Course: Machine Learning
Assignment: Week 2 _ Programming Assignment

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1. Method

In this assignment, we approximate the Runge function

$$f(x) = \frac{1}{1 + 25x^2}, \qquad x \in [-1,1]$$

using a feed-forward neural network.

- **Data**: 200 evenly spaced points were sampled for training, and another 200 for validation.
- **Model**: A fully connected network with architecture 1→64→64→1, using Tanh activation in hidden layers.
- **Training setup**:
  - Loss function: Mean Squared Error (MSE)
  - Optimizer: Adam, learning rate = 0.01
  - Epochs: 800
- **Evaluation**: We compared the network predictions with the true function, plotted training/validation loss curves, and computed errors.

2. Results

Figure 1. True function vs neural network prediction

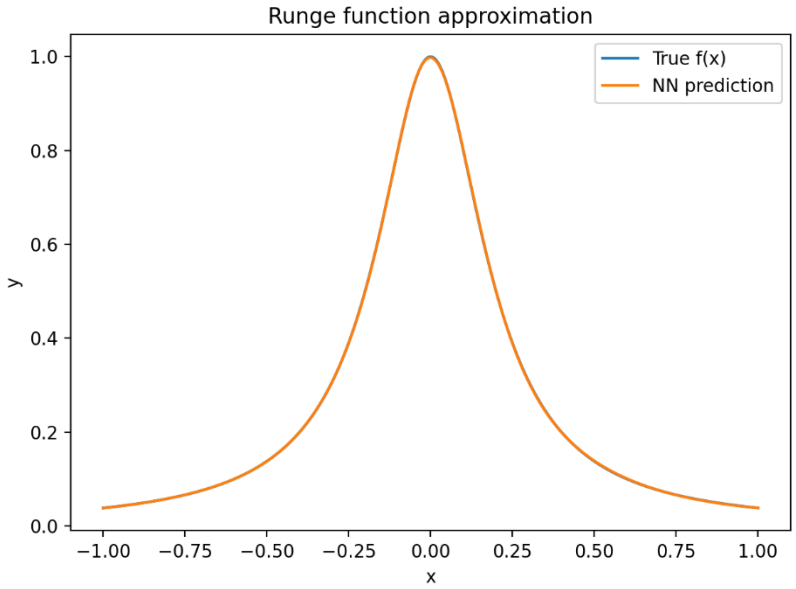
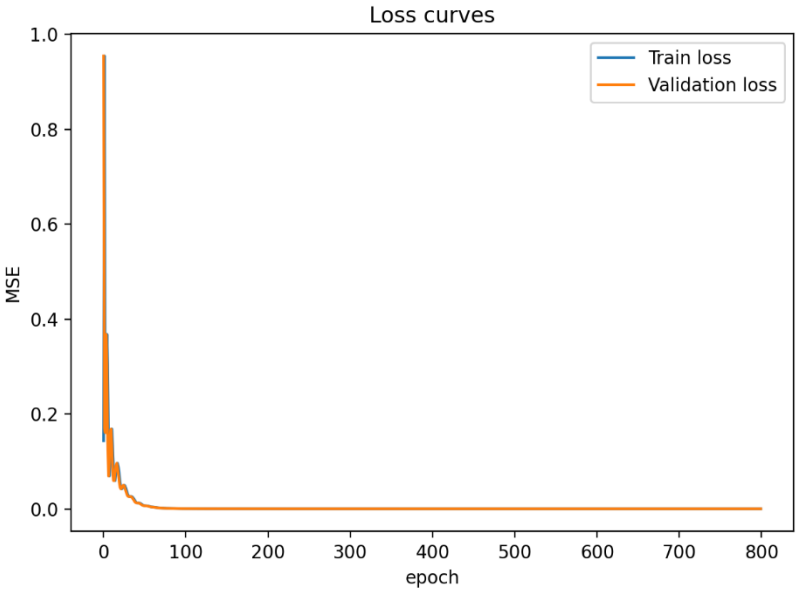


Figure 2. Training and validation loss curves



Error metrics (Validation set):

- Mean Squared Error (MSE): *6.650753334724868e-07*
- Maximum absolute error: *0.002384185791015625*

3. Discussion

The neural network successfully approximated the Runge function, especially around the center of the domain. Slightly larger errors were observed near the boundaries ($x=\pm1$), which is consistent with the Runge phenomenon.

The training and validation loss curves showed stable convergence without severe overfitting. The low MSE and maximum error indicate that the chosen network architecture and training setup were effective.

Possible improvements could include:

- Increasing the number of hidden neurons or layers.
- Trying alternative activation functions (ReLU, sigmoid).
- Using regularization (dropout, weight decay) to further stabilize training.

4. References

- Course lecture notes and assignment instructions.
- OpenAI. (2025)。ChatGPT (GPT-5) 取自https://chat.openai.com/