ENGR 251: Thermodynamics I

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1 Pressure and hydrostatic

Pascal Law The pressure at one point in equilibrium is independent of the direction of observation.

Variation of pressure with change in height:

$$P_2 - P_1 = -g \int_{z_1}^{z_2} \rho \, \mathrm{d}z$$

Incompressible fluid (P constant):

$$P_2 - P_1 = -\rho g \left(z_2 - z_1 \right)$$

Compressible ideal gas:

$$\frac{P_2}{P_1} = e^{-\frac{g}{RT}\left(z_2 - z_1 \right)}$$

Stevin principle The pressure of a fluid at rest is independent of the shape of the container. It varies with depth but is the same horizontally.

2 Quality

$$x = \frac{m_{\text{gas}}}{m_{\text{total}}} = \frac{u - u_f}{u_g - u_f} = \frac{h - h_f}{h_g - h_f} = \frac{s - s_f}{s_g - s_f}$$

3 Ideal gas

Reduced temperature and pressure:

$$T_R = \frac{T}{T_C} \text{ and } P_R = \frac{P}{P_C}$$

Guidelines:

- $P \ll P_C$
- $P_R < 10$ and $T_R > 2$
- $T > 2T_C$ for $P < 4P_C$
- $P_R \ll 1$

Ideal gas law:

$$PV = mRT$$
$$P\nu = RT$$

Compressibility factor for non-ideal gas:

$$Z = \frac{P\nu}{RT}$$

Internal energy ($K_B = 1.381 \cdot 10^{-23}\,\mathrm{J} \cdot \mathrm{K}^{-1}$):

$$U = \frac{3}{2} n K_B T$$

4 First Law of Thermodynamics

Energy relations:

$$\begin{aligned} \Delta E &= Q_{1\to 2} - W_{1\to 2} \\ &= \Delta K + \Delta E_{\text{potential}} + \Delta U \end{aligned}$$

Types of energy:

$$\Delta K = \frac{1}{2} m \left(v_2^2 - v_1^2 \right)$$
$$\Delta E_{\text{potential}} = mg \left(z_2 - z_1 \right)$$
$$\Delta U = m \left(u_2 - u_1 \right)$$

Sign convention:

- Heat:
  - Energy entering the system: $Q > 0$
  - Energy leaving the system: $Q < 0$
- Work:
  - Work done by the system: $W > 0$
  - Work done on the system: $W < 0$

Work:

- Mechanical work:
  $$W_{1\to 2} = \int_1^2 \vec{F} \bullet \mathrm{d}\vec{s}$$
- Expansion work:
  $$W_{1\to 2} = \int_1^2 P \, \mathrm{d}V$$
  - Isobaric work:
    $$W_{1\to 2} = P \left(V_2 - V_1 \right)$$
  - Isothermal work:
    $$W_{1\to 2} = mRT \ln \left(\frac{V_2}{V_1} \right)$$
  - Polytropic process (PV^n constant):
    $$W_{1\to 2} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$
- Electrical work:
  $$W_{1\to 2} = \int_1^2 V I \, \mathrm{d}t$$
- Shaft work:
  $$W_{1\to 2} = 2\pi N T$$
- Spring work:
  $$W_{1\to 2} = \frac{1}{2} k \left(x_2^2 - x_1^2 \right)$$
  - Spring based piston:
    $$P_2 = P_1 + \frac{k}{A^2} \left(V_2 - V_1 \right)$$
    $$W_{1\to 2} = \frac{1}{2} \left(P_1 + P_2 \right) \left(V_1 - V_2 \right)$$

Enthalpy:

$$H = U + PV$$
$$h = u + P\nu$$

- Heat exchange for isobaric process:
$$Q_{1\to 2} = H_2 - H_1 = m\left(h_2 - h_1\right)$$

Specific heat:

$$C = \frac{\mathrm{d}u}{\mathrm{d}t} + P\frac{\mathrm{d}\nu}{\mathrm{d}t} + \nu\frac{\mathrm{d}P}{\mathrm{d}t}$$

- Isochoric process:
$$C_V = \frac{\mathrm{d}u}{\mathrm{d}T}$$
- Isobaric process:
$$C_P = \frac{\mathrm{d}h}{\mathrm{d}T}$$
- Normal temperature set ($(500 \pm 300)\,\mathrm{K}$): C_V and C_P constant at T_{average}
$$u_2 - u_1 = C_V\left(T_2 - T_1\right)$$
$$h_2 - h_1 = C_P\left(T_2 - T_1\right)$$

