AERO 201: Introduction to Flight and Aerospace Systems

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1 Formulas

Bernoulli equation Any increase in fluid velocity corresponds to a decrease in pressure (Venturi effect)

$$\mathrm{d}P = -\rho v \mathrm{d}v$$
$$\iff P_1 + \frac{1}{2}\rho_1 {v_1}^2 = P_2 + \frac{1}{2}\rho_2 {v_2}^2 = P_0$$

P[Pa]: flow pressure
$v[\mathrm{m \cdot s^{-1}}]$: flow velocity
$\rho[\mathrm{kg \cdot m^{-3}}]$: flow density

Continuity equation Mass can neither be created nor destroyed, thus

$$\dot{m}_1 = \dot{m}_2$$
$$\iff A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

$\dot{m}[\mathrm{kg \cdot s^{-1}}]$: mass flow
$A[\mathrm{m^2}]$: flow area
$v[\mathrm{m \cdot s^{-1}}]$: flow velocity
$\rho[\mathrm{kg \cdot m^{-3}}]$: flow density

Excess power

$$P_{\mathrm{excess}} = v(T - D) = W v_{\mathrm{climb}}$$

P_{excess}[W]: excess power
$v[\mathrm{m \cdot s^{-1}}]$: aircraft velocity
T[N]: thrust
D[N]: drag
W[N]: aircraft weight
$v_{\mathrm{climb}}[\mathrm{m \cdot s^{-1}}]$: rate of climb

Density in non-isothermal region

$$\rho = \rho_{\mathrm{ref}} \left(\frac{T}{T_{\mathrm{ref}}}\right)^{-\frac{g}{aR_{\mathrm{air}}}+1}$$

$\rho[\mathrm{kg \cdot m^{-3}}]$: density at the given temperature
$\rho_{\mathrm{ref}}[\mathrm{kg \cdot m^{-3}}]$: reference density
T[K]: given temperature
T_{ref}[K]: reference temperature
$g = 9.81\,\mathrm{m \cdot s^{-2}}$
$a[\mathrm{K \cdot m^{-1}}]$: temperature gradient
$R_{\mathrm{air}} = 287\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$: specific gas constant

Density in isothermal region

$$\rho = \rho_{\mathrm{ref}} e^{-\frac{g}{R_{\mathrm{air}} T}(h - h_{\mathrm{ref}})}$$

ρ[Pa]: density at the given height
ρ_{ref}[Pa]: reference density
$g = 9.81\,\mathrm{m \cdot s^{-2}}$
$R_{\mathrm{air}} = 287\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$: specific gas constant
T[K]: temperature
h[m]: altitude
h_{ref}[m]: reference altitude

Drag

$$D = \frac{1}{2}\rho v^2 S_{\mathrm{ref}} c_D$$

$\rho[\mathrm{kg \cdot m^{-3}}]$: air density at the given altitude
$v[\mathrm{m \cdot s^{-1}}]$: speed of the body relative to air
$S_{\mathrm{ref}}[\mathrm{m^2}]$: wing area
c_D: drag coefficient

Drag polar The aerodynamic data for an aircraft is presented in from of a drag polar

$$c_D = c_{D,0} + \frac{{c_L}^2}{\pi e \mathrm{AR}}$$

c_D: drag coefficient
$c_{D,0}$: parasitic (form and friction) drag coefficient at zero lift
c_L: lift coefficient
e: Oswald efficiency factor
AR: aspect ratio

Dynamic pressure Dynamic pressure is the pressure developed bu the forward motion of a body.

$$P = \frac{1}{2}\rho v^2$$

P[Pa]: dynamic pressure
$\rho[\mathrm{kg \cdot m^{-3}}]$: air density at the given altitude
$v[\mathrm{m \cdot s^{-1}}]$: speed of the body relative to air

