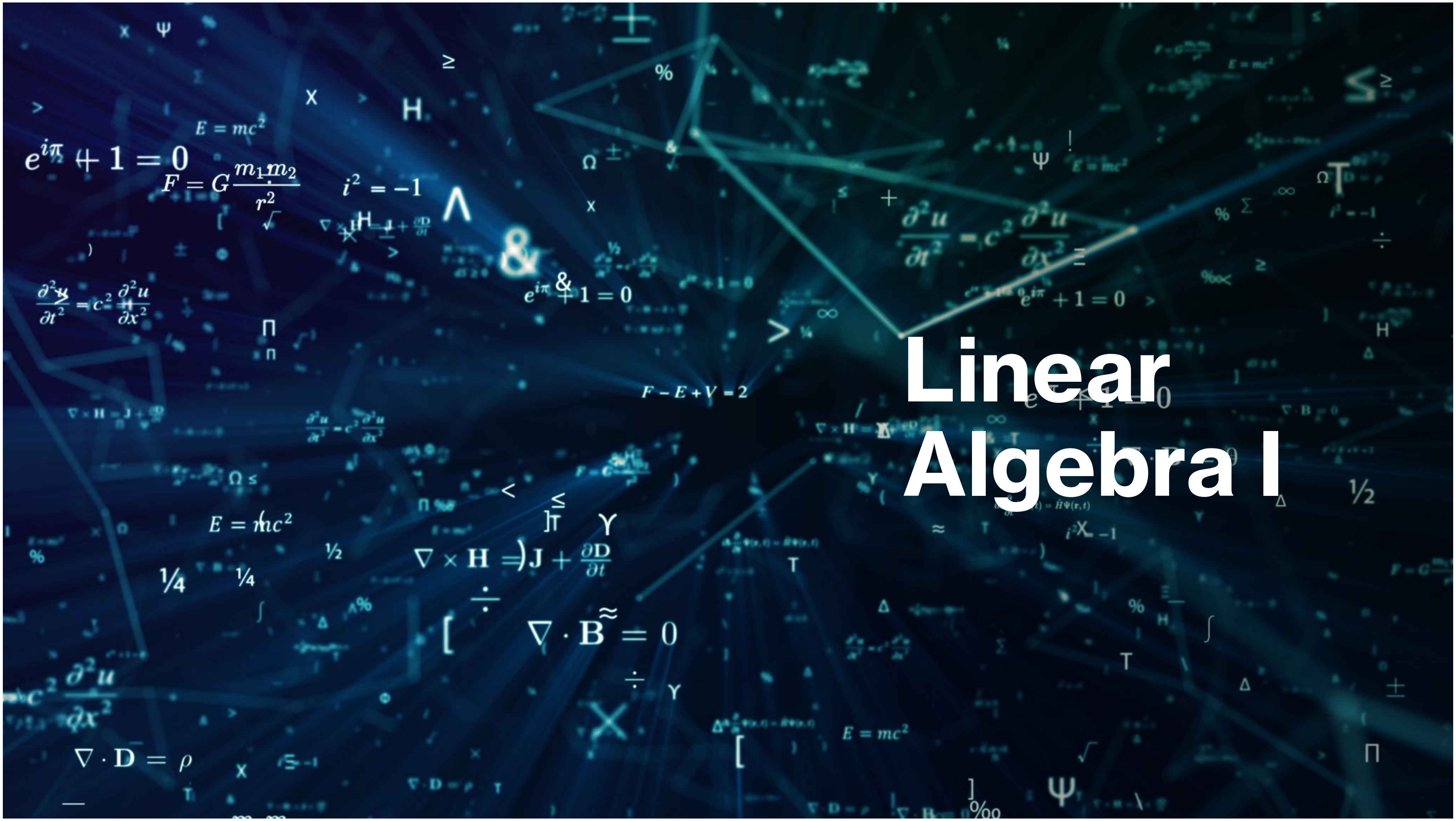
Linear Algebra I



Data Representation

- For example, We have the following data

Age	GPA	Hours Studied
20	3.4	15
21	3.6	10
19	3.2	18

- How do we represent the data?
- Every **row** is a vector representing one student.
  Every **column** is a vector representing a feature.

Data Representation

- Suppose we have the grades obtain for each student as well.

Age	GPA	Hours Studied	Grades
20	3.4	15	80
21	3.6	10	50
19	3.2	18	90

- How can we train from this data to predict the grades of any students whose features are known?

Linear Algebra

- Linear algebra is the study of vectors and certain rules associated with vectors.

Vectors

- Linear algebra is the study of vectors and certain rules associated with vectors.
- The vectors many of us know from school are called “geometric vectors”, which are usually denoted by a small arrow above the letter, e.g., $\vec{x}$ and $\vec{y}$.
- We will discuss more general concepts of vectors and use a bold letter to represent them, e.g., $\boldsymbol{x}$ and $\boldsymbol{y}$.
- One major idea in mathematics is the idea of “closure”.
- This is the question: Does the resultant of the addition of two vectors belong from the same set or outside the set? Does the resultant of the scalar multiplication of a vectors with a scalar belong from the same set or outside the set?