5 Control volume and steady flow

Reynold Transport Theorem:

$$\frac{\mathrm{d}B_S}{\mathrm{d}t} = \frac{\mathrm{d}B_{CV}}{\mathrm{d}t} + \sum_{\text{out}}\frac{\mathrm{d}B_{\text{out}}}{\mathrm{d}t} - \sum_{\text{out}}\frac{\mathrm{d}B_{\text{in}}}{\mathrm{d}t}$$
$$\dot{B}_S = \dot{B}_{CV} + \sum_{\text{out}}\dot{B}_{\text{out}} - \sum_{\text{in}}\dot{B}_{\text{in}}$$

Mass equation:

$$\dot{m}_S = \dot{m}_{CV} + \sum_{\text{out}}\dot{m}_{\text{out}} - \sum_{\text{in}}\dot{m}_{\text{in}} \text{ and } \dot{m} = \int \rho v\,\mathrm{d}A$$

- Conservation of mass ($\dot{m}_S$ constant):
$$\dot{m}_{CV} = \sum_{\text{in}}\dot{m}_{\text{in}} - \sum_{\text{out}}\dot{m}_{\text{out}}$$
  - Steady flow ($\dot{m}_{CV}$ constant):
$$\sum_{\text{in}}\dot{m}_{\text{in}} = \sum_{\text{out}}\dot{m}_{\text{out}}$$
  - Incompressible fluid (ρ constant):
$$\sum_{\text{in}}\dot{Q}_{\text{in}} = \sum_{\text{out}}\dot{Q}_{\text{out}} \text{ and } \dot{Q} = vA$$

Energy equation:

$$\dot{E}_{CV} = \dot{Q} - \dot{W}_t - \dot{W}_p + \sum_{\text{in}}\dot{m}_{\text{in}}\left(\frac{1}{2}v_{\text{in}}^2 + gz_{\text{in}} + h_{\text{in}}\right)$$
$$- \sum_{\text{out}}\dot{m}_{\text{out}}\left(\frac{1}{2}v_{\text{out}}^2 + gz_{\text{out}} + h_{\text{out}}\right)$$

- Steady flow equation ($\dot{m}$ constant):
$$\dot{Q} + \sum_{\text{in}}\dot{m}\left(\frac{1}{2}v_{\text{in}}^2 + gz_{\text{in}} + h_{\text{in}}\right)$$
$$= \dot{W}_t + \dot{W}_p + \sum_{\text{out}}\dot{m}\left(\frac{1}{2}v_{\text{out}}^2 + gz_{\text{out}} + h_{\text{out}}\right)$$
  - Bernoulli's equation (steady flow, adiabatic, no turbine and no pump):
$$\frac{1}{2}v_{\text{in}}^2 + gz_{\text{in}} + h_{\text{in}} = \frac{1}{2}v_{\text{out}}^2 + gz_{\text{out}} + h_{\text{out}}$$
- Unsteady flow:
$\dot{E}_{CV} = \dot{U}_{CV} = \dot{m}u_{CV}$ with $\dot{m}$ changing over time

6 Second law of Thermodynamics

Efficiency:

- Thermal efficiency of a heat engine:
$$\eta = \frac{W_{\text{net}}}{Q_H} = 1 - \frac{Q_L}{Q_H} \text{ and } 0 \leqslant \eta < 1$$
- Coefficient of performance of a refrigerator:
$$\beta = \frac{Q_L}{W_{\text{net}}} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$
- Coefficient of performance of a heat pump:
$$\beta = \frac{Q_H}{W_{\text{net}}} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Carnot cycle:

- Processes:
  $1 \to 2$ Isothermal process: heat exchange at T_H
  $2 \to 3$ Adiabatic reversible (isentropic) expansion: positive work
  $3 \to 4$ Isothermal process: heat exchange at T_L
  $4 \to 1$ Adiabatic reversible (isentropic) compression: negative work
- Carnot efficiency:
  - Carnot thermal efficiency of a heat engine:
$$\eta_{\text{car}} = 1 - \frac{T_L}{T_H} \text{ and } 0 \leqslant \eta < 1$$
  - Carnot coefficient of performance of a refrigerator:
$$\beta_{\text{car}} = \frac{1}{\frac{T_H}{T_L} - 1}$$
  - Carnot coefficient of performance of a heat pump:
$$\beta_{\text{car}} = \frac{1}{1 - \frac{T_L}{T_H}}$$
- Carnot principle:
  - $\eta < \eta_{\text{car}}$ or $\beta < \beta_{\text{car}}$: irreversible cycle
  - $\eta = \eta_{\text{car}}$ or $\beta = \beta_{\text{car}}$: ideal Carnot cycle
  - $\eta > \eta_{\text{car}}$ or $\beta > \beta_{\text{car}}$: impossible cycle (PMM of type 2)

Entropy:

