ESTIMATING HAMILTONIAN IN A 3+0 SCHWARZCHILD METRIC WITH QUANTUM COMPUTERS USING VQE ALGORITHM

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ABSTRACT & BRIEF EXPLANATION

The advances on quantum computers are increasing rapidly. Algorithms on these computers are ***logarithmically faster***. In order to catch this era of computing we need to understand the algorithms of the quantum information. Via using quantum information algorithms it is possible to make simulations to a given ***Hamiltonian*** that we are looking for.[1,2]

The topic that I finalize my project is using in black-hole metrics. What interesting is near in ***Schwarzchild radius*** wave function ***vanishes*** to ***zero***. Which arises some questions like ‘Is quantum mechanics reversable?’ and ‘Can an ***information*** be ***destroyed***?’. In this work ***VQE*** may help us to see the relations between ***radius*** and ***energy***. [1,3]

The image retrieved from IBMQ

INTRODUCTION

What is quantum computing?

Quantum computers have far different architecture than the classical computers. In quantum computers we make ***circuit designs*** to execute our programs. These circuits are consisting of both ***classical registers*** and ***quantum registers***. In other words, we use qubits (quantum bits) in our logic designs. [4]

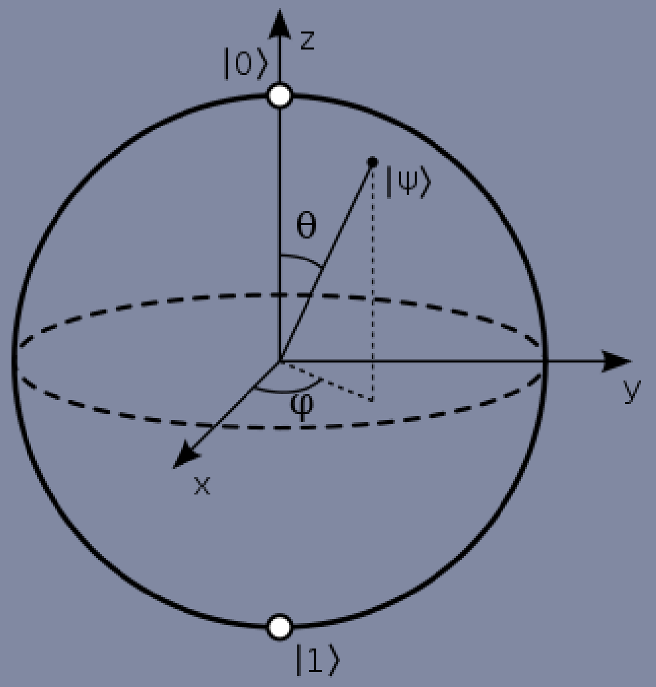


Fig1. Bloch Sphere Representation of Qubit

INTRODUCTION

What is Qubit?

Qubits uses quantum phenomena of ***superposition*** and ***entanglement*** which gives quantum computers superior ***advantage*** when it compared with classical computer. It natively shows linear combination of states which spans in ***Hilbert Space***. [4]

INTRODUCTION

Superposition gives quantum computers superior computing power

Superposition allows quantum algorithms to process information in a fraction of the time it would take even the fastest classical systems to solve certain problems.

- The amount of information a qubit system can represent grows exponentially. Information that 500 qubits can easily represent would not be possible with even more than 2^500 classical bits.

