# PECAN: A Deterministic Certified Defense Against Backdoor Attacks

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## **Abstract**

Neural networks are vulnerable to backdoor poisoning attacks, where the attackers maliciously poison the training set and insert triggers into the test input to change the prediction of the victim model. Existing defenses for backdoor attacks either provide no formal guarantees or come with expensive-to-compute and ineffective probabilistic guarantees. We present PECAN, an efficient and certified approach for defending against backdoor attacks. The key insight powering PECAN is to apply off-the-shelf test-time evasion certification techniques on a set of neural networks trained on disjoint partitions of the data. We evaluate PECAN on image classification and malware detection datasets. Our results demonstrate that PECAN can (1) significantly outperform the state-of-the-art certified backdoor defense, both in defense strength and efficiency, and (2) on real backdoor attacks, PECAN can reduce attack success rate by order of magnitude when compared to a range of baselines from the literature.

#### 1. Introduction

Deep learning models are vulnerable to *backdoor* poisoning attacks (Saha et al., 2020; Turner et al., 2019), which assume that the attackers can maliciously *poison* a small fragment of the training set before model training and add *triggers* to inputs at test time. As a result, the prediction of the victim model that was trained on the poisoned training set will diverge in the presence of a trigger in the test input.

Effective backdoor attacks have been proposed for various domains, such as image recognition (Gu et al., 2017), sentiment analysis (Qi et al., 2021), and malware detection (Severi et al., 2021). For example, Severi et al. (2021) can break malware detection models as follows: The attacker *poisons* a small portion of benign software in the training set by modifying the values of the most important features so that

the victim model recognizes these values as evidence of the benign prediction. At test time, the attacker inserts a *trigger* by changing the corresponding features of malware to camouflage it as benign software and thus making it bypass the examination of the victim model. Thus, backdoor attacks are of great concern to the safety and security of deep learning models and systems, particularly as training data is gathered from different sources, e.g., via web scraping.

Several works have studied defenses against various types of attacks. We identify two limitations with these defenses. First, many existing approaches only provide empirical defenses that are specific to certain attacks and do not generalize to *all* backdoor attacks. Second, existing certified defenses—i.e., approaches that produce robustness certificates—are either unable to handle backdoor attacks, or are probabilistic (instead of deterministic), and therefore expensive and ineffective in practice.

**Why certification?** A defense against backdoor attacks should construct effective certificates (proofs) that the learned model can indeed defend against backdoor attacks. Empirical defenses (Geiping et al., 2021a; Liu et al., 2018) do not provide certificates, can only defend against specific attacks, and can be bypassed by new unaccounted-for attacks (Wang et al., 2020b; Koh et al., 2022). Certification has been successful at building models that are provably robust to trigger-less poisoning attacks and evasion attacks, but models trained to withstand such attacks are still weak against backdoor attacks. The trigger-less attack (Zhu et al., 2019; Shafahi et al., 2018; Aghakhani et al., 2021; Geiping et al., 2021b) assumes the attacker can poison the training set but cannot modify the test inputs, e.g., adding triggers, while the evasion attack (Madry et al., 2018) assumes the attacker modifies the test inputs but cannot poison the training set. Existing certified defenses against trigger-less and evasion attacks, e.g., DPA (Levine & Feizi, 2021) and CROWN-IBP (Zhang et al., 2020), cannot defend against backdoor attacks as they can either defend against the poison in the training data or the triggers at test time, but not both. As we show in the experiments, we can break these certified defenses using a backdoor attack (Section 5.3).

Why determinism? It is desirable for a certified defense to be *deterministic* because probabilistic defenses (Zhang et al., 2022b; Weber et al., 2020) typically require one to

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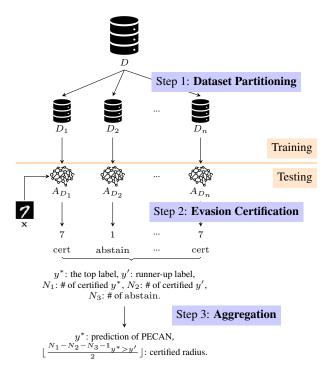


Figure 1. An overview of our approach PECAN.

retrain thousands of models when performing predictions for a single test input. Retraining can be mitigated by Bonferroni correction, which allows reusing the trained models for a fixed number of predictions. However, retraining is still necessary after a short period, making it hard to deploy these defenses in practice. On the other hand, deterministic defenses (Levine & Feizi, 2021; Wang et al., 2022b) can reuse the trained models an arbitrary number of times when producing certificates for different test inputs. Furthermore, probabilistic defenses for backdoor attacks, e.g., BagFlip (Zhang et al., 2022b), need to add noise to the training data, resulting in low accuracy for datasets that cannot tolerate too much noise when training (Section 5.2).

**PECAN** In this paper, we propose PECAN (Partitioning data and Ensembling of Certified neurAl Networks), a deterministic certified defense against backdoor attacks for neural networks. The key insight underlying PECAN is that we can take any off-the-shelf technique for evasion certification and use it to construct a certified backdoor defense. This insight results in a simple implementation and allows us to seamlessly leverage future advances in evasion certification algorithms. Specifically, PECAN trains a set of neural networks on disjoint partitions of the dataset, and then applies evasion certification to the neural networks. By partitioning the dataset, we analytically bound the number of poisoned data seen per neural network; by employing evasion certification, we bound the number of neural networks that are robust in the face of triggers. Using this information, we efficiently derive a backdoor-robustness guarantee.

Figure 1 illustrates the workflow of PECAN. In Step 1, inspired by *deep partition aggregation* (Levine & Feizi, 2021), PECAN deterministically partitions a dataset into multiple disjoint subsets. This step ensures that a poisoned data item only affects a single partition. In Step 2, PECAN trains an ensemble of neural networks, one on each partition. At test time, PECAN performs evasion certification to check which neural networks are immune to triggers; those that are not immune (or that cannot be proven immune) abstain from performing a prediction. Finally, in Step 3, PECAN aggregates the results of the ensemble and produces a prediction together with a robustness certificate: the percentage of the poisoned data in the training set that the training process can tolerate, the *certified radius*.

We evaluate PECAN on two three datasets, MNIST, CI-FAR10, and EMBER. First, we show that PECAN outperforms or competes with BagFlip, the state-of-the-art probabilistic certified defense against backdoor attacks. Furthermore, BagFlip takes hours to compute the certificate, while PECAN only takes a few seconds. Second, when we evaluate PECAN against a concrete known backdoor attack (Severi et al., 2021), PECAN reduces the attack success rate to 1.85%, while DPA and CROWN-IBP fail to defend against the backdoor attack on 18.05% and 15.24% of the cases, respectively. The results show that PECAN can defend against a known backdoor attack while other baselines, such as DPA and CROWN-IBP, cannot.

## 2. Related Work

Deep learning models are vulnerable to backdoor attacks (Saha et al., 2020; Turner et al., 2019). Although many empirical defenses (Geiping et al., 2021a; Liu et al., 2018) have been proposed, recent works (Wang et al., 2020b; Koh et al., 2022) show that new attacks can break these empirical defenses. Therefore, certified defense is crucial for defending against backdoor attacks.

Certified defenses against backdoor attacks Existing certification approaches provide probabilistic certificates by extending randomized smoothing (Cohen et al., 2019; Dvijotham et al., 2020; Lee et al., 2019), originally proposed to defend against adversarial evasion attacks, to defend against backdoor attacks. BagFlip (Zhang et al., 2022b) is the state-of-the-art model-agnostic probabilistic defense against feature-flipping backdoor attacks. Wang et al. (2020a); Weber et al. (2020) proposed backdoor-attack defenses that are also model-agnostic, but are less effective than BagFlip. PECAN is deterministic and therefore less expensive and more effective than these defenses. Probabilistic defenses are model-agnostic; while PECAN is evaluated on neural networks, it can work for any machine learning model as long as a deterministic evasion certification approach of the model is available. Weber et al. (2020) proposed a deterministic de-randomized smoothing approach for kNN classifiers. Their approach computes the certificates using an expensive dynamic programming algorithm, whereas PECAN's certification algorithm has constant time complexity.

Certified defenses against trigger-less attacks Many approaches provide certificates for trigger-less attacks. Jia et al. (2021) use bootstrap aggregating (Bagging). Chen et al. (2020) extended Bagging with new selection strategies. Rosenfeld et al. (2020) defend against label-flipping attacks on linear classifiers. Differential privacy (Ma et al., 2019) can also provide probabilistic certificates for triggerless attacks. DPA (Levine & Feizi, 2021) is a deterministic defense that partitions the training set and ensembles the trained classifiers. Wang et al. (2022b) proposed FA, an extension of DPA, by introducing a spread stage. A conjecture proposed by Wang et al. (2022a) implies that DPA and FA are asymptotically optimal defenses against trigger-less attacks. Chen et al. (2022) proposed to compute collective certificates, while PECAN computes sample-wise certificates. Jia et al. (2020); Meyer et al. (2021); Drews et al. (2020) provide certificates for nearest neighborhood classifiers and decision trees. The approaches listed above only defend against trigger-less attacks, while PECAN is a deterministic approach for backdoor attacks.

Certified defenses against evasion attacks There are two lines of certified defense against evasion attacks: complete certification (Wang et al., 2021; Zhang et al., 2022a; Katz et al., 2019) and incomplete certification (Xu et al., 2020; Zhang et al., 2021; Singh et al., 2019). The complete certified defenses either find an adversarial example or generate proof that all inputs in the given perturbation space will be correctly classified. Compared to the complete certified defenses, the incomplete ones will abstain from predicting if they cannot prove the correctness of the prediction because their techniques will introduce over-approximation. The complete approaches do not have over-approximation issues but require expensive verification algorithms such as branch and bound. Our implementation of PECAN uses an incomplete certified approach CROWN-IBP (Zhang et al., 2020) because it is the best incomplete approach, trading off between efficiency and the degree of over-approximation.

## 3. Problem Definition

Given a dataset  $D = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$ , a (test) input  $\mathbf{x}$ , and a machine learning algorithm A, we write  $A_D$  to denote the machine learning model learned on dataset D by the algorithm A, and  $A_D(\mathbf{x})$  to denote the output label predicted by the model  $A_D$  on input  $\mathbf{x}$ . We assume the algorithm will behave the same if trained on the same dataset across multiple runs. This assumption can be guaranteed by fixing the random seeds during training.

We are interested in certifying that if an attacker has poisoned the dataset, the model we have trained on the dataset will still behave "well" on the test input with maliciously added triggers. Before describing what "well" means, we need to define the *perturbation spaces* of the dataset and the test input, i.e., what possible changes the attacker could make to the dataset and the test input.

**Perturbation space of the dataset** Following Levine & Feizi (2021), we define a *general* perturbation space over the dataset, allowing attackers to delete, insert, or modify training examples in the dataset. Given a dataset D and a *radius*  $r \geq 0$ , we define the *perturbation space* as the set of datasets that can be obtained by deleting or inserting up to r examples in D:

$$S_r(D) = \left\{ \widetilde{D} \mid |D \ominus \widetilde{D}| \le r \right\},$$

where  $A \ominus B$  is the symmetric difference of sets A and B. Intuitively, r quantifies how many examples need to be deleted or inserted to transform from D to  $\widetilde{D}$ .

**Example 3.1.** If the attacker modifies one training example  $\mathbf{x} \in D$  to another training example  $\widetilde{\mathbf{x}}$  to form a poisoned dataset  $\widetilde{D} = (D \setminus \{\mathbf{x}\}) \cup \{\widetilde{\mathbf{x}}\}$ . Then  $\widetilde{D} \in S_2(D)$  but  $\widetilde{D} \notin S_1(D)$  because  $S_r(D)$  considers one modification as one deletion and one insertion.

Note that we assume a more general perturbation space of the training set than the one considered by Zhang et al. (2022b); Weber et al. (2020); Wang et al. (2020a); our work allows inserting and deleting examples instead of just modifying existing training examples.

**Perturbation space of the test input** We write  $\pi(\mathbf{x})$  to denote the set of perturbed examples that an attacker can transform the example  $\mathbf{x}$  into. Formally, the perturbation space  $\pi(\mathbf{x})$  can be defined as the  $l_p$  norm ball with radius s around the test input  $\mathbf{x}$ ,

$$\pi(\mathbf{x}) = \{\widetilde{\mathbf{x}} \mid \|\mathbf{x} - \widetilde{\mathbf{x}}\|_p \le s\}$$

**Example 3.2.** BagFlip (Zhang et al., 2022b) considers the  $l_0$  feature-flip perturbation  $F_s(\mathbf{x})$ , which allows the attacker to modify up to s features in an input  $\mathbf{x}$ ,

$$F_s(\mathbf{x}) = \{\widetilde{\mathbf{x}} \mid ||\mathbf{x} - \widetilde{\mathbf{x}}||_0 \le s\}$$

**Threat models** Next, we define what type of guarantees we are interested in our learning algorithm and model. We consider backdoor attacks, where the attacker can perturb both the training set and the test input. For the training set, we assume we are given a perturbation space  $S_r(D)$  of the training set D with a radius  $r \geq 0$ . For the test input, we assume a perturbation space  $\pi(\mathbf{x})$  of the test input  $\mathbf{x}$  with a given  $l_p$  norm and the radius s.

We say that an algorithm A is robust to a **backdoor attack** on a backdoored test input  $\widetilde{\mathbf{x}}$  if the algorithm trained on any perturbed dataset  $\widetilde{D}$  would predict the backdoored input  $\widetilde{\mathbf{x}}$  the same as  $A_D(\mathbf{x})$ . Formally,

$$\forall \widetilde{D} \in S_r(D), \ \widetilde{\mathbf{x}} \in \pi(\mathbf{x}). \ A_{\widetilde{D}}(\widetilde{\mathbf{x}}) = A_D(\mathbf{x})$$
 (1)

**Remark 3.1.** When r = 0, Eq 1 degenerates to evasion robustness, i.e.,  $\forall \widetilde{\mathbf{x}} \in \pi(\mathbf{x})$ .  $A_D(\widetilde{\mathbf{x}}) = A_D(\mathbf{x})$ , because  $S_0(D) = \{D\}$ .

Given a large enough radius r, an attacker can always change enough inputs and succeed at breaking robustness. Therefore, we will typically focus on computing the maximal radius r for which we can prove that Eq 1 for given perturbation spaces  $S_r(D)$  and  $\pi(\mathbf{x})$ . We refer to this quantity as the *certified radius*.

Certified guarantees This paper aims to design a certifiable algorithm A, which can defend against backdoor attacks, and to compute the certified radius of A. In our experiments (Section 5.2), we suppose a given benign dataset D and a benign test input  $\mathbf{x}$ , and we certifiably quantify the robustness of the algorithm A against backdoor attacks by computing the certified radius.

In Section 5.3, we also experiment with how the certifiable algorithm A defends the backdoor attacks if a poisoned dataset  $\widetilde{D}$  and a test input  $\widetilde{\mathbf{x}}$  with malicious triggers are given, but the clean data is unknown. We theoretically show that we can still compute the certified radius if the clean data D and  $\mathbf{x}$  are unknown in Section 4.3.

## 4. The PECAN Certification Technique

Our approach, which we call PECAN (Partitioning data and Ensembling of Certified neurAl Networks), is a deterministic certification technique that defends against backdoor attacks. Given a learning algorithm A, we show how to automatically construct a new learning algorithm  $\bar{A}$  with certified backdoor-robustness guarantees (Equation (1)) in Section 4.1. In Section 4.2, we prove the certified backdoor-robustness guarantees (Equation (1)) provided by  $\bar{A}$ . We further discuss how  $\bar{A}$  can defend against a backdoored dataset and formally justify our discussion in Section 4.3.

#### 4.1. Constructing Certifiable Algorithm $\bar{A}$

The key idea of PECAN is that we can take any off-the-shelf technique for evasion certification and use it to construct a certified backdoor defense. Intuitively, PECAN uses the evasion certification to defend against the possible triggers at test time, and it encapsulates the evasion certification in deep partition aggregation (DPA) (Levine & Feizi, 2021) to defend against training set poisoning.

Given a dataset D, a test input x, and a machine learning

algorithm A, PECAN produce a new learning algorithm  $\bar{A}$  as described in the following steps (shown in Figure 1),

**Dataset Partitioning** We partition the dataset D into n disjoint sub-datasets, denoted as  $D_1, \ldots, D_n$ , using a hash function that deterministically maps each training example into a sub-dataset  $D_i$ . Train n classifiers  $A_{D_1}, \ldots, A_{D_n}$  on these sub-datasets.

**Evasion Certification** We certify whether the prediction of each classifier  $A_{D_i}$  is robust under the perturbation space  $\pi(\mathbf{x})$  by any evasion certification approach for the learning algorithm, e.g., CROWN-IBP for neural networks (Xu et al., 2020). Formally, the certification approach determines whether the following equation holds,

$$\forall \widetilde{\mathbf{x}} \in \pi(\mathbf{x}). \ A_{D_i}(\mathbf{x}) = A_{D_i}(\widetilde{\mathbf{x}}) \tag{2}$$

We denote the output of each certification as  $A_{D_i}^{\pi}(\mathbf{x})$ , which can either be  $A_{D_i}^{\pi}(\mathbf{x}) = \text{cert}$ , meaning Eq 2 is certified. Otherwise,  $A_{D_i}^{\pi}(\mathbf{x}) = \text{abstain}$ , meaning the certification approach cannot certify Eq 2.

**Aggregation** We compute the top label  $y^*$  by aggregating all predictions from  $A_{D_i}(\mathbf{x})$ . Concretely,  $y^* \triangleq \underset{y \in \mathcal{C}}{\operatorname{argmax}} \sum_{i=1}^n \mathbb{1}_{A_{D_i}(\mathbf{x})=y}$ , where  $\mathcal{C} = \{0,1,\ldots\}$  is the set

of possible labels. Note that if a tie happens when taking the argmax, we break ties deterministically by setting the smaller label index as  $y^*$ . We denote the runner-up label as y' as  $\underset{y \in \mathcal{C} \land y \neq y^*}{\operatorname{argmax}} \sum_{i=1}^n \mathbb{1}_{A_{D_i}(\mathbf{x}) = y}$ . We count the number

of certified predictions equal to  $y^*$  as  $N_1$ , the number of certified predictions equal to y' as  $N_2$ , and the number of abstentions as  $N_3$ ,

$$\begin{split} N_1 &= \sum_{i=1}^n \mathbb{1}_{A_{D_i}(\mathbf{x}) = y^* \wedge A_{D_i}^\pi(\mathbf{x}) = \text{cert}}, \\ N_2 &= \sum_{i=1}^n \mathbb{1}_{A_{D_i}(\mathbf{x}) = y' \wedge A_{D_i}^\pi(\mathbf{x}) = \text{cert}}, \\ N_3 &= \sum_{i=1}^n \mathbb{1}_{A_{D_i}^\pi(\mathbf{x}) = \text{abstain}}. \end{split}$$

We set the prediction  $\bar{A}_D(\mathbf{x})$  as  $y^*$ . We compute the certified radius r in the following two cases. If  $N_1-N_2-N_3-\mathbb{1}_{y^*>y'}<0$ , we set r as  $\diamond$ , i.e., a value denoting no certification. In this case, PECAN cannot certify that  $\bar{A}$  is robust to evasion attacks even if the dataset is not poisoned. Otherwise, we compute r as  $\lfloor \frac{N_1-N_2-N_3-\mathbb{1}_{y^*>y'}}{2} \rfloor$ . A special case is r=0, when PECAN can certify  $\bar{A}$  is robust to evasion attacks, but cannot certify that it is robust if the dataset is poisoned.

We note that the computation of the certified radius is equivalent to DPA when no classifier abstains, i.e.,  $N_3 = 0$ ,

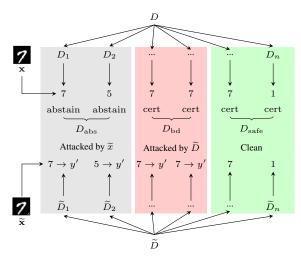


Figure 2. An illustration of the proof of Theorem 4.1. It shows the worst case for PECAN, where the attacker can change all predictions in  $D_{\rm abs}$  and  $D_{\rm bd}$  to the runner-up label y'. Note that we group  $D_{\rm abs}$ ,  $D_{\rm bd}$ , and  $D_{\rm safe}$  together to ease illustration.

#### 4.2. Proving the Soundness of PECAN

In this section, we show that the prediction  $\bar{A}_D(\mathbf{x})$  and the certified radius r satisfy the certified backdoor-robustness guarantees (Equation (1)) by proving the following theorem.

**Theorem 4.1** (Soundness of PECAN). Given a dataset D and a test input  $\mathbf{x}$ , PECAN computes the prediction  $\bar{A}_D(\mathbf{x})$  and the certified radius as r. Then, either  $r = \diamond$  or

$$\forall \widetilde{D} \in S_r(D), \ \widetilde{\mathbf{x}} \in \pi(\mathbf{x}). \ \bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}}) = \bar{A}_D(\mathbf{x})$$
 (3)

*Proof.* For any poisoned dataset  $\widetilde{D}$ , we partition  $\widetilde{D}$  into n sub-datasets  $\{\widetilde{D}_1,\ldots,\widetilde{D}_n\}$  according to  $\{D_1,\ldots,D_n\}$  from the clean dataset D. Note that we can determine such a correspondence between  $D_i$  and  $\widetilde{D}_i$  because our hash function is deterministic and only depends on each training example. We further divide  $\{D_1,\ldots,D_n\}$  into three disjoint parts  $D_{\rm abs},D_{\rm bd}$ , and  $D_{\rm safe}$  in the following way,

- $D_{\mathrm{abs}} = \{D_i \mid A_{D_i}^{\pi}(\mathbf{x}) = \mathrm{abstain}\}$  are the subdatasets, on which A abstains from making the prediction on  $\mathbf{x}$ . From the definition of  $N_3$ , we have  $|D_{\mathrm{abs}}| = N_3$ . Intuitively,  $D_{\mathrm{abs}}$  contains the subdatasets that can possibly be attacked by the test input  $\widetilde{\mathbf{x}}$  with malicious triggers.
- $D_{\mathrm{bd}}$  are the sub-datasets on which A does not abstain and are also poisoned, i.e., each of them has at least one training example removed or inserted. Even though we do not know the exact sub-datasets in  $D_{\mathrm{bd}}$ , we know  $|D_{\mathrm{bd}}| \leq r$  because  $\widetilde{D} \in S_r(D)$  constrains that there are at most r such poisoned sub-datasets.
- $D_{\text{safe}} = \{D_i \mid D_i = \widetilde{D}_i \wedge A_{D_i}^{\pi}(\mathbf{x}) = \text{cert}\}\$ contains the clean sub-datasets, on which A does not abstain.

We denote the numbers of the original top prediction  $y^*$  and the original runner-up prediction y' on the backdoored data  $\widetilde{D}$  and  $\widetilde{\mathbf{x}}$  as  $\widetilde{N}_{y^*}$  and  $\widetilde{N}_{y'}$ , respectively. Formally,

$$\widetilde{N}_{y^*} = \sum_{i=1}^n \mathbb{1}_{A_{\widetilde{D}_i}(\widetilde{\mathbf{x}}) = y^*}, \quad \widetilde{N}_{y'} = \sum_{i=1}^n \mathbb{1}_{A_{\widetilde{D}_i}(\widetilde{\mathbf{x}}) = y'}$$

Next, we prove Eq 3 for any backdoored data  $\widetilde{D}$  and  $\widetilde{\mathbf{x}}$  by showing that

$$\widetilde{N}_{y^*} \ge \widetilde{N}_{y'} + \mathbb{1}_{y^* > y'} \tag{4}$$

We prove Eq 4 by showing a lower bound of  $\widetilde{N}_{y^*}$  is  $N_1-r$  and an upper bound of  $\widetilde{N}_{y'}$  is  $N_2+r+N_3$ . Together with the definition of r, we can prove Eq 4 because we have,

$$\begin{split} &\widetilde{N}_{y^*} - \widetilde{N}_{y'} - \mathbbm{1}_{y^* > y'} \\ \ge & N_1 - r - \left(N_2 + r + N_3\right) - \mathbbm{1}_{y^* > y'} \\ = & N_1 - N_2 - 2r - N_3 - \mathbbm{1}_{y^* > y'} \\ = & N_1 - N_2 - 2 \lfloor \frac{N_1 - N_2 - N_3 - \mathbbm{1}_{y^* > y'}}{2} \rfloor - N_3 - \mathbbm{1}_{y^* > y'} \\ \ge & N_1 - N_2 - \left(N_1 - N_2 - N_3 - \mathbbm{1}_{y^* > y'}\right) - N_3 - \mathbbm{1}_{y^* > y'} \\ = & 0. \end{split}$$

Note that the second last line holds iff  $N_1 - N_2 - N_3 - \mathbb{1}_{y^* > y'} \ge 0$ . Otherwise, we have  $r = \diamond$ .