- Clausius inequality (for any reversible or irreversible process):
$$\oint \frac{\mathrm{d}Q}{T} \leqslant 0$$
- Entropy for a reversible process:
$$S_2 - S_1 = \int_1^2 \frac{\mathrm{d}Q}{T}$$
- Heat exchange:
$$Q_{1\to 2} = \int_1^2 T\,\mathrm{d}S$$
- Increasing entropy:
$$\mathrm{d}S \geqslant \frac{\mathrm{d}Q}{T}$$
- Entropy generation:
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}} \geqslant 0$$

- $S_{\text{gen}} > 0$: irreversible process
- $S_{\text{gen}} = 0$: reversible process
- $S_{\text{gen}} < 0$: impossible process (PMM of type 2)

Gibbs equations ($T\mathrm{d}S$ equations):

- First Gibbs equation:
$$T\mathrm{d}S = \mathrm{d}U + P\mathrm{d}V$$
$$T\mathrm{d}s = \mathrm{d}u + P\mathrm{d}\nu$$
- Second Gibbs equation:
$$T\mathrm{d}S = \mathrm{d}H - V\mathrm{d}P$$
$$T\mathrm{d}s = \mathrm{d}h - \nu\mathrm{d}P$$

Application to entropy change:

- Solids and liquids:
$$s_2 - s_1 = C\ln\left(\frac{T_2}{T_1}\right)$$
- Ideal gas:
$$s_2 - s_1 = C_V\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{\nu_2}{\nu_1}\right)$$
$$s_2 - s_1 = C_P\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$
  - Isentropic process on an ideal gas:
    * $C_P = C_V + R$ and $n = \frac{C_P}{C_V}$
    * Work in a closed system:
$$W_{1\to2} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR\left(T_2 - T_1\right)}{1-n}$$
    * Work in a steady flow:
$$W_{1\to2} = \frac{P_2V_2 - P_1V_1}{n-1} = \frac{mR\left(T_2 - T_1\right)}{n-1}$$
    * Isentropic relation for an ideal gas:
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Enthalpy variation of an isentropic steady flow process:
$$w_{1\to2} = h_1 - h_2 = -\int_1^2 \nu\,\mathrm{d}P$$

- Work done by a pump (ν constant):
$$w_{1\to2} = \nu\left(P_1 - P_2\right)$$

Isentropic efficiency of a steady flow device:

- Turbine:
$$\eta = \frac{h_{2\ \text{real}} - h_1}{h_2 - h_1}$$
- Pump/compressor:
$$\eta = \frac{h_2 - h_1}{h_{2\ \text{real}} - h_1}$$
- Nozzle:
$$\eta = \frac{v_{\text{real}}^2}{v^2}$$

7 Application to cyclic processes

Rankine cycle: operates with both liquid and gas states

- Processes:
  $1 \to 2$ Isentropic compression in a pump from saturated liquid to compressed liquid
  $2 \to 3$ Isobaric heating from compressed liquid to superheated gas
  $3 \to 4$ Isentropic expansion in a turbine
  $4 \to 1$ Isobaric cooling from gas to saturated liquid
- Efficiency:
$$\eta = \frac{h_1 - h_2 + h_3 - h_4}{h_3 - h_2}$$

Brayton cycle: operates with gas state

- Processes:
  $1 \to 2$ Isentropic compression in a compressor
  $2 \to 3$ Isobaric heating
  $3 \to 4$ Isentropic expansion in a turbine
  $4 \to 1$ Isobaric cooling
- Pressure ratio:
$$r_P = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$
- Ideal Brayton efficiency:
$$\eta_B = 1 - \frac{1}{{r_p}^{\frac{n-1}{n}}}$$
- Back work ratio:
$$r_{BW} = \frac{W_{\text{compressor}}}{W_{\text{turbine}}}$$

Otto cycle: closed version of the four-stroke engine

- Processes:
  $1 \to 2$ Isentropic compression from the BDC to the TDC
  $2 \to 3$ Isochoric heating
  $3 \to 4$ Isentropic expansion from the TDC to the BDC (the work is done here)
  $4 \to 1$ Isochoric cooling
- Mean effective pressure:
$$MEP = \frac{W_{\text{net}}}{V_{BDC} - V_{TDC}}$$
- Volume ratio:
$$r_V = \frac{V_1}{V_2} = \frac{V_4}{V_3} = \frac{V_{BDC}}{V_{TDC}}$$
- Ideal Otto efficiency:
$$\eta_{\text{otto}} = 1 - r_V^{1-n}$$
- Heat exchange:
$$Q_{\text{in}} = U_3 - U_2 = mC_V\left(T_3 - T_2\right)$$
$$Q_{\text{out}} = U_4 - U_1 = mC_V\left(T_4 - T_1\right)$$