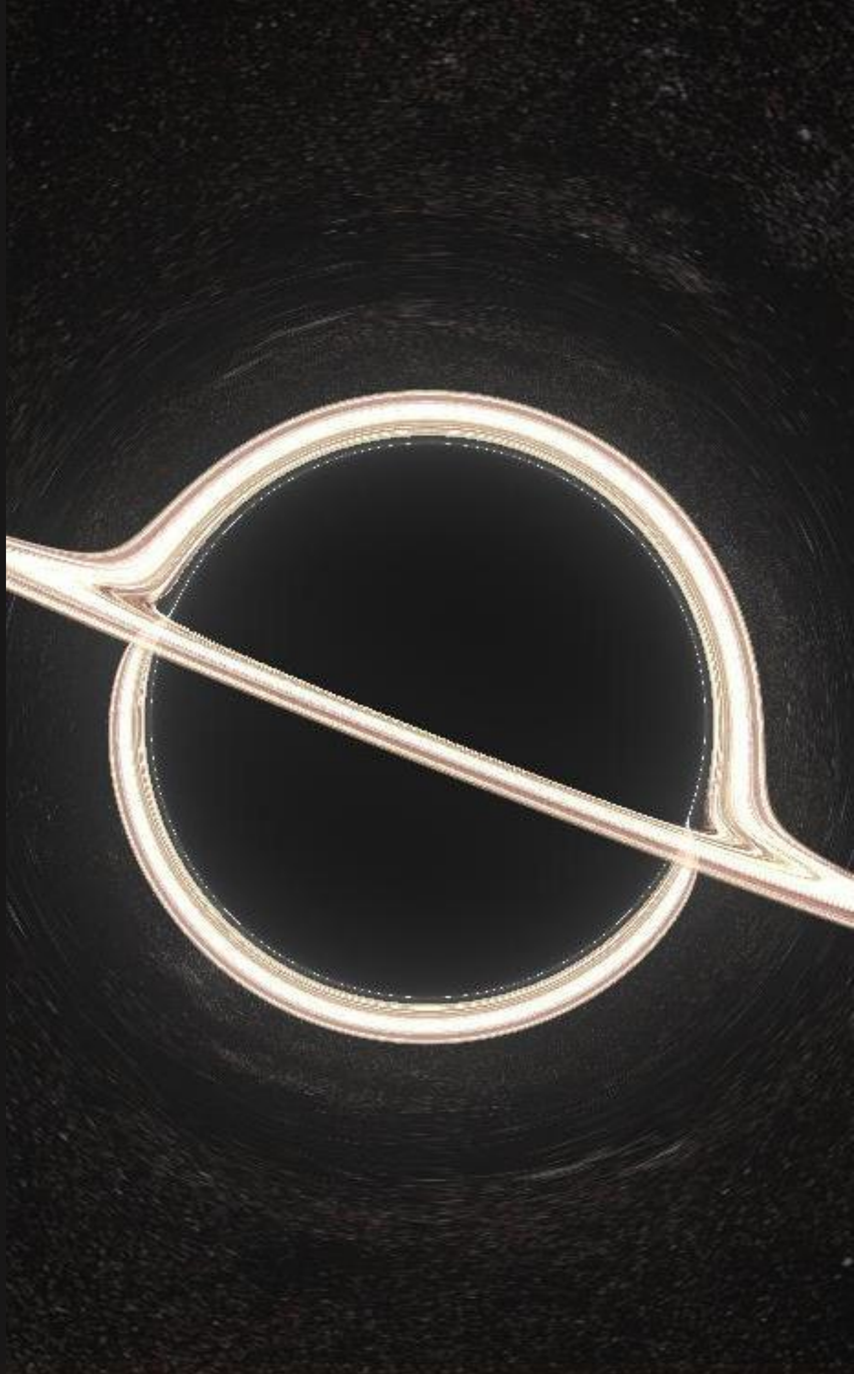
- It would take a classical computer millions of years to find the prime factors of a 2,048-bit number. Qubits could perform the calculation in just minutes.

Fig2. Advantages of Quantum Computers with given examples by Microsoft Azure

SCHWARZCHILD BLACK-HOLE

- Which describes *non-rotating* Black-Holes
- Space-time curvature becomes *infinite* at *r=0*
- *Time-dilation* effects becomes *infinite* on the spherical surface of event horizon.
- The Schwarzchild Radius (event horizon) is

  $\mathrm{R} = 2GM/c^2$ [6]



SCHWARZCHILD METRIC

This ***metric*** is an ***exact solution*** to the ***Einstein's Field Equations*** that explains the ***gravitational field*** caused by a ***spherical mass***. [3]