As shown in Figure 2, the lower bound of  $\widetilde{N}_{y^*}$  can be computed by noticing that 1) the attacker can change any prediction in  $D_{\rm bd}$  from  $y^*$  to another label because these datasets are poisoned, 2) the attacker can change any prediction in  $D_{\rm abs}$  to another label because CROWN-IBP cannot certify the prediction under the evasion attacks, and 3) the attacker cannot change anything in  $D_{\rm safe}$  because of the guarantee of CROWN-IBP and  $D_{\rm safe}$  is not poisoned,

$$\forall D_i \in D_{\text{safe}}, \widetilde{\mathbf{x}} \in \pi(\mathbf{x}). \ A_{D_i}(\mathbf{x}) = A_{D_i}(\widetilde{\mathbf{x}}) = A_{\widetilde{D}_i}(\widetilde{\mathbf{x}})$$

The upper bound of  $N_{y'}$  can be computed by noticing that 1) the attacker can change any prediction in  $D_{\rm bd}$  to y', 2) the attacker can change any prediction in  $D_{\rm abs}$  to y', and 3) the attacker cannot change anything in  $D_{\rm safe}$ .

We complete the proof by showing that the best attack strategy of the attacker is to change the prediction of  $\bar{A}$  to the runner-up label y'. If the attacker chooses to change the prediction of  $\bar{A}$  to another label y'', denoted the counts as  $\widetilde{N}_{y''}$ , then the upper bound of  $\widetilde{N}_{y''}$  will be always smaller or equal to  $\widetilde{N}_{y'}$ .

#### 4.3. PECAN under the Backdoored Data

The above algorithm and proof of PECAN assume that a clean dataset  $\widetilde{D}$  and a clean test example  $\mathbf{x}$  are already given. However, we may be interested in another scenario where the poisoned dataset  $\widetilde{D} \in S_r(D)$  and the input example

 $\widetilde{\mathbf{x}} \in \pi(\mathbf{x})$  with malicious triggers are given, and the clean data D and  $\mathbf{x}$  are unknown. In other words, we want to find the maximal radius r such that  $\bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}}) = \bar{A}_D(\mathbf{x})$  for any D and  $\mathbf{x}$  that can be perturbed to  $\widetilde{D}$  and  $\widetilde{\mathbf{x}}$  by the perturbation  $S_r$  and  $\pi$ , respectively. Formally,

$$\forall D, \mathbf{x}. \ \widetilde{D} \in S_r(D) \land \widetilde{\mathbf{x}} \in \pi(\mathbf{x}) \implies \bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}}) = \bar{A}_D(\mathbf{x})$$
 (5)

Intuitively, Eq 5 is the symmetrical version of Eq 1. Owing to the symmetrical definition of  $S_r$  and  $\pi$ , if we apply PECAN to the given poisoned data  $\widetilde{D}, \widetilde{\mathbf{x}}$ , then the prediction  $A_{\widetilde{D}}(\widetilde{\mathbf{x}})$  and the certified radius r satisfy the certified backdoor-robustness guarantee (Eq 5). The following theorem formally states the soundness of PECAN under the backdoored data. We prove Theorem 4.2 in Appendix A.

**Theorem 4.2** (Soundness of PECAN under the backdoored data). Given a dataset  $\widetilde{D}$  and a test input  $\widetilde{\mathbf{x}}$ , PECAN computes the prediction  $\bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}})$  and the certified radius as r. Then, either  $r = \diamond$  or Eq 5 holds.

## 5. Experiments

We implemented PECAN in Python and provided the implementation in the supplementary materials. In our evaluation, we use CROWN-IBP, implemented in auto-LiRPA (Xu et al., 2020), as the evasion defense approach for neural networks. We also use CROWN-IBP to train the classifiers in the dataset partitioning step since the classifiers trained by CROWN-IBP can improve the certification rate in the evasion certification step.

In Section 5.2, we evaluate the effectiveness and efficiency of PECAN by comparing it to BagFlip (Zhang et al., 2022b), the state-of-the-art probabilistic certified defense against backdoor attacks. In Section 5.3, we evaluate the effectiveness of PECAN under the backdoor attack (Severi et al., 2021) for malware detection and compare PECAN to other baselines, DPA and CROWN-IBP.

#### 5.1. Experimental Setup

**Datasets** We conduct experiments on MNIST, CIFAR10, and EMBER (Anderson & Roth, 2018) datasets. MNIST is an image classification dataset containing 60,000 training and 10,000 test examples. CIFAR10 is an image classification dataset containing 50,000 training and 10,000 test examples. EMBER is a malware detection dataset containing 600,000 training and 200,000 test examples. Each example is a vector containing 2,351 features of the software.

**Models** For image classification datasets MNIST and CI-FAR10, we train fully-connected neural networks with four layers for PECAN, while BagFlip uses CNN and ResNet for MNIST and CIFAR10, respectively. We do not use CNN and ResNet because CROWN-IBP used in PECAN has a higher abstention rate for deeper and more complex neu-

ral network structures. We use the same fully-connected neural network for EMBER as in related works (Zhang et al., 2022b; Severi et al., 2021). We use the same data augmentation for PECAN and other baselines.

**Metrics** For each test input  $x_i$ ,  $y_i$ , the algorithm A will predict a label and the certified radius  $r_i$ . In this section, we assume that the attacker had *modified* R% examples in the training set. We denote R as the *modification amount*. We summarize all the metrics used as follows,

Certified Accuracy denotes the percentage of test examples that are correctly classified and whose certified radii are no less than R, i.e.,  $\frac{1}{m}\sum_{i=1}^m \mathbb{1}_{\bar{A}_D(\mathbf{x}_i)=y_i\wedge\frac{r_i}{|D|}\geq 2R\%}$ , where m and |D| are the sizes of test set and training set, respectively. Notice that there is a factor of 2 on the modification amount R because  $S_r(D)$  considers one modification as one insertion and one deletion, as illustrated in Example 3.1.

Normal Accuracy denotes the percentage of test examples that are correctly classified by the algorithm without certification, i.e.,  $\frac{1}{m} \sum_{i=1}^m \mathbb{1}_{\bar{A}_D(\mathbf{x}_i) = y_i}$ .

Attack Success Rate (ASR). In Section 5.3, we are interested in how many test examples are certified but wrongly classified by the classifier, i.e.,  $\frac{1}{m}\sum_{i=1}^{m}\mathbb{1}_{\bar{A}_D(\mathbf{x}_i)\neq y_i\wedge\frac{r_i}{|D|}\geq 2R\%}$ . We denote the above quantity as the attack success rate. We note that a prediction can still be incorrect even if it is certified by PECAN because the classifier can have incorrect predictions even when the data is clean.

Abstention Rate is computed as  $\frac{1}{m} \sum_{i=1}^{m} \mathbb{1}_{\frac{r_i}{|D|} < 2R\%}$ .

