Energy Analytics

Lecture 5: Risk in Energy Systems

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In this lecture:

- Different types of risk for energy market participants
- Value at Risk
- Expected Shortfall

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Different types of risk

- **Risk for the consumer of power**
  - Involuntary loss of power, either localised or widespread – this is a blackout
  - May be caused by breakdown of the electricity system or by insufficient power available
  - It is hard to estimate the cost of this.
- **Risk for companies**
  - This applies for all the firms involved in the energy ecosystem
  - Primarily financial – the risk of large losses

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Value of Lost Load

- **The Value of Lost Load (VoLL)** is the estimated amount that customers receiving electricity with firm contracts would be willing to pay to avoid a disruption.
- Very hard to estimate. Depends on
  - What sort of customer?
  - When does the supply disruption occur?
  - How long does it last for?
- Different VOLL values are used for different purposes. In the UK for forward estimates of generation capacity required, VOLL is set at £17,000 /MWh. But for short term use (within the balancing market) a nominal figure of £6000 /MWh is used (though on 20/7/2022 power from an interconnector was purchased at £9725 /MWh)

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Blackout on 9 August 2019

- When energy systems fail costs are very large.


Britain comes to a standstill as power cut causes chaos

» Trains, traffic lights and airports hit as National Grid outages affect a million across England and Wales

» 'It felt apocalyptic': rush-hour panic as London goes dark

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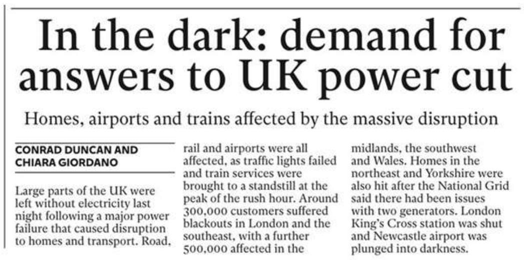
Britain hit by power cut chaos

Blackout affects railways, roads and homes as failing generators trip the National Grid


In the dark: demand for answers to UK power cut

Homes, airports and trains affected by the massive disruption

CONRAD DUNCAN AND CHIARA GIORDANO

Large parts of the UK were left without electricity last night following a major power failure that caused disruption to homes and transport. Road, rail and airports were all affected, as traffic lights failed and train services were brought to a standstill at the peak of the rush hour. Around 300,000 customers suffered blackouts in London and the southeast, with a further 500,000 affected in the midlands, the southwest and Wales. Homes in the northeast and Yorkshire were also hit after the National Grid said there had been issues with two generators. London King's Cross station was shut and Newcastle airport was plunged into darkness.

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What happened?

- Begins with a lightening strike.
- Some problems with software at Little Barford gas generator caused it to trip, which led to a drop in frequency
- Then Hornsea offshore wind farm turned off
- This meant sufficiently large drop in supply that frequency dropped enough to trigger automatic "demand disconnection"

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Taiwan: Massive power outage affects five million households

3 March 2022: Major cities across Taiwan including the capital Taipei have seen widespread power failures after a reported accident at a power plant. The nation's economic affairs minister, Wang Meihua, said an accident had occurred at a power plant in southern Taiwan. State-run power operator Taipower said there had been an incident with a transformer at the Xingda power plant in the southern city of Kaohsiung, and that they were activating backup sources of power.

Local media outlets had earlier reported chaotic scenes at road junctions as traffic lights failed to function. Traffic police had been dispatched to direct vehicles and fire trucks deployed across cities to deal with emergencies such as rescuing people trapped in lifts.