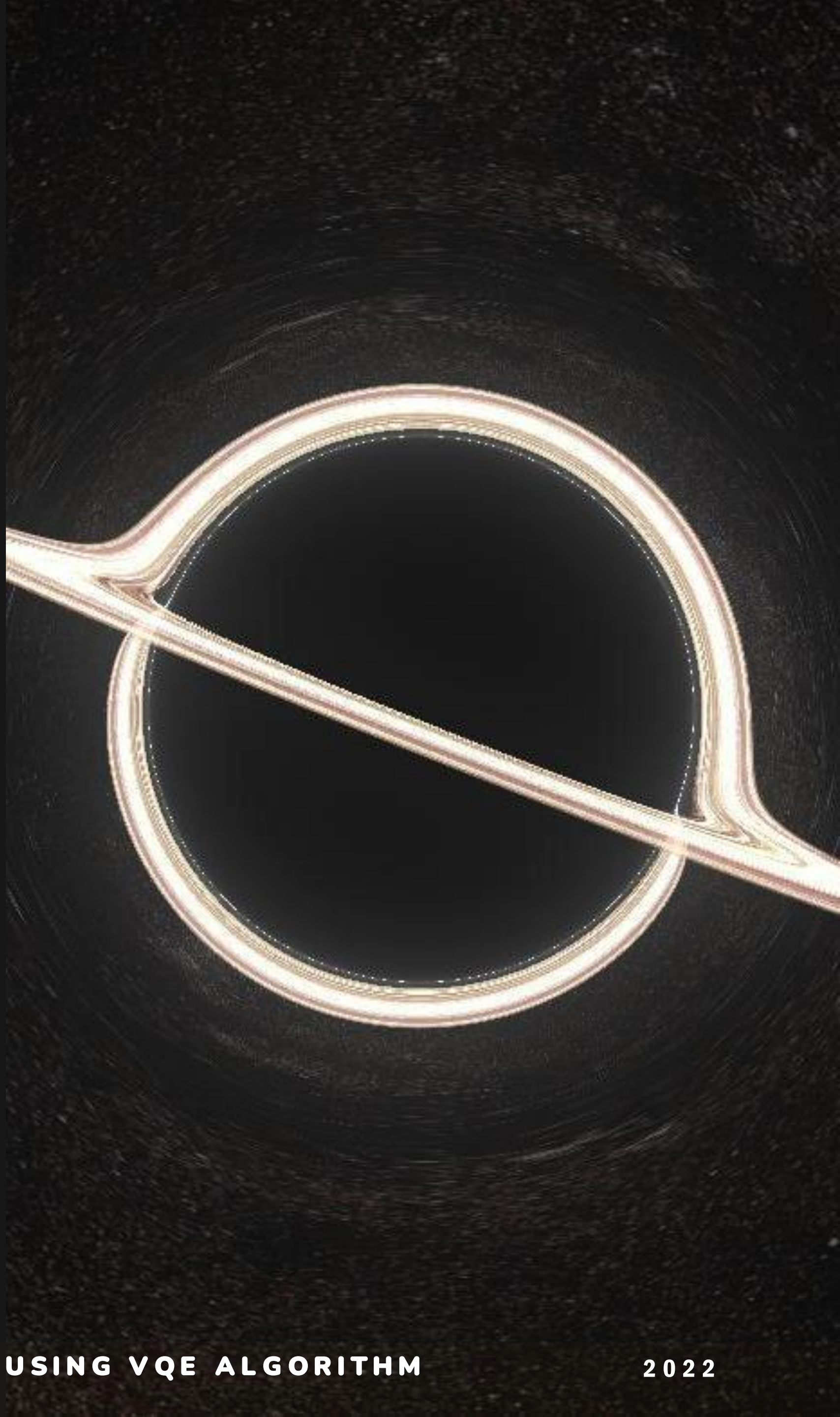
$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right)$$

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

$$x = r \sin\theta \cos\phi, y = r \sin\theta \sin\phi, z = r \cos\theta$$

$$ds^2 = \frac{(1 - GM/2r)^2}{(1 + GM/2r)^2} dt^2 - (1 + GM/2r)^4 (dx^2 + dy^2 + dz^2)$$

$$\gamma_{ij} = (1 + \frac{GM}{2r})^4 (dx^2 + dy^2 + dz^2)$$



FINAL HAMILTONIAN THAT WE WANT TO DEAL WITH:

