2021

Microphotonics

CAD-LAB: Fourier Optics

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Contents

* HELP

Once have been refered you can delete this part.

If you want to write an inline equation, code like this $E = mc^2$.

If you want to write a displayed equation without numbering, code like this

$$E = mc^2$$

If you want to write a numbering displayed equation, code like this

$$E = mc^2 \tag{1}$$

and you can quote the equation by using Eq.1.

Tips: The software `Mathpix` can convert images and PDFs to LaTeX format which is highly recommended by me.

List items like this:

1. contents
2. contents

If you want to add one or more images as Figure 1 shown, you can copy the code as following.

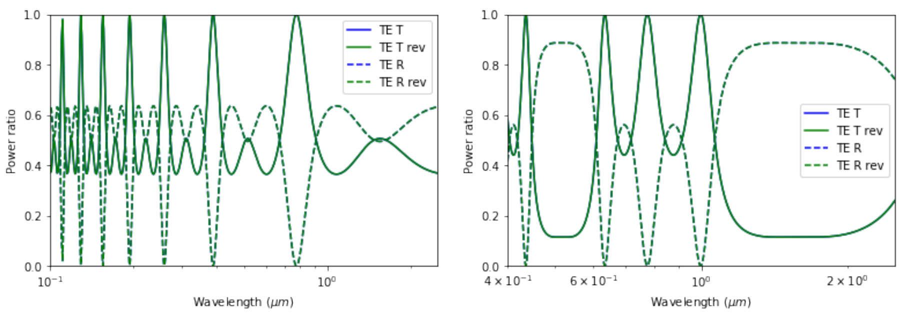

Figure 1: Here to write the caption of the figure.

Some details to notice:

1. **Function name** and **units** should be presented in the Roman Regular Font. We can do this by using

the sign backslash such as $\log(x)$, $\sin(x)$, $\cos(x)$, etc.

2. When writing source codes in LaTeX, make sure that the one-line code length **does not** exceed one page(or 1/2 screen width on computer). Too long in width makes a worse reading experience.

3. Everytime when the new task comes, you are allowed to copy the whole template file and rename it into the task name, then modify on the copy. Please don't modify the code directly on the template.

4. Don't forget enter a space after the punstuation marks.

1 TASK 1

Suppose that the circle aperture lies in the (x', y') plane and the observed point lies in the (x, y) plane. The distance between (x', y') and (x, y) can be written as binominal series:

$$\begin{aligned} R &= \sqrt{(x-x')^2 + (y-y')^2 + z^2} = \sqrt{\rho^2 + z^2} \\ &= z\sqrt{1 + \frac{\rho^2}{z^2}} \\ &= z\left[1 + \frac{\rho^2}{2z^2} - \frac{1}{8}\left(\frac{\rho^2}{z^2}\right)^2 + \cdots\right] \\ &= z + \frac{\rho^2}{2z} - \frac{\rho^4}{8z^3} + \cdots \end{aligned} \tag{2}$$

with $\rho = \sqrt{(x-x')^2 + (y-y')^2}$.

Fresnel approximation is to assum that the third term of the series is small enough to ignore, which means

$$\frac{k\rho^4}{8z^3} \ll 2\pi \Rightarrow z \gg \left(\frac{\rho^4}{8\lambda}\right)^{1/3} \tag{3}$$

Meanwhile the second term cannot be ignored:

$$\frac{k\rho^2}{2z^2} > 2\pi \Rightarrow z < \frac{\rho^2}{2\lambda} \tag{4}$$

Fraunhofer approximation assums that the second term is small enough as

$$\frac{k\rho^2}{2z^2} \ll 2\pi \Rightarrow z \gg \frac{\rho^2}{2\lambda} \tag{5}$$

Assum diameter of the observed plane and the circle apperture are the same. Put $\lambda = 1.55 \times 10^{-6}m$, $r = \frac{\rho}{2} = 1.5 \times 10^{-4}m$ into equations above, we get z distance condition of Fresnel diffraction $8.68 \times 10^{-4}m \ll z < 2.90 \times 10^{-2}m$ and of Fraunhofer diffraction $z \gg 2.90 \times 10^{-2}m$.

