ESTIMATING HAMILTONIAN IN A 3+0 SCHWARZCHILD METRIC WITH QUANTUM COMPUTERS USING VQE ALGORITHM

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Abstract

The advances on quantum computers are increasing rapidly. Algorithms on these computers are logarithmically faster. In order to catch this era of computing we need to understand the algorithms of the quantum information. Via using quantum information algorithms it is possible to make simulations to a given Hamiltonian that we are looking for. In other words, we can use the logic operations to solve Hamiltonian with VQE technique. [1,2]

In this work we tried to use quantum computers and its algorithms in the astrophysical subjects. The topic that I finalize my project is using in black hole metrics. What interesting is near in Schwarzchild radius wave function vanishes to zero. Which arises some questions like 'Is quantum mechanics reversable?' and 'Can an information be destroyed?'. In this work VQE may help us to see the relations between radius and energy. [1,3]

Introduction

What is quantum computing?

Quantum computers have far different architecture than the classical computers. In quantum computers we make circuit designs to execute our program. These circuits are consisting of both classical registers and quantum registers. In other words, we use qubits (quantum bits) in our logic designs. [4]

What is Qubit?

Qubits uses quantum phenomena of superposition and entanglement which gives quantum computers superior advantage when it compared with classical computer. It natively shows linear combination of states which spans in Hilbert Space. [4]

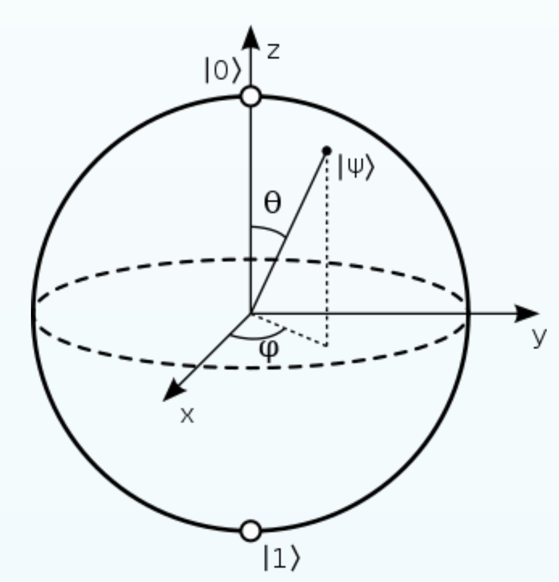

Fig1. Bloch Sphere Representation of Qubit

Superposition gives quantum computers superior computing power

Superposition allows quantum algorithms to process information in a fraction of the time it would take even the fastest classical systems to solve certain problems.

- The amount of information a qubit system can represent grows exponentially. Information that 500 qubits can easily represent would not be possible with even more than 2^500 classical bits.
- It would take a classical computer millions of years to find the prime factors of a 2,048-bit number. Qubits could perform the calculation in just minutes.

Fig2. Advantages of Quantum Computers with given examples by Microsoft Azure

Schwarzchild Metric:

This metric is an exact solution to the Einstein's Field Equations that explains the gravitational field caused by a spherical mass. [3]

To construct our Hamiltonian on non-rotating black hole we use the following metric. [3]

$$ds^2 = \left(1-\frac{2GM}{r}\right)dt^2 - \left(1-\frac{2GM}{r}\right)^{-1}dr^2 - r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

By applying given coordinates [3]

$$x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$$

$$r = \left(x^2+y^2+z^2\right)^{1/2}$$

The Isotropic rectangular metric as follows [3]

$$ds^2 = \frac{(1-GM/2r)^2}{(1+GM/2r)^2}dt^2 - (1+GM/2r)^4(dx^2+dy^2+dz^2)$$

Since The Schwarzchild solution is a static solution, our metric becomes time independent. [1,3]

$$\gamma_{ij} = (1+\frac{GM}{2r})^4(dx^2+dy^2+dz^2)$$

Using Jacobi's principle of least action, we obtain the Hamiltonian that we need to simulate. [1]

$$H^d = \frac{1}{2}(1+\frac{GM}{2r})^{1/4}[\frac{(p_x^d)^2}{2}+\frac{(p_y^d)^2}{2}+\frac{(p_z^d)^2}{2}]$$

Variational Principle:

The solution of Scrödinger's equation as follows,

$$\hat{H}\psi = E\psi.$$

The energy eigen values can be found by looking expectation values by given ansatzes,

$$\tilde{E} = \frac{\langle\phi|\hat{H}|\phi\rangle}{\langle\phi|\phi\rangle}$$

When the ansatzes changes the energy eigen values are changing up to the minimum value of Ground State eigen value. Hence, If we minimize the graph of Eigenvalue vs the parameter in the ansatz wave function, we obtain the energy eigen values. [5,6]

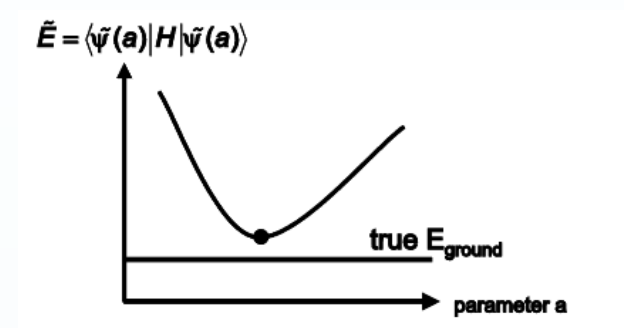

Fig3. Eigenvalues vs Ansatz's Parameter

$$E[\psi] = \frac{\langle\psi|\mathcal{H}|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0$$

Fig4. The inequality that indicates the minimum value is the Ground State value

Methodology

Variational Quantum Eigensolver (VQE)

This hybrid algorithm uses variational principle that calculates the eigenvalues to the Quantum Computer with given optimized ansatz waves by the classical computers. We used two optimizers for ansatzes SLSQP (Sequential Least SQuares Programming) and SPSA (Simultaneous Perturbation Stochastic Approximation. Ansatz consists of RY & CZ gates.

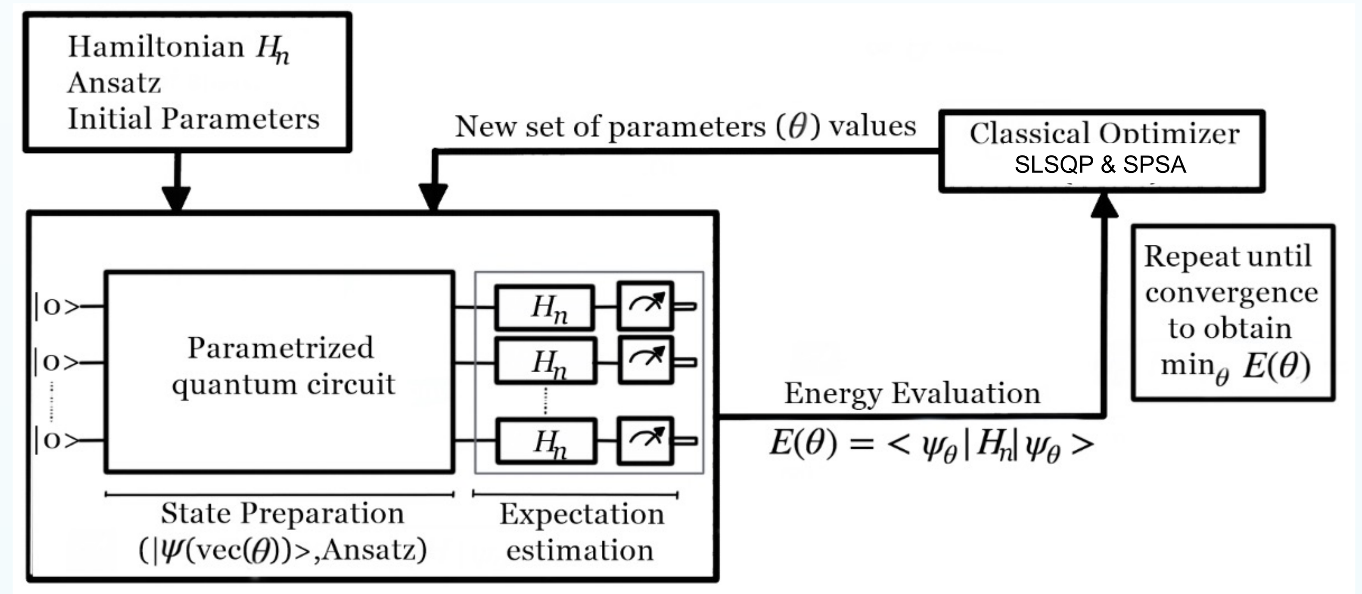

Fig5. VQE algorithm diagram

Decomposing Hamiltonian

In order to use VQE algorithm in IBM Qiskit we need to decompose Hamiltonian into Pauli Matrices. For 4 qubit example,

$$H = \sum C_{\mu,\nu,\lambda,\theta} \otimes \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\lambda \otimes \sigma_\theta \qquad C_{\mu,\nu,\lambda,\theta} = \frac{1}{16}tr(\sum H * \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\lambda \otimes \sigma_\theta)$$