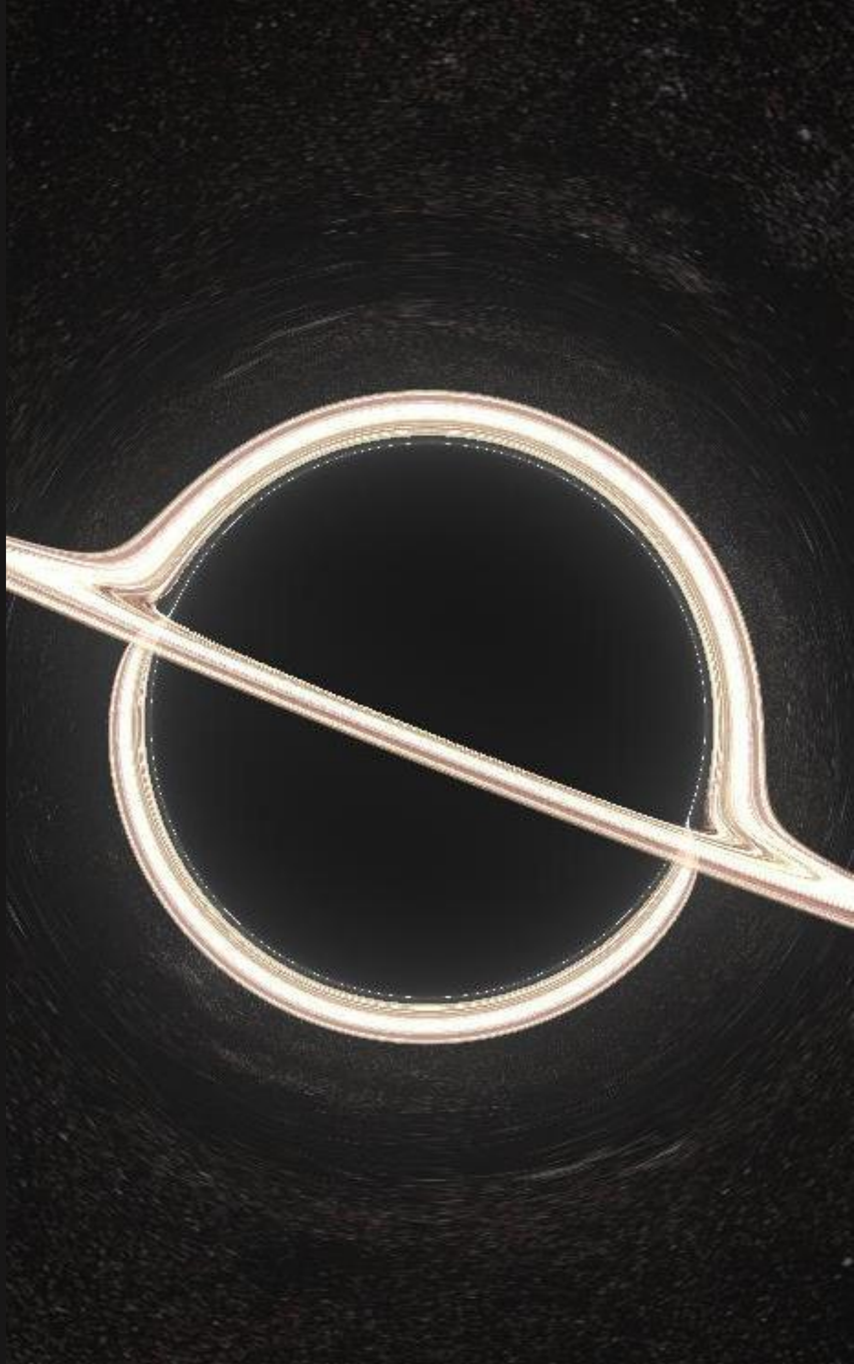
$$\gamma_{ij} = (1 + \frac{GM}{2r})^4 (dx^2 + dy^2 + dz^2)$$

$$H(q,p) = \frac{1}{2} g^{ij} p_i p_j$$

Free particle

$$\hat{H} = \frac{1}{2}(1 + GM/2r)^{1/4} (\hbar^2 \hat{\nabla}^2)$$

The equations are retrieved from Kohli's article [3]