1.1

As Fig2 shown, with z becoming larger, the intensity of first level circle takes more advantage than other level circles. And when $z > 0.029m$, the diffraction shape matches inferrence, which is consistent with calcuation.

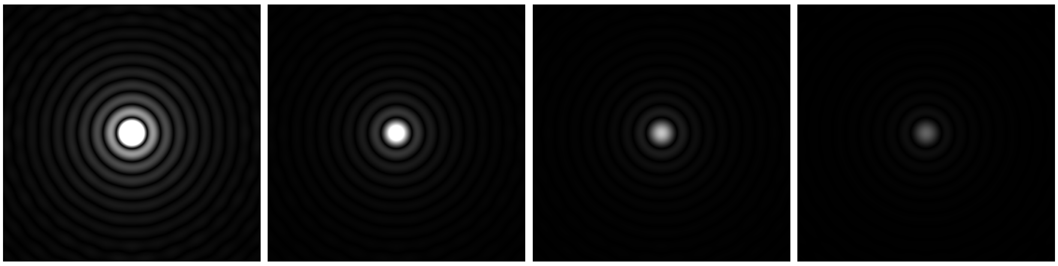

Figure 2: Fraunhofer diffraction when $z = 0.01m, 0.029m, 0.058m$ and $0.116m$.

1.2

According to the calculation, Fresnel approximation is valid when $8.68 \times 10^{-4}m \ll z < 2.90 \times 10^{-2}m$. So we take $z = 0.0001m, 0.001m, 0.01m$ and $0.1m$ to simulate in both Fresnel diffraction and Fraunhofer diffraction.

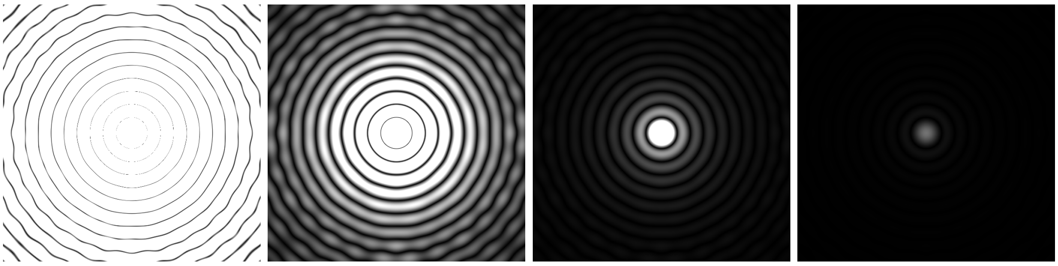

Figure 3: Fraunhofer diffraction when $z = 0.0001m, 0.001m, 0.01m$ and $0.1m$.

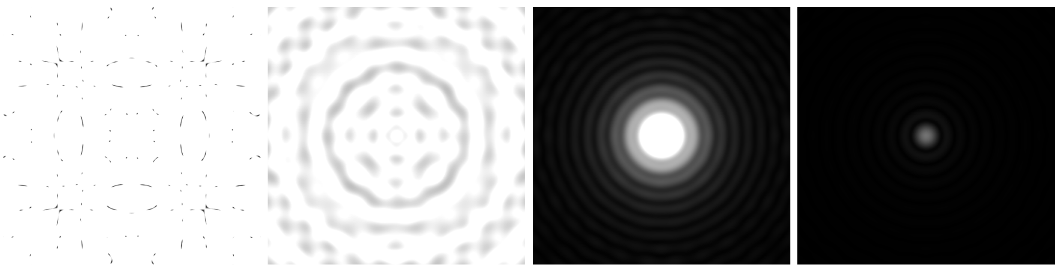

Figure 4: Fresnel diffraction when $z = 0.0001m, 0.001m, 0.01m$ and $0.1m$.

From 3, we can see that Fraunhofer diffraction shapes behave similarly, with several circles surrounding the center and darkening from the inside out. However, with z decreases near to the threshold of $8.68 \times 10^{-4}m$, Fresnel diffraction shapes distort and finally become unrecognizable, consistent with the calculation.

