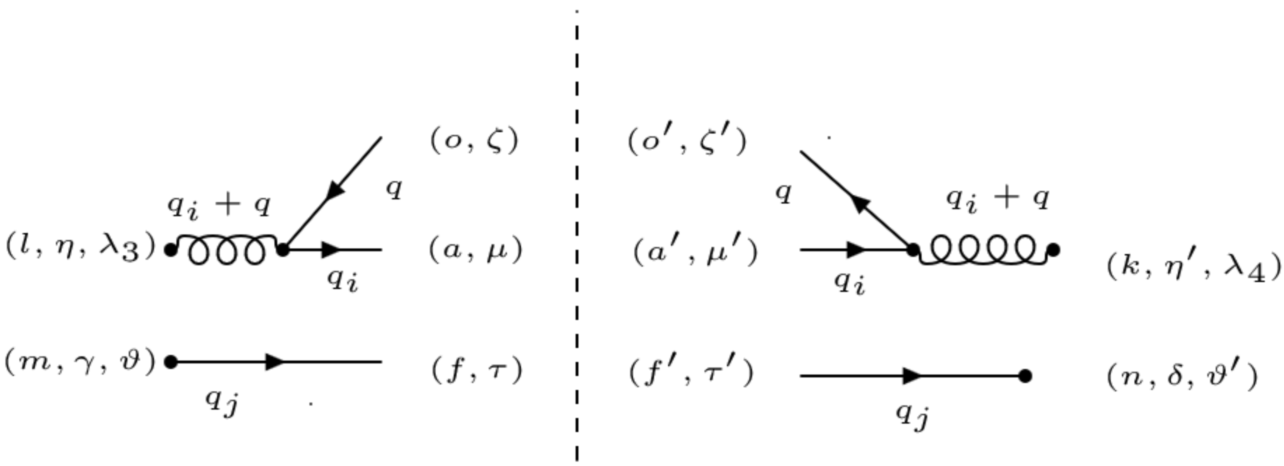
2.4 $|M^2|$

Chapter 3

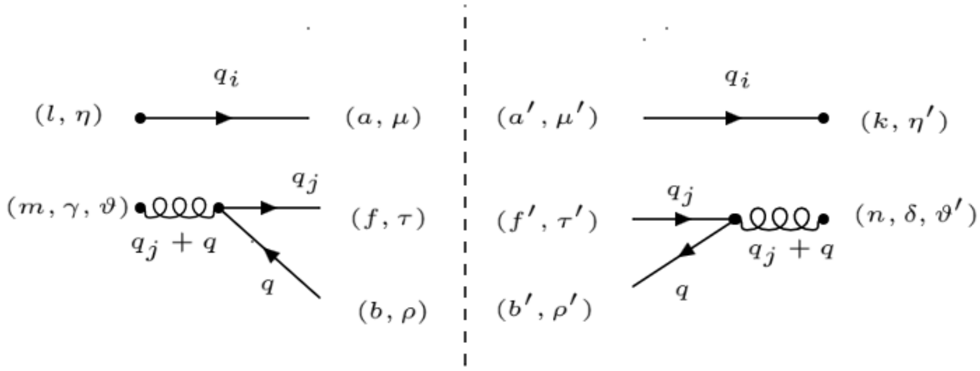
Quark gluon quark emission kernel



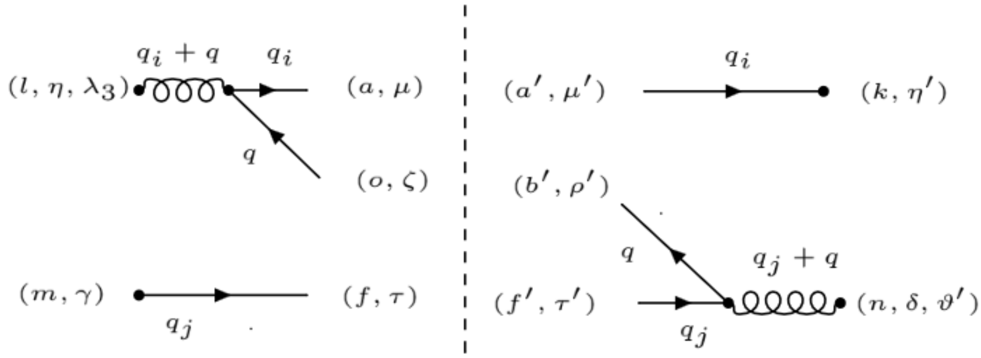
3.1 Gluon-Emitter Quark loop



3.2 Gluon-Spectator Quark loop

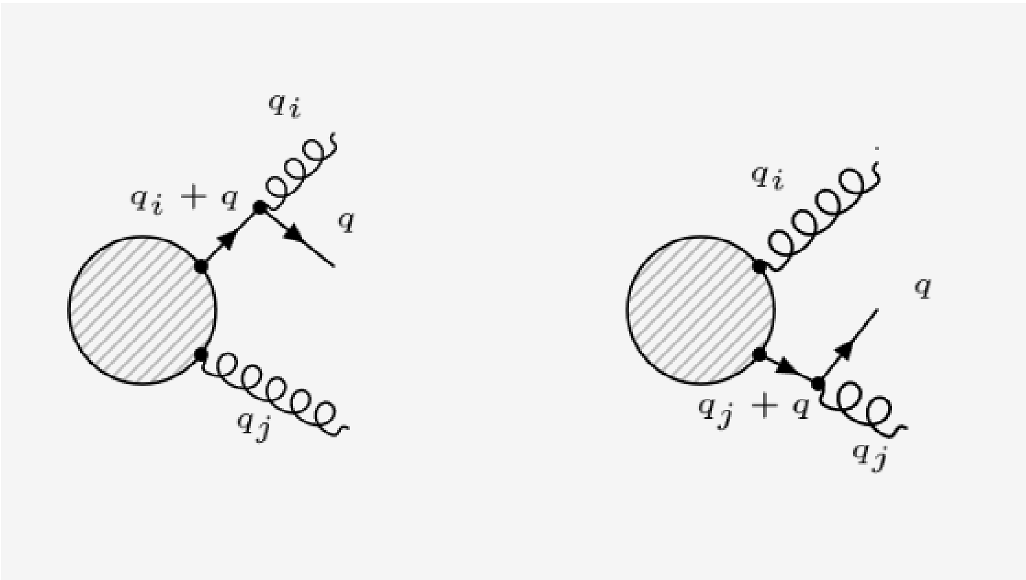


3.3 Gluon-Emitter Quark loop

