Spacecraft Formation Flying Project

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1 Introduction

In recent years, there is a trend amongst mission designers to design formation flying missions. Such missions distribute the tasks usually performed by a single, monolithic, expensive spacecraft among two or more small, cost-effective cooperating satellites flying in formation. A good example to this is the TerraSAR-X, TanDEM-X mission, which produced Earth Digital Elevation Model of unprecedented accuracy.

Therefore, it is interesting to understand how it is possible to realize a formatin flying mission in practice and how it is possible to satisfy the mission requirements in a strongly perturbed orbit, such as the Low Earth Orbit (LEO).

2 Dynamical system

To facilitate the analysis for this project, we will assume the relative distance between the satellites is small. This allows us to linearize the dynamics of the secondary spacecraft (follower) relative to the trajectory of the primary satellite (leader). These linearized equations are the so-called Hill-Clohessy-Wiltshire (HCW) equations:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x &= 0 \\ \ddot{y} + 2n\dot{x} &= 0 \\ \ddot{z} + n^2 z &= 0 \end{cases} \tag{1}$$

where x, y, z are the radial, along-track and cross-track coordinates of the follower with respect to the leader in [km], and n is the mean motion of the primary spacecraft. Here

$$n = \sqrt{\frac{398600}{6778^3}} \ [1/\mathrm{s}] \tag{2}$$

Furthermore, we assume we have three cold-gas thrusters that are constantly aligned with the principal axis of the RTN frame that are being used to maneuver the secondary vehicle. Finally, we have GPS receivers on-board of both satellites, from which we can get the relative difference in the three directions of the RTN frame. From this, our linear system can be written as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{u}$$

Where $\mathbf{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$, $\mathbf{y} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ and $\mathbf{u} = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^T$