1.3

In theory, the position of amplitude zero's goes as

$$r_0 = \frac{1.22\lambda z}{2r} \tag{6}$$

When $z = 1m$, we get $r_0 = 0.0063m$, correspondent with Fig5

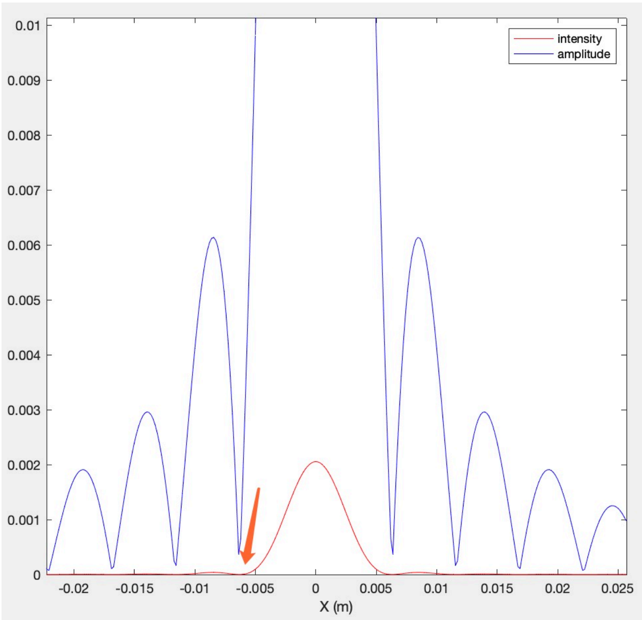

Figure 5: Airy pattern when $z = 1m$.

2 TASK 2

Let $z = 0.05m$. (Being too far will cause a sharp decrease in intenstity, unsuitable for observation)

Keep width unchange as $0.0001m$, and change height in $0.0001m, 0.0005m, 0.001m$ and $0.005m$. The diffraction patterns are shown as below. When height increases, the diffraction stripes in the perpendicular direction become longer and brighter. If height tends to infinity, the diffraction pattern will transform to the slit diffraction. We can see it from

$$I\left(x_0, y_0\right) = \left(\frac{\ell_x \ell_y}{\lambda z}\right)^2 \operatorname{sinc}^2\left(\frac{\pi \ell_x x_0}{\lambda z}\right) \operatorname{sinc}^2\left(\frac{\pi \ell_y y_0}{\lambda z}\right) \tag{7}$$

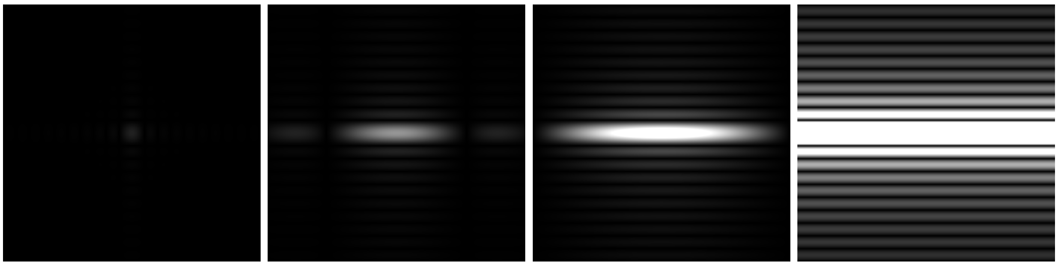

Figure 6: Fraunhofer diffraction patterns as height at $0.0001m, 0.0005m, 0.001m$ and $0.005m$.

3 TASK 3

The beamwidth function $w(z)$ can be written as

$$w(z) = \sigma\sqrt{1 + \left(\frac{z}{z_\mathrm{R}}\right)^2} \tag{8}$$

with $z_\mathrm{R} = \frac{\pi\sigma^2}{\lambda}$.

Apply $\sigma_1 = 3.9 \times 10^{-7}m$ and $\sigma_2 = 3.63 \times 10^{-6}m$ into function 8, we can draw the plot as below.

