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AI 5002 – Mathematical Foundations of AI
Homework # 1

Matrix Multiplication and Its Effect

You are given the vertices of a simple 2D square.

$$(0,0), (0,2), (2,0), (2,2)$$

Your task is to:

a) Plot the original square in the xy-plane.

b) Represent each of the vertices as vectors in the form of a column vector.

c) Suppose we have a matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Use matrix multiplication to calculate the new values of the vertices.

d) Plot the new values to reveal the transformed square.

e) What do you interpret about the new shape?

f) Suppose we have a matrix $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Use matrix multiplication to calculate the new values of the vertices.

g) Plot the new values to reveal the transformed square.

h) What do you interpret about the new shape?

Use the following code to check the plot of the new shapes in Python.

```python
import numpy as np
import matplotlib.pyplot as plt

# Original square
square = np.array([[0,2,2,0], [0,0,2,2]])

# matrix A
A = np.array([[0, -1], [1,0]])
A1=A@square
# matrix B
B = np.array([[1, 2], [0, 1]])
B1=B@square
# Plot
plt.figure(figsize=(10,5))
plt.subplot(131)
plt.plot(square[0], square[1], 'r-'); plt.title("Original")
plt.subplot(132)
plt.plot(A1[0], A1[1], 'g-'); plt.title("Rotated 90°")
plt.subplot(133)
plt.plot(B1[0], B1[1], 'b-'); plt.title("Tilted (k=1.5)")
plt.tight_layout()
plt.show()
```