#### 5.2. Effectiveness and Efficiency of PECAN

We evaluate the effectiveness and efficiency of PECAN on MNIST, CIFAR10, and EMBER under the backdoor attack with the  $l_0$  feature-flip perturbation  $F_1$ , which allows the attacker to modify up to one feature in an example. We compare PECAN to BagFlip, the state-of-the-art probabilistic certified defense against  $l_0$  feature-flip backdoor attacks. Moreover, we note that PECAN needs to construct harder proofs than BagFlip because their definitions of perturbation space are different, as discussed in Appendix B.1.

In Appendix B.2, we evaluate the effectiveness of PECAN against the perturbation space with the  $l_{\infty}$  norm.

Summary of the results PECAN achieves significantly higher certified accuracy than BagFlip on CIFAR10 and EMBER. PECAN achieves competitive results on MNIST compared to BagFlip. PECAN has similar normal accuracy as BagFlip for all datasets. PECAN is more efficient than BagFlip at computing the certified radius.

**Setup** We use the same hyper-parameters for BagFlip as reported in their paper for all datasets. For PECAN, we vary

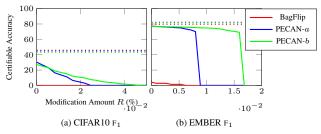


Figure 3. Comparison to BagFlip on CIFAR10 and EMBER, showing the normal accuracy (dotted lines) and the certified accuracy (solid lines) at different modification amounts R. For CIFAR10: a=50 and b=100. For EMBER: a=200 and b=400.

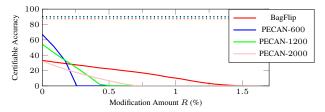


Figure 4. Comparison to BagFlip on MNIST, showing the normal accuracy (dotted lines) and the certified accuracy (solid lines) at different modification amounts R.

n, the number of partitions, to ensure a fair comparison between BagFlip. Appendix B.1 presents a detailed discussion of hyper-parameter settings for BagFlip and PECAN. We denote PECAN with different settings of n as PECAN-n.

BagFlip achieves meaningful results only on MNIST, where we also tune the parameter n of PECAN to 2000 to achieve the same certified accuracy of BagFlip at R=0 and compare their results following the practice in related works (Jia et al., 2021; 2020).

**Results** Figure 3 shows the comparison between PECAN and BagFlip on CIFAR10 and EMBER. **PECAN achieves** significantly higher certified accuracy than BagFlip across all modification amounts R and the similar normal accuracy as BagFlip for both datasets.

BagFlip performs poorly on CIFAR10 and EMBER because these two datasets cannot tolerate the high level of noise that the BagFlip algorithm adds to the training data. Specifically, BagFlip can add 20% noise to the training data of MNIST, i.e., a feature (pixel) in a training example will be flipped to another value with 20% probability. However, for CIFAR10 and EMBER, this probability has to be decreased to 5% to maintain normal accuracy.

Figure 4 shows the comparison between PECAN and BagFlip on MNIST. **PECAN achieves competitive results compared to BagFlip.** We find that two approaches have similar normal accuracy. Comparing PECAN-600 and PECAN-1200 with BagFlip, we find that 1) PECAN-600 and PECAN-1200 achieves higher certified accuracy than BagFlip when  $R \in [0, 0.25]$  and  $R \in [0, 0.17]$ , respec-

tively, and 2) BagFlip has non-zero certified accuracy when  $R \in [0.5, 1.5]$ , where the certified accuracy of PECAN-600 and PECAN-1200 is zero. Comparing PECAN-2000 with BagFlip, we find that BagFlip outperforms PECAN-2000 across all modification amounts R.

We argue that the gap of certified accuracy between PECAN-2000 and BagFlip mainly comes from the different definitions of the perturbation spaces as discussed in Appendix B.1. Moreover, the root cause of this difference is owing to the probabilistic nature of BagFlip.

PECAN is more efficient than BagFlip at computing the certified radius. PECAN computes the certified radius in a constant time complexity via the closed-form solution in the aggregation step. However, in our experiment of the MNIST dataset, BagFlip requires 8 hours to prepare a lookup table because BagFlip does not have a closed-form solution for computing the certified radius.

#### 5.3. PECAN under the Backdoored Data

We evaluate the effectiveness of PECAN under the back-door attack (Severi et al., 2021) for malware detection on the EMBER dataset. We do not compare PECAN to BagFlip because BagFlip has poor certified accuracy on EMBER, as shown in Figure 3. We also evaluate other baselines, DPA and CROWN-IBP, which do not aim to defend against back-door attacks. DPA is the certified defense against trigger-less attacks, and CROWN-IBP is the certified defense against evasion attacks. We also present the results of the victim classifiers without any defense. Appendix B.4 shows that the empirical defense spectral signatures (Tran et al., 2018) cannot defend against the backdoor attack.

**Summary of the results** PECAN reduces the ASR of the victim model on the test set with malicious triggers from 41.33% to 1.85%, while the other baselines fail to defend against the backdoor attack. Being the most conservative, PECAN has the highest abstention rate.

**Setup** We use Severi et al. (2021) to generate backdoored data by modifying 0.1% training examples and adding triggers into the test inputs that should be labeled as malware to fool the victim model to predict the malware with malicious triggers as benign software (non-malware). We generate three poisoned datasets  $\widetilde{D}_1, \widetilde{D}_2, \widetilde{D}_3$  and their corresponding test sets with triggers by perturbations  $F_1$ ,  $F_2$ , and  $F_3$ , which allow the attacker to modify up to one, two, and three features in an example, respectively.

We report the results of all approaches on the malware test sets with triggers and the malware test sets without triggers, i.e., the original malware test set. The results on the non-malware test sets without triggers can be found in Appendix B.3. ASR on malware is a much more critical metric

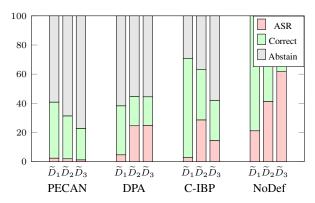


Figure 5. Results of PECAN, DPA, CROWN-IBP (C-IBP), and vanilla model without defense (NoDef) trained on three poisoned EMBER datasets when evaluated on the malware test set with malicious triggers. We note that NoDef does not have abstention rates because it does not use any defense.

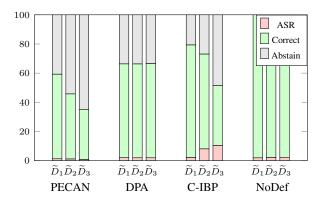


Figure 6. Results of PECAN, DPA, C-IBP, and NoDef when evaluated on the (original) malware test set without malicious triggers.

than the ASR on non-malware, because the former shows how many pieces of malware can bypass the classifier.

For PECAN and DPA, we show their results at modification amount R=0.1%. We show CROWN-IBP results against the perturbations  $F_1$ ,  $F_2$ , and  $F_3$  regardless of R because CROWN-IBP does not consider R.

Results Figures 5 and 6 show the ASR, accuracy, and abstention rate of all the approaches on the malware test set with and without triggers, respectively. Table 1 in the appendix shows the detailed numbers. Note that PECAN is the only certified approach for backdoor attacks. The results of other baselines can be seen as empirical because DPA and CROWN-IBP certify a different goal, and NoDef has no defense.

PECAN can defend against the backdoor attack on the EMBER dataset. Figures 5 and 6 show that PECAN has the lowest ASR 1.85% and 1.03% on both malware test sets with and without triggers on average, compared to DPA (18.05%, 1.98%), CROWN-IBP (15.24%, 6.82%), and NoDef (41.33%, 2.12%).

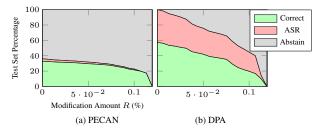


Figure 7. Comparison between PECAN and DPA trained on  $\tilde{D}_3$  across all modification amount R when evaluated on the malware test set with triggers.

DPA and CROWN-IBP fail to defend against the backdoor attack. The average ASR of DPA and CROWN-IBP on the malware test set with triggers are 18.05% and 15.24% in Figure 5, respectively, meaning that many malware with triggers can bypass their defenses. The average ASR of DPA on the malware test set without triggers, 1.98%, is much lower than its ASR on the one with triggers, 18.05%, which shows that DPA successfully defends against triggerless attacks when the test input does not have any trigger. CROWN-IBP has high ASR on both the malware test sets with and without triggers, as CROWN-IBP cannot defend against the poison in the training sets.

**PECAN** has higher abstention rates than other approaches. On average, PECAN abstains from 50.41% predictions compared to DPA (34.73%) and CROWN-IBP (26.44%). We further compare the accuracy, ASR, and abstention rate of PECAN and DPA across all modification amount R when trained on  $\widetilde{D}_3$  in Figure 7. The results on  $\widetilde{D}_1$  and  $\widetilde{D}_2$  are shown in Appendix B.5. We can observe that PECAN has a much lower ASR than DPA across all modification amounts. Meanwhile, Figure 7 shows that the certification of PECAN might be over-conservative because the ASR is low (3.17%) even when we regard  $\widetilde{D}_3$  as non-poisoned (when R=0), yet  $\widetilde{D}_3$  is actually poisoned.

#### 6. Conclusion, Limitations, and Future Work

We presented PECAN, a deterministic certified approach to effectively and efficiently defend against backdoor attacks. We foresee many future improvements to PECAN. First, we implemented PECAN as a certified defense specialized for neural networks because the evasion certification step, CROWN-IBP, is limited to neural networks. However, we can replace CROWN-IBP with an evasion certification approach for another machine learning model to get a corresponding backdoor defense for that model. Second, we adopt the idea of deep partition aggregation (DPA) to design the partition and aggregation steps in PECAN. We can improve these steps by using finite aggregation (FA) (Wang et al., 2022b), which extends DPA and gives higher certified accuracy. Third, during the certification of evasion attacks, we need to propagate the abstraction of the same test input

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through thousands of neural networks that have different weights but the same architecture. Sharing the propagation results among different neural networks (Fischer et al., 2022) can greatly improve the efficiency of PECAN and may enable using complete certification methods.

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## A. Proof of Theorem 4.2

*Proof.* Theorem 4.1 tells that either  $r = \diamond$  or the following equation holds,

$$\forall D' \in S_r(\widetilde{D}), \ \mathbf{x}' \in \pi(\widetilde{\mathbf{x}}). \ \bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}}) = \bar{A}_{D'}(\mathbf{x}')$$
 (6)

By the symmetrical definition of  $S_r$  and  $\pi$ , we have

$$\forall D. \ \widetilde{D} \in S_r(D) \implies D \in S_r(\widetilde{D}) \tag{7}$$

$$\forall \mathbf{x}.\ \widetilde{\mathbf{x}} \in \pi(\mathbf{x}) \implies \mathbf{x} \in \pi(\widetilde{\mathbf{x}}). \tag{8}$$

Then, for all possible clean data D and x, we have

$$\widetilde{D} \in S_r(D) \wedge \widetilde{\mathbf{x}} \in \pi(\mathbf{x})$$
 $\Longrightarrow D \in S_r(\widetilde{D}) \wedge \mathbf{x} \in \pi(\widetilde{\mathbf{x}})$  (By Eq 7 and Eq 8)
 $\Longrightarrow \bar{A}_{\widetilde{D}}(\widetilde{\mathbf{x}}) = \bar{A}_D(\mathbf{x})$  (By Eq 6)

## **B.** Experiment

#### **B.1. Detailed Setup of Section 5.2**

Following the BagFlip paper (Zhang et al., 2022b), we set k, the number of training examples in a bag used in BagFlip, as 100, 1000, and 3000 for the MNIST, CIFAR10, and EMBER dataset, respectively. For PECAN, we vary n, the number of partitions, according to the value of k in BagFlip by setting  $n = \frac{|D|}{k}$ .

BagFlip defines their perturbation space  $S'_r(D)$  that is different from PECAN,

$$S_r'(D) = \left\{ \widetilde{D} \mid \max(|D \setminus \widetilde{D}|, |\widetilde{D} \setminus D|) \leq r \right\},$$

where  $A \setminus B$  is the set difference, i.e., the elements in A but not in B. Notice that with the same radius r, the above definition gives a larger  $S_r'(D)$  than  $S_r(D)$  as the following example shows.

**Example B.1.** If the attacker modifies one training example  $\mathbf{x} \in D$  to another training example  $\widetilde{\mathbf{x}}$  to form a poisoned dataset  $\widetilde{D} = D \setminus \{\mathbf{x}\} \cup \{\widetilde{\mathbf{x}}\}$ . Then  $\widetilde{D} \in S_2(D)$  but  $\widetilde{D} \notin S_1(D)$  because  $S_r(D)$  considers one modification as one deletion and one insertion. However, we have  $\widetilde{D} \in S_1'(D)$ .

Chen et al. (2022) show that  $S_r'(D)$  works when the approach uses non-deterministic sub-sampling (Jia et al., 2021; Zhang et al., 2022b). However, the certification of deterministic approaches only works under the definition of  $S_r(D)$ .

We adjust the computation of certified accuracy for BagFlip as  $\frac{1}{m}\sum_{i=1}^m \mathbb{1}_{\bar{A}_D(\mathbf{x}_i)=y_i\wedge\frac{r_i}{|D|}\geq R\%}$  by removing the factor 2 on R. Thus, we are also interested in the performance of PECAN when  $n=\frac{|D|}{2k}$  to compensate the removed factor 2.

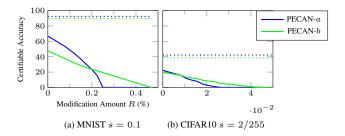


Figure 8. Results of PECAN on CIFAR10 and EMBER, showing the normal accuracy (dotted lines) and the certified accuracy (solid lines) at different modification amounts R. For MNIST: a=600 and b=1200. For CIFAR10: a=50 and b=100.

## B.2. Evaluation on the $l_{\infty}$ Perturbation Space

Setup As the CROWN-IBP used in PECAN can handle  $\pi$  with different  $l_p$  norms, PECAN can handle different  $l_p$  norms as well. We evaluate PECAN on the  $l_\infty$  norm with distance s=0.1 and s=2/255 on MNIST and CIFAR10, respectively, because the  $l_\infty$  norm is widely applied to evaluate the robustness of image classifiers. In this experimental setting, we use two CNN models for MNIST and CIFAR10 because CROWN-IBP works better for CNN on  $l_\infty$  norm than on  $l_0$  norm. For training on MNIST and CIFAR10, we train on s=0.2 and s=5/255 but test on s=0.1 and s=2/255 to overcome the overfitting issue when s is small, following the practice in the original paper of CROWN-IBP.

For the experiments on  $l_0$  (Sections 5.2 and 5.3), we set the  $\kappa_{start}=0$  and  $\kappa_{end}=0$  for CROWN-IBP. For the experiments on  $l_{\infty}$ , we set the  $\kappa_{start}=1$  and  $\kappa_{end}=0$  for CROWN-IBP.

**Results** Figure 8 shows the results of PECAN against  $l_{\infty}$  perturbation space. The results show that PECAN achieves certified accuracy similar to  $F_1$  as shown in Figures 3 and 4.

# B.3. Comparison to DPA, CROWN-IBP, and NoDef on the Non-Malware Test Set without Trigger

Figure 9 shows that NoDef has the lowest ASR of 2.70% on the non-malware set without trigger than all three defenses because the backdoor attack does not aim to attack the prediction of non-malware. However, PECAN still achieves the lowest ASR of 5.73% compared to DPA (7.85%) and CROWN-IBP (6.68%).

#### **B.4.** Comparison to Spectral Signatures

We followed the experiment in Severi et al. (2021) to filter out poisoned examples in the training dataset  $\widetilde{D}_3$ . After removing the top 15% outliers in the non-malware training set, we observe that only 14% (84 out of 600) of the poison is removed. Then we train a new model using the filtered training set. We find the ASR of the new model on the

Table 1. Results of PECAN, DPA, CROWN-IBP (C-IBP) and vanilla model without defense (NoDef) trained on three backdoored EMBER datasets. Malware with triggers is the backdoored test data that should be labeled as malware. Malware w/o triggers is the original test data that should be labeled as malware. Non-malware w/o triggers is the original test data that should be labeled as non-malware.

	Test sets	Malware with triggers				Malware w/o triggers				Non-Malware w/o triggers			
	Approaches	PECAN	DPA	C-IBP	NoDef	PECAN	DPA	C-IBP	NoDef	PECAN	DPA	C-IBP	NoDef
$\widetilde{D}_1$	ASR. (↓) Correct Pred. (↑)	2.38%	4.68%		21.15%	1.27% 58.01%	2.00%	2.10%	1.92%	6.48% 73.82%	7.81%	9.41% 83.66%	2.94%
	Abstention Rate		61.75%		N/A					19.70%		6.93%	N/A
$\widetilde{D}_2$	ASR. (↓) Correct Pred. (↑) Abstention Rate	29.33%	24.61% 20.12% 55.27%	34.78%	58.83%	1.12% 44.62% 54.27%		65.01%	97.84%	65.95%	79.03%	6.11% 90.68% 3.21%	2.64% 97.36% N/A
$\widetilde{D}_3$	ASR. (↓) Correct Pred. (↑) Abstention Rate	21.48%	24.87% 19.59% 55.54%	27.59%	38.33%	0.71% 34.45% 64.84%	64.58%			54.40%		4.51% 87.91% 7.59%	2.51% 97.49% N/A

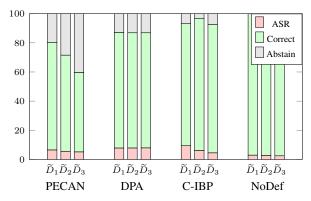


Figure 9. Results of PECAN, DPA, CROWN-IBP (C-IBP), and vanilla model without defense (NoDef) trained on three poisoned EMBER datasets when evaluated on the (original) non-malware test set without triggers.

malware set with triggers, the malware set without triggers, and the non-malware set without triggers are 48.55%, 1.38%, and 8.91%, respectively. These ASRs are all higher than PECAN's 1.19%, 0.71%, and 5.16% on the three parts of the test set.

## B.5. Comparison to DPA on $\widetilde{D}_1$ and $\widetilde{D}_2$

Figures 10 and 11 show the comparison between PECAN and DPA on  $\widetilde{D}_1$  and  $\widetilde{D}_2$ . We can observe that PECAN has much higher ASRs than DPA across all modification amounts on  $\widetilde{D}_1$  and  $\widetilde{D}_2$ .

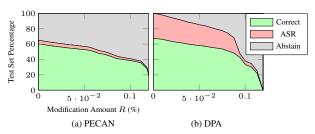


Figure 10. Comparison between PECAN and DPA trained on  $\widetilde{D}_1$  across all modification amount R when evaluated on the malware test set with triggers.

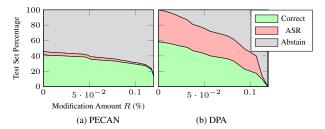


Figure 11. Comparison between PECAN and DPA trained on  $\bar{D}_2$  across all modification amount R when evaluated on the malware test set with triggers.