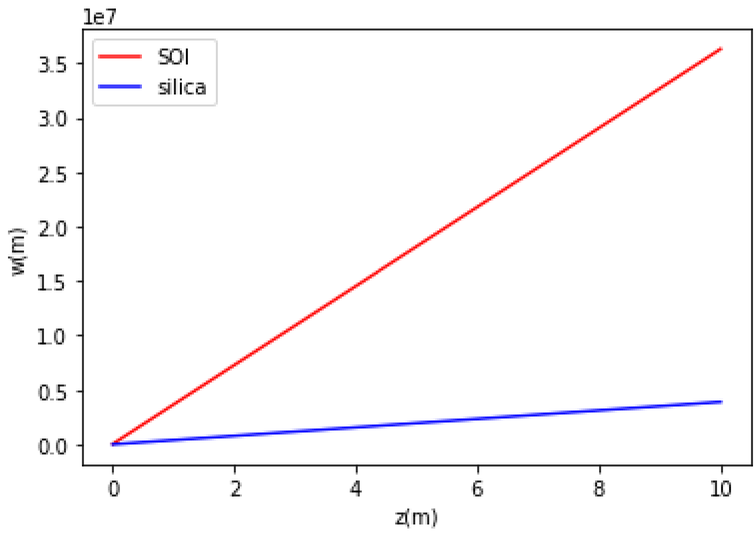

Figure 7: The beamwidth function of SOI and silica

4 TASK 4

Opening circular aperture is a low-pass filter, which can filter out the high frequency components and only allow the low frequency components through. As Fig8 shown, the high frequencies of the oscillation are filtered out, leaving only the low frequencies. In Fig9, the 2D image also loses its original shape and became blurry around the edges.

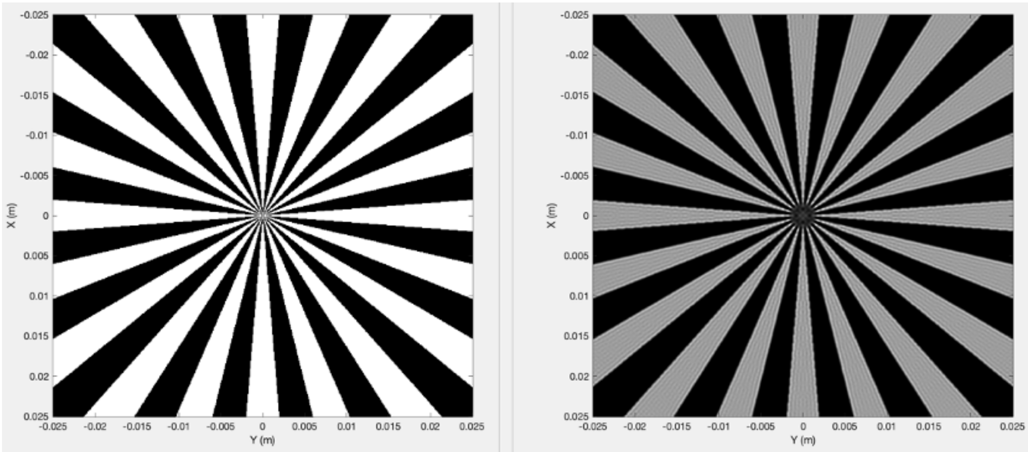

Figure 8: Opening circular aperture graph(1d)

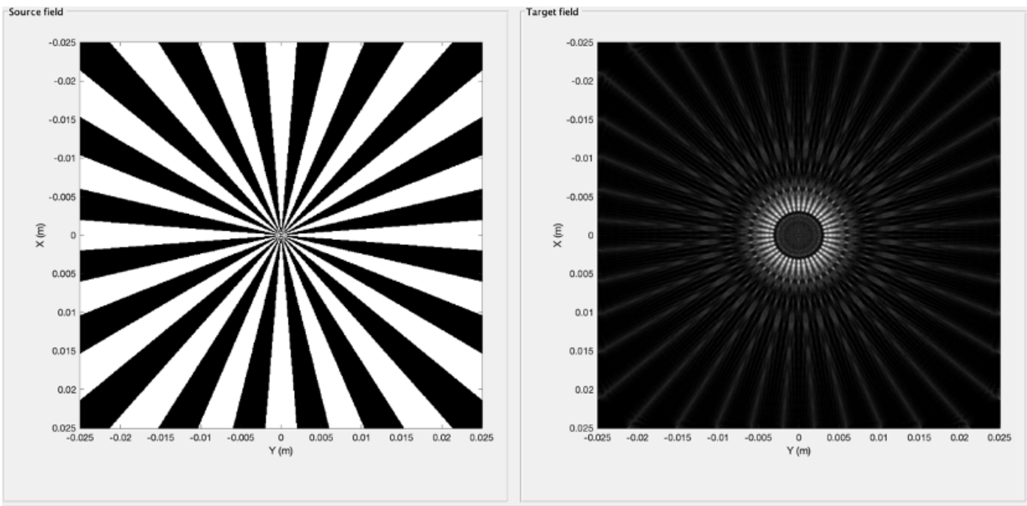

Figure 9: Opening circular aperture graph(2d)