Equivalent air speed Equivalent speed on sea level

$$v_e = v\sqrt{\frac{\rho}{\rho_0}}$$

$v_e[\mathrm{m \cdot s^{-1}}]$: equivalent air speed
$v[\mathrm{m \cdot s^{-1}}]$: speed of the body relative to air
$\rho[\mathrm{kg \cdot m^{-3}}]$: air density at the given altitude
$\rho_0[\mathrm{kg \cdot m^{-3}}]$: air density at sea level

Flow Flow of a fluid in a pump

$$Q = \omega V$$

$Q[\mathrm{m^3 \cdot s^{-1}}]$: flow
$\omega[\mathrm{rad \cdot s^{-1}}]$: shaft speed
$V[\mathrm{m^3}]$: volume of fluid displaced

Lift

$$L = \frac{1}{2}\rho v^2 S_{\mathrm{ref}} c_L$$

$\rho[\mathrm{kg \cdot m^{-3}}]$: air density at the given altitude
$v[\mathrm{m \cdot s^{-1}}]$: speed of the body relative to air
$S_{\mathrm{ref}}[\mathrm{m^2}]$: wing area
c_L: lift coefficient

Mach number The mach number is used to identify different aerodynamic flow regimes (subsonic, sonic and supersonic)

$$M = \frac{v}{a} = \frac{v}{\sqrt{\gamma R T}}$$

$v[\mathrm{m \cdot s^{-1}}]$: true air speed
$a[\mathrm{m \cdot s^{-1}}]$: speed of sound
γ: isentropic coefficient, $\gamma = 1.4$ at $T = 300\,\mathrm{K}$
$R_{\mathrm{air}} = 287\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$: specific gas constant
$T[\mathrm{K}]$: air temperature

Orbit equation The minimal orbit velocity is $v_{\mathrm{orbit}} \approx 7.9 \cdot 10^3\,\mathrm{m \cdot s^{-1}}$ and the escape velocity is $v_{\mathrm{escape}} \approx 11.2 \cdot 10^3\,\mathrm{m \cdot s^{-1}}$

$$\begin{aligned} r &= \frac{p}{1 + e\cos(\theta - C)} \\ &= \frac{\frac{(r^2\dot{\theta})^2}{GM}}{1 + A\left(\frac{(r^2\dot{\theta})^2}{GM}\right)\cos(\theta - C)} \end{aligned}$$

$e = 0$: path is a circle
$e < 1$: path is an ellipse
$e = 1$: path is a parabola
$e > 1$: path is a hyperbola

Power required Power required from the thrust required and the velocity of the aircraft (level, constant speed flight):

$$P = Tv = \sqrt{\frac{2W^3 {c_D}^2}{\rho S_{\mathrm{ref}} {c_L}^3}}$$

$P[\mathrm{W}]$: power required
$T[\mathrm{N}]$: thrust required
$v[\mathrm{m \cdot s^{-1}}]$: velocity relative to the air
$W[\mathrm{N}]$: aircraft weight
c_D: drag coefficient
$\rho[\mathrm{kg \cdot m^{-3}}]$: air density
$S_{\mathrm{ref}}[\mathrm{m^2}]$: wing area
c_L: lift coefficient

Pressure in non-isothermal region

$$P = P_{\mathrm{ref}}\left(\frac{T}{T_{\mathrm{ref}}}\right)^{-\frac{g}{aR_{\mathrm{air}}}}$$

$P[\mathrm{Pa}]$: pressure at the given temperature
$P_{\mathrm{ref}}[\mathrm{Pa}]$: reference pressure
$T[\mathrm{K}]$: given temperature
$T_{\mathrm{ref}}[\mathrm{K}]$: reference temperature
$g = 9.81\,\mathrm{m \cdot s^{-2}}$
$a[\mathrm{K \cdot m^{-1}}]$: temperature gradient
$R_{\mathrm{air}} = 287\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$: specific gas constant

Pressure in isothermal region

$$P = P_{\mathrm{ref}} e^{-\frac{g}{R_{\mathrm{air}}T}(h - h_{\mathrm{ref}})}$$

$P[\mathrm{Pa}]$: pressure at the given height
$P_{\mathrm{ref}}[\mathrm{Pa}]$: reference pressure
$g = 9.81\,\mathrm{m \cdot s^{-2}}$
$R_{\mathrm{air}} = 287\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$: specific gas constant
$T[\mathrm{K}]$: temperature
$h[\mathrm{m}]$: altitude
$h_{\mathrm{ref}}[\mathrm{m}]$: reference altitude

Range Range is the max distance the aircraft can make with a given amount of fuel:

$$R = \frac{v}{C}\frac{L}{D}\ln\left(\frac{W_{\mathrm{cruise,start}}}{W_{\mathrm{cruise,end}}}\right)$$

$R[\mathrm{m}]$: range
$v[\mathrm{m \cdot s^{-1}}]$: aircraft velocity
$C[\mathrm{s^{-1}}]$: specific fuel consumption
$L[\mathrm{N}]$: lift
$D[\mathrm{N}]$: drag
$W[\mathrm{N}]$: weight

Reynolds number Reynolds number is used to characterize the flow (laminar, transient, turbulent)

$$\mathrm{Re}_x = \frac{\rho v x}{\mu}$$

Re_x: Reynolds number
$\rho[\mathrm{kg \cdot m^{-3}}]$: free stream density
$v[\mathrm{m \cdot s^{-1}}]$: free stream velocity
$x[\mathrm{m}]$: characteristic length
$\mu[\mathrm{kg \cdot m^{-1} \cdot s^{-1}}]$: free stream viscosity

Rocket equation The rocket equation relates the burnout velocity v_b of a rocket vehicle to the specific impulse I_{sp} associated with teh engine and teh mass ration of the initial mass m_i and the final mass m_f:

$$\begin{aligned} \frac{m_i}{m_f} &= \exp\left(\frac{v_b}{gI_{\mathrm{sp}}}\right) \\ \iff v_b &= gI_{\mathrm{sp}}\ln\left(\frac{m_i}{m_f}\right) \end{aligned}$$

Take-off length

$$L_{\mathrm{TO}} = \frac{mv^2}{2T} \approx \frac{1.44W^2}{g\rho S c_{L,\max} T}$$

L_{TO}[m]: take-off length
m[kg]: aircraft mass
v[$\mathrm{m\cdot s^{-1}}$]: aircraft take-off speed
T[N]: thrust
W[N]: aircraft weight
ρ[$\mathrm{kg\cdot m^{-3}}$]: air density
S[$\mathrm{m^2}$]: wing area
$c_{L,\max}$: max lift coefficient

Thrust Thrust equation for generic propulsive device

$$T = (\dot{m}_{\mathrm{air}} + \dot{m}_{\mathrm{fuel}})v_{\mathrm{out}} + P_{\mathrm{out}}A_{\mathrm{out}} - \dot{m}_{\mathrm{air}}v_{\mathrm{in}} - P_{\mathrm{in}}A_{\mathrm{in}}$$

T[N]: thrust generated
$\dot{m}$[$\mathrm{kg\cdot s^{-1}}$]: mass flow
v[$\mathrm{m\cdot s^{-1}}$]: flow velocity
P[Pa]: flow pressure
A[$\mathrm{m^2}$]: flow area

Thrust required Thrust required for a given aircraft at a given altitude caries with its velocity (level, constant speed flight)

$$T = W\frac{c_D}{c_L}$$

Weight

$$W_{\mathrm{TO}} = W_{\mathrm{empty}} + W_{\mathrm{fixed}} + W_{\mathrm{fuel}}$$

W_{TO}[N]: take-off gross weight, MTOW
W_{empty}[N]: empty weight, OWE
W_{fixed}[N]: fixed weight, payload
W_{fuel}[N]: fuel weight

2 Unit conversion

3 International standard atmosphere

Atmosphere model (oxygen supply at 12 500 ft or more for more than 30 min):