$$where\ \mu, \nu, \lambda, \theta = X, Y, Z, I$$

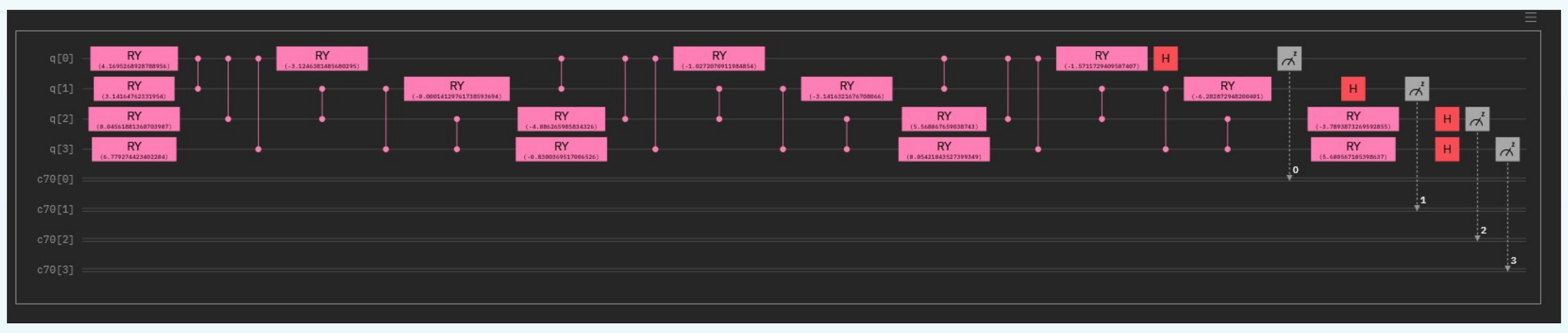

Fig6. VQE algorithm 4 qubit logic design created via Qiskit for harmonic oscillator

Results and Conclusion

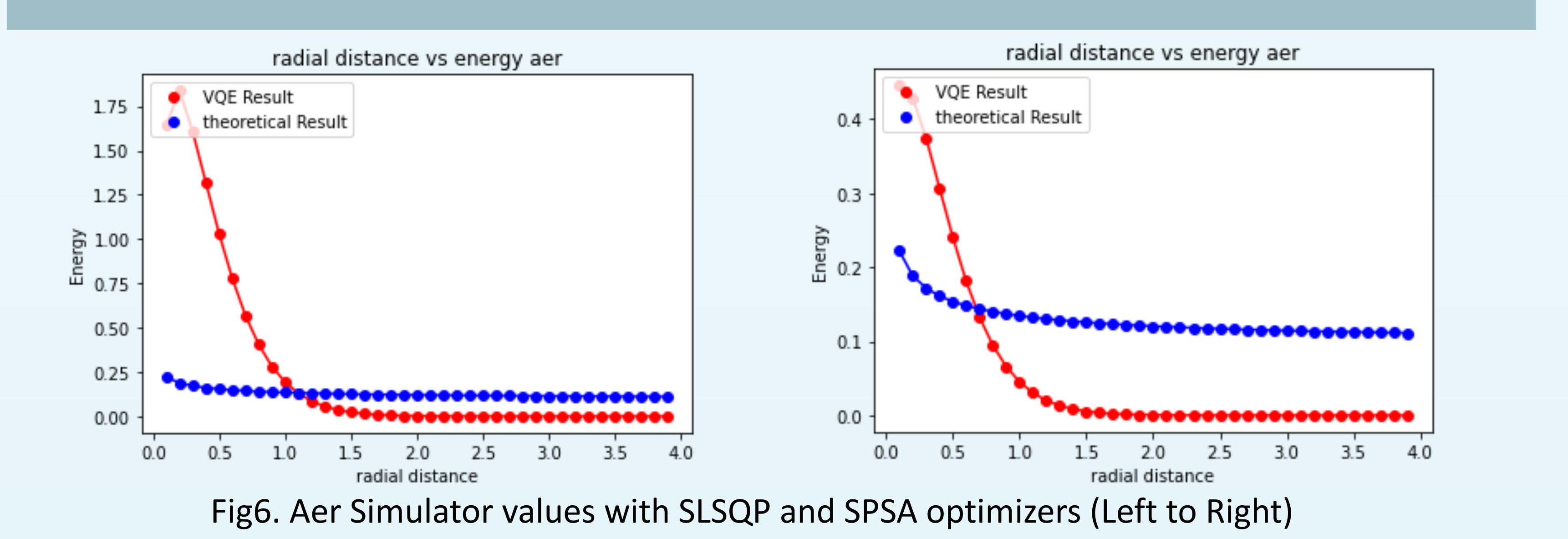

Fig6. Aer Simulator values with SLSQP and SPSA optimizers (Left to Right)

In this project we use different ansatz and different optimizers. Results show us that there are some deviations in VQE when the radial distances become smaller than 1 and we can relatively see $E = a(b + c/r)^{1/4}$ relation when we have done the fitting with 5 solar mass. (a= 0.808, b=-0.065 , c=0.255)

Different structure of ansatz can be used to improve this work which can be seen on the 1st reference. For source codes: https://github.com/Tihulu/400-Project-Cagil-Benibol

Bibliography & Acknowledgements

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