Blocking one is a high-pass filter, which can filter out the low frequency components and only allow the high frequency components to pass through. It can be seen from Fig10 that the high frequency part of the oscillation is retained, while the low frequency part is filtered out. In Fig11, the 2D image retains the edges of the original image, with Lenas figure clearly visible and the smooth brightness changes disappearing.

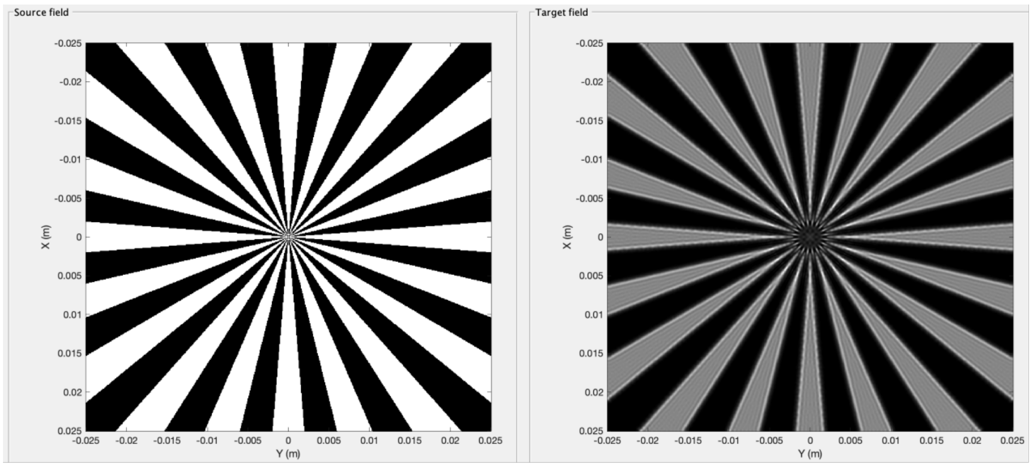

Figure 10: Blocking circular aperture graph(1d)

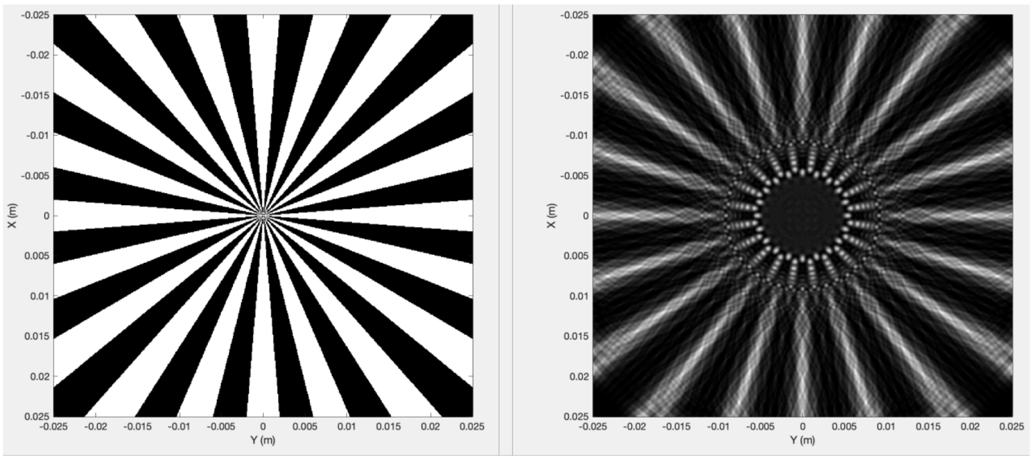

Figure 11: Blocking circular aperture graph(2d)