ISA temperature (T [K], h [km]):
0 - 11 km: $T = -6.5h + 288.16$
11 - 25 km: $T = 216.66$
25 - 47 km: $T = 3(h-25) + 216.66$
47 - 53 km: $T = 282.66$
53 - 79 km: $T = -4.5(h-53) + 282.66$
79 - 90 km: $T = 165.66$
90 - 105 km: $T = 4(h-90) + 165.66$

Table 1: Unit conversion EES, SI and others

	EES	SI
m	1 slug	14.5939 kg
F	1 lb	4.44822 N
l	1 ft	0.3048 m
P	1 lb · ft^{-2}	47.8803 Pa
T	1 °R	5/9·R K
ρ	1 slugs · ft^{-3}	515.378 kg · m^{-3}
v	1 ft · s^{-1}	0.3048 m · s^{-1}
R	1 ft · lb · slugs^{-1} · °R^{-1}	0.167226 J · kg^{-1} · K^{-1}

	Others
m	32.1740 lb
l	12 in, 1/5280 mi, 1/6076 nmi
P	1/144 psi
T	5/9·R - 273.15 °C, R - 459.67 °F
ρ	32.1740 lb · ft^{-3}
v	3600/5280 mi · h^{-1}, 3600/6076 kn

4 Common acronyms

AFDS: aircraft flight direction system
AC: advisory circulars, example of acceptable means, but not the only means, of demonstrating compliance with regulations and standards
AD: airworthiness directives, legally enforceable regulation to correct unsafe conditions in a product
AFDX: aviation full duplex
AGB: accessory gear box
AR: aspect ratio

$$AR = \frac{b^2}{S_{\mathrm{ref}}}$$

AR: aspect ratio
b[m]: wing span
S_{ref}[$\mathrm{m^2}$]: wing area

ARINC: aeronautical radio incorporated
ARP: aircraft recommended practice
ASI: air speed indicator
ATA: Air Transport Association
ATC: air traffic control
BPR: bypass ratio

$$BPR = \frac{\dot{m}_{\mathrm{fan}}}{\dot{m}_{\mathrm{core}}}$$

BPR < 2: Low
2< BPR < 4: Medium
BPR > 4: High
BPR = 12: Ultra-high
EASA: European Aviation Safety Agency
CAR: Canadian Aviation Regulations
EBHA: electrical backup hydraulic actuator
EDP: engine driven pump
EHA: electro-hydraulic actuator
EHSA: electro-hydraulic servo actuator
EICAS: engine indicating and crew alerting system
EMA: electro-mechanical actuator
EMC: electro-magnetic compatibility

EMI: electro-magnetic interference
EMP: electrical motor pump
FAA: Federal Aviation Administration
FAR: Federal Aviation Regulations
FADEC: full authority digital engine control system
FBW: fly-by-wire
FCU: fuel control unit, controls thrust generated by the engine
FCU: flight control unit
FMS: flight management system
HF: high frequency
HMA: hydro-mechanical actuator
IATA: International Air Transport Association
IEEE: Institute of Electrical and Electronics Engineers
IDG: integrated drive generator
IMA: integrated modular architecture
ISA: International Standard Atmosphere
ITT: inter turbine temperature
JAR: Joint Aviation Regulations
LCC: life-cycle cost
LRU: line replaceable units
LRM: line replaceable modules
MCDU: multipurpose control and display unit
MLG: main landing gear
MTOW: maximum takeoff weight
NLG: nose landing gear
NACA: National Advisory Committee for Aeronautics
NH: rotation speed of the engine high pressure shaft
NF: rotation speed of the engine free turbine shaft
OAT: outside air temperature
OBOGS: on-board oxygen generation system
OEM: original equipment manufacturer
OWE: operational weight empty (structure propulsion, systems, instruments, avionics)
Payload: crew, equipment, passengers, baggage, food, drinks, cargo, missiles bombs, ammunition
PSR: primary surveillance radar
PTU: power transfer unit (transfer power from a hydraulic system to another one)
RAT: ram air turbine
RNP: required navigation performance
SELCAL: selective call
SFC: specific fuel consumption

$$\frac{m_{\text{fuel consumed}}}{\dot{W}_{\text{brake horse}} t}$$

> $m_{\text{fuel consumed}}$: mass of fuel consumed
> $\dot{W}_{\text{brake horse}}$: for propeller engines, power provided to the propeller
> t[h]: duration of the fuel consumption

