PHYS 205: Electricity and Magnetism

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Constants

$$\begin{aligned} c &= 3.00 \cdot 10^{8}\,\mathrm{m} \cdot \mathrm{s}^{-1} \\ N_A &= 6.02 \cdot 10^{23}\,\mathrm{mol}^{-1} \\ m_{\text{proton}} &= 1.672 \cdot 10^{-27}\,\mathrm{kg} \\ m_{\text{neutron}} &= 1.674 \cdot 10^{-27}\,\mathrm{kg} \\ m_{\text{electron}} &= 9.11 \cdot 10^{-31}\,\mathrm{kg} \\ e &= 1.602 \cdot 10^{-19}\,\mathrm{C} \\ k_e &= \frac{1}{4\pi\epsilon_0} = 8.8976 \cdot 10^{9}\,\mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{C}^{-2} \\ \epsilon_0 &= 8.8542 \cdot 10^{-12}\,\mathrm{C}^2 \cdot \mathrm{N}^{-1} \cdot \mathrm{m}^{-2} \\ \mu_0 &= 4\pi \cdot 10^{-7}\,\mathrm{T} \cdot \mathrm{m} \cdot \mathrm{A}^{-1} \end{aligned}$$

1 Electric forces and fields

Conservation of charge:

$$\sum q_{\text{initial}} = \sum q_{\text{final}}$$

Quantization of charge:

$$q = \left(N_{\text{proton}} - N_{\text{electron}}\right) e$$

Electric force between two point-charges:

$$F_E = k_e \frac{|q_1|\,|q_2|}{r^2}$$

Electric field:

- Point-charge:

$$\vec{E} = \frac{\vec{F}_E}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

- Uniform electric field:

$$E = \frac{|\sigma|}{2\epsilon_0}$$

- Two parallel plates:

$$E = \frac{|\sigma|}{\epsilon_0}$$

2 Continuous charge distribution

Charge densities:

- Linear charge density:

$$\lambda = \frac{Q}{l}$$

- Area charge density:

$$\sigma = \frac{Q}{A}$$

- Volume charge density (insulator):

$$\rho = \frac{Q}{V}$$

- Varying charge density:

$$Q_{\text{tot}} = \int \mathrm{d}q = \int \lambda\left(x\right)\mathrm{d}x$$

Electric field of a continuous charge distribution:

$$\vec{E} = \int \mathrm{d}\vec{E} = k_e \int \frac{\mathrm{d}q}{r^2}\hat{r}$$

Electric flux:

- Constant electric field:

$$\Phi_E = \vec{E} \bullet \vec{A} = EA\cos\theta$$

- Gauss's law:

$$\Phi_E = \oint \vec{E} \bullet \mathrm{d}\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

3 Electrical potential

Work:

- Uniform electric field:

$$W_{1\to 2} = q\vec{E} \bullet \Delta\vec{r} = qE\Delta r\cos\theta$$

- Non-uniform electric field:

$$W_{1\to 2} = q\int_1^2 \vec{E} \bullet \mathrm{d}\vec{s}$$

Electric potential energy:

- Uniform electric field:

$$\Delta U = -W_{1\to 2} = -q\vec{E} \bullet \Delta\vec{r} = -\mathrm{qE}\Delta r\cos\theta$$

- Non-uniform electric field:

$$\Delta U = -W_{1\to 2} = -q\int_1^2 \vec{E} \bullet \mathrm{d}\vec{s}$$

- Two point-charges:

$$U = k_e \frac{q_1 q_2}{r}$$

Potential:

- Point-charge:

$$V = \frac{U}{q_0} = k_e \frac{q}{r}$$

- Potential difference of a point charge:

$$\Delta V = \frac{\Delta U}{q_0}$$

- Uniform electric field:

$$\Delta V = -\vec{E} \bullet \Delta\vec{r} = -E\Delta r\cos\theta$$

- Non-uniform electric field:

$$\Delta V = -\int_1^2 \vec{E} \bullet \mathrm{d}\vec{s}$$

  - Electric field from potential:

$$E = -\frac{\mathrm{d}V}{\mathrm{d}s}$$

- Continuous charge distribution:

$$V = \int \mathrm{d}V = k_e \int \frac{\mathrm{d}q}{r}$$

4 Capacitance and dielectric

Capacitance:

$$C = \frac{Q}{\Delta V_C}$$

- Parallel plate capacitor:

$$C = \frac{\epsilon_0 A}{d}$$

Capacitor in circuits:

- Parallel:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots = \sum_i C_i$$

- Series:

$$C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} = \frac{1}{\sum_i \frac{1}{C_i}} = \left(\sum_i \frac{1}{C_i} \right)^{-1}$$

Energy related to a capacitor:

- Energy stored in a capacitor:

$$U_C = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V_C = \frac{1}{2} C \left(\Delta V_C\right)^2$$

- Energy stored in the electric field between the two plates:

$$U_C = \frac{1}{2} \epsilon_0 A d E^2$$

- Energy density of the electric field:

$$u_E = \frac{U_C}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

Dielectric:

$$E_1 = E_0 - E_{\text{induced}}$$

$$\kappa = \frac{E_0}{E_1}$$

$$C_1 = \kappa C_0$$

Electrostatic breakdown:

$$V = E_{\max} d$$

5 Current and resistance

Current:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Drift speed (speed of the electron in a conductor):

$$v_d = \frac{I}{A e n_e}$$

Ohm's law:

$$J = \frac{I}{A} = v_d e n_e$$

- Conductivity of a material:

$$\sigma = \frac{J}{E}$$

Magnetic flux:

- Uniform magnetic field:

$$\Phi_B = \vec{A} \bullet \vec{B} = AB \cos\theta$$

- Non-uniform magnetic field:

$$\Phi_B = \int \vec{B} \bullet \mathrm{d}\vec{A}$$

Faraday's law:

- Single loop induced EMF:

$$\Delta V_{\text{induced}} = \left| \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \right|$$

- Multiple loops induced EMF:

$$\Delta V_{\text{induced}} = N \left| \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \right|$$

- AC generators:

$$\Delta v = ABN\omega \sin \omega t$$

10 Inductance

Self-inductance:

$$L = \frac{\Delta V}{\left|\frac{\mathrm{d}I}{\mathrm{d}t}\right|} = \frac{\Phi_B}{I}$$

Inductance of a solenoid:

$$L = \frac{\mu_0 A N^2}{l}$$

RL circuits:

- Time constant:

$$\tau = \frac{L}{R}$$

- Decreasing current:

$$I\left(t\right) = I_{\max} e^{-\frac{t}{\tau}} = \frac{\Delta V_R}{R} e^{-\frac{t}{\tau}}$$

- Increasing current:

$$I\left(t\right) = I_{\max} \left(1 - e^{-\frac{t}{\tau}}\right) = \frac{\Delta V_R}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$$

- Power dissipation:

$$P_L = LI \frac{\mathrm{d}I}{\mathrm{d}t}$$

- Energy stored in the magnetic field of and inductor:

$$U_L = \frac{1}{2} L I^2$$

LC circuits with a charged capacitor:

- Angular frequency:

$$\omega = \sqrt{\frac{1}{LC}}$$

- Natural frequency:

$$f_{\text{natural}} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

- Charge:

$$Q\left(t\right) = Q_{\max} \cos \omega t$$

- Current:

$$I\left(t\right) = -I_{\max} \sin \omega t = -\omega Q_{\max} \sin \omega t$$

11 AC circuits

Angular frequency:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Instantaneous voltage: