Fluvial Evolution of Riverbeds with Cellular Automata

Group XI: River's Eleven

Marvin Frommer, Wessel Beumer, Paul Jungnickel, Tika van Bennekum

Complex System Simulation, January 2025

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Introduction

Braided Rivers

Necessary Conditions for Braided Rivers

- High sediment supply from surrounding landscapes or glaciers.
- Steep channel gradients that promote fast-moving water.
- Easily erodible banks, often composed of sand and gravel.

Methods

The paper our system is based on

FIG. 2 Images from successive times in one run of the model, showing 22×200 cells. Flow is from top to bottom. The left-hand image of each pair shows topography (minus the average slope), while the right-hand image shows discharge. The colour schemes for discharge and topography are shown at left. Topography is shown in shaded relief with lighting from the bottom (downstream). In this run we used: $n=1$ (see text and Box 1); sediment-transport rule 3 (see Box 1) with $m=2.5$ and $C=3$ times the average slope; and the lateral transport rule (see Box 1). The images are separated by 300 iterations. Channel migration occurs around location A, between times 3 and 4. A bar develops at location B by time 1, and the flow becomes shallow and develops multiple channels as it passes over the bar. By time 2, the bar has caused a split in the flow. By time 3, channel switching and new bar development just upstream of location B have cut off most of the water from the first bar. But by time 4 the new bar has caused another channel split, redirecting some flow back in the original direction. Then by time 5, the flow has reworked part of the new bar, cutting a channel across it at higher elevations than exist in some areas around the bar, as we have observed in real braided streams in the laboratory. Bar development and reworking also occur at location C.

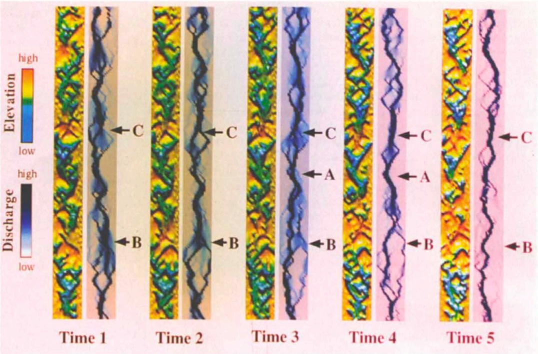
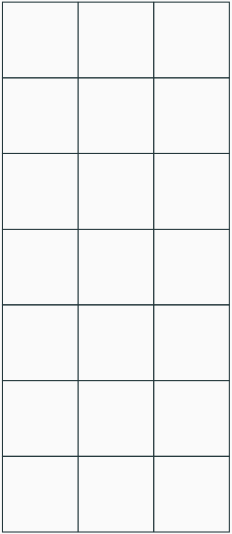


Figure 1: B. Murrey and C. Paola, 1994 & 1997.

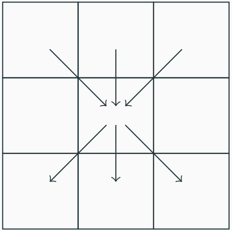
https://doi.org/10.1038/371054a0

The paper

- Rivers are complicated
- Simple model
- Continuous Cellular automata
- Gradual slope



Rules



On each step:

- All water flows down
- Distribution by hight
    - Favor downhill flow
- Sediment-transport
    - Water flow
    - Local slope
    - Erosion constant

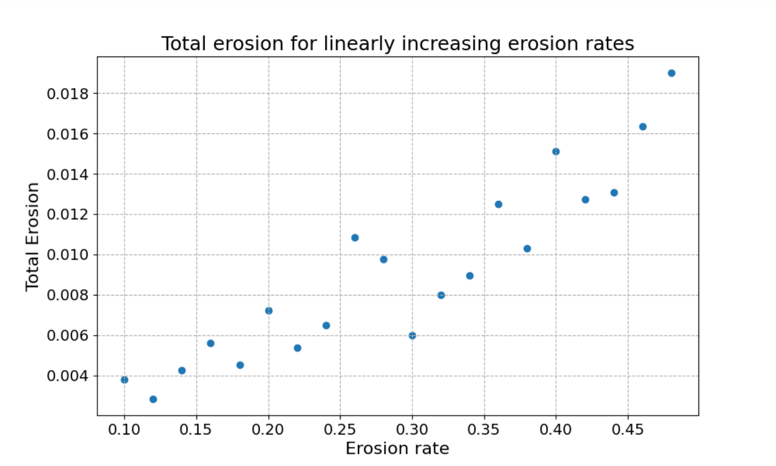
Video 3D visualization of our system

Lane's relation

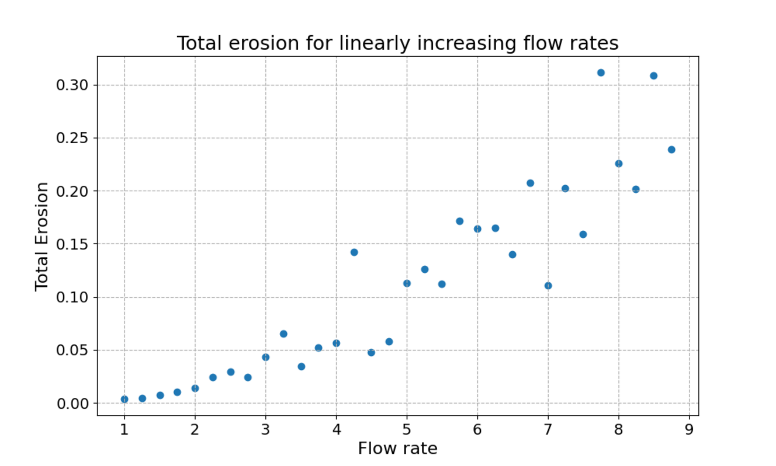
$$\text{Sediment Flow} \times \text{Sediment Size} \propto \text{Water Flow} \times \text{Slope}$$

Experiments

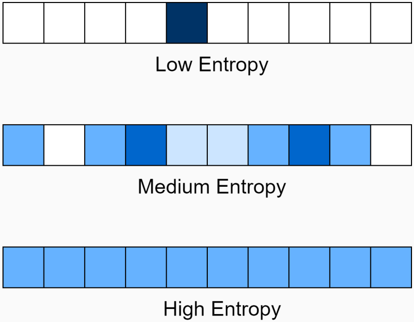
Does our system follow Lane's relation?



Does our system follow Lane's relation?



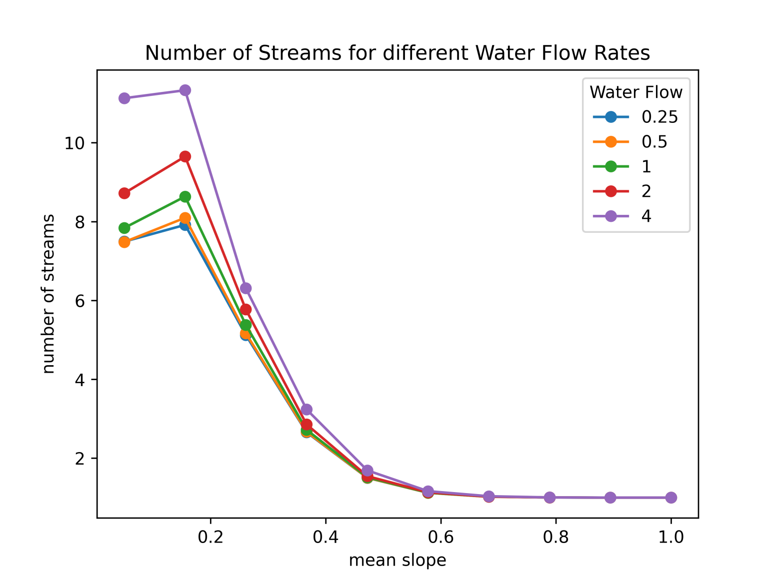
Measuring the complexity of the simulation states



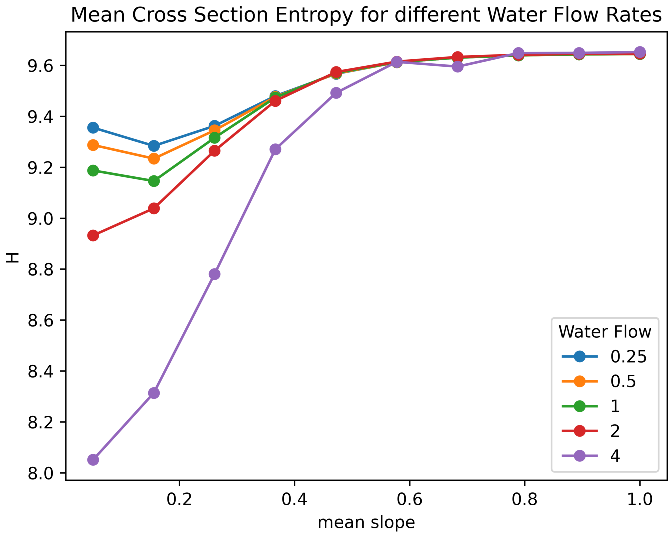
- Measure 1:
  Count the number of disjunct streams in a cross section
- Measure 2:
  Compute the Entropy of the water distribution

$$H := -\sum_{i=1}^{N} P(x_i) log_2(P(x_i))$$

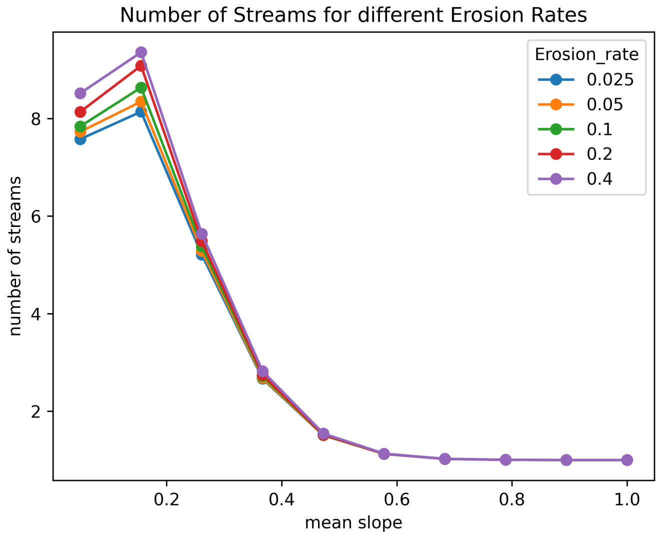
Analyzing the Number of Streams



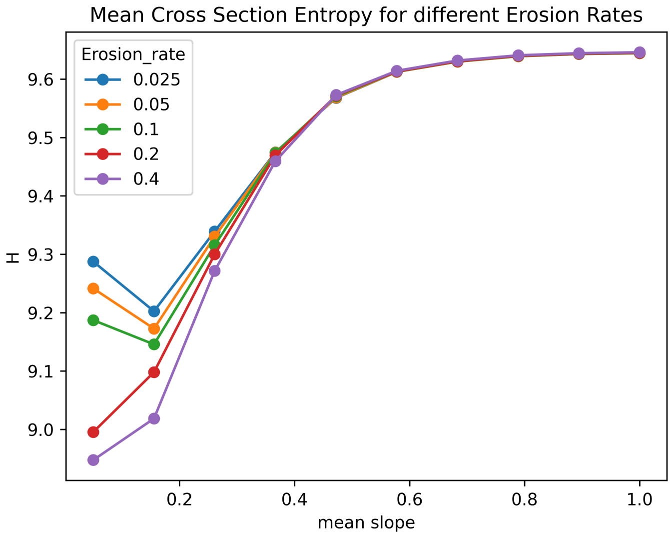
Analyzing the Entropy of water distributions



Analyzing the Number of Streams



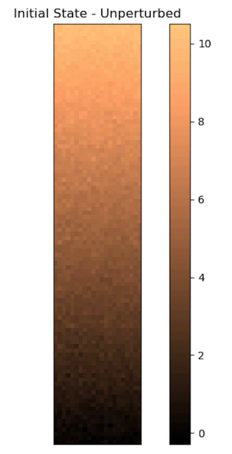
Analyzing the Entropy of water distributions

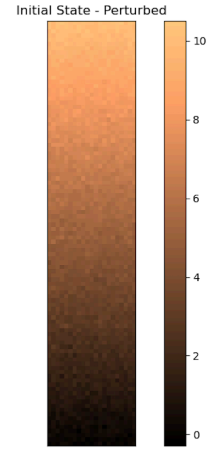


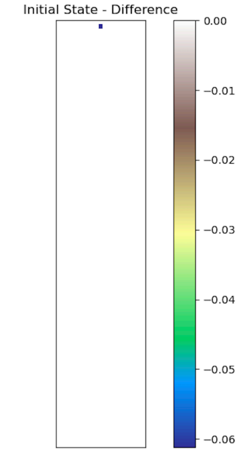
Sensitivity to Initial Conditions

- Objective:
  - Investigate how much a minimal perturbation impacts the development of the system.
- Perturbation Method:
  - Adjust the terrain height in the cell directly beneath the water source by a maximum of **±0.1%**.
- Analysis:
  - Conduct multiple simulations with varying perturbations.
  - Measure the average absolute difference over time compared to the initial state.
- Expectation:
  - Divergence increases significantly over time, following a non-linear trend.

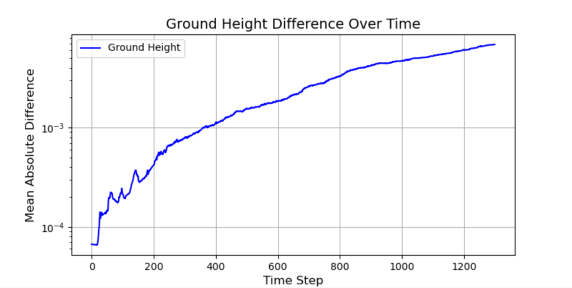
Sensitivity to Initial Conditions

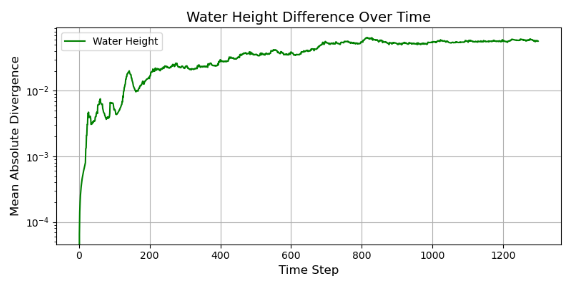




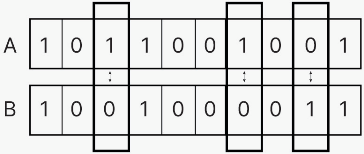


Sensitivity to Initial Conditions - Results





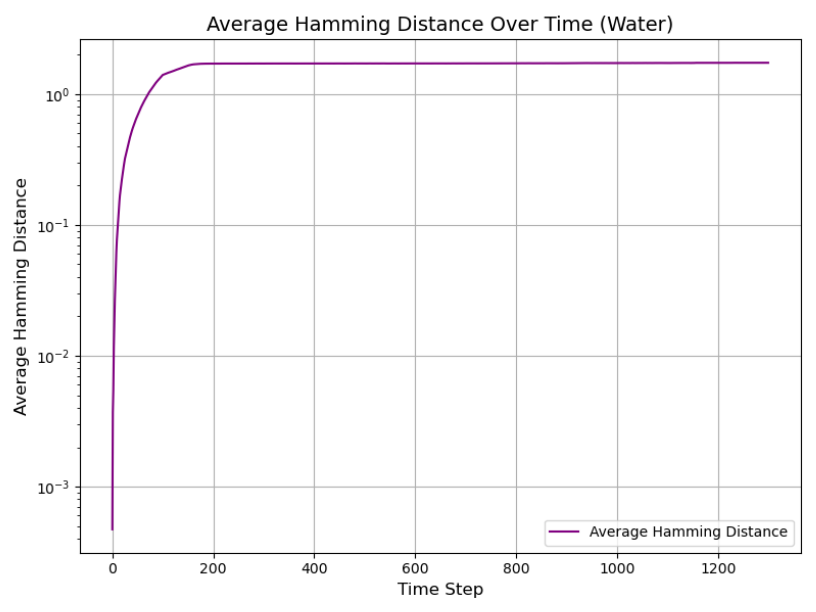
Sensitivity to Initial Conditions - Hamming Distance



$$D_H = \sum_{i=1}^{n} (x_i \oplus y_i)$$

- D_H: Hamming distance
- x_i, y_i: Binary states at position i
- $\oplus$: XOR operation (1 if different, 0 if the same)
- n: Total number of positions

Sensitivity to Initial Conditions - Results



Future work

Future work

- More types of rivers
- Experiment with different rule sets

Thank you!