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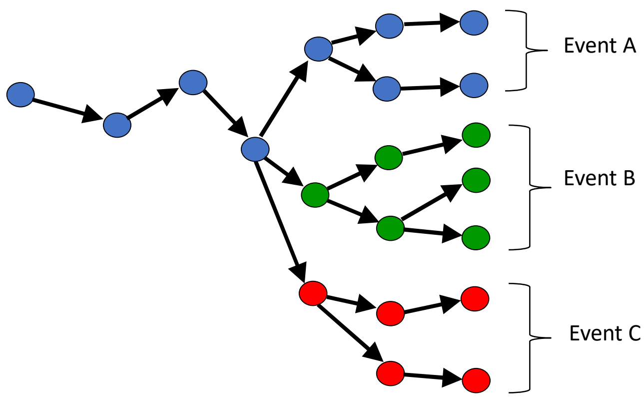
A risk event:

- The best way to start thinking about ‘risk’ events is to think about **sets** of possible future occurrences.



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Event risk

- Intersection risk occurs when several occurrences combine to give a bad outcome. E.g. If a power cut occurs at the same time as the failure of an emergency generator this leaves a hospital without power.

Intersection risk relates to A **and** B **and** C occurring at once

- Union risk occurs when any of a number of different things can give a failure. E.g. there are many different things that can go wrong causing a failure in a rocket launch just before takeoff.

Union risk relates to one or more of A **or** B **or** C occurring

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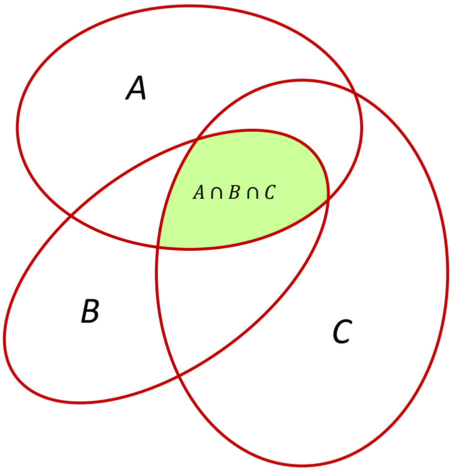
Venn diagram

- An event is always a subset of the set of all possible realisations.
- Venn diagrams are the natural way to represent these sets.
- Intersection risk of many events can be very small unless the events line up.



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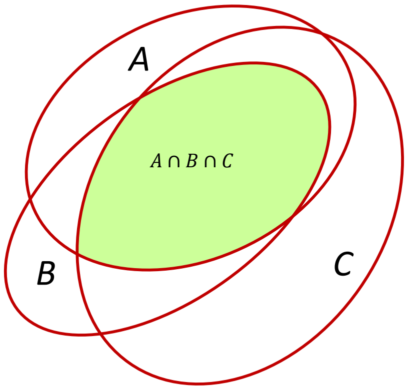

Positive correlation makes intersection risk larger and union risk smaller

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Event risk with continuous variables

- We can get event risk with continuous quantities when there are critical values: e.g. when a transmission line reaches its capacity.
- If X is the unknown random variable, we might be interested in the event "$X > 100$".
- The event $\max(X, Y) > 100$ is the **union** of the events $X > 100$ and $Y > 100$.
- The event $\max(X, Y) < 100$ is the **intersection** of the events $X < 100$ and $Y < 100$.

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Effect of correlation

- For part of a distribution network let X be the maximum power requirement in hour 1, let Y be the maximum power requirement during hour 2 .
- Problems occur if either X or Y is greater than 100MW.
- Both X and Y have a normal distribution with mean 70 and standard deviation 10.
- If X and Y are positively correlated is the probability of the event "$\max(X, Y) > 100$" greater or less than if they were independent?

Careful! max or min ? ">" or "<" ?

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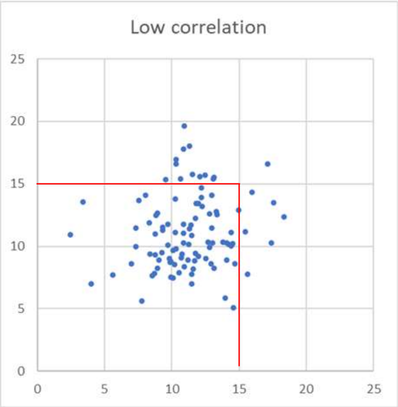
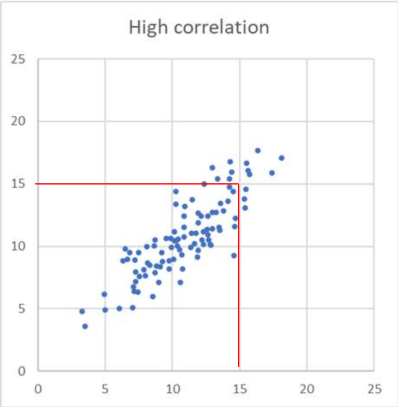
What this looks like in a scatter plot

- How many points where max(X,Y)>15 ?
- 19 points with low correlation 14 points with high correlation





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Value at Risk

- Value at Risk (VaR) is simply the quantile of the distribution of **losses**. If L is the random variable giving uncertain loss, then

$$\begin{aligned} \mathrm{VaR}_\alpha &= \text{minimum } x \text{ such that } \Pr(L > x) \le 1-\alpha \\ &= \text{minimum } x \text{ such that } \Pr(L \le x) \ge \alpha \\ &= \text{minimum } x \text{ with } F(x) \ge \alpha \end{aligned}$$

- Thus a 95% VaR is the value x with the property that losses are more than x only one time in 20 (sometimes this is referred to as VaR with a 95% confidence level).

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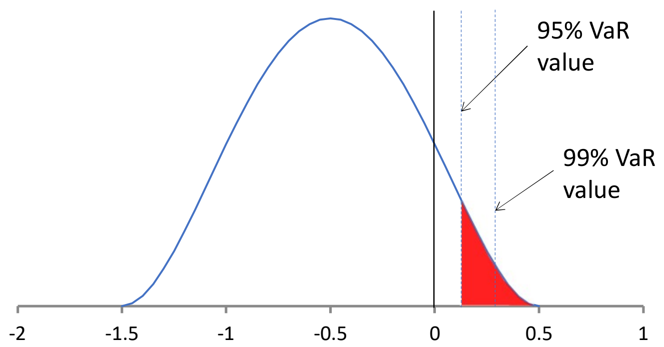
VaR from the Loss distribution

- Easiest to think of the density function for **losses.**
- Most of the time we make profits. i.e. the loss is negative.



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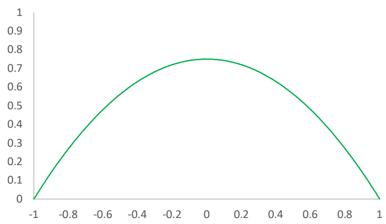
An example



- Suppose the density is
$$f(x) = \frac{3}{4}(1 - x^2)$$
over the range -1 to 1 (units could be \$100,000).
- Integrate to get the cumulative distribution
$$F(x) = \int_{-1}^{x} f(t)dt = \left[\frac{3}{4}\left(t - \frac{t^3}{3}\right)\right]_{-1}^{x}$$
$$= \frac{3}{4}\left(x - \frac{x^3}{3}\right) - \frac{3}{4}\left(-1 + \frac{1}{3}\right)$$
$$= \frac{3}{4}x - \frac{x^3}{4} + \frac{1}{2}$$

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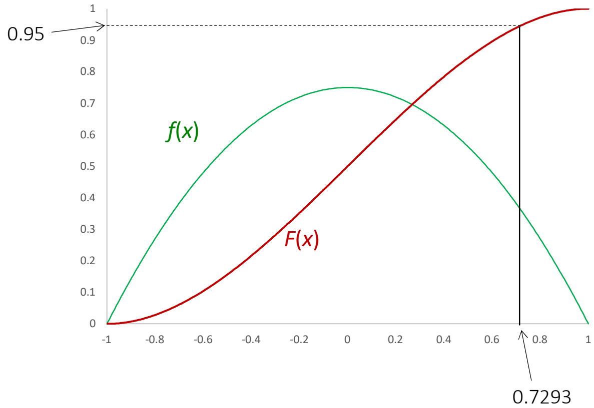
Calculations

