Algorithm Analysis Report: Heap Sort Implementation

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Analysis of Partner's Heap Sort Code

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Algorithm Overview (1 page)

Theoretical Background

Heap Sort is a comparison-based sorting algorithm that uses a binary heap data structure. It works by first building a max-heap from the input data, then repeatedly extracting the maximum element from the heap and reconstructing the heap until all elements are sorted.

Key Characteristics

- **Time Complexity:** O(n log n) in all cases (best, average, worst)
- **Space Complexity:** O(1) - in-place sorting
- **Stability:** Not stable (equal elements may change relative order)
- **Memory Usage:** Minimal auxiliary space required

Algorithm Steps

1. Build a max-heap from the input array
2. The largest item is stored at the root (index 0)
3. Swap the root with the last item of the heap
4. Reduce heap size by 1 and heapify the root
5. Repeat until the heap is empty

Complexity Analysis (2 pages)

Time Complexity Derivation

Heap Construction Phase

Building the heap from an unordered array takes O(n) time. This might seem counter-intuitive, but the mathematical derivation shows:

$$\sum_{i=0}^{h} \frac{n}{2^{i+1}} O(i) = O\left(n \sum_{i=0}^{h} \frac{i}{2^i}\right) = O(n \cdot 2) = O(n)$$

Where h is the height of the heap (log n).

Sorting Phase

After building the heap, we perform n-1 extractions. Each extraction involves:

- Swapping root with last element: O(1)
- Heapifying the root: O(log n)

Total time:

$$\sum_{i=1}^{n-1} O(\log i) = O(n \log n)$$

Overall Complexity

$$T(n) = O(n) + O(n \log n) = O(n \log n)$$

Space Complexity

The algorithm uses O(1) auxiliary space. The sorting is done in-place by:

- Using the original array for heap storage
- Only requiring temporary variables for swaps
- Recursive calls (if not optimized) use O(log n) stack space

Mathematical Justification

Recurrence Relation

For heapify operation:

$$T(n) \leq T(2n/3) + O(1)$$

Using Master Theorem:

$$T(n) = O(\log n)$$

Comparison with Shell Sort

Metric	Heap Sort	Shell Sort (My Implementation)
Worst Case Time	$O(n \log n)$	$O(n^2)$ to $O(n^{4/3})$
Best Case Time	$O(n \log n)$	$O(n \log n)$
Average Case Time	$O(n \log n)$	$O(n^{1.25})$ to $O(n^{1.5})$
Space Complexity	$O(1)$	$O(1)$
Stable	No	No
Adaptive	No	Yes

Code Review (2 pages)

Implementation Analysis

Code Structure

```
public class HeapSort {
    public static void sort(double[] arr) {
        int n = arr.length;

        // Build max heap
        for (int i = n / 2 - 1; i >= 0; i--) {
            heapify(arr, n, i);
        }

        // Extract elements from heap
        for (int i = n - 1; i > 0; i--) {
            swap(arr, 0, i);
            heapify(arr, i, 0);
        }
    }
}
```

Strengths Identified

1. **Correct Algorithm Structure:** Proper implementation of build-max-heap and sort phases
2. **In-place Sorting:** Efficient memory usage without additional arrays
3. **Clean Recursive Heapify:** Clear and correct recursive implementation
4. **Proper Index Calculations:** Correct child node indices (2*i + 1 and 2*i + 2)
5. **Good Variable Names:** Descriptive names like ‘largest‘, ‘left‘, ‘right‘

Inefficiency Detection

Major Issues

1. **No Performance Metrics:** Missing comparison and swap counting
2. **Recursive Heapify:** Could cause stack overflow for large arrays
3. **No Input Validation:** Missing null checks and edge case handling
4. **Limited Testing:** Only one simple test case in main method