THE VARIATIONAL PRINCIPLE

The solution of Scrödinger's equation as follows,

$$\hat{H}\psi = E\psi.$$

The energy eigen values can be found by looking expectation values by given ansatzes,

$$\tilde{E} = \frac{\langle\phi|\hat{H}|\phi\rangle}{\langle\phi|\phi\rangle}$$

When the ansatzes changes the energy eigen values are changing up to the minimum value of Ground State eigen value. Hence, If we minimize the graph of Eigenvalue vs the parameter in the ansatz wave function, we obtain the energy eigen values. [5,7]

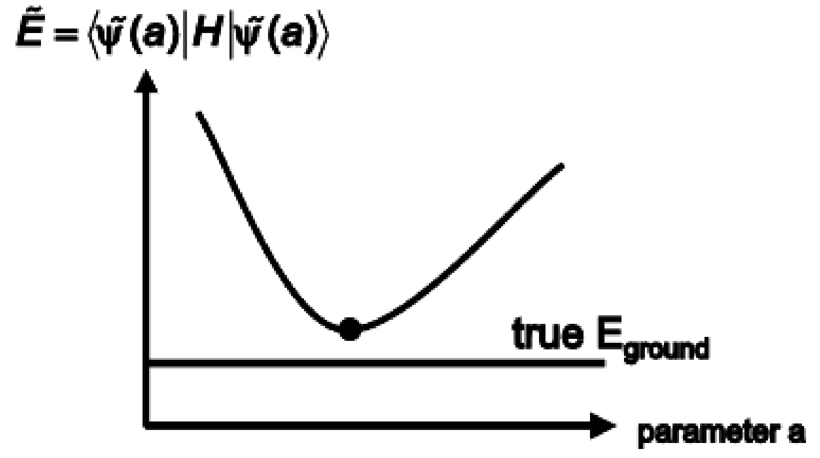


Fig3. Eigenvalues vs Ansatz's Parameter

$$E[\psi] = \frac{\langle\psi|\mathcal{H}|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0$$

Fig4. The inequality that indicates the minimum value is the Ground State value

METHODOLOGY

VARIATIONAL QUANTUM EIGENSOLVER (VQE)