- Definition: $\mathrm{VaR}_{0.95} = $ smallest x with $\Pr(L > x) \leq 0.05$
- So $1 - F(x) = 0.05$ i.e. $F(x) = 0.95$



Numerically we find
$$F(0.7293) = 0.95$$
So if units are \$100,000 we get $\mathrm{VaR}_{.95}$= \$72,930

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A discrete distribution

- Need to take care with discrete distributions. An example:
  - loss of \$11,000 probability 0.02
  - loss of \$10,500 probability 0.02
  - loss of \$10,000 probability 0.02
  - loss of \$9,000 probability 0.04
  - loss of \$0 probability 0.9
- What is 95% VaR? \$10,500 or \$10,000?
- This is where the "minimum *x* such that ..." in the definition comes in.

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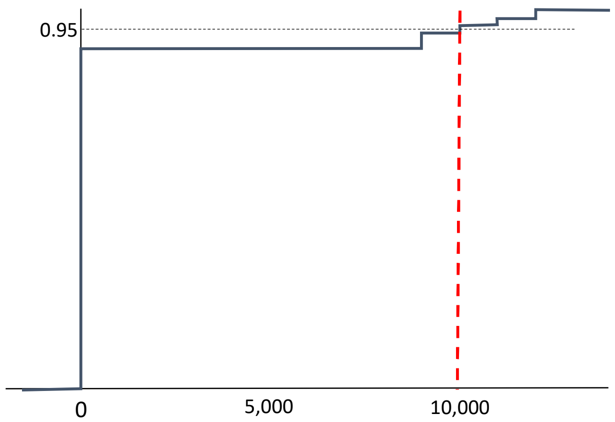
VaR for the discrete example

- 95% VaR is the **first** point where the graph of *F* reaches 0.95. Here it is \$10,000.

Important if there is a horizontal section at 0.95



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Options for VaR calculation

- A historical approach looks at the investments subject to market risk and looks at actual historical figures on how the prices have varied (including underlying securities for derivative positions). Need to take at least a year's data.
- Parametric approach assumes a distribution (e.g. log normal) for price movements and then uses variances and covariances to tie everything together into a VaR number.
- A Monte Carlo approach uses a computer simulation to model everything and come up with a synthetic historical series (which may run for a long period in order to estimate risks better).

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Problems with VaR

- VaR deals with more or less everyday risks (99% daily VaR means something worse happens twice a year) – so VaR does not track extreme risks. It does not say anything about what is happening in the far right tail of the distribution of losses.
- VaR encourages gaming the system where a manager takes on an asymmetric risk that improves performance without changing VaR (e.g. selling a credit default swap – essentially insurance that a firm does not fail). This is possible because making extreme tail events worse will not effect VaR, and this can win us something in average performance.

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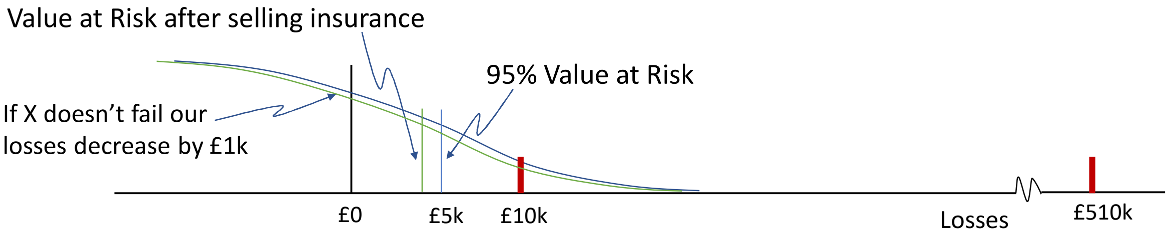
VaR sometimes encourages more risk

- With prob 1/1000 a counterparty X fails and we lose £10,000. Suppose that, taking account of this, our 95% Value at Risk is £5k.