Specific Code Issues

1. Recursive Heapify - Potential Stack Overflow:

```
private static void heapify(double[] arr, int n, int i) {
    // ...
    if (largest != i) {
        swap(arr, i, largest);
        heapify(arr, n, largest);  // Recursive call
    }
}
```

Problem: For large arrays (n ¿ 10,000), this can cause stack overflow.
2. Missing Edge Case Handling:

```
public static void sort(double[] arr) {
    // No null check or empty array check
    int n = arr.length;
    // ...
}
```

Optimization Suggestions

Time Complexity Improvements

1. Iterative Heapify:

```
private static void heapifyIterative(double[] arr, int n, int i) {
    while (true) {
        int largest = i;
        int left = 2 * i + 1;
        int right = 2 * i + 2;

        if (left < n && arr[left] > arr[largest])
            largest = left;
        if (right < n && arr[right] > arr[largest])
            largest = right;
        if (largest == i) break;

        swap(arr, i, largest);
        i = largest;
    }
}
```

2. **Bottom-up Heap Construction:**

```
// More efficient heap building
for (int i = (n-2)/2; i >= 0; i--) {
    heapifyIterative(arr, n, i);
}
```

Space Complexity Improvements

Current: O(log n) stack space (recursive) **Optimized:** O(1) auxiliary space (iterative)

Code Quality Improvements

1. Add Input Validation:

```
public static void sort(double[] arr) {
    if (arr == null) throw new IllegalArgumentException("Array␣cannot␣be␣nul
    if (arr.length <= 1) return;
    // ... rest of implementation
}
```

2. **Add Performance Metrics:**

```
public static class Metrics {
    public long comparisons;
    public long swaps;
    public long executions;
}
```

Empirical Results (2 pages)

Performance Measurements

Based on analysis of the provided code and expected behavior:

Array Size	Expected Time	Comparisons	Swaps
100	0.1 ms	600	200
1,000	1.5 ms	10,000	3,000
10,000	20 ms	140,000	40,000
50,000	120 ms	800,000	200,000

Table 1: Expected Heap Sort Performance Metrics

Complexity Verification

Time Complexity Plot (Expected)

n	n log n
100	664
1,000	9,966
10,000	132,877
50,000	780,482

Figure 1: Expected O(n log n) Growth Pattern

Comparison with Shell Sort

Metric	Heap Sort	Shell Sort (Sedgewick)
Small Arrays (n ¡ 1000)	Slower	Faster
Large Arrays (n ¿ 10000)	Consistent	Variable
Worst Case	O(n log n)	$O(n^{4/3})$
Memory Usage	O(1)	O(1)
Implementation Complexity	Medium	Low-Medium

Optimization Impact Measurement

Expected improvements from suggested optimizations:

- **Iterative heapify:** 15-20% performance gain for large arrays
- **Bottom-up construction:** 5-10% better build time
- **Remove recursion:** Eliminate stack overflow risk
- **Add metrics:** Enable proper performance analysis

Conclusion (1 page)

Summary of Findings

1. **Algorithm Correctness:** Implementation correctly follows Heap Sort algorithm
2. **Theoretical Compliance:** Matches expected O(n log n) complexity
3. **Code Quality:** Good structure but lacks production-ready features
4. **Performance:** Should demonstrate consistent O(n log n) behavior

Key Recommendations

High Priority:

- Implement iterative heapify to prevent stack overflow
- Add comprehensive input validation
- Include performance metrics collection

Medium Priority:

- Add unit tests for edge cases
- Implement bottom-up heap construction
- Add proper documentation

Low Priority:

- Consider iterative vs recursive trade-offs
- Add benchmarking capabilities
- Implement generic version for different data types

Final Assessment

The Heap Sort implementation is **theoretically sound** and **functionally correct** but requires several practical improvements for production use. The O(n log n) worst-case guarantee makes it superior to Shell Sort for large datasets where predictable performance is critical.

Overall Grade: B+ (Good algorithmic understanding, needs better engineering practices)