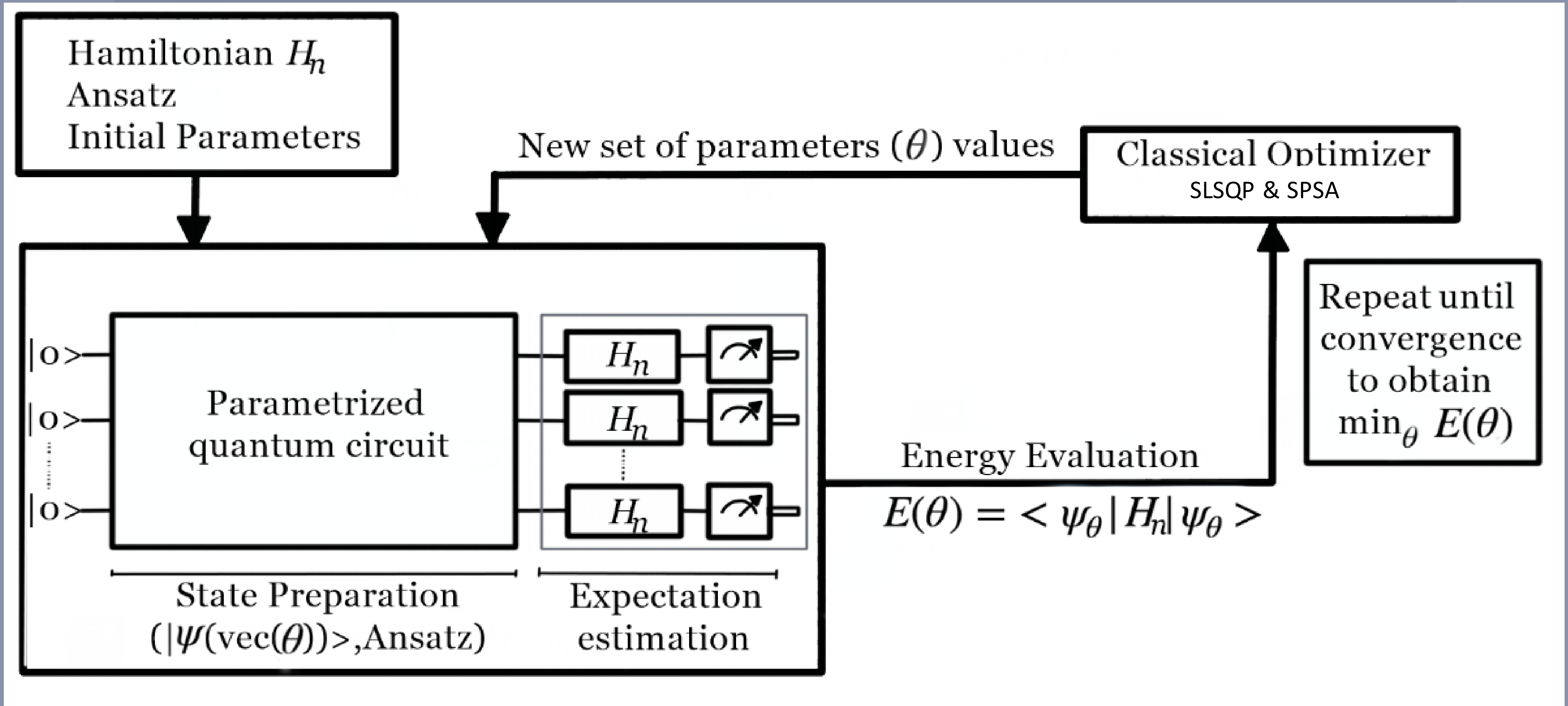


Fig5. VQE algorithm diagram

METHODOLOGY

VARIATIONAL QUANTUM EIGENSOLVER (VQE)

The position basis is used for Hamiltonian

$$x^d = \sqrt{\frac{\pi}{2N}} \begin{bmatrix} -N/2 & 0 & 0 & . & 0 \\ 0 & (-N/2)+1 & 0 & . & 0 \\ 0 & . & . & . & . \\ 0 & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & . & (N/2)-1 \end{bmatrix}$$

$$p^d = \frac{\sqrt{\pi}}{8\sqrt{2}} \begin{bmatrix} -2 & -2-2i & -2 & -2+2i \\ -2+2i & -2 & -2-2i & -2 \\ -2 & -2+2i & -2 & -2-2i \\ -2-2i & -2 & -2+2i & -2 \end{bmatrix}$$

$$p^d = (F^d)^{-1} x^d F^d$$

METHODOLOGY

VARIATIONAL QUANTUM EIGENSOLVER (VQE)

Decomposing Hamiltonian

In order to use VQE algorithm in IBM Qiskit we need to decompose Hamiltonian into Pauli Matrices. For 4 qubit example,

$$C_{\mu,\nu,\lambda,\theta} = \frac{1}{16} tr\left(\sum H \otimes \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\lambda \otimes \sigma_\theta\right)$$

$$H = \sum C_{\mu,\nu,\lambda,\theta} * \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\lambda \otimes \sigma_\theta$$