- We double down and sell insurance on X failing, 50 other companies buy this insurance we pay each of them £10,000 if X fails, in return for a premium of \$20. If X does not fail we gain £1000, if it does fail we lose an extra £500,000.
- Rather than losing £10k we may lose £510k, but otherwise we gain £1000. There is much more risk, but VaR improves as well as expected profit.

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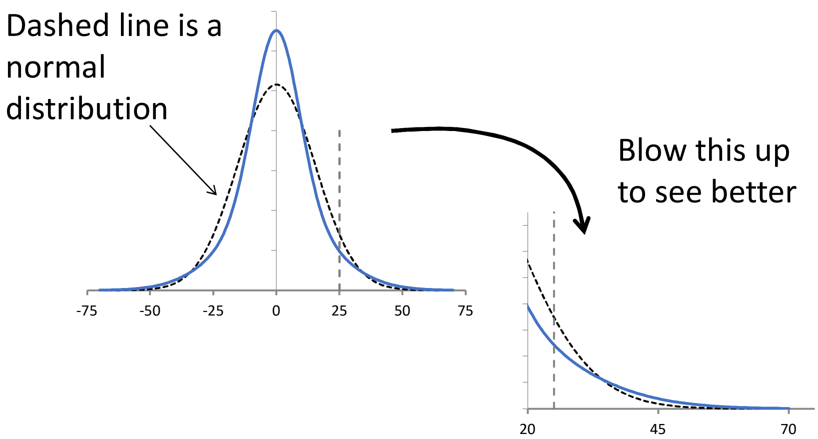
Beyond Value at Risk

- Different behaviours can lead to the same VaR



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Expected shortfall

- One approach is to look at different confidence levels for VaR. The solid line distribution with the fatter tails has a 99% VaR of 41, while the 99% VaR for the normal distribution is 36.
- An alternative is to use *Expected Shortfall,* (sometimes called “tail value at risk” or “conditional value at risk”).
- The 95% VaR answers the question “What is the minimum loss amongst the 5% of worst outcomes?” But the 95% expected shortfall value answers the question “What is the average loss amongst the 5% of worst outcomes?”
- Given a sample of points from the distribution, $\mathrm{ES}_{0.98}(\mathrm{X})$ is estimated from the average of the worst 2% of the sample.

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A formula for Expected Shortfall

- The expected shortfall for X is

$$\mathrm{ES}_\alpha(X) = \mathrm{E}[X|X > \mathrm{VaR}_\alpha(X)]$$

i.e. the expected loss conditional on the VaR level being exceeded (the average value over that part of the distribution which is greater than $\mathrm{VaR}_\alpha(X)$).

- This means

$$\mathrm{ES}_\alpha(X) = \frac{1}{\Pr(X > \mathrm{VaR}(X))} \int_{\mathrm{VaR}(X)}^{\infty} x f(x) dx$$

$$= \frac{1}{1-\alpha} \int_{\mathrm{VaR}(X)}^{\infty} x f(x) dx$$

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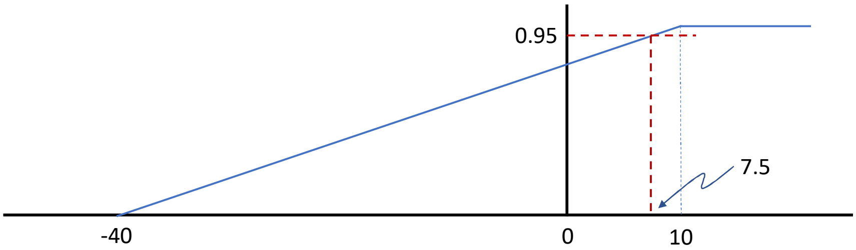
An example

- Returns are equally likely to be anywhere from a loss of 10k to a profit of 40k. What is the value of expected shortfall at 0.95 level?
- Work in 1000s of pounds. Losses follow a uniform distribution between -40 and $+10$. So $f(x) = 1/50$ over this range.
- Thus $F(x) = (x + 40)/50$ and $\text{VaR}_{0.95}(X) = 7.5$.



