Week 8: Written Assignment

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1. Show that the Sliced Score Matching (SSM) loss can also be written as:

$$L_{\text{SSM}} = \mathbb{E}_{x\sim p(x)}\mathbb{E}_{v\sim p(v)}\left[\|v^T S(x;\theta)\|^2 + 2v^T\nabla_x(v^T S(x;\theta))\right].$$

Derivation:
The original score matching loss (Hyvärinen, 2005) is given by:

$$L_{\text{SM}} = \mathbb{E}_{x\sim p(x)}\left[\frac{1}{2}\|S(x;\theta)\|^2 + \nabla_x \cdot S(x;\theta)\right].$$

Sliced score matching introduces a random direction vector v sampled from a distribution $p(v)$, typically the isotropic Gaussian $v \sim \mathcal{N}(0, I)$, to reduce computational cost.

The sliced loss can be written as:

$$L_{\text{SSM}} = \mathbb{E}_{x,v}\left[\frac{1}{2}(v^T S(x;\theta))^2 + v^T\nabla_x(v^T S(x;\theta))\right].$$

Expanding the squared term and removing the $\frac{1}{2}$ factor (absorbed into scaling), we obtain:

$$L_{\text{SSM}} = \mathbb{E}_{x,v}\left[\|v^T S(x;\theta)\|^2 + 2v^T\nabla_x(v^T S(x;\theta))\right].$$

Hence, the given expression is an equivalent formulation of the sliced score matching loss, expressing the directional projection of the score function and its divergence term.

2. Briefly explain SDE (Stochastic Differential Equation)

A **Stochastic Differential Equation (SDE)** describes the time evolution of a random variable that is influenced by both deterministic and stochastic components:

$$dx = f(x,t)\,dt + g(x,t)\,dW_t,$$

where:

- $f(x,t)$: drift term (deterministic trend),
- $g(x,t)$: diffusion term (random fluctuation strength),
- W_t: Wiener process or Brownian motion.

In the context of score-based generative modeling, an SDE defines the process that gradually perturbs data into noise (forward process) and the reverse SDE is used for generating new data samples by integrating backward from noise to data.

3. Unanswered Questions

- Why does score matching allow training without knowing the true data density $p(x)$?

References.

- Course lecture notes and assignment instructions.
- OpenAI. (2025)ChatGPT (GPT-5) From https://chat.openai.com/