

# Reductions

Handout: Oct 16, 2020 12:00 AM

Due: Nov 2, 2020 3:30 PM

## Exercise 2 -- Ind-CPA Security of El Gamal PKE

[Open Task](#)

## Exercise 2 - Hardness of DDH implies ElGamal PKE is IND-CPA-secure

Again, we consider  $\mathbb{G}$  to be a cyclic group of prime order  $p$ . In this assignment, we want to show that if DDH holds, then the ElGamal PKE scheme is IND-CPA-secure. Your task is to implement the `decideDDH` method in the `Solution2` java class:

```
public class Solution2 implements IDDHIndCPAElgamalReduction {

    @Override
    public boolean decideDDH(DDHChallenge ddhChallenge, IElgamalIndCPAAdversary adversary) {
        //your code here
        return true if the ddhChallenge is an honestly generated DDH tuple, false otherwise;
    }

}
```

As in Exercise 1, you will have to initialize and invoke the `IElgamalIndCPAAdversary`. Recall that the DDH problem means you are given a generator  $g$  and you are asked to distinguish between  $(g, g^x, g^y, g^{xy})$  and  $(g, g^x, g^y, g^z)$ , where  $x, y, z \leftarrow_r \mathbb{Z}_p$ . If the challenge is an honestly generated DDH tuple (i.e. of the form  $(g, g^x, g^y, g^{xy})$ ), you should output `true`. Otherwise, output `false`.

As in Exercise 1, your reduction is only allowed to call the method `adversary.solveIndCPAChallenge(ciphertext)` at most **once** (meaning that we ask that your reduction is tight).

The `init` and `solve` methods of the adversary are defined similarly to Exercise 1. One key difference is that now we have a new method:

```
public CandidateMessagePair<IGroupElement> getCandidateMessages();
```

This method should be called after you have already called `init`. It models the stage in the IND-CPA game when the adversary picks its plaintext challenges. Here, the adversary will generate two plaintexts (of type `IGroupElement`) and return them to your reduction. If `init` has not been called before, `getCandidateMessages` will return null. Calling this method will cause this instance to save copies of both messages. Calling this method a second time will cause this instance to overwrite its last saved values.

The output value is a `CandidateMessagePair` which contains two `IGroupElements`. The first entry is `message0` (the first message the adversary has chosen) and the second one is `message1` (the second message the adversary has chosen).

To retrieve the adversary's guess on whether `message0` or `message1` have been encrypted, you should run the following method of the adversary:

```
public int solveIndCPAChallenge(ElgamalCiphertext ciphertext);
```

You should only call this method after you have called `init` and `getCandidateMessages` and supplied a ciphertext of an instance of ElGamal's PKE scheme whose public key and group generator are consistent with the parameters of your `init` call.

If the ciphertext is a valid encryption of `message0`, the first message returned by `getCandidateMessages`, then this method should return 0.

If ciphertext is a valid encryption of `message1`, the second message returned by `getCandidateMessages`, then this method should return 1.

*Note* If neither of the above cases does occur, then this method will return 0 or 1 (without any guarantees on which value will be returned). If `init` or `getChallengeMessages` has never been called before, this method will also return 0 or 1 (without any guarantees which value will be returned).

## Evaluation

When you submit your solution, our tests will run the reduction 350 times per adversary. Your reduction should win, on average, 75% of the games (meaning that the theoretical maximum advantage of your reduction should be 1/2). Your reduction passes our test if the advantage we measure is at least 25% (i.e. if it wins at least 62,5% of all games against each adversary).