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Calculation of Expected Shortfall

- Now

$$\text{ES}_\alpha(X) = \frac{1}{1-\alpha}\int_{\text{VaR}(X)}^{\infty} x f(x) dx$$

- So

$$\text{ES}_{0.95}(X) = \frac{1}{0.05}\int_{7.5}^{10} \frac{x}{50} dx = \frac{1}{0.05}\left[\frac{x^2}{100}\right]_{7.5}^{10}$$

$$= \frac{1}{0.05}\left(1 - \frac{56.25}{100}\right) = 8.75$$

- This is the average loss of the worst 5% of outcomes.

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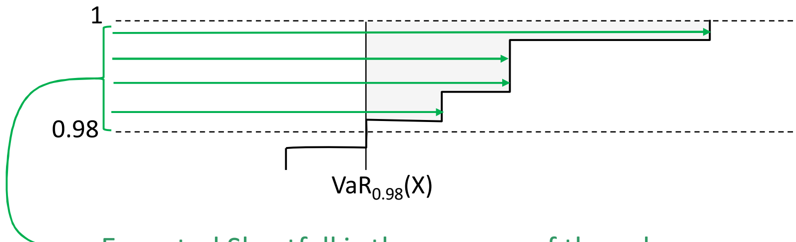
Expected shortfall for discrete distribution

- We can write

$$\mathrm{ES}_\alpha(X) = \mathrm{VaR}_\alpha(X) + \frac{1}{1-\alpha}\int_{\mathrm{VaR}(X)}^{\infty} \big(x - \mathrm{VaR}(X)\big) f(x)dx$$

- A discrete example



Expected Shortfall is the average of these losses

$$\mathrm{ES}_{0.98}(X) = \mathrm{VaR}_{0.98}(X) + (1/0.02) \times (\text{shaded area})$$

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A different approach to Expected Shortfall

- Write $\quad W(z) = z + \dfrac{1}{1-\alpha}\displaystyle\int_z^{\infty} (x-z) f(x)dx$

$$= z + \frac{1}{1-\alpha}\int_{-\infty}^{\infty} \max(x-z,0)\, f(x)dx$$

- Then our previous formula shows that $W(\mathrm{VaR}_\alpha)$ is the Expected Shortfall when losses have density $f(x)$.
- It turns out (first shown by Rockafellar and Uryasev) that the expected shortfall is the minimum value of $W(z)$ over choices of z.

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The formula for a sample

- We can convert the integral into a sum when there are a finite number of observations. Take a sample of losses $\{L_1(y), L_2(y), L_3(y)..., L_N(y)\}$ depending on a decision variable y.
- Then Expected Shortfall is

$$\min_z W(z) = \min_z \left\{ z + \frac{1}{1-\alpha} \int_{-\infty}^{\infty} \max(x-z,0)\, f(x)dx \right\}$$

$$= \min_z \left\{ z + \frac{1}{1-\alpha} \sum_{i=1}^{N} \frac{1}{N} \max(L_i(y) - z, 0) \right\}$$

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Optimal decisions involving risk

- Suppose we want to choose a decision variable y to maximize
  (Expected profit) - γ(Expected Shortfall)
- An equivalent objective is to choose y to minimize:
  (Expected losses) + γ(Expected Shortfall)
- Then we solve

$$\min_{z,y} \left\{ \sum_{i=1}^{N} \frac{1}{N} L_i(y) \; + \gamma \left(z + \frac{1}{1-\alpha} \sum_{i=1}^{N} \frac{1}{N} \max(L_i(y) - z, 0) \right) \right\}$$

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Next time:

01

Using copulas in risk modelling

02

Different types of financial contracts that can be used to reduce risk

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Risk calculations with contracts for differences

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