SSR: secondary surveillance radar
TCCA: Transport Canada's Civil Aviation
TGT: turbine gas temperature
TRL: technology readiness level
TRU: transformer rectifier unit
TSFC: thrust specific fuel consumption

$$\frac{m_{\text{fuel consumed}}}{F_{\text{thrust generated}} t}$$

> $m_{\text{fuel consumed}}$[kg]: mass of fuel consumed
> $F_{\text{thrust generated}}$[N]: force generated by the thrust
> t[h]: duration of the fuel consumption

VFG: variable frequency generator
VHF: very high frequency
VSCFG: variable speed constant frequency generator
WTB: wing tip brake

5 Materials and structures

Categories and function of aircraft structures:
Primary structure Load-bearing, stress and safety (wing box, fuselage, load paths)
Secondary structure Shaping, contour, surface (fairings, engine nacelle, nose cone)
Innter structural configuration Partitioning (payload, cockpit, system installation zones, safety-critical zones, segregation)

Structural layout of the aircraft fuselage:
External skin Carries shear from external transverse, torsional and cabin pressure loads
Tranverse elements Bulkheads support high concentrated loads in strategic zones (wings, empennage, landing gear) and distribute it into the skin; Frames maintain geometric integrity, support small loads
Longitudinal elements Longerons carry major portion of axial loads from bending; stringer carry residual axial loads from bending

Wing structure layout:
Multi-rib transport aircraft with high aspect ratio and relatively large thickness
Multi-spar high speed fighter aircraft, thin and highly loaded wings

Table 2: Material properties

	Strength/weight	Cost
Al-alloy	good	reasonable
Steel	too heavy	reasonable
Titanium	better than Al	very high
Composites	very good	depends
	Temperature sensitivity	Corrosion
Al-alloy	good	good
Steel	good	stainless steel: good
Titanium	very good	good
Composites	bad	very good

6 Aviation alphabet

A: ALPHA
B: BRAVO
C: CHARLIE

Table 3: Fastener types

Fastener	Advantages	Disadvantages
Rivets	Low cost, low weight, flush surface possible, high rigidity	Permanent, limited static shear, low tension, noisy installation
Bolts	Removable, high shear, high tension	no flush surface, high weight, high cost
Solid rivet	Low cost, low weight, good shear strength, automation, good clamp-up	Low tension strength, access to both side, permanent, noisy installation, incompatible with composite
Blind rivet	One side installation, low cost, low weight	Low tension strength, low shear strength, permanent, moderate clamp-up, poor fatigue, moderate reliability
Blind Bolt	One side installation, good shear strength	Low tension strength, high cost and weight, permanent, moderate clamp-up, poor fatigue, moderate reliability
Hi-Lites and Hi-Loks	High shear strength, high clamp-up, high reliability, moderate tension strength, high interference	Access to both side, permanent, noisy installation, moderate cost and weight, very high cost for Taper-Lok

D: DELTA
E: ECHO
F: FOXTROT
G: GOLF
H: HOTEL
I: INDIA
J: JULIET
K: KILO
L: LIMA
M: MIKE
N: NOVEMBER
O: OSCAR
P: PAPA
Q: QUEBEC
R: ROMEO
S: SIERRA
T: TANGO
U: UNIFORM
V: VICTOR
W: WHISKEY
X: XRAY
Y: YANKEE
Z: ZULU