$$where\ \mu,\nu,\lambda,\theta = X, Y, Z, \mathrm{I}$$

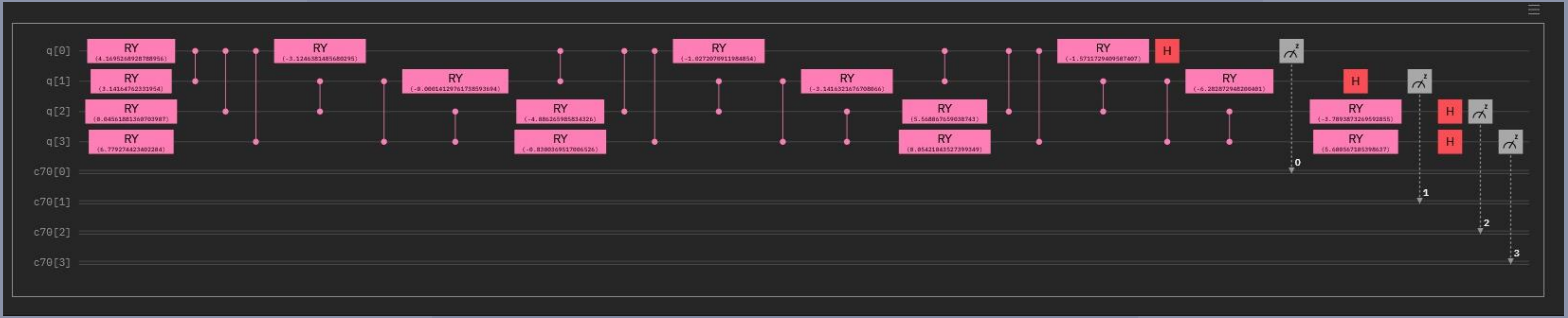


Fig6. VQE algorithm 4 qubit logic design created via Qiskit for harmonic oscillator

RESULTS

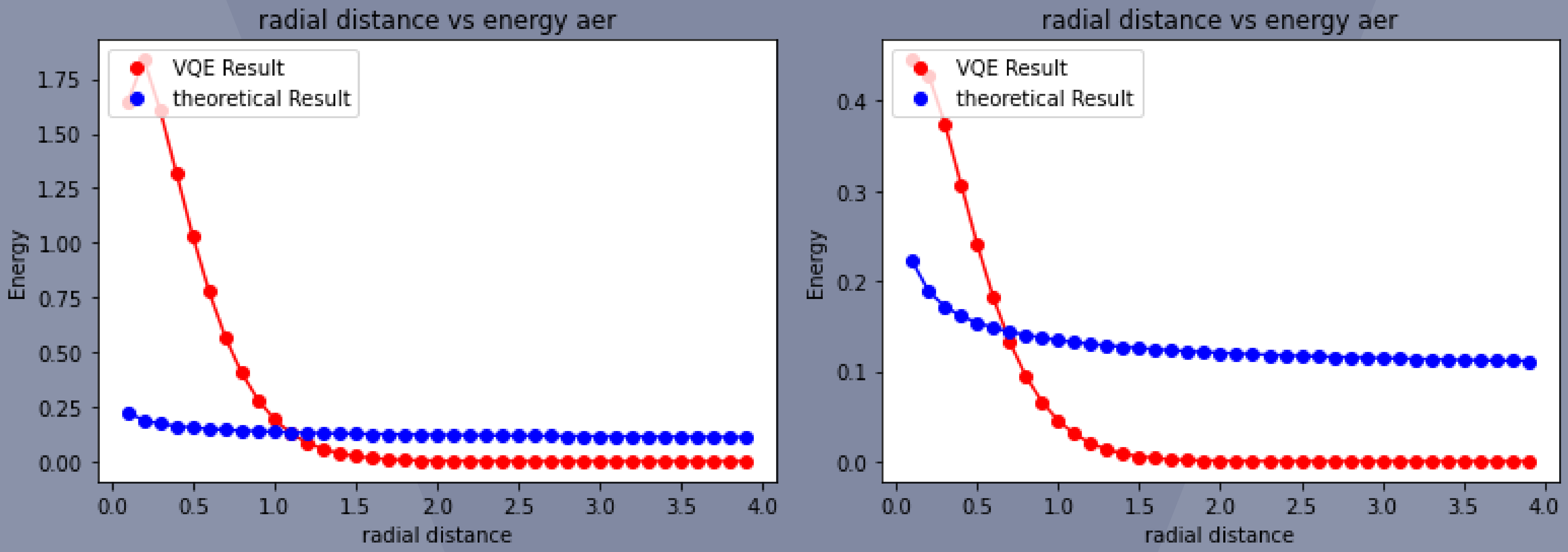


Fig6. Aer Simulator values with SLSQP and SPSA optimizers (Left to Right)

DISCUSSION

In this project we use different ansatz and different optimizers. Results show us that there are some ***deviations*** in VQE when the radial distances become smaller than 1 and we can relatively see $\mathbf{E = a(b + c/r)^{1/4}}$ relation when we have done the fitting with 5 solar mass. (a= 0.808, b=-0.065 , c=0.255)
Different structure of ansatz can be used to improve this work which can be seen on the 1st reference.