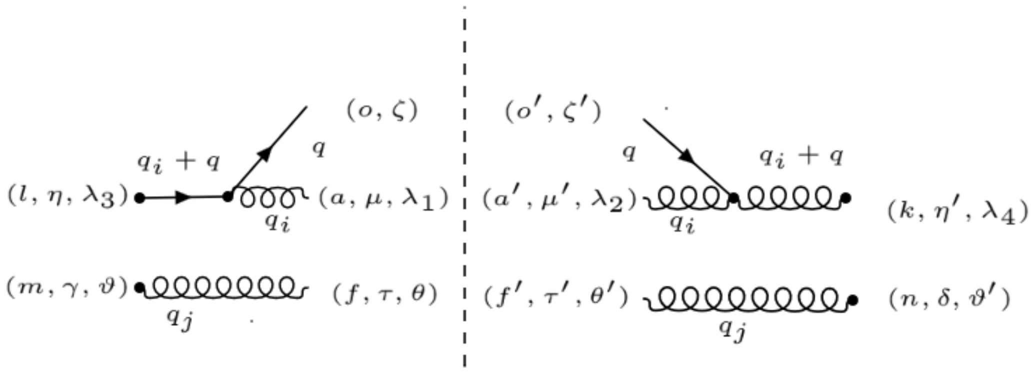


Chapter 4

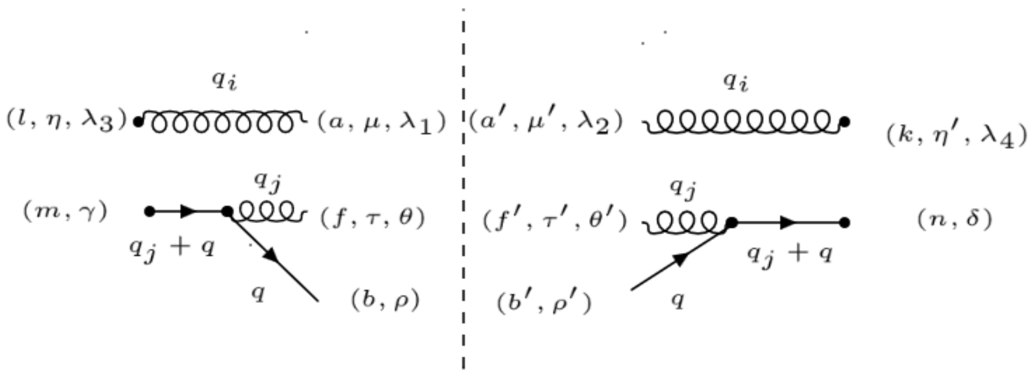
Gluon quark quark emission kernel



4.1 M_1



4.2 M_2


q_i
(l, η, λ_3)
(a, μ, λ_1)
(a', μ', λ_2)
q_i
(k, η', λ_4)
(m, γ)
q_j
(f, τ, θ)
(f', τ', θ')
q_j
(n, δ)
$q_j + q$
q
(b, ρ)
(b', ρ')
q
$q_j + q$

4.3 $M1M_2^\dagger$