Assignment 2

$n_L = 1$

$\nabla_a{}^{[L]}(x)$

(3.1) $a^{[1]} = x \in \mathbb{R}^{n_1}$

(3.2) $a^{[l]} = \sigma(W^{[l]} a^{[l-1]} + b^{[l]}) \in \mathbb{R}^{n_i}$

$$\nabla a^{[L]}(x) = \left(\frac{\partial a^{[L]}(x)}{\partial x_1}, \frac{\partial a^{[L]}(x)}{\partial x_2}, \dots, \frac{\partial a^{[L]}(x)}{\partial x_{n_1}} \right)$$

Let $z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]} \Rightarrow a^{[l]} = \sigma z^{[l]}$

$$\delta^{[l]} := \frac{\partial a^{[L]}}{\partial z^{[l]}}$$

$$\delta_j^{[l]} = \frac{\partial a^{[L]}}{\partial z_j^{[l]}} = \sum_k \underbrace{\frac{\partial a^{[L]}}{\partial z_k^{[l+1]}}}_{\delta_k^{[l+1]}} \cdot \underbrace{\frac{\partial z_k^{[l+1]}}{\partial a_j^{[l]}}}_{W_{kj}^{[l+1]}} \cdot \underbrace{\frac{\partial a_j^{[l]}}{\partial z_j^{[l]}}}_{\sigma'(z_j^{[l]})}$$

$$\delta^{[L]} = \sigma'(z^{[L]}) = \sigma(z^{[L]})(1 - \sigma(z^{[L]}))$$

$$\delta^{[l]} = (W^{[l+1]})^T \delta^{[l+1]} \odot \sigma'(z^{[l]})$$

Thus, $\delta_j^{[l]} = \sigma'(z_j^{[l]}) \sum_k W_{kj}^{[l+1]} \delta_k^{[l+1]}$

$$\nabla a^{[L]}(x) = (W^{[2]})^T \delta^{[2]} \xrightarrow[\text{Input , layer 1}]{z^{[2]} = W^{[2]} x + b^{[2]}} \frac{\partial a^{[L]}}{\partial x_j} = \sum_i \underbrace{\frac{\partial a^{[L]}}{\partial z_i^{[2]}}}_{\delta_i^{[2]}} \cdot \underbrace{\frac{\partial z_i^{[2]}}{\partial x_j}}_{W_{ij}^{[2]}} = \sum_i \delta_i^{[2]} W_{ij}^{[2]}$$

pseduo code

```
a[1] = x
for l in range(2, L+1)
    z[l] = W[l] @ a[l-1] + b[l]
    a[l] = sigmoid(z[l])

δ[L] = sigmoid_derivative(z[L])
for l in range(L-1, 1, -1)
    δ[l] = (W[l+1].T @ δ[l+1]) * sigmoid_derivative(z[l])

grad_x = W[2].T @ δ[2]
return grad_x
```