For source codes: https://github.com/Tihulu/400-Project-Cagil-Benibol

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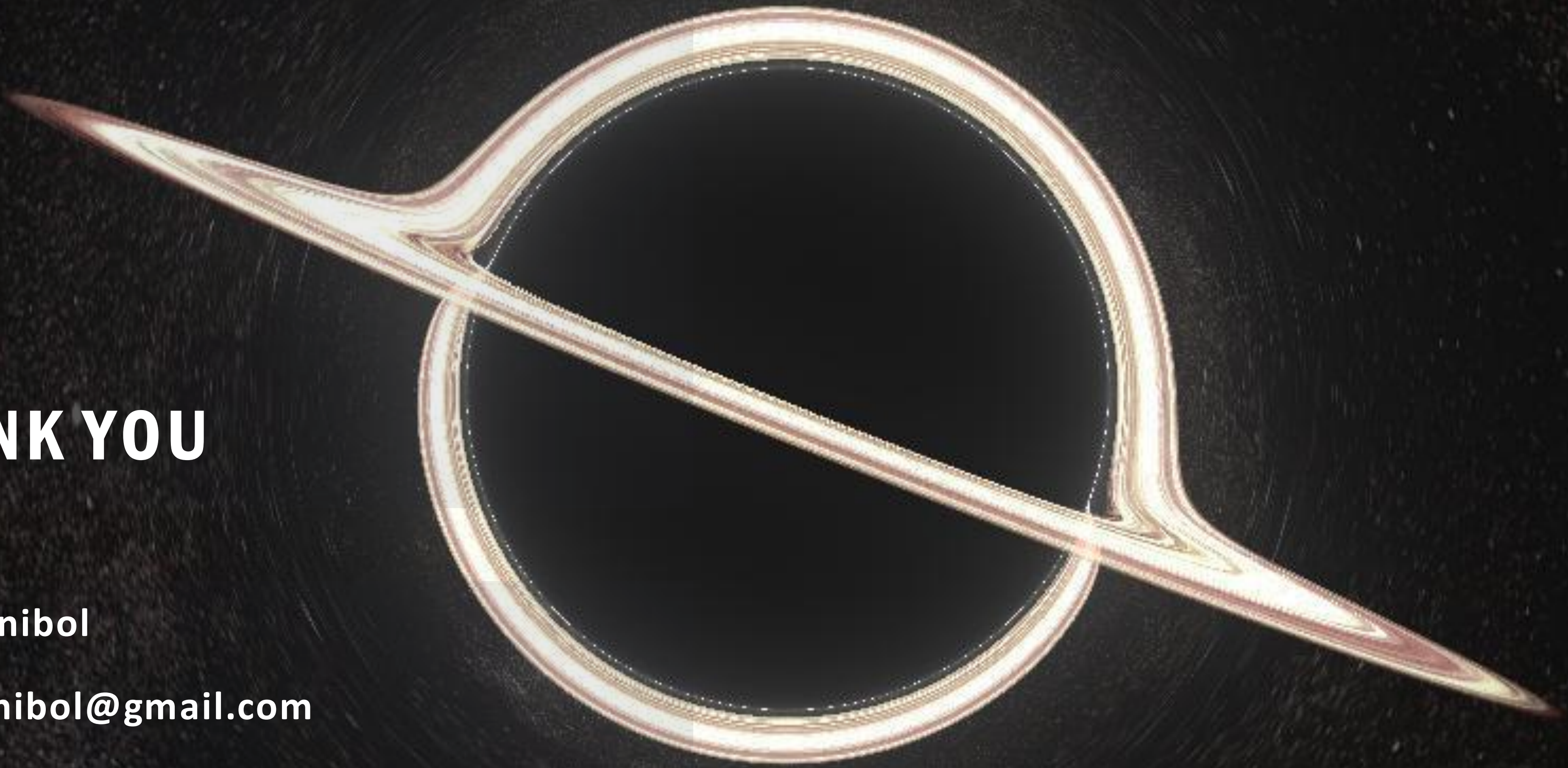

THANK YOU

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