Vectors

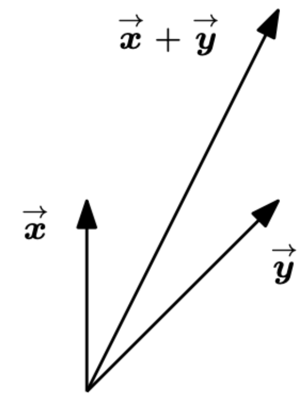
- In general, vectors are special objects that can be added together and multiplied by scalars to produce another object of the same kind.

Vectors

Examples:

1. Geometric vectors. This example of a vector may be familiar from high school mathematics and physics. Two geometric vectors can be added, such that $\vec{x} + \vec{y} = \vec{z}$ is another geometric vector. Furthermore, multiplication by a scalar $\lambda\vec{x}$, $\lambda \in \mathbb{R}$, is also a geometric vector.

2. Polynomials are also vectors; see Figure 2.1(b): Two polynomials can be added together, which results in another polynomial; and they can be multiplied by a scalar λ ∈ R, and the result is a polynomial as well.



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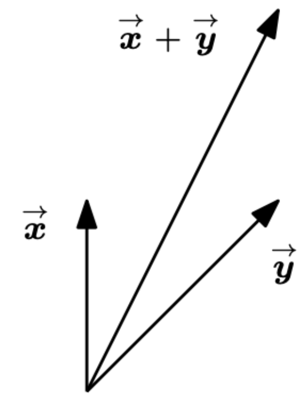

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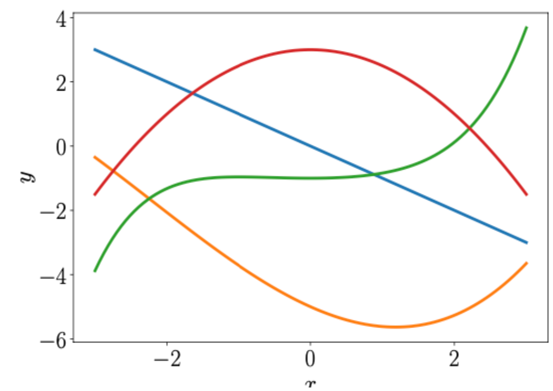

Vectors

Examples:

3. Audio signals are vectors. Audio signals are represented as a series of numbers. We can add audio signals together, and their sum is a new audio signal. If we scale an audio signal, we also obtain an audio signal. Therefore, audio signals are a type of vector, too.

4. Elements of $\mathbb{R}^n$ (tuples of n real numbers) are vectors. $\mathbb{R}^n$ is more abstract than polynomials. For instance,

$$\boldsymbol{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in \mathbb{R}^n$$

   is an example of a triplet of numbers. Adding two vectors $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^n$ component-wise results in another vector: $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c} \in \mathbb{R}^n$. Moreover, multiplying $\boldsymbol{a} \in \mathbb{R}^n$ by $\lambda \in \mathbb{R}^n$ results in a scaled vector $\lambda\boldsymbol{a} \in \mathbb{R}^n$.

Matrices

An array of numbers is called a matrix.

A matrix with n number of rows and m number of columns is known as an $n \times m$ matrix.

Matrices

Example:

Data can often be represented or abstracted as an $n \times d$ data matrix, with n rows and d columns, where rows correspond to entities in the dataset, and columns represent attributes or properties of interest.

$$
\mathrm{D} = \begin{array}{c} \\ \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \vdots \\ \boldsymbol{x_n} \end{array}
\begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_d \end{array} \\
\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nd} \end{pmatrix} \end{array}
$$

Matrices

Example:.

$$
\begin{array}{c}
 \\
\mathbf{D} = \begin{array}{c} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \vdots \\ \boldsymbol{x_n} \end{array}
\end{array}
\begin{array}{c}
\begin{array}{cccc} X_1 & X_2 & \cdots & X_d \end{array} \\
\begin{pmatrix}
x_{11} & x_{12} & \cdots & x_{1d} \\
x_{21} & x_{22} & \cdots & x_{2d} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{nd}
\end{pmatrix}
\end{array}
$$

where $\boldsymbol{x_i}$ denotes the ith row, which is a d-tuple given as

$$\boldsymbol{x_i} = (x_{i1}, x_{i2}, \dots, x_{id})$$

And $\boldsymbol{X_j}$ denotes the *j*th column, which is an n-tuple given as

$$\boldsymbol{X_j} = (x_{1j}, x_{2j}, \dots, x_{nj})$$

Matrices

- Depending on the application domain, rows may also be referred to as **entities, instances, examples, records, transactions, objects, points, feature-vectors, tuples**, and so on.
- Likewise, columns may also be called **attributes, properties, features, dimensions, variables, fields,** and so on.
- The number of instances n is referred to as the **size** of the data, whereas the number of attributes d is called the **dimensionality** of the data.
- The analysis of a single attribute is referred to as **univariate analysis**.
- The simultaneous analysis of two attributes is called **bivariate analysis.**
- The simultaneous analysis of more than two attributes is called **multivariate analysis.**

Matrices

The following are the operations of matrices

1. **Matrix addition** $M + N$ (only possible of both matrices have same size)
2. **Scalar multiplication** λM ($\lambda \in \mathbb{R}$)
3. **Matrix multiplication** MN (only possible if the number of column of the first matrix is equal to the number of rows of the second matrix).

If M is a $m \times p$ matrix and N is a $p \times n$ matrix then MN